3rd Assignment. Frank Wolfe algorithm

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Introduction

In this work we present a solution for equilibrium traffic assignment problem. The algorithm is based on the concepts of simplicial decomposition, regularization and partial linearization. In this assigment, we consider an integrable linear delay cost function. For link (i, j), the equation is: $s_{ij} = c_{ij} + d_{ij}x$, where c_{ij},d_{ij} are taken from the corresponding values c, d in the data set.

We have an instance of 7 nodes, and consider the nodes 1-3 as origins and 6-7 as destinations. So we have the next o-d pairs:

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(1,6)
(1,7)
(3,6)
(3,7)
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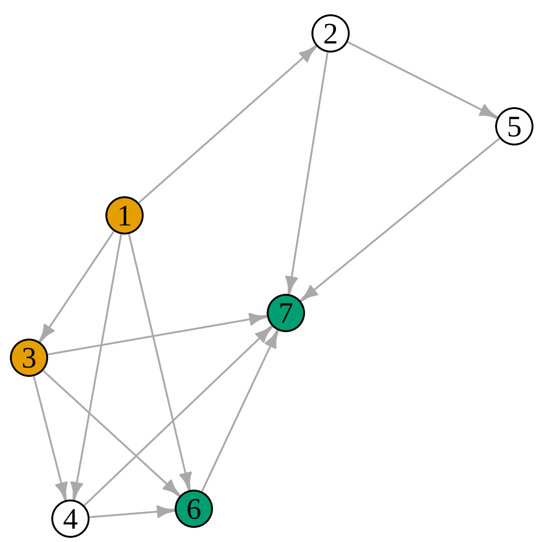
Graphical representation of the network

In the next graph, we represented the network associated with our instance. Green colors are destinations and orange colors origins:



Idea of the algorithm

We formulate the problem in terms of paths (routes) between o-d pairs. The simplicial decomposition approach can be viewed as a column generation approach. Consider a subset of k paths , we solve a master problem (MP) and evaluate this solution to other problem. If optimal path flows are optimal in MP are also optimal in other problem. The idea is to evaluate the arc flows and the gradient of the objective function. By the Caratheodory theorem, any feasible point of bounded polytype can be expressed as a convex combination of its extreme points, and it's obtained by the MP.

The quadratic function to minimize is: $\sum c_{ij}\ x + \sum \frac{1}{2} d_{ij}\ x^2$ subject to supply the o-d pairs flow in the network.

Steps of the algorithm:

1) Solve subproblem (gradient of the objective function): $c_{ij} + d_{ij}\ x$.
2) Update working sets: Add new vertex and remove with information of α (MP variable) the smaller baricentric coordinate (small value of α). When we say small is in terms of zero or close to zero.
3) Update best lower bound (BLB) and calculate gap. *If some criteria = STOP*
4) Solve MP: Minimize quadratic function subject to actual working set, or in other words, solve problem with a convex combination of extreme points obtaining in previous steps. *Go to step 1.*

In step 3, we calculate BLB as: $max(BLB, f(x^v) + \nabla_x f(x^v)^T (\hat{x}^v - x^v))$ and gap as: $\frac{f(x^v - BLB)}{BLB}$. We stop if $gap < 0.005$ or iteration number=500.

We start algorithm with BLB = $-\infty$

Procedure information

- nº iter
- Funcio objectiu Q
- relgap value
- step length ?
- nº vertexs
- Size of working sets Wx, Ws

Graphical representation of procedure

- iter vs lower bound
- iter vs log(relgap)

Direct solution

Printout of the iterations