

## Evaluation rules for $L23_{rr}$ (big-step semantics)

$$\begin{array}{c}
 \frac{}{n \Downarrow n} \quad \frac{}{\mathbf{T} \Downarrow \mathbf{T}} \quad \frac{}{\mathbf{F} \Downarrow \mathbf{F}} \quad \frac{}{() \Downarrow ()} \quad \frac{}{[\lambda x \tau_1 t_1] \Downarrow [\lambda x \tau_1 t_1]} \\
 \\
 \frac{t_1 \Downarrow [\lambda x \tau_1 t_{11}] \quad t_2 \Downarrow v_2 \quad \frac{}{[x \mapsto v_2] t_1 \Downarrow v_3}}{(t_1 t_2) \Downarrow v_3} \\
 \\
 \frac{t_1 \Downarrow [\lambda f \tau_1 t_{11}] \quad \frac{}{[f \mapsto (\mathbf{fix} [\lambda f \tau_1 t_{11}])] t_{11} \Downarrow v_1}}{(\mathbf{fix} t_1) \Downarrow v_1} \\
 \\
 \frac{t_1 \Downarrow v_1 \quad \frac{}{[x \mapsto v_1] t_2 \Downarrow v_2}}{\{x t_1 t_2\} \Downarrow v_2} \quad \frac{t_1 \Downarrow \mathbf{T} \quad t_2 \Downarrow v_2}{[t_1 ? t_2 : t_3] \Downarrow v_2} \quad \frac{t_1 \Downarrow \mathbf{F} \quad t_3 \Downarrow v_3}{[t_1 ? t_2 : t_3] \Downarrow v_3} \\
 \\
 \frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 + t_2] \Downarrow n_1 + n_2} \quad \frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 - t_2] \Downarrow n_1 - n_2} \quad \frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 * t_2] \Downarrow n_1 * n_2} \\
 \\
 \frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 == t_2] \Downarrow n_1 = n_2} \quad \frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 < t_2] \Downarrow n_1 < n_2} \quad \frac{t_1 \Downarrow \mathbf{T}}{!t_1 \Downarrow \mathbf{F}} \quad \frac{t_1 \Downarrow \mathbf{F}}{!t_1 \Downarrow \mathbf{T}} \\
 \\
 \frac{t_1 \Downarrow v_1 \dots t_k \Downarrow v_k}{[\sim \mathbf{l}_1 t_1 \dots \sim \mathbf{l}_k t_k] \Downarrow [\sim \mathbf{l}_1 v_1 \dots \sim \mathbf{l}_k v_k]} \quad \frac{t_1 \Downarrow [\dots \sim \mathbf{l}_i v_i \dots]}{(\sim \mathbf{l}_i t_1) \Downarrow v_i}
 \end{array}$$

## Typing rules for $L23_{rr}$

$$\begin{array}{c}
\frac{}{\Gamma \vdash n : \mathbb{I}} \quad \frac{}{\Gamma \vdash \mathbf{T} : \mathbb{B}} \quad \frac{}{\Gamma \vdash \mathbf{F} : \mathbb{B}} \quad \frac{}{\Gamma \vdash () : \mathbb{U}} \\
\\
\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1} \quad \frac{\Gamma, x : \tau_1 \vdash t_1 : \tau_2}{\Gamma \vdash \{\mathbf{lam} \ x \ \tau_1 \ t_1\} : \tau_1 \rightarrow \tau_2} \quad \text{check} \\
\\
\frac{\Gamma \vdash t_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash t_2 : \tau_3 \quad \tau_3 <: \tau_1}{\Gamma(t_1 \ t_2) : \tau_2} \quad \frac{\Gamma \vdash t_1 : \tau_1 \rightarrow \tau_1}{\Gamma \vdash (\mathbf{fix} \ t_1) : \tau_1} \\
\\
\frac{\Gamma \vdash t_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash t_2 : \tau_2}{\Gamma \vdash \{x \ t_1 \ t_2\} : \tau_2} \\
\\
\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \tau_2 \quad \Gamma \vdash t_3 : \tau_3 \quad \tau_4 = s(\tau_2, \tau_3)}{\Gamma \vdash [t_1 ? t_2 : t_3] : \tau_4} \\
\\
\frac{\Gamma \vdash t_1 : \mathbb{I} \quad \Gamma \vdash t_2 : \mathbb{I}}{\Gamma \vdash [t_1 + t_2] : \mathbb{I}} \quad \text{Likewise the other arithmetic operators.} \\
\\
\frac{\Gamma \vdash t_1 : \mathbb{I} \quad \Gamma \vdash t_2 : \mathbb{I}}{\Gamma \vdash [t_1 == t_2] : \mathbb{B}} \quad \frac{\Gamma \vdash t_1 : \mathbb{I} \quad \Gamma \vdash t_2 : \mathbb{I}}{\Gamma \vdash [t_1 < t_2] : \mathbb{B}} \quad \frac{\Gamma \vdash t_1 : \mathbb{B}}{\Gamma \vdash !t_1 : \mathbb{B}} \\
\\
\frac{\Gamma \vdash t_1 : \tau_1 \ \dots \ \Gamma \vdash t_k : \tau_k}{\Gamma \vdash [\sim \mathbf{l}_1 \ t_1 \ \dots \ \sim \mathbf{l}_k \ t_k] : (\sim \mathbf{l}_1 \ \tau_1 \ \dots \ \sim \mathbf{l}_k \ \tau_k)} \quad \frac{\Gamma \vdash t_1 : (\dots \sim \mathbf{l}_i \ \tau_i \ \dots)}{\Gamma \vdash (\sim \mathbf{l}_i \ t_1) : \tau_i} \\
\\
s(\tau_1, \tau_2) = \begin{cases} \tau_2 & \tau_1 <: \tau_2 \\ \tau_1 & \tau_2 <: \tau_1 \\ \text{fail} & \text{otherwise} \end{cases}
\end{array}$$