

## Rules for writing duals for a generic LP.

Primal Problem:

$$\begin{aligned}
 & \text{Min}_x \quad d_1^\top x_1 + d_2^\top x_2 + d_3^\top x_3 \\
 (P) \quad & \begin{array}{ll} A_1 x_1 + A_2 x_2 + A_3 x_3 & \stackrel{(1)}{\geq} a \mid y_1 \\ B_1 x_1 + B_2 x_2 + B_3 x_3 & = b \mid y_2 \\ C_1 x_1 + C_2 x_2 + C_3 x_3 & \stackrel{(2)}{\leq} c \mid y_3 \\ x_1 \geq 0; x_2 \leq 0; x_3 \text{ free} \end{array} \\
 & \Downarrow \\
 & \text{Max}_y \quad a^\top y_1 + b^\top y_2 + c^\top y_3 \\
 (D) \quad & \begin{array}{ll} A_1^\top y_1 + B_1^\top y_2 + C_1^\top y_3 & \leq d_1 \leftarrow -x_1 (x_1 \geq 0) \\ A_2^\top y_1 + B_2^\top y_2 + C_2^\top y_3 & \geq d_2 \leftarrow -x_2 (x_2 \leq 0) \\ A_3^\top y_1 + B_3^\top y_2 + C_3^\top y_3 & = d_3 \leftarrow -x_3 \text{ free} \\ y_1 \stackrel{(1)}{\geq} 0; y_3 \stackrel{(2)}{\leq} 0; y_2 \text{ free} \end{array}
 \end{aligned}$$

Putting in Standard Form (P) and (D)

$$\begin{pmatrix} A_1 & -A_2 & A_3 & -A_3 & -I_a & 0 \\ B_1 & -B_2 & B_3 & -B_3 & 0 & 0 \\ C_1 & -C_2 & C_3 & -C_3 & 0 & I_c \end{pmatrix} \begin{pmatrix} x_1 \\ x_2' \\ x_3^+ \\ x_3^- \\ \sigma_a \\ \sigma_c \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$x_1, x_2', x_3^+, x_3^-, \sigma_a, \sigma_c \geq 0; \quad (x_2' = -x_2)$$

$$\Downarrow \\
 \begin{pmatrix} A_1^\top & B_1^\top & C_1^\top \\ -A_2^\top & -B_2^\top & -C_2^\top \\ A_3^\top & B_3^\top & C_3^\top \\ -A_3^\top & -B_3^\top & C_3^\top \\ -I_a & 0 & 0 \\ 0 & 0 & I_c \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \leq \begin{pmatrix} d_1 \\ -d_2 \\ d_3 \\ -d_3 \\ 0 \\ 0 \end{pmatrix}$$