An Introduction to Finance

Business Intelligence per i Servizi Finanziari 2023-2024

Antonio Candelieri

Buy and selling stocks

- Buy and sell orders are regulated by the market authority and have different forms to allow for (as well as forbid!) different trading strategies

- Three common forms for buy and sell orders:
  - **Market order**
  - **Limit order**
  - **Stop order**

Market order

- Order sent to the market to execute the trade on the security immediately
- Used when execution of the trade is a priority over the price
- Price when order is executed (aka «filled») is the best possible at the time of execution: so it might be different (better or worse) than the price at the time order was sent

Limit order

- Sent to the market for a given or better price at which to buy/sell the security
- The trade is executed only when a matching offer for the security shows up
- This type of order gives the trader the control on the price but not on the time of execution

Stop order

- Held by the broker and sent to market only when the price of the security reaches a specified amount (*stop price*)
- When the pre-set price is reached, the stop order can be turned into a market or a limit order (as specified by the investor)
- Used to put a limit to possible losses (*stop loss order*) or to take a pre-set percentage of profits (*stop gains* or *profit taking order*)

Stop vs Limit orders

- There is a subtle yet important difference between a stop and a limit order:

- Both demand a certain price (i.e. stop and limit price), but only limit order is sent immediately to the market (even it might not get executed)

Example of stop loss order with a limit. *Consider to have 200 shares with a price of 40 € each. You decide to sell if the price goes below 35 € but restraining your losses to no more than 7 € per share.*

«To sell the 200 shares at the limit price of 33 € and activate this (limit) order when the price <= 35 €. Keep this order active until the end of next month»

Research topic

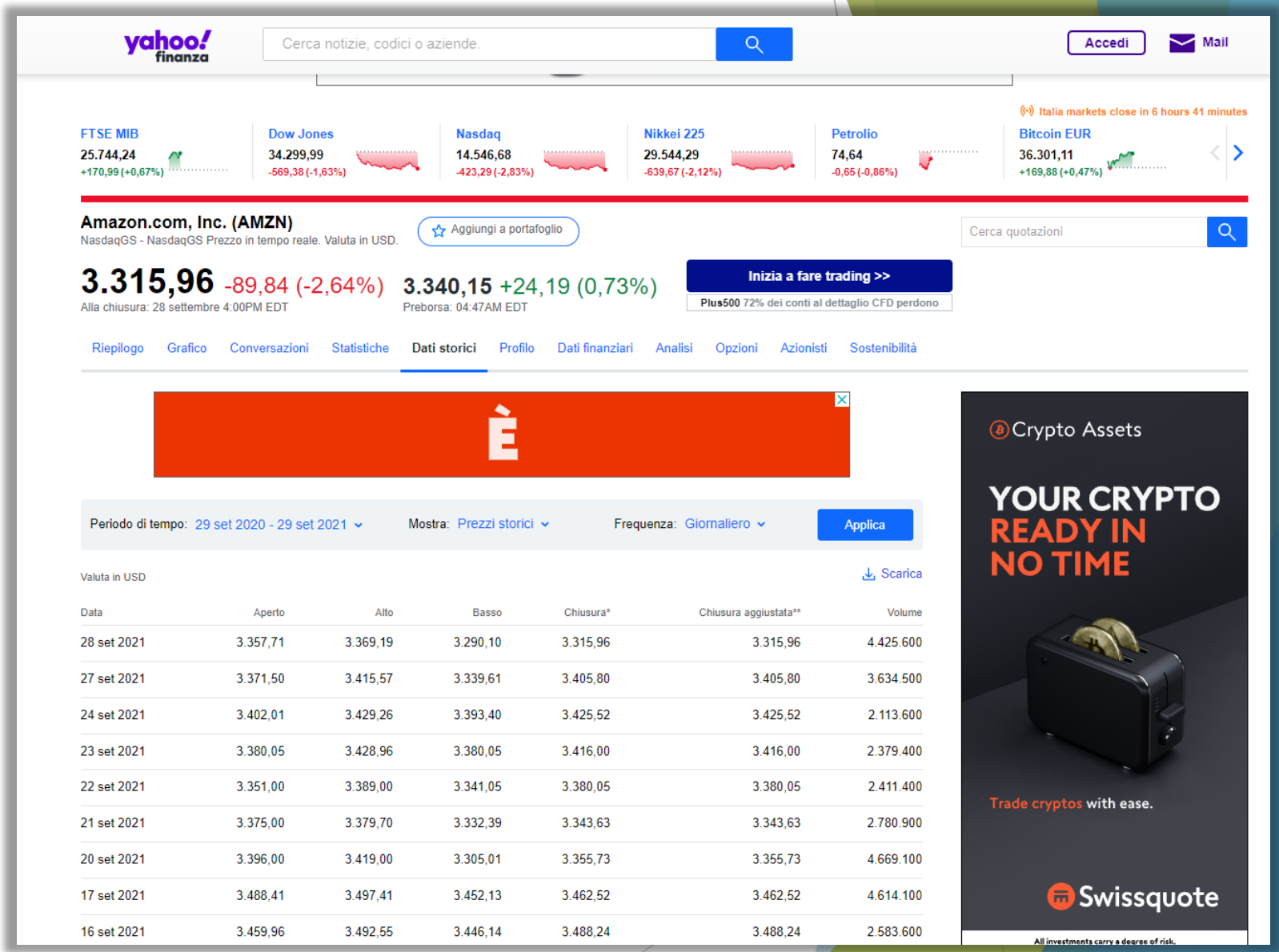
- How to establish the stop price in stop orders, in particular stop loss orders
- We would like to be certain that when our stop order triggers we are really minimizing our losses, not missing on future gains
- *We will see, during the course, that «pricing» is a recurrent optimization problem in finance…*

Stock quotes data

- Several providers for stock quotes data, freely viewable
  - **Google's Finance:** https://www.google.com/finance
  - **Yahoo! Finance:** https://finance.yahoo.com/
  - **…**

OHLC data

- Historical data (usually daily is the finer scale)
  - Open, High, Low, Close
  - + Adjusted Close
  - + Volume



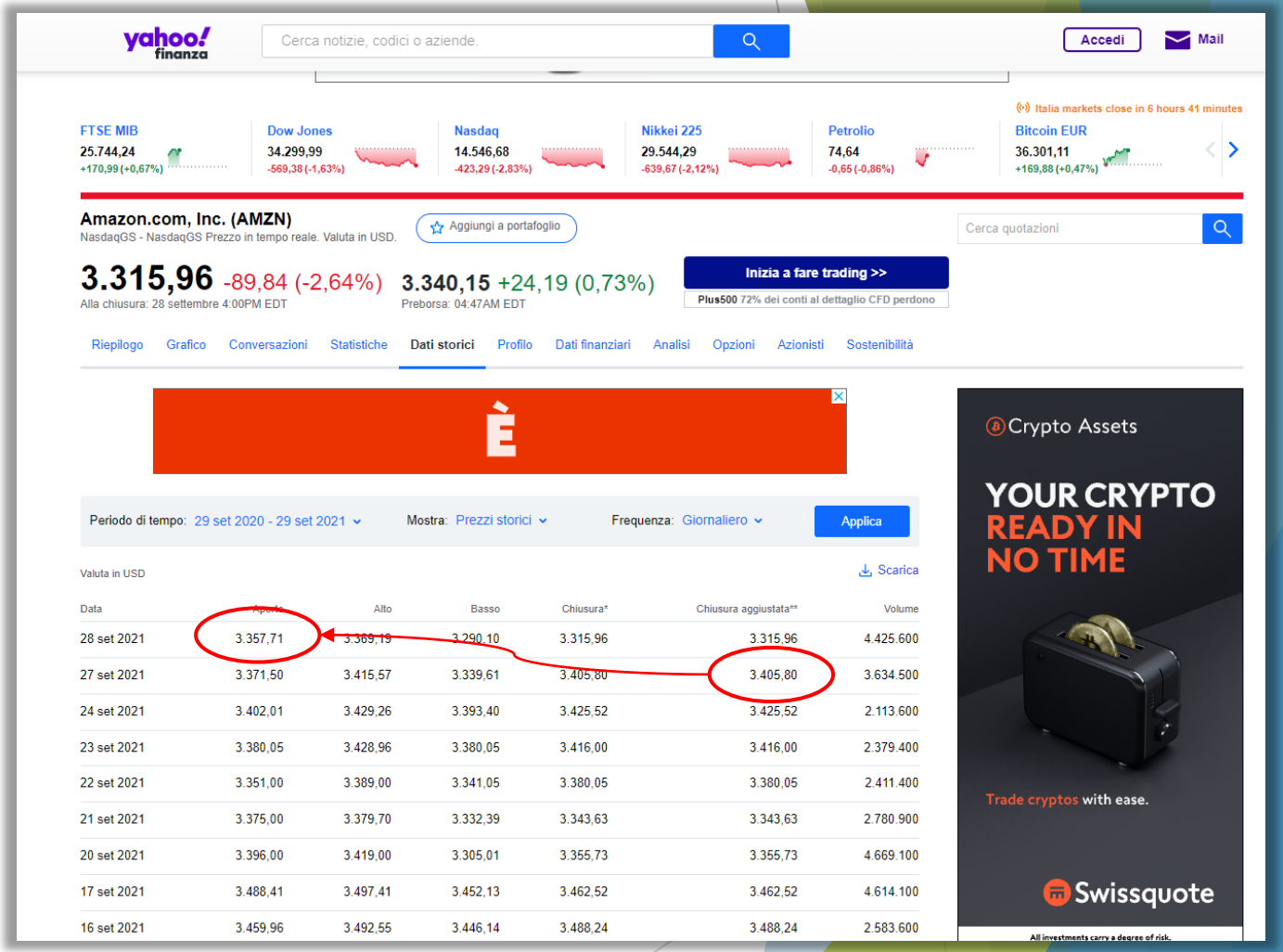
Historical quotes/prices

- **Ticker**: the company's id for the market (e.g. AAPL for Apple Inc.)
- **Open**: the price of the stock at the opening time of the market session
- **Close**: the current price of the stock, if the market is in session, or the last price traded when market closes
- **High**: max price reached by the stock between the Open and Close price
- **Low**: min price reached by the stock between the Open and Close price
- **Adjusted Close**: closing price adjusted to include dividend payments plus any other corporate action that affects the stock price (e.g. splits and rights offerings)
- **Volume**: number of shares traded between Open and Close time

- **Dividend**: a cash payment given periodically by some companies reflecting profits returned to shareholders

Errors in the data???

- These are not errors…
- Usually Open is not equal to the previous Close, due to some «particular» trades which may significantly alter the market and are put on hold by the market authority and executed after the market closes
- **Volume** is defined as the total number of shares that are flowing in the market, regardless of their direction (buy/sell)
- Detailed info about Volume is usually unknown on cost-free channels but investors can have access to these infos through private brokerage services



Payoff and profit of stocks

- Consider a simple situation with no dividends and no transaction costs
- Buy one share of a stock at a time $t = t_0$ for a price of S_0 and sell it at a later time $t = T$ for a price of S_T
- Thus, the payoff is S_T (it will be m x S_T in the case of m shares)

- However, the profit obtained can not be just the result of discounting the initial investment to the payoff, because there could be cash flows at different points in time, and there is some risk involved in the investment which has to be taken into consideration!
- Thus, assuming continuous compounding at a constant interest rate r, the profit for one share of a stock bought at time t_0 for S_0 and sold at time T for S_T is:

$$S_T + D_T - C(S_0)e^{r\,T}$$

Stock indices

- An index is a mathematical function that measures the changes in a representative group of data points
- In the stock market the data points are the prices of stocks, and a stock index tracks the changes in the value of a selected group of stocks that supposedly represent the market or an industrial sector
- It is a general reference of the market's value trend and the general state of health of the economy
- For the majority of investors, it represents a benchmark for the performance of their investments
- As it does not represent one company, but a hypothetical basket of companies, it cannot be traded by the investors; it is only a reference

Stock indices

- Stock indices in use worldwide today are composed by either one of the following two methods:
  - **Price weighted**: only the price of each component stock is considered (e.g., Dow Jones Industrial Average DJIA, NYSE ARCA Tech 100, and the Amex Major Market)
  - **Capitalization-weighted**: considers the market capitalization of each stock composing the index; that is, the stock price times the number of shares outstanding (e.g. NASDAQ Composite, FTSE 100, Russell 2000, CAC 40 and IBEX 35)

- In a price-weighted index a significant change in the price of a single component may heavily influence the value of the index, regardless the size of the company (i.e., *shares outstanding*)
- In a capitalization-weighted index a small shift in price of a big company will heavily influence the value of the index

A price-weighted index

Example 1.2 **(A price weighted index)** The Dow Jones Industrial Average, conceived by Charles Dow and Edward Jones (1896), is computed by the following formula

$$\text{DJIA}_t = \frac{\sum_{i=1}^{30} S_{i,t}}{D} \tag{1.8}$$

where $S_{i,t}$ is the price of stock i at time t and D is the Dow Divisor, a constant included in the equation since 1928 to adjust, or rather stabilized, the average in case of stock splits, spinoffs or similar structural changes; as of July 2 of 2010 the divisor is 0.132129493.[6] □

A capitalization weighted index

Example 1.3 **(A capitalization weighted index)** The Madrid Stock Exchange principal index, IBEX35, is computed by the following formula[7]

$$\text{IBEX}_t = \text{IBEX}_{t-1} \times \frac{\sum_{i=1}^{35} Cap_{i,t}}{\sum_{i=1}^{35} Cap_{i,t-1} \pm J} \tag{1.9}$$

where IBEX_t is the value of the index at time t, $Cap_{i,t}$ is the free float market capitalization of company i at time t, and J a coefficient used to adjust the index, similar in nature as the D in the DJIA equation. □

Why indices are important?

- A way that investors have to gauge the performance of their stock investments is to compare their price behavior with another reference stock's price history, or usually with the history of the market index
- More precisely, the comparison is made between the cumulative ***rate of benefits*** that one obtains throughout the investment period with the cumulative rate of benefits that one would have obtained in the same time period if the investment would have been made in the reference stock, or (hypothetically) in the market index
- Note that it is important to consider the successive sums of rates of benefits because under this factor of price variation all stocks are measured at a same scale, so comparisons make sense
- For any given price series $\{P_t : t \geq 0\}$ the rate of benefit (also known as ***return***), from one time instant to the next, is given by

$$R_t = (P_t / P_{t-1}) - 1$$

- *We will discuss about returns more in detail…*

Options and other Derivatives

- An **option** is a contract that investors can buy for a fee to have **the opportunity**, but **not the obligation**, to trade an asset at a future date and at a given price
- *Exercise date*, *expiration date* or *maturity*: date for making the trade
- *Exercise price* or *strike price*: price of the asset as written in the option
- **Call** (**Put**) option: if the trade in the option is to buy (sell) the asset
- *Exercising the option* means the trade specified in the option is carried out by the exercise date and at the exercise price

Different styles of options

- **European**: may only be exercised on expiration date
- **American**: may be exercised on any trading day on or before the expiration date
- **Bermudan**: may be exercised only on predetermined dates before expiratrion (somewhat between Europen and American)
- **Asian**: the payoff is determined by the average price of the underlying asset during some period of time within the life of the option
- **Barrier**: may only be exercised if the price of the underlying asset passes a certain level (i.e. a barrier) during a certain period of time

Different styles of options

- *European* and *American* options are classified as *vanilla*, which means that the option is characterized by direct and simple payoff conditions
- Other options involving more complex conditions are named *exotic*
- Moreover, Asian and Barrier are *path-dependent* because for calculating their payoffs it is needed to known their price history
- For *path-dependent* options, the pricing problem is further complicated!

Reading options quotes

- The price of an option is also named ***premium*** and is given on a per share basis

- An option on a stock correspondes to 100 shares
  - *For instance, if an option on stocks of a company (e.g. AAPL) has a quoted premium of 2.64$, the total amount to pay is 2.64$ x 100 = 264$. This entitles the holder to buy or sell 100 shares of the company at maturity*

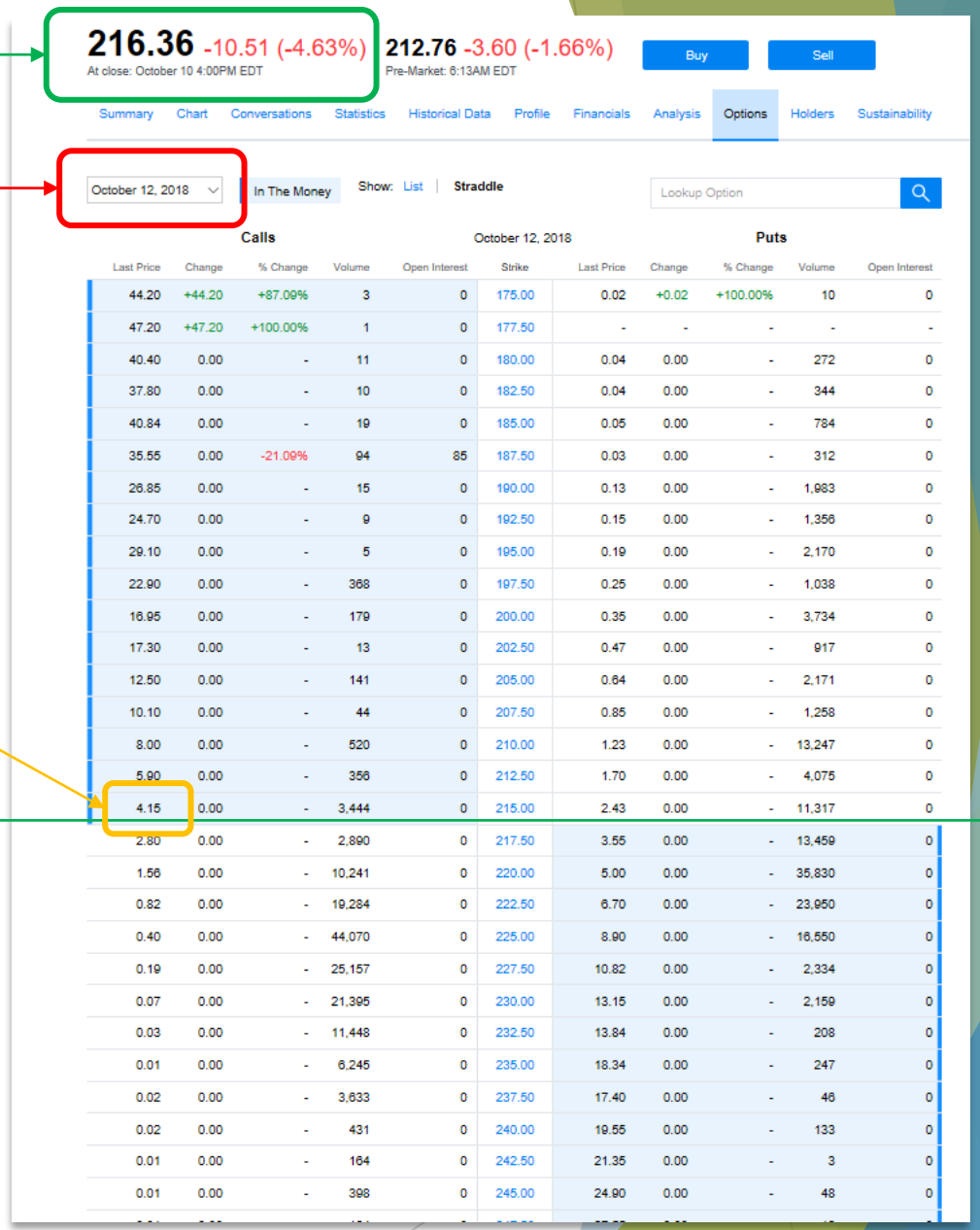
- The options are organized in series of different premiums, for a fixed maturity date, and quoted at the exchange market using the following standard nomenclature:

  Security Simbol + Expiration Date + Type (call/put) + Strike Price

current price

maturity

Last quoted
premium for the call



216.36 -10.51 (-4.63%)
At close: October 10 4:00PM EDT
212.76 -3.60 (-1.66%)
Pre-Market: 6:13AM EDT
Buy
Sell

Summary | Chart | Conversations | Statistics | Historical Data | Profile | Financials | Analysis | Options | Holders | Sustainability

October 12, 2018 | In The Money | Show: List | Straddle | Lookup Option

Calls — October 12, 2018 — Puts

Last Price	Change	% Change	Volume	Open Interest	Strike	Last Price	Change	% Change	Volume	Open Interest
44.20	+44.20	+87.09%	3	0	175.00	0.02	+0.02	+100.00%	10	0
47.20	+47.20	+100.00%	1	0	177.50	-	-	-	-	-
40.40	0.00	-	11	0	180.00	0.04	0.00	-	272	0
37.80	0.00	-	10	0	182.50	0.04	0.00	-	344	0
40.84	0.00	-	19	0	185.00	0.05	0.00	-	784	0
35.55	0.00	-21.09%	94	85	187.50	0.03	0.00	-	312	0
26.85	0.00	-	15	0	190.00	0.13	0.00	-	1,983	0
24.70	0.00	-	9	0	192.50	0.15	0.00	-	1,356	0
29.10	0.00	-	5	0	195.00	0.19	0.00	-	2,170	0
22.90	0.00	-	368	0	197.50	0.25	0.00	-	1,038	0
16.95	0.00	-	179	0	200.00	0.35	0.00	-	3,734	0
17.30	0.00	-	13	0	202.50	0.47	0.00	-	917	0
12.50	0.00	-	141	0	205.00	0.64	0.00	-	2,171	0
10.10	0.00	-	44	0	207.50	0.85	0.00	-	1,258	0
8.00	0.00	-	520	0	210.00	1.23	0.00	-	13,247	0
5.90	0.00	-	356	0	212.50	1.70	0.00	-	4,075	0
4.15	0.00	-	3,444	0	215.00	2.43	0.00	-	11,317	0
2.80	0.00	-	2,890	0	217.50	3.55	0.00	-	13,459	0
1.56	0.00	-	10,241	0	220.00	5.00	0.00	-	35,830	0
0.82	0.00	-	19,284	0	222.50	6.70	0.00	-	23,950	0
0.40	0.00	-	44,070	0	225.00	8.90	0.00	-	16,550	0
0.19	0.00	-	25,157	0	227.50	10.82	0.00	-	2,334	0
0.07	0.00	-	21,395	0	230.00	13.15	0.00	-	2,159	0
0.03	0.00	-	11,448	0	232.50	13.84	0.00	-	208	0
0.01	0.00	-	6,245	0	235.00	18.34	0.00	-	247	0
0.02	0.00	-	3,633	0	237.50	17.40	0.00	-	46	0
0.02	0.00	-	431	0	240.00	19.55	0.00	-	133	0
0.01	0.00	-	164	0	242.50	21.35	0.00	-	3	0
0.01	0.00	-	398	0	245.00	24.90	0.00	-	48	0

Observe that:

- the premium changes the further away the strike is from the current price
- call options are generally more expensive than put options

Payoff and profit of options

- The payoff of an option depends on the possibility of exercising or not
  - Payoff is the difference between the exercise price and the asset price at maturity if option is exercised, otherwise 0

- Let denote with P_T the price of the asset at date T and K the strike price, so the payoff for a call contract of a vanilla option is:

$$max(P_T - K , 0)$$

since we would exercise the option only if $P_T > K$

- By analogous reasonoing, the payoff for a put contract of a vanilla option is:

$$max(K - P_T , 0)$$

Payoff and profit of options

- Payoff for path dependent options depends on the conditions tying the strike price to past prices of the asset

- For an Asian call option the payoff can be:

$$max(A(T_0,T) - K, 0)$$

where $A(T_0,T)$ is the average price of the asset from date T_0 to date T

- Other variants can be obtained by using wieghted or geometric average

Computing profit on options

- To compute profit on options we must consider the fee, or the commission given at the time of settling the agreement (price of the option)

- This price, always positive and denoted by $C(P_0, T)$ depends on:
  - P_0 : price of the underlaying asset at initial time T_0
  - T : time set for maturity

- as with stocks, we should subtract the price of the option from the payoff, but considering that these amounts are given and received at different points in time, and that we could have done a safe investment instead, we must include the possible gains given by a risk-free asset earning a constant interest rate r for the period $\tau = T - t_0$ and continuously compounded

- Therefore, under all these considerations, the profit of a call option is given by:

$$max(P_T - K, 0) - C(P_0, T)\, e^{r\tau}$$

Just a simple exercise!

- *Starting from the previous equations related to payoff computation of an Asian call option and profit computation, try to define the corresponding equations for an Asian put option*

Why using options?

- There are at least 2 advantages in using options instead of buying common stocks directly:
  - We can buy (or reserve to buy at later date) more shares with less money (as we only pay for a premium which is always lower than the price of the underlying stock) → *otherwise options make no sense!*
  - We can set, in advance, a limit on losses, while we can still make, theoretically, unlimited profits

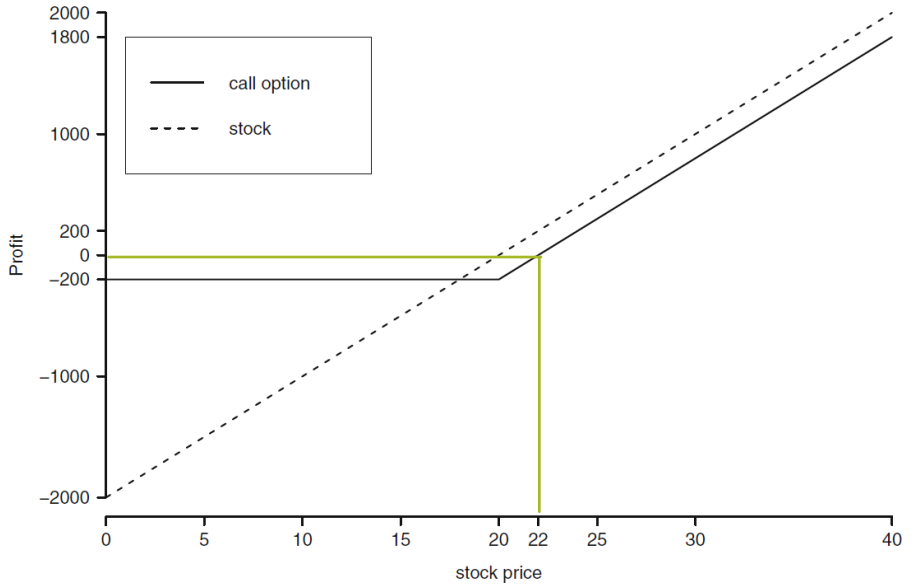
- To better understand the benefits of options it is good to analyze some *investment scenarios* and the resulting *profit charts*

Scenario 1

Example 1.6 Suppose that common stock of company XYZ is trading today at €20, and a call option on XYZ with strike $K = 20$ and a month of maturity is selling for $C_0 = 2$, or €200 ($= 2 \times 100$). Buying one call option would have at expiration a loss on the downside of the price of the stock limited to −200, since we would not exercise the option. On the contrary, buying the stock could have a future loss of −2000, in the worst scenario where the stock losses 100 % of its value. Table 1.2 shows a comparison of profits for the call versus holding directly 100 shares of the stock, for different prices of the stock. Figure 1.5 shows the corresponding profit graphs for the call (solid line) and the stock (dashed line) with their evolution of value as described in the table. □

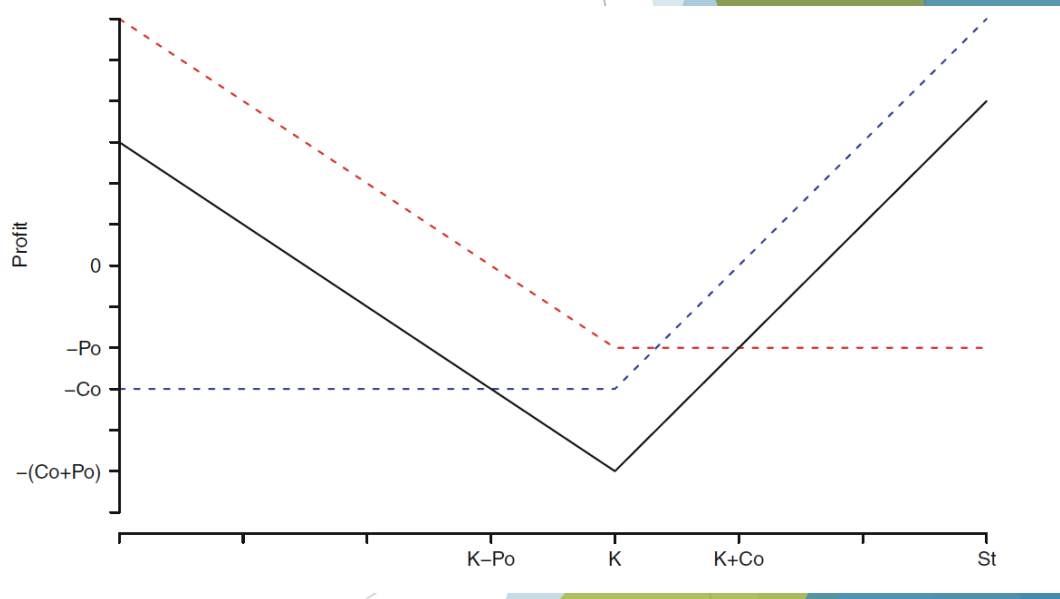
Table 1.2 Profits for the call option and for holding 100 shares of the stock

Stock price	Profit for 100 shares	Call	
		Price	Profit
5	−1500	0	−200
10	−1000	0	−200
15	−500	0	−200
20	0	0	−200
22	200	2	0
25	500	5	300
30	1000	10	800
40	2000	20	1800



Scenario 2 (Straddle strategy)

Example 1.7 The *straddle* strategy consists on buying a call and a put options on a stock with same strike price and expiration date. Figure 1.6 presents the profit graph (solid line) of this strategy resulting from the sum of the two profit graphs (dashed lines), one for the call (in blue) with premium C_0 and the other for the put (in red) with premium P_0, and common strike price K. If the price of the stock at maturity, S_T, is close to the strike price K (i.e $|S_T - K| < C_0 + P_0$) then the straddle strategy produces a loss, whereas if S_T moves far away from K, in either positive or negative direction, then there could be significant profits. Thus, a straddle strategy should be used if one expects in the future a significant variation of the price of the stock but is uncertain about the direction it will move. □



Option Pricing: an overview

- In an option contract, both *parties assume a financial risk*:
  - the holder (of the option) assumes the payment of the premium which could be a financial loss if the option is not exercised;
  - if the option is exercised, the writer incurs in a loss for financing the underlying asset at a price worse than the market price at the time of delivery

- Consider, for example, a European call option on a stock, if at exercise time *T*:
  - $S_T \leq K$, the holder of the option would not exercise it and loses the premium;
  - $S_T > K$ the option would be exercised, and the writer must deliver a share of the stock for K units of cash, losing the difference with the real price S_T

- Thus, the problem of pricing an option is to determine a price that is deemed fair for the writer and the holder; that is, an amount of cash that the buyer should be willing to pay, and the seller should charge and not systematically lose

Futures and Forward

- They are derivatives, differing from options basically in their terms of ***liability***
- Two parties subscribe an obligation to trade a specific asset (one party to buy it and the other to sell it), for a specified price (the ***delivery* price**) and at a specified future date (for the forward contract) or time period (e.g. a month, for the futures contract).
- The asset specified in the contract is usually any **commodity** (i.e., agricultural products or raw materials), currencies, or bonds, but could also be any stock
- Forward contracts are not traded on exchange markets while futures contracts are traded on exchanges. Thus, the delivery price of forwards is settled by the two parties involved, while in futures contracts is marked to the market on a regular basis
- A forward contract can be used by an investor (the buyer) to lock in a future purchase of some asset from a supplier (the seller) at a price settled today, without actually paying the full amount up until the date of expiration or maturity on the contract.

Payoff and Profit of Futures and Forward

- If K is the delivery price of the asset and P_T is the price at exercise date T, the payoff for the buyer on one unit of the asset is $P_T - K$, while the payoff for the seller on one unit of the asset is $K - P_T$

- Since there are no fees for writing a forward or futures contract, nor initial investment, the profit function for either party is the same as the payoff, and this can be positive or negative at maturity

- As with options, one important problem is to assign a value to these contracts that should be considered fair for both parties involved!
  - At the time of entering any of these contracts the obvious fair value for both parties is zero, which means that the delivery price is equal to the price of the underlying asset. This is because neither party, with the current information at hand, is assuming any risk
  - But as time progresses, while the delivery price remains the same, the price of the asset may change producing an imbalance in the odds of profits for both parties