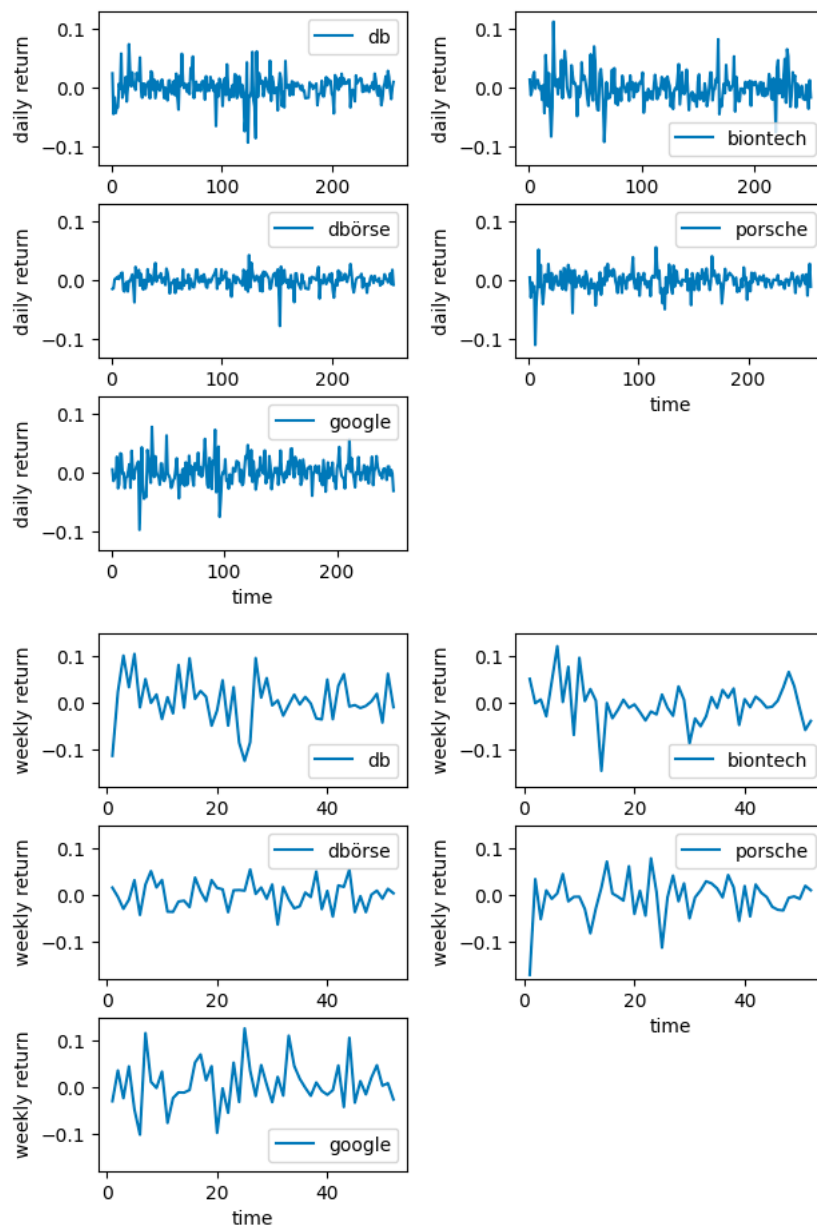


I did only print the outputs here, when it was explicitly written to do so. You find all the results in the provided code.

### Exercise 1a

I chose to evaluate the data of the past year and evaluated the following companies: Deutsche Bank, Biontech, Deutsche Börse, Porsche, Google.



### Exercise 1b

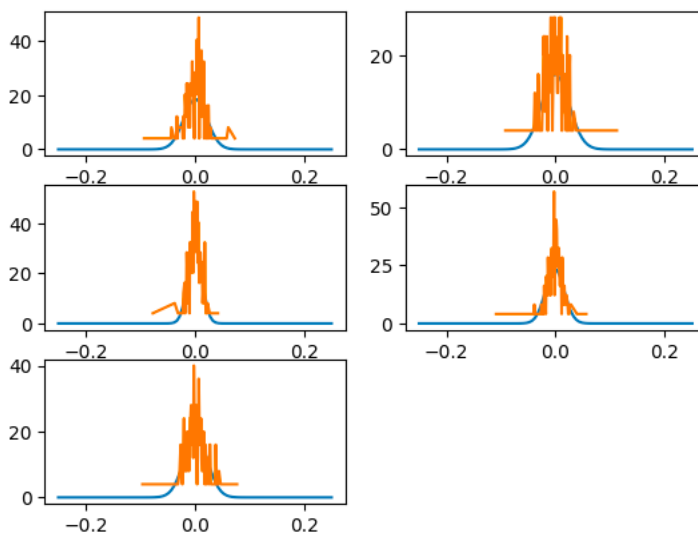
db\_daily\_mean: 0.0007702932370513637

db\_weekly\_mean: 0.004339099753338295  
 biontech\_daily\_mean: -0.000352979434944665  
 biontech\_weekly\_mean: -0.002254000204807188  
 porsche\_daily\_mean: -0.0015397334518382772  
 porsche\_weekly\_mean: -0.005766768506563269  
 dbörse\_daily\_mean: -0.00010191563226261088  
 dbörse\_weekly\_mean: 0.0003808141656413158  
 google\_daily\_mean: 0.0007702932370513637  
 google\_weekly\_mean: 0.0007702932370513637  
 db\_daily\_vol: 0.021474053388312056  
 db\_weekly\_vol: 0.04970409213566631  
 biontech\_daily\_vol: 0.02506202373140432  
 biontech\_weekly\_vol: 0.04385937736704859  
 porsche\_daily\_vol: 0.016888909081598263  
 porsche\_weekly\_vol: 0.04275574495374037  
 dbörse\_daily\_vol: 0.012628215177437474  
 dbörse\_weekly\_vol: 0.027629501206997847  
 google\_daily\_vol: 0.021474053388312056  
 google\_weekly\_vol: 0.021474053388312056

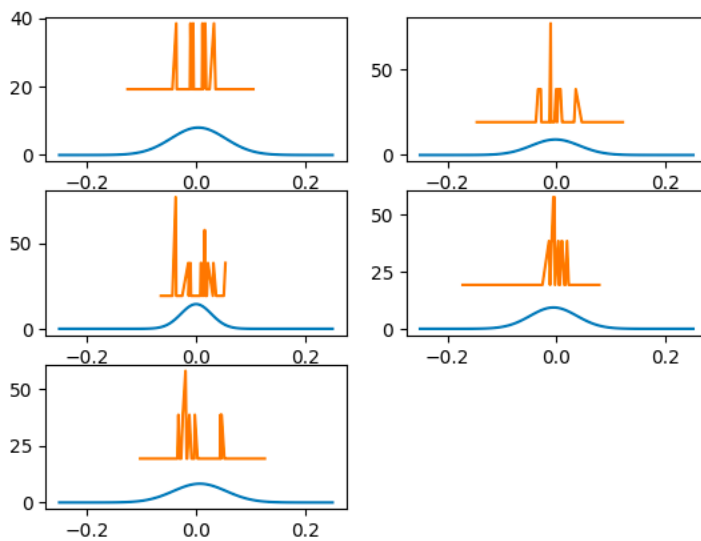
### Exercise 1c

Order: Deutsche Bank, Biontech, Deutsche Börse, Porsche, Google

empirical densities of daily returns vs. fitted normal



empirical densities of weekly returns vs. fitted normal



While for the densities of the daily returns, the assumption seems reasonable, the weekly returns have heavier tails than estimated by the normal distribution. Regarding Biontech and Porsche this becomes the most obvious.

## Exercise 2a

To verify the results, I checked that the diagonals equal the squared vol from 1)

cov3

```
[ [ 2.47049678e-03 -7.01158123e-05 -1.28490239e-04]
  [-7.01158123e-05  1.92364498e-03  8.20789355e-05]
  [-1.28490239e-04  8.20789355e-05  7.63389337e-04]]
```

cov4

```
[ [ 2.47049678e-03 -7.01158123e-05 -1.28490239e-04 -1.97663336e-04]
  [-7.01158123e-05  1.92364498e-03  8.20789355e-05 -1.06013278e-04]
  [-1.28490239e-04  8.20789355e-05  7.63389337e-04  4.44353077e-04]
  [-1.97663336e-04 -1.06013278e-04  4.44353077e-04  2.34718608e-03]]
```

cov5

```
[ [ 2.47049678e-03 -7.01158123e-05 -1.28490239e-04 -1.97663336e-04
   1.04344448e-03]
  [-7.01158123e-05  1.92364498e-03  8.20789355e-05 -1.06013278e-04
   -3.02192053e-04]
  [-1.28490239e-04  8.20789355e-05  7.63389337e-04  4.44353077e-04
   1.76438578e-04]
  [-1.97663336e-04 -1.06013278e-04  4.44353077e-04  2.34718608e-03
   4.47586359e-04]
  [ 1.04344448e-03 -3.02192053e-04  1.76438578e-04  4.47586359e-04
   1.82805373e-03]]
```

### Exercise 2b

# The variance of the minimum variance portfolio has to (not strictly) decline in the number of assets  $n$ . This is because larger  $n$  increases the set of possible portfolios and includes all possible portfolios from  $n-1$ . Hence due to monotonicity the minimal variance is decreasing in  $n$ .

### Exercise 2c

# By the same argument as above follows that the efficient frontier curve with  $n$  assets must be equal or strictly higher than the corresponding curve with  $n-1$  assets. For any given volatility the efficient frontier portfolio's return with  $n$  assets has to be equal or higher than the corresponding one with  $n-1$  assets.

### Exercise 2d

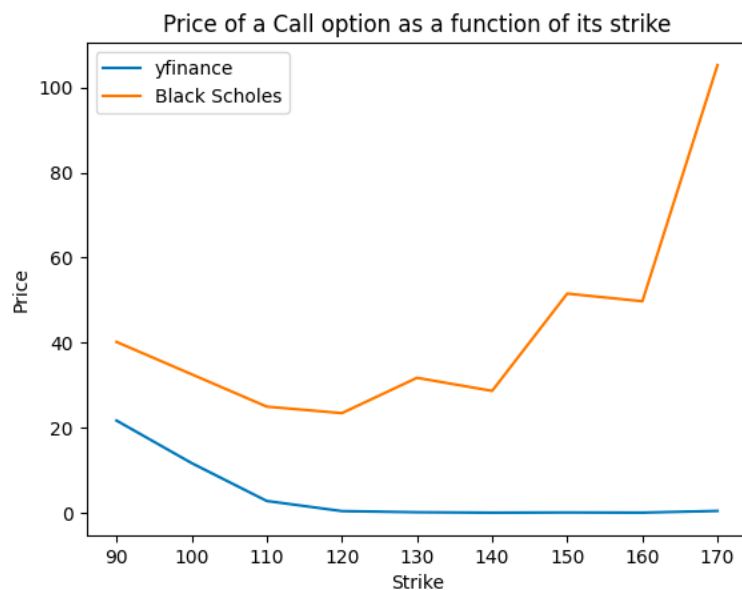
# The risk can be anywhere between larger or equal zero (portfolio only consisting of risk free investment). Risk larger than the maximal volatility of the assets can be achieved by negative positions. The expected return can be anywhere above the risk free interest rate  $r_0$ , since infinitely high risk implies infinitely high expected return.

### Exercise 3a

# As the risk free interest rate I chose the current deposit rate given by the EZB, since the central bank is in charge of stabilizing and assuring financial stability.

### Exercise 3b

Note: As a sanity check I fixed the volatility and then plotted; the BS curve was declining as it should.



**Exercise 3c**

I didn't manage to implement a method to find the zero of the function.