

**Corresponding Manuscript:** Conditional Operation of Hole Spin Qubits above 1 K — [Notebook: S5\_Two\_Tone\_varied\_pump-time\_J38.7MHZ\_v040925\_v3.nb]

**Authors:** M.J. Carballido | ORCID: <https://orcid.org/0000-0001-7385-8284>

T. Patlatiuk

**Project mirror:** <https://github.com/carmig00/Publications-OpenAccess-Code-Conditional2Q>

**License:** CC BY-NC-SA 4.0 International – <https://creativecommons.org/licenses/by-nc-sa/4.0/>

**How to run:** Evaluate top-to-bottom (Evaluation → Evaluate Notebook).

**Outputs:** HDF5 (.h5) written to ./exports.

**Figure mapping:** This notebook reproduces the supplementary information's 2-tone experiments which were expanded by an additional dimension. That is, the duration of the pump tone, at fixed frequency  $f_1$  to  $f_4$ , was additionally varied.

#### Comment/Note:

-The fast qubit (high  $f_{\text{Rabi}}$ ), at lower Larmor frequency, is the one located physically on the right and is represented by pink/purple color tones, here qubit 1.

- The slow qubit (low  $f_{\text{Rabi}}$ ), at higher Larmor frequency, is the one located physically on the left and is represented by orange/yellow color tones, here qubit 2.

(\*  $f_{\text{Rabi}}$ , MHz \*)

(\* We assume this to be the bare Rabi frequency of the two ideally isolated spin qubits. Sometimes also referred to as bare Rabi frequency, for  $J=0$  \*)

$f_{R1t} = 24;$

$f_{R2t} = 13;$

(\* Experimentally chosen pulse durations to induce a  $\pi/2$  flip for qubit 1 and qubit 2, see supplementary material for details on the calibration \*)

(\* units in  $\mu\text{s}$  \*)

$Tp1 = 20 \times 10^{-3};$  (\* corresponding to 20 ns \*)

$Tp2 = 30 \times 10^{-3};$  (\* corresponding to 30 ns \*)

(\* Effective exchange interaction  $J$ , MHz\*)

$Jval = 38.7;$

(\*Larmor frequencies,

$2\pi \times \text{MHz}$ . These values were settled on after running the 2-tone simulations.\*)

$\omega_{z1} = 2\pi \times 2075;$

$\omega_{z2} = 2\pi \times 2270;$

(\* Empirical frequency choices for  $f_1$ - $f_4$ , see supplementary information. \*)

(\*  $2\pi \times \text{MHz}$  \*)

$\omega_{xexp1} = 2\pi \times 2057;$

$\omega_{xexp2} = 2\pi \times 2094;$

$\omega_{xexp3} = 2\pi \times 2257;$

$\omega_{xexp4} = 2\pi \times 2284;$

```

(* Theta is the SO-angle. In an accurate and complete microscopic description,
his accounts for the angle by which the spin is rotated due to SOI,
i.e. the angle in the rotation matrix R. To avoid over-
fitting this angle is set to 0 as we consider an effective J,
J_eff between the two qubits *)
 $\theta_{SO} = 0;$ 
(* Phases of the drives. *)
 $\varphi_1 = 0;$ 
 $\varphi_2 = 0;$ 

(* Populations:
These are the states that are blocked in PSB and which we initialize in. *)
 $\psi_{0a} = \{1, 0, 0, 0\};$ 
 $\psi_{0b} = \{0, 0, 0, 1\};$ 

(*NOTE: block not used here. Originally for testing time scans.*)
resTg = 100;
(*initial time*)
Tg0 = 0;
(*final time, 66.6ns*)
Tgf = 3 * 1 / 30;
Tgarr = Table[t, {t, Tg0, Tgf,  $\frac{Tgf - Tg0}{resTg - 1}$  }];

(* SO-vector *)
{nx, ny, nz} = {1, 0, 0};

(*Effective exchange interaction*)
(*Relevant component parallel with respect to external magnetic field B.*)
Jpar[J_] = 2 Pi J (nz2 + (1 - nz2) Cos[ $\theta_{SO}$ ]);
(* $\lambda x_{00} = 2$  Pi 30; (*Russ/Burkard resonance condition for driving*)*)

(*2-Qubit Hamiltonian*)
HQ[ $\omega_{z1in}$ _,  $\omega_{z2in}$ _] = KroneckerProduct[PauliMatrix[3]  $\frac{\omega_{z1in}}{2}$ , PauliMatrix[0]] +
KroneckerProduct[PauliMatrix[0], PauliMatrix[3]  $\frac{\omega_{z2in}}{2}$ ];
Rso = {
{nx2 + (1 - nx2) Cos[ $\theta_{SO}$ ],
nx ny - nx ny Cos[ $\theta_{SO}$ ] - nz Sin[ $\theta_{SO}$ ], nx nz - nx nz Cos[ $\theta_{SO}$ ] + ny Sin[ $\theta_{SO}$ ] },

```

$$\left\{ \begin{aligned} & \{ nx\, ny - nx\, ny \cos[\theta_{so}] + nz \sin[\theta_{so}], \, ny^2 + (1 - ny^2) \cos[\theta_{so}], \\ & ny\, nz - ny\, nz \cos[\theta_{so}] - nx \sin[\theta_{so}] \}, \, \{ nx\, nz - nx\, nz \cos[\theta_{so}] - ny \sin[\theta_{so}], \\ & 2 \sin\left[\frac{\theta_{so}}{2}\right] \left( nx \cos\left[\frac{\theta_{so}}{2}\right] + ny\, nz \sin\left[\frac{\theta_{so}}{2}\right] \right), \, nz^2 + (1 - nz^2) \cos[\theta_{so}] \} \end{aligned} \right\};$$

(\*Exchange Hamiltonian\*)

$$HJ[J\_] = \frac{J}{4} \text{Sum}[\text{KroneckerProduct}[\text{PauliMatrix}[i], (\text{Rso.Array}[\text{PauliMatrix}, 3])[i]], \{i, 1, 3\}];$$

(\*turn on time of pulses\*)

$$dt0 = 0.001;$$

(\*wait time between pulses\*)

$$Tw = 0.001;$$

(\*driving amplitudes, here normalised to 2 Pi and equal to each other. One could choose to e.g. drive one qubit harder than the other one. (this could lead to over-fitting)\*)

$$\lambda x10 = 2 \text{ Pi}; \quad (*\text{Drive amplitude Q1}*)$$

$$\lambda x20 = 2 \text{ Pi}; \quad (*\text{Drive amplitude Q2}*)$$

(\*first pulse for a time Tflip1\*)

$$\lambda x1[t\_]=\lambda x10 \left( \frac{1}{2} + \frac{1}{2} \text{Erf}\left[\frac{T\text{flip1}-t}{dt0}\right] \right);$$

(\*second pulse after a time Tflip1+Tw\*)

$$\lambda x2[t\_]=\lambda x20 \left( \frac{1}{2} + \frac{1}{2} \text{Erf}\left[\frac{t-T\text{flip1}-Tw}{dt0}\right] \right);$$

(\* Drive Hamiltonian \*)

$$H\lambda[t\_]=\lambda x1[t] \cos[\omega x1 t + \phi 1]$$

$$\begin{aligned} & (fR1 \text{KroneckerProduct}[\{nx, ny, nz\}.Array[\text{PauliMatrix}, 3], \text{PauliMatrix}[0]] + fR2 \\ & \text{KroneckerProduct}[\text{PauliMatrix}[0], \{nx, ny, nz\}.Array[\text{PauliMatrix}, 3]]) + \lambda x2[t] \cos[ \\ & \omega x2 t + \phi 2] (fR1 \text{KroneckerProduct}[\{nx, ny, nz\}.Array[\text{PauliMatrix}, 3], \text{PauliMatrix}[0]] + \\ & fR2 \text{KroneckerProduct}[\text{PauliMatrix}[0], \{nx, ny, nz\}.Array[\text{PauliMatrix}, 3]]); \end{aligned}$$

(\*Total Hamiltonian\*)

$$Htot[t\_ , J\_ , \omega z1in\_ , \omega z2in\_ ] = HQ[\omega z1in, \omega z2in] + HJ[J] + H\lambda[t];$$

$$\rho Sf =$$

$$\begin{aligned} & \text{Function}\left[\{\psi 0, \psi ro, T\text{flip1in}, T\text{flip2in}, \omega z1in, \omega z2in, \omega x1in, \omega x2in, fR1in, fR2in, Jin\}, \text{sub} = \right. \\ & \quad \{T\text{flip1} \rightarrow T\text{flip1in}, T\text{flip2} \rightarrow T\text{flip2in}, \omega x1 \rightarrow \omega x1in, \omega x2 \rightarrow \omega x2in, fR1 \rightarrow fR1in, fR2 \rightarrow fR2in\}; \\ & \quad Tfin = T\text{flip1} + Tw + T\text{flip2} /. \text{sub}; \\ & \quad \psi fin[t\_]=\{a[t], b[t], c[t], d[t]\} /. \text{NDSolve}[ \end{aligned}$$

```

{Htot[t, Jin,  $\omega$ z1in,  $\omega$ z2in].{a[t], b[t], c[t], d[t]} ==
  i D[{a[t], b[t], c[t], d[t]}, t] /. sub,
  a[0] ==  $\psi$ 0[[1]], b[0] ==  $\psi$ 0[[2]], c[0] ==  $\psi$ 0[[3]], d[0] ==  $\psi$ 0[[4]],
  {a[t], b[t], c[t], d[t]}, {t, 0, Tfin},
  Method → "PDEDiscretization" → {"MethodOfLines",
    "SpatialDiscretization" → "FiniteElement", MaxCellMeasure → 0.0005 / fR1}] [[1]];
(*readout signal*)
 $\sum_{i=1}^{\text{Dimensions}[\psi_{ro}][[1]]} \text{Abs}[(\psi_{ro}[[i]].\psi_{fin}[Tfin])]^2];$ 

(*Two-tone spectroscopy 2D, averaged over different Larmor frequencies*)

(*To simulate noise, we averaged over different Larmor
frequencies sampled from a random, normal distributed set.*)
(*sigma of normal distribuion 2*Pi*MHz *)
 $\sigma\omega$ z1 = 25  $\times$  2 Pi;
 $\sigma\omega$ z2 = 25  $\times$  2 Pi;
(* Resolution of the array of random Larmor frequencies MHz*)
res $\omega$ z1 = 150; (* 150 *)
res $\omega$ z2 = 150; (* 150 *)
(* Arrays of random Larmor freqs. MHz*)
 $\omega$ z1arr = Abs[RandomVariate[NormalDistribution[ $\omega$ z1,  $\sigma\omega$ z1], {res $\omega$ z1}]];
 $\omega$ z2arr = Abs[RandomVariate[NormalDistribution[ $\omega$ z2,  $\sigma\omega$ z2], {res $\omega$ z2}]];

(* 2Pi rotation of the slover qubit to define upper time bound.*)
t1max = 1 / fR2t;
rest1 = 40; (* 40 *)
t1arr = Table[t1val, {t1val, 0, t1max, t1max / (rest1 - 1)}];

(*Resolution of frequency scanned (x-axis) MHz*)
res $\omega$  = 80; (* 160 *)
(*initial freq*)
 $\omega$ in = 2  $\pi$  1800;
(*final freq*)
 $\omega$ fin = 2  $\pi$  2600;
(*array of freqs. scanned over, MHz*)
 $\omega$ arr = Table[t, {t,  $\omega$ in,  $\omega$ fin, ( $\omega$ fin -  $\omega$ in) / (res $\omega$  - 1)}];

(* We performed the experiment with the fixed frequency
guesses introduced in the supplementary information (and above),
denoted as  $\omega$ xexp1 to  $\omega$ xexp4 above. We further use  $\omega$ z1 and  $\omega$ z2 that we extracted as fit
parameters from the previous two-tone scans. The values for the Larmor frequencies
 $\omega$ z1 and  $\omega$ z2 found herein are the same ones as used in the two-tone scans.*)

```

```

(* We read out in the 0,1,0,0 and *)
(* 0,0,1,0 states (anti-parallel) which are not blocked in PSB *)
(*NOTE: For the probe tones, we apply the Pi pulse Tp1 for
f1 and f2 and Tp2 for f3 and f4. The pump tones follow the array*)

resF1vs2D =
ParallelTable[( $\rho$ Sf[ $\psi$ 0a, {{0, 1, 0, 0}}, {0, 0, 1, 0}}, t1arr[[k]], Tp1,  $\omega$ z1arr[[j]],  $\omega$ z2arr[[j]],
 $\omega$ xexp1, warr[[i]], fR1t, fR2t, Jpar[Jval]] +  $\rho$ Sf[ $\psi$ 0b, {{0, 1, 0, 0}}, {0, 0, 1, 0}},
t1arr[[k]], Tp1,  $\omega$ z1arr[[j]],  $\omega$ z2arr[[j]],  $\omega$ xexp1, warr[[i]], fR1t, fR2t, Jpar[Jval]]) / 2,
{j, 1, res $\omega$ z1}, {k, 1, rest1}, {i, 1, res $\omega$ }}; // AbsoluteTiming
resF2vs2D =
ParallelTable[( $\rho$ Sf[ $\psi$ 0a, {{0, 1, 0, 0}}, {0, 0, 1, 0}}, t1arr[[k]], Tp1,  $\omega$ z1arr[[j]],  $\omega$ z2arr[[j]],
 $\omega$ xexp2, warr[[i]], fR1t, fR2t, Jpar[Jval]] +  $\rho$ Sf[ $\psi$ 0b, {{0, 1, 0, 0}}, {0, 0, 1, 0}},
t1arr[[k]], Tp1,  $\omega$ z1arr[[j]],  $\omega$ z2arr[[j]],  $\omega$ xexp2, warr[[i]], fR1t, fR2t, Jpar[Jval]]) / 2,
{j, 1, res $\omega$ z1}, {k, 1, rest1}, {i, 1, res $\omega$ }}; // AbsoluteTiming
resF3vs2D =
ParallelTable[( $\rho$ Sf[ $\psi$ 0a, {{0, 1, 0, 0}}, {0, 0, 1, 0}}, t1arr[[k]], Tp2,  $\omega$ z1arr[[j]],  $\omega$ z2arr[[j]],
 $\omega$ xexp3, warr[[i]], fR1t, fR2t, Jpar[Jval]] +  $\rho$ Sf[ $\psi$ 0b, {{0, 1, 0, 0}}, {0, 0, 1, 0}},
t1arr[[k]], Tp2,  $\omega$ z1arr[[j]],  $\omega$ z2arr[[j]],  $\omega$ xexp3, warr[[i]], fR1t, fR2t, Jpar[Jval]]) / 2,
{j, 1, res $\omega$ z1}, {k, 1, rest1}, {i, 1, res $\omega$ }}; // AbsoluteTiming
resF4vs2D =
ParallelTable[( $\rho$ Sf[ $\psi$ 0a, {{0, 1, 0, 0}}, {0, 0, 1, 0}}, t1arr[[k]], Tp2,  $\omega$ z1arr[[j]],  $\omega$ z2arr[[j]],
 $\omega$ xexp4, warr[[i]], fR1t, fR2t, Jpar[Jval]] +  $\rho$ Sf[ $\psi$ 0b, {{0, 1, 0, 0}}, {0, 0, 1, 0}},
t1arr[[k]], Tp2,  $\omega$ z1arr[[j]],  $\omega$ z2arr[[j]],  $\omega$ xexp4, warr[[i]], fR1t, fR2t, Jpar[Jval]]) / 2,
{j, 1, res $\omega$ z1}, {k, 1, rest1}, {i, 1, res $\omega$ }}; // AbsoluteTiming

Export["C:\\Users\\export\\two_tone_2D_average_Larmor_sigma" <>
TextString[ $\sigma$  $\omega$ z1] <> "MHz_J" <> TextString[Jval] <> "MHz_v170925_v1.h5",
{resF1vs2D, resF2vs2D, resF3vs2D, resF4vs2D,  $\omega$ z1arr / (2 Pi),  $\omega$ z2arr / (2 Pi),
warr / (2 Pi), t1arr}, {"Datasets", {"resF1vs2D", "resF2vs2D",
"resF3vs2D", "resF4vs2D", "fR1arr", "fR2arr", "farr", "t1arr"}}];

Out[8]=
{3510.87, Null}

Out[9]=
{2862.44, Null}

Out[10]=
{3272.62, Null}

Out[11]=
{3256., Null}

```