Assignment 2. Questionnaire

SOLVING A NETWORK DESIGN PROBLEM USING BENDERS DECOMPOSITION.

The problem to solve can be stated as:

$$
\begin{array}{ll}
Min_{\ x,y} & \sum_{\ell\in K} c^{\ell\top} x^{\ell} + f^{\top} y \\
(1) & B x^{\ell} = t^{\ell}, \ \ \ell \in K \\
(2) & x^{\ell}_a \le \rho\, y_a, \ \ \ell \in K, \ a \in \hat{A} \\
(3) & x^{\ell} \ge 0 \\
 & y \in \{0,1\}^{|\hat{A}|}
\end{array}
$$

(For the notation and meaning of the variables, please refer to the Assignment's Description)

As stated in the Assignment's Description, at each iteration of the algorithm a Master Problem is solved and for the resulting value $\bar{y}$ of the decision variables, a subproblem is solved. The Master Problem's structure at iteration M will be:

$$
\begin{array}{lll}
Min_{\ y,z} & z & \\
s.t: & z \ge f^{\top} y + \sum_{\ell\in K} (\mathcal{T}^{\ell} + F y)^{\top} \hat{\theta}^{\ell,s}, \ \ s = 1,2,3,...M & \rightarrow \ \ (\bar{z}, \bar{y}) \\
 & y \in Y = \{0,1\}^{|\hat{A}|} &
\end{array}
$$

(In some cases, due to the data, no extreme rays are generated)

In its primal form the Subproblem is:

$$
\begin{array}{ll}
z_D = f^{\top} \bar{y} + Min_{\ x} & \sum_{\ell\in K} c^{\ell\top} x^{\ell} \\
(1) & B x^{\ell} = t^{\ell}, \ \ \ell \in K \\
(2) & x^{\ell}_a \le \rho\, \bar{y}_a, \ \ \ell \in K, \ a \in \hat{A} \\
(3) & x^{\ell} \ge 0
\end{array}
$$

For delivering the assignment the following items will be addressed:

1. Solve the problem directly, i.e., without using Benders decomposition and report the resulting solution:
   (a) Objective function value in its tow components: *a)* investment costs $f^{\top} y$, *b)* exploitation costs $\sum_{\ell\in K} c^{\ell\top} x^{\ell}$.
   (b) List of candidate links $a \in \hat{A}$ which will be included in the solution of the problem, i.e., links $a \in \hat{A}$ for which $y_a = 1$
2. Solve the problem using Benders decomposition. Once the algorithm has been satisfactorily implemented, report the problem's solution starting from two different initial solutions for the binary decision variables y. These two point are: a) $y_a = 0, \ \forall a \in \hat{A}$, b) $y_a = 1, \ \forall a \in \hat{A}$.

   In both cases report always a summary of the iterations carried out by the algorithm by means of a table or list in which, for each iteration the following appears reported:

   - Objective function's value $\bar{z}$ of Master Problem.
   - Objective function's value for the Subproblem z_D resulting from decision variables $\bar{y}$, as well as the investment and exploitation costs.

   Finally report what led the algorithm to stop and the final solution in the form of the subset of links within the candidates in $a \in \hat{A}$ that have been included and excluded. Report also the value of the flows on the network links for the final network's configuration.

It is required to present the AMPL's code developed in the implementation and a report, of up to 12 pages in length, answering properly the previous questions.