$$
\begin{aligned}
\text{(FFLP)}\quad \text{Min}\quad & \tilde z = (-1,0,2)\tilde x_1 + (1,2,3)\tilde x_2 \in \mathcal{F}_C \\
\text{s.t.}\quad & (2,5,8)\tilde x_1 + (3,4,10)\tilde x_2 <\simeq (1,3,6) \\
& (4,5,7)\tilde x_1 + (0,5,15)\tilde x_2 <\simeq (2,3,6) \\
& \tilde x_1, \tilde x_2 >\simeq 0
\end{aligned}
$$

There are two fuzzy variables, and two constraints with triangular fuzzy numbers, this is a $1-$polygonal case.

In [1]:
```
M = 2 # no. constraints
N = 2 # no. fuzzy variables
K = 3 # fuzzy "size" (triangular)

# Objective function (costs vector)
C1 = array(c(-1,0,2,1,2,3),c(K,N))
print("Objective function")
C1

# Constraints (matrix of coeff.):
A1= array(c(2,4,5,5,8,7,
            3,0,4,5,10,15),c(M,K,N))
print("Matrix of coeff., 1st component")
# First component of the Triangular Fuzzy Number
A1[,1,]
# Second component of the Triangular Fuzzy Number
print("Matrix of coeff., 2nd component")
A1[,2,]
# Third component of the Triangular Fuzzy Number
print("Matrix of coeff., 3rd component")
A1[,3,]




# another simple option is to define them as a matrix,
# but, for algorithm purposes, this structure suits better

# RHS (right hand side), or Vector of coefficients:
B1 = matrix(c(1,3,6,
              2,3,6),
            nrow=M,ncol=K,
            byrow = TRUE)

print("Vector of coefficients")
B1
```

```
[1] "Objective function"
```

-1	1
0	2
2	3

```
[1] "Matrix of coeff., 1st component"
```

2	3
4	0

```
[1] "Matrix of coeff., 2nd component"
```

5	4
5	5

```
[1] "Matrix of coeff., 3rd component"
```

8	10
7	15

```
[1] "Vector of coefficients"
```

1	3	6
2	3	6

Following the multiobjective linear problem (MOLP) associated is formulated, where the multiobjective function $f=(f_1,f_2,f_3):\mathbb{R}^6\to\mathbb{R}^3$ is a vector-valued function, evaluated on $x=(x_1^-,\hat x_1,x_1^+,x_2^-,\hat x_2,x_2^+)\in\mathbb{R}^6$,

$$
\begin{aligned}
\text{(MOLP)}\ \text{Min}\ f(x) \quad &= \quad (-x_1^+ + x_2^-, 2\hat x_2, 2x_1^+ + 3x_2^+) \in \mathbb{R}^3 \\
s.t.\quad & \\
2x_1^- + 3x_2^- &\le 1, \\
5\hat x_1 + 4\hat x_2 &\le 3, \\
8x_1^+ + 10x_2^+ &\le 6, \\
4x_1^- &\le 2, \\
5\hat x_1 + 5\hat x_2 &\le 3, \\
7x_1^+ + 15x_2^+ &\le 6, \\
x_n^- - \hat x_n &\le 0, \quad n=1,2 \\
\hat x_n - x_n^+ &\le 0, \quad n=1,2 \\
x_n^-, \hat x_n, x_n^+ &\ge 0, \quad n=1,2
\end{aligned}
$$

There are $M_2 = 3M + 3N = 10$ constraints, and $N_2 = 3N = 6$ variables.

In [2]:
```
# MOLP associated
M2 = M*K+N*(K-1) # total no. of constraints
N2 = N*K # total no. of vars
```

Equivalently, the above problem in matrix notation results as

$$
\begin{aligned}
\text{(MOLP)}\ \text{Min}\ & Cx \in \mathbb{R}^3 \\
s.t.\ & Ax \le b, \\
& x \ge 0
\end{aligned}
$$

where,

$$
C = \begin{pmatrix} 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 3 \end{pmatrix}, \quad
A = \begin{pmatrix} 2 & 0 & 0 & 3 & 0 & 0 \\ 0 & 5 & 0 & 0 & 4 & 0 \\ 0 & 0 & 8 & 0 & 0 & 10 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 5 & 0 \\ 0 & 0 & 7 & 0 & 0 & 15 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}, \quad
b = \begin{pmatrix} 1 \\ 3 \\ 6 \\ 2 \\ 3 \\ 6 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \text{and} \quad
x = \begin{pmatrix} x_1^- \\ \hat x_1 \\ x_1^+ \\ x_2^- \\ \hat x_2 \\ x_2^+ \end{pmatrix} \in \mathbb{R}^6
$$

In [3]:
```
## MULTIOBJECTIVE LP associated:

# Matrix of coefficients


# simplest way: direct manually definition of the matrix
#A2 = matrix( c(2 ,   0  ,  0  ,  3  ,  0  ,  0
#                0    5  ,  0 ,   0 ,   4   , 0
#                0    0  ,  8  ,  0  ,  0  , 10
#                4    0  ,  0 ,  0  ,  0  ,  0
#                0    5  , 0  ,  0   , 5   , 0
#                0  ,  0  ,  7 ,   0 ,   0  , 15
#                1  , -1  ,  0 ,   0  ,  0  ,  0
#                0  ,  1  , -1  ,  0  ,  0 ,   0
#                0  ,  0  ,  0  ,  1  , -1  ,  0
#                0 ,   0  ,  0  ,  0  ,  1  , -1),
#            nrow = M2,ncol=N2,byrow=TRUE)

# Or automatic definition, which requires some programming
A2 = matrix(0,nrow = M2,ncol=N2)
# See Eq.(19) + Remark 3
for(m in 1:M){
  # M*K equivalent constraints
  for(n in 1:N){
    if(K%%2==0){ # gral. k-polygonal Fuzzy Numbers
      for(k in 1:(K/2)){ # see Remark 3
        ifelse(A1[m,k,n]>=0, A2[K*(m-1)+k,K*(n-1)+k] <- A1[m,k,n], A2[K*m+1-k,K*n+1-k] <- A1[m,k,n])
      }
      for(k in (K/2+1):K){
        ifelse(A1[m,k,n]<=0, A2[K*m+1-k,K*n+1-k] <- A1[m,k,n] , A2[K*(m-1)+k,K*(n-1)+k] <- A1[m,k,n])
      }
    }
    else{ # k-polygonal Fuzzy Numbers when a^{-}_k = a^{+}_k (i.e. triangular, etc)
      for(k in 1:((K-1)/2)){ # see Remark 3
        ifelse(A1[m,k,n]>=0, A2[K*(m-1)+k,K*(n-1)+k] <- A1[m,k,n], A2[K*m+1-k,K*n+1-k] <- A1[m,k,n])
      }
      A2[K*(m-1)+(K+1)/2,K*(n-1)+(K+1)/2] = A1[m,(K+1)/2,n]
      for(k in ((K+3)/2):K){
        ifelse(A1[m,k,n]<=0, A2[K*m+1-k,K*n+1-k] <- A1[m,k,n] , A2[K*(m-1)+k,K*(n-1)+k] <- A1[m,k,n])
      }
    }
    # N*(K-1) constraints to ensure fuzzy numbers (ordering between components)
    for(k in 1:(K-1)){
      A2[M*K + (K-1)*(n-1)+k,K*(n-1)+k] = 1 # k=i
      A2[M*K + (K-1)*(n-1)+k,K*(n-1)+k+1] = -1 # k=i+1
    }
  }
}


# check it
A2
```

2	0	0	3	0	0
0	5	0	0	4	0
0	0	8	0	0	10
4	0	0	0	0	0
0	5	0	0	5	0
0	0	7	0	0	15
1	-1	0	0	0	0
0	1	-1	0	0	0
0	0	0	1	-1	0
0	0	0	0	1	-1

In [4]:
```
# Vector of coefficients
B2 = rbind(matrix(t(B1)),matrix(0,N*(K-1)))

# check it
B2
```

1
3
6
2
3
6
0
0
0
0

In [5]:
```
# Coefficient function, or costs vector associated
C2 = matrix(0,nrow=K,ncol = N*K)
# See Eq.(18) + Remark 3
for(n in 1:N){
  if(K%%2==0){ # gral. k-polygonal Fuzzy Numbers
    for(k in 1:(K/2)){
      ifelse(C1[k,n]>=0, C2[k,K*(n-1)+k] <- C1[k,n], C2[K+1-k,K*n+1-k] <- C1[k,n])
    }
    for(k in (K/2+1):K){
      ifelse(C1[k,n]<=0, C2[K+1-k,K*n+1-k] <- C1[k,n], C2[k,K*(n-1)+k] <- C1[k,n])
    }
  }
  else{ # k-polygonal Fuzzy Numbers when a^{-}_k = a^{+}_k (i.e. triangular, etc)
    for(k in 1:((K-1)/2)){
      ifelse(C1[k,n]>=0, C2[k,K*(n-1)+k] <- C1[k,n], C2[k,K*n+1-k] <- C1[k,n])
    }
    C2[(K+1)/2,K*(n-1)+(K+1)/2] = C1[(K+1)/2,n]
    for(k in ((K+3)/2):K){
      ifelse(C1[k,n]<=0, C2[k,K*n+1-k] <- C1[k,n], C2[k,K*(n-1)+k] <- C1[k,n])
    }
  }
}

# check it
C2
```

0	0	-1	1	0	0
0	0	0	0	2	0
0	0	2	0	0	3

In [6]:
```
# # Weak Pareto Solution for (MOLP)

# install.packages("lpSolve")
library("lpSolve")

# Some weight (arbitrary)
W = c(1,0,0) #rep(1,times=K)
W
W%*%C2
f.obj = W%*%C2 #c(t(C2))
f.con = A2
f.rhs = B2
f.dir <- rep("<=",times=M2)
# Now run.
WP.Sol = lp ("min", f.obj, f.con, f.dir, f.rhs)
#WP.Sol = array(lp ("min", f.obj, f.con, f.dir, f.rhs)$solution,c(Fu,N))

print("Weak Pareto Solution (WPS)")
WP.Sol$solution

print("WPS objective value")
WP.Sol$objval

#check it
f.obj%*%WP.Sol$solution
```

1 0 0

0	0	-1	1	0	0

```
[1] "Weak Pareto Solution (WPS)"
```

0 0 0.75 0 0 0

```
[1] "WPS objective value"
```

-0.75

-0.75

In [7]:
```
# Corresponding non dominated Fuzzy Solution for (FFLP)
x1.fsol = WP.Sol$solution[1:3]
x2.fsol = WP.Sol$solution[4:6]

x1.fsol
x2.fsol
```

0 0 0.75

0 0 0

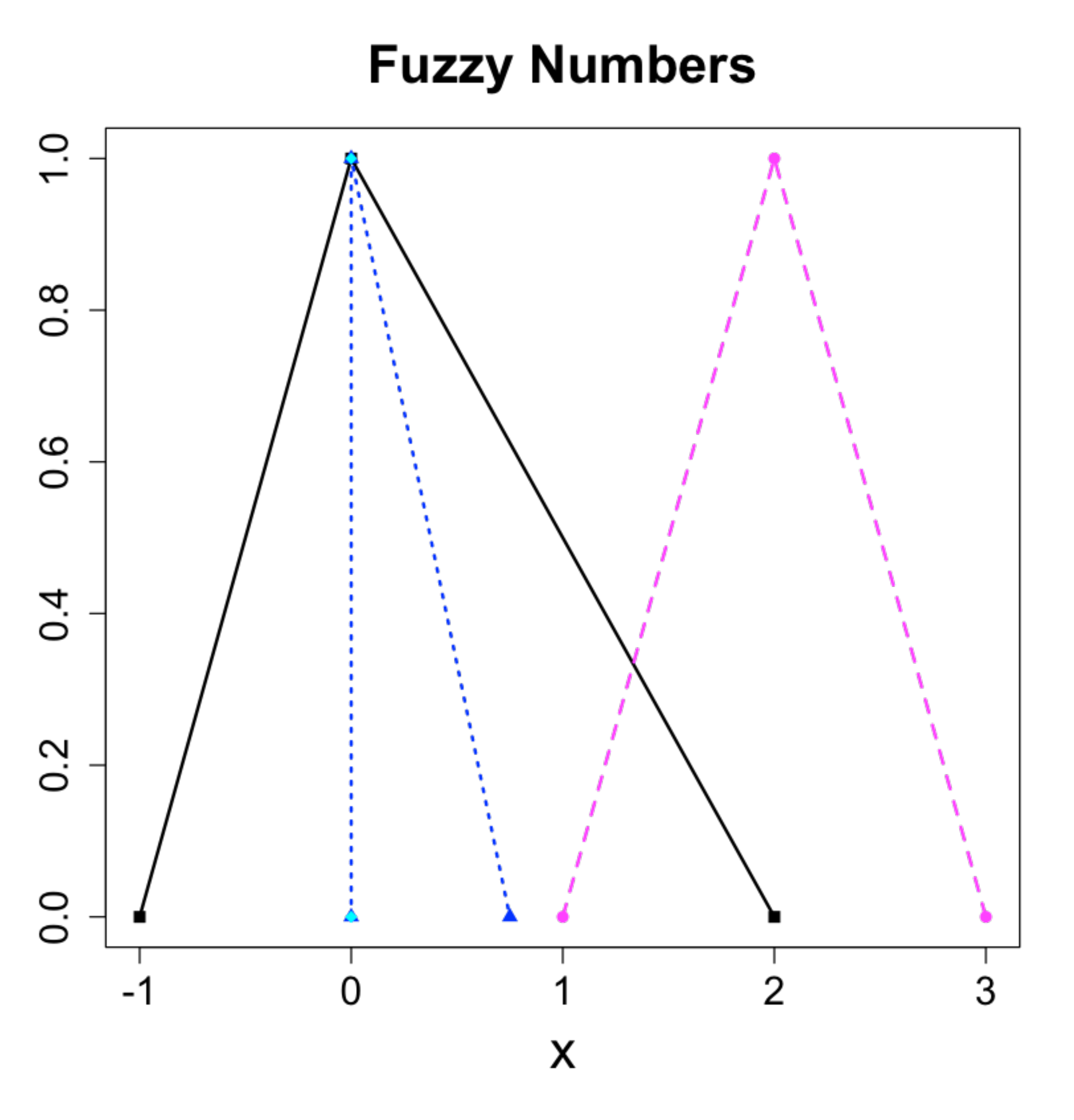
In [8]:
```
#Example of plotting Fuzzy Numbers

# Objective function (costs vector)
plot(C1[1:3],c(0,1,0),type = "l",lwd=2,col="black" ,
       xlim = c(-1,3),
       main="Fuzzy Numbers", ylab="",
       xlab = "x",
       cex.lab=2,cex.axis=1.5,cex.main=2)
# adding new points/lines (fuzzy number representation) to the previous plot
points(C1[1:3],c(0,1,0),lwd=2,#linewidth
       col= "black",# define color
       pch=15) # point character

lines(C1[4:6],c(0,1,0),lty=2,#line type
      lwd=2,col= "magenta")
points(C1[4:6],c(0,1,0),lwd=2,col= "magenta",pch=16)

# Fuzzy solutions:
lines(x1.fsol,c(0,1,0),lty=3,lwd=2,col= "blue")
points(x1.fsol,c(0,1,0),lwd=2,col= "blue",pch=17)

#lines(x2.fsol,c(0,1,0),lty=4,lwd=2,col= "cyan")
points(x2.fsol,c(0,1,0),lwd=2,col= "cyan",pch=18)
```



In [ ]: