Return forecasting through ARMA extensions, ARCH & GARH

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Testing for stationarity

- In practice, time series of asset prices (as well as of interest rates) are often **non-stationary** → because there is not a price value to which they revert.

- The **augmented Dickey-Fuller** test (**ADF** test) is a statistical test for non-stationarity of a time series.

- Formally, the ADF performs the following regression:

$$X_t = \alpha + \beta t + \varphi X_{t-1} + \theta_1 \Delta X_{t-1} + \theta_2 \Delta X_{t-2} + \cdots + \theta_{p-1} \Delta X_{t-p+1} + \boldsymbol{\varepsilon_t}$$

- where α is a constant, β is a coefficient on a trend, and p refers to the number of lags used in the model
- The constraint $\alpha = \beta = 0$ implies a **random walk**, where $\alpha \neq 0$ and $\beta = 0$ implies a **random walk with drift**
- The order *p* is usually decided using the **Aikaike Information Criterion** (**AIC**) and **Bayesian Information Criterion** (**BIC**).
- The ADF test check for the value of φ → if $\varphi = 0$ then non-stationarity, if $\varphi < 0$ then stationarity

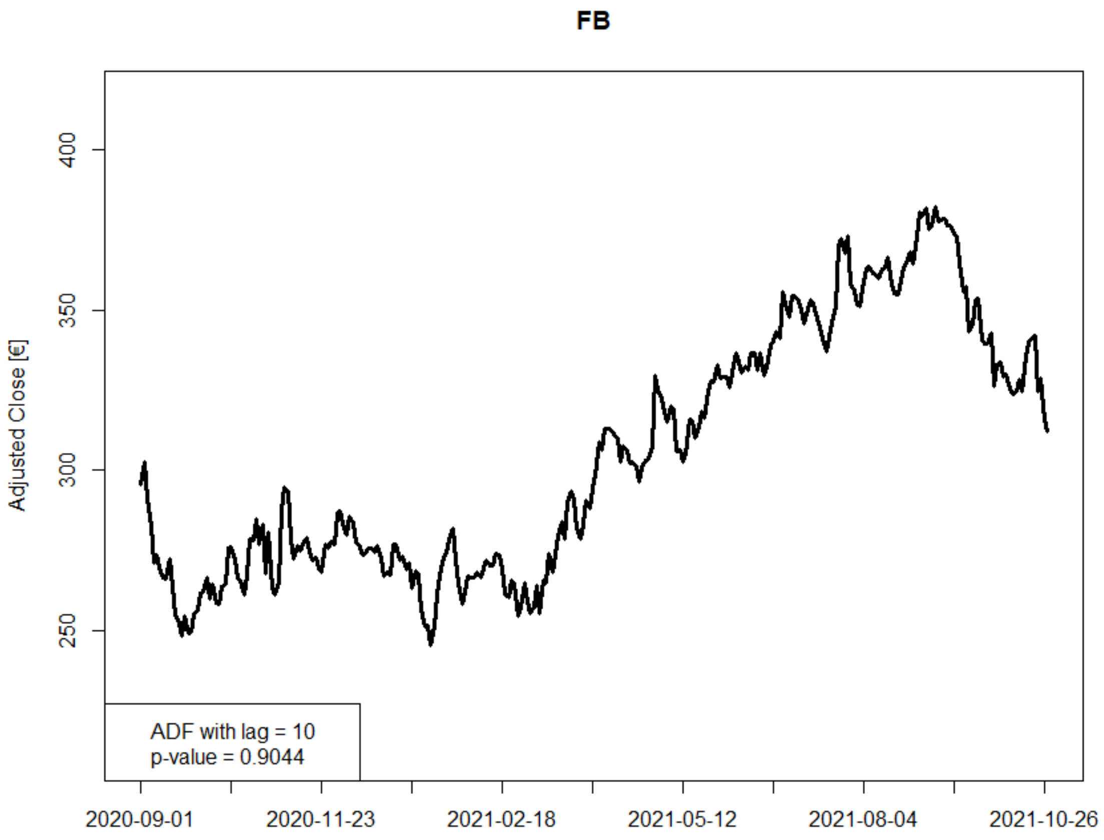
Handling non-stationarity

- Identifying the correct transformation to make the time series stationary is not always simple

- Some "heuristics" have been proposed:
  - Lag-1 autocorrelation close to zero or negative → no need for higher-order differencing
  - Positive autocorrelation up to 10+ lags → the series probably needs higher-order differencing
  - Lag-1 autocorrelation < -0.5 → the series may be over-differenced
  - Slightly over- or –underdifferencing can be corrected ith AR or MA terms

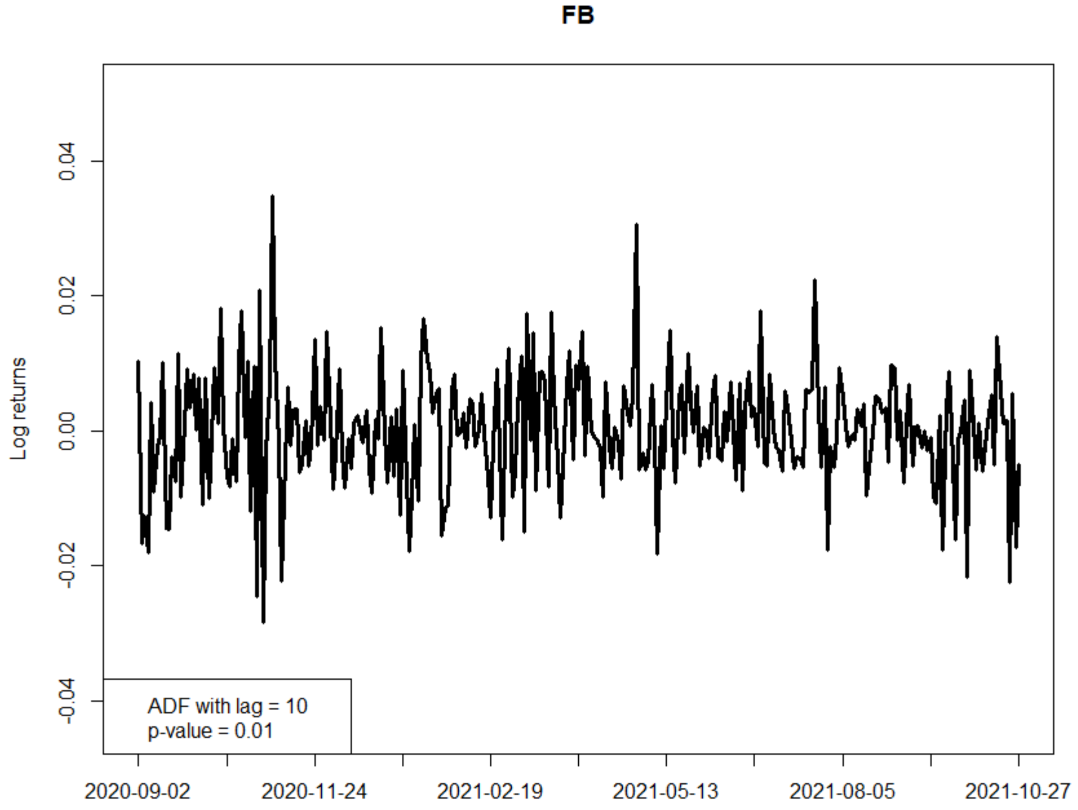
Testing for stationarity

As an example, the asset price time series reported in the picture is **non-stationary** (i.e., ADF *p*-value>0.05)



Testing for stationarity

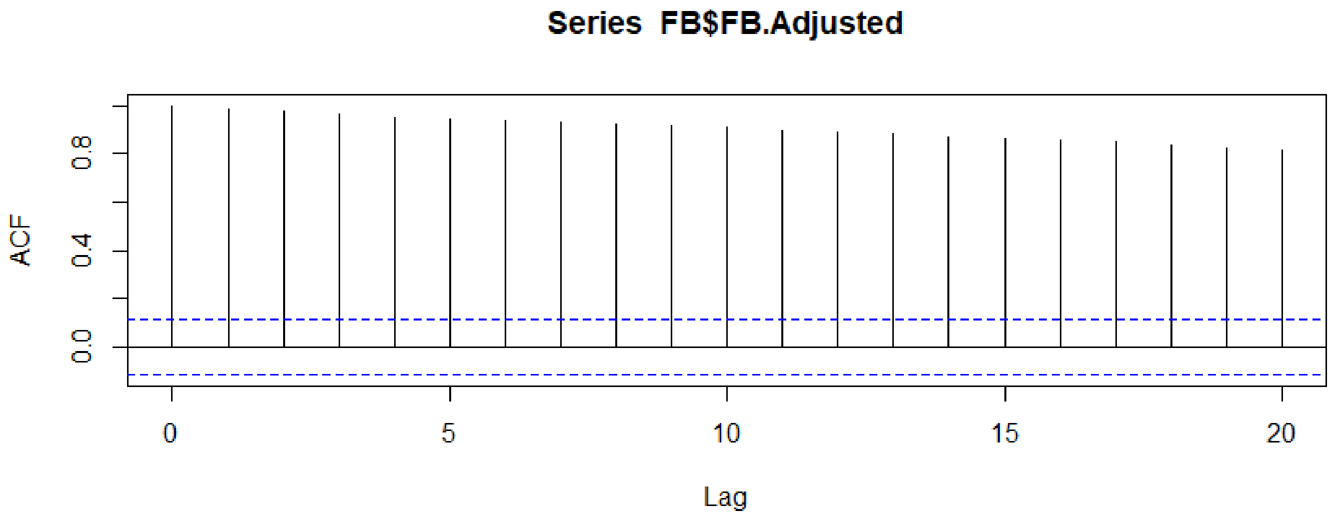
- Remind that the asset log-return series is given by: $r_t = p_t - p_{t-1} = \log P_t - \log P_{t-1} = \log \frac{P_t}{P_{t-1}}$
- If we consider this transformed series this is (usually) stationary (ADF *p*-value<0.05)

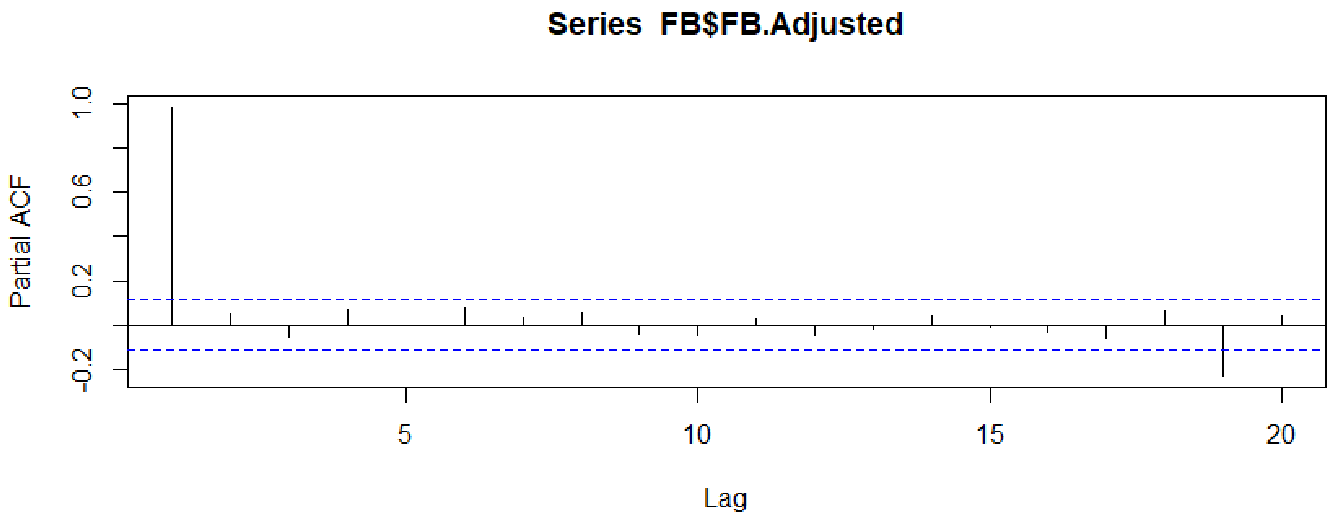


Testing for stationarity

- ARIMA is an extension of ARMA approach, to model autocorrelations in the data
  - AR(p) → autoregressive term with order *p*
  - I(d) → integrated term with order *d*
  - MA(q) → moving average with order *q*

- Plotting both the autocorrelation function (ACF) and the partial autocorrelation function (PACF) to identify significant autocorrelation at different lags.





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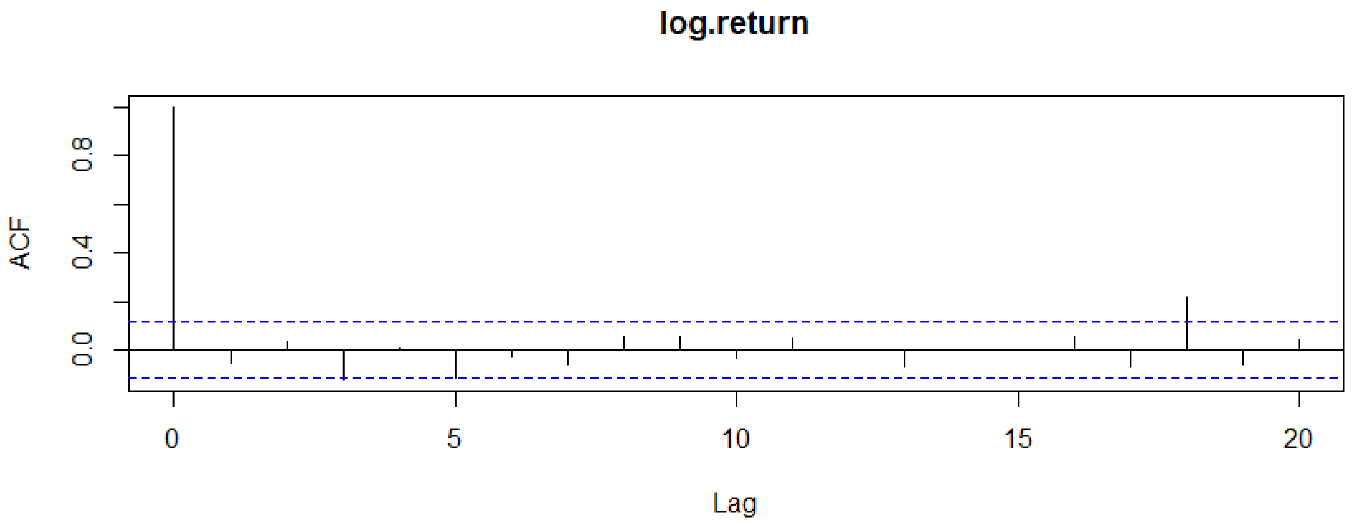

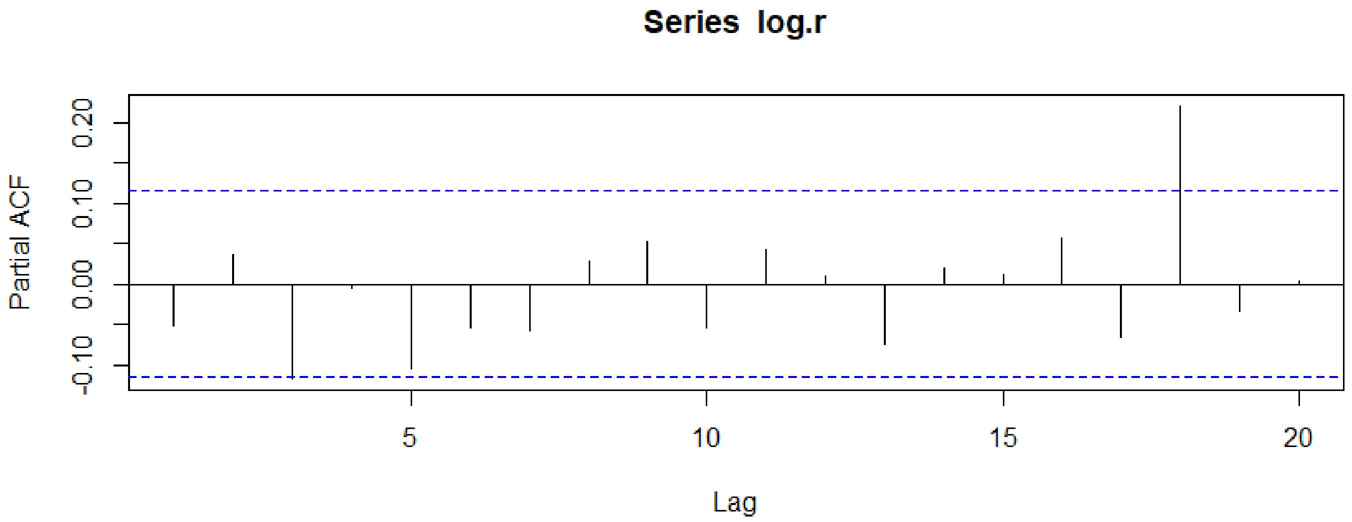

How to identify the order(s)

For the AR term, if an AR model of order $\bar{p}$ is the correct model:

- the ACF will show a significant serial correlation up to lag $\bar{p}$.
- the PACF will not reflect correlation for lags beyond $\bar{p}$

For the MA term, if a MA model of order $\bar{q}$ is the correct model:

- the ACF will show significant coefficients for values up to lag $\bar{q}$.
- the PACF would show a similar behaviour (contrary to the AR model)

Since AR(p) and MA(q) terms interact, the information provided by ACF and PACF is no longer reliable and can only be used as a starting point. → Using validation (for instance measuring AIC or BIC)

ARIMAX & SARIMAX

ARIMAX: **ARIMA** with e**X**ogenous inputs

- It allows to add further input variables (e.g., sentiment indicators, information from fundamental analysis, …)
- Although it is anyway a linear regression model, it is more difficult to interpret

SARIMAX: **S**easonal **ARIMAX**

- For time series with seasonal effects, we can include AR and MA terms modelling seasonality/periodicity
- The ARIMAX(p,d,q) becomes SARIMAX(p,d,q)x(P,D,Q)
- For instance, when using monthly data and seasonal effect length is 1 year, the seasonal AR and MA terms would reflect this specific relation (i.e., P=Q=12)

Forecasting volatility

- Predicting volatility is a crucial topic in finance

- Volatility is uasually not constant over time

- Changes in volatility (i.e., variance) implies challenges for time series forecasting through ARIMA, as they assume stationarity

- Two approaches to model volatility to predict changes in variance
  - the **ARCH** (autoregressive conditional heteroskedasticity) model
  - the **GARCH** (generalized autoregressive conditional heteroskedasticity) model

The ARCH model

- **Heteroskedasticity** is the technical term for changes in a variable's variance

- The **ARCH(*p*)** model is simply an AR(*p*) model which is applied to the variance of the residuals (i.e., errors) of a time series model

- This makes the variance at time *t* conditional on lagged observations of the variance

- Thus, the error terms ε_t are the residuals of a linear model such as ARIMA, and split into a time-dependent standard deviation σ_t and a disturbance z_t

The GARCH model

- The ARCH model is relatively simple, but often <u>requires many parameters</u> to capture the volatility patterns of an asset-return series

- A **GARCH(*p*,*q*)** model applies to a log-return series and assume an ARMA(p,q) model for the error of the variance terms ε_t

How to build a volatility forecasting model

- Developing a volatility forecasting model for an asset-returns time series consists of 4 steps:
  1. Build an ARMA model of the financial time series (by using ACF and PACF, or validation)
  2. Test the residuals of the model for ARCH/GARCH effects (again by using ACF and PACF, or validation, <u>for the series of the squared residuals</u>)
  3. Specify a volatility model if serial effects are significant, and jointly estimate the mean and volatility equations
  4. Validate the model carefully and refine it, if needed.

- When applying volatility forecasting to return series, the serial dependence may be limited so that a constant mean may be used instead of an ARMA model.

The ML4T workflow

- **ML4T** stands for **Machine Learning for Trading**: and end-to-end perspective of the process of ***designing***, ***simulating***, and ***evaluating*** a ***trading strategy driven by ML algorithms***

- A realistic simulation of a strategy needs to faithfully represents how security markets operate and how trades are executed
  - The institutional details of exchanges must be considered (e.g., available order types, price determination, …)
  - Accurate performance measurements
  - Methodological aspects to carefully consider in order to avoid biased results and false discoveries, leading to poor investment decisions

How to backtest a ML-driven strategy

- Source and prepare: ***market***, ***fundamental*** and ***alternative*** data
- Engineer predictive factors and features
- Design, tune, and evaluate ML models to generate trading signals
- Decide on trades based on these signals
- Size individual positions in the portfolio
- Simulate the resulting trades triggered using historical market data
- Evalute how the resulting positions would have performed

Some backtesting pitfalls and how to avoid them

- <u>**Getting the right data**</u>
  - **Look-ahead bias: use only point-in-time data.** We need to guarantee that conclusions are based only on point-in-time that does not inadvertently include information from the future
  - **Survivorship bias: track your historical universe.** This bias occurs when the backtest contains only securities that are currently active while omitting assets that have disappeared over time
  - **Outlier control: do not exclude realistic extremes**
  - **Sample period: try to represent relevant future scenarios**

- <u>**Getting the simulation right**</u>
  - Transactions costs
  - Timing of decisions

- <u>**Getting the statistics right**</u>
  - …being not too optimistic!