The MATH130 Student's Guide to Chen and Duong's MATH130 notes.

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Chapter 1

Copyright and Terms of Use, and Other Bits

1.1 Copyright

I chose a NoDerivs suffix as it is trivial to take this work, and push "compile" button in any LaTeX editor and generate a textbook to take to a publisher. It would be unfair (on myself, as well as everyone who helped) to allow that to happen. Should you wish to generate a textbook, please contact the author to discuss textbook licensing.

If you wish to make ammendments to details, by all means, fork the code, and send me a patch containing your changes, or a "pull request" (or both). You will get a mention in the acknowledgements, and if a publisher wants a textbook you will also get a reward commensurate with the changes you provide.

1.2 Terms and conditions of use

This work is an open source project. It strives to be technically correct and accurate, though, it may not be.

1.3 A cautionary word

I've failed MATH130 at least twice. I've been pissed with the Maths department. I've written most of this book whilst ropable at them for not doing it themselves. This may not be the best resource, but they haven't really done a good enough job of organizing their resources, so resources are limited. Please attend lectures, tutorials and practicals. Go ask your teachers, tutors, professors, lecturers, and friends questions and be active in classes. It is said the only way to learn math is to do math. If you say "screw math", it *will* screw you[1].

1.4 Typesetting

This book has been typeset in LaTeX using a Lenovo X201 Tablet running Microsoft Windows 7, Eclipse Indigo, TeXlipse and MikTex 2.9 64-bit beta. I've been drinking copious amounts of energy drinks and coffee; if you would like to become a sponsor and have your name mentioned here, let me know.

The source is presently available from https://github.com/carneeki/Grokking-MATH130. You should be able to find updates of the PDF there too.

1.5 Introduction

In 1999 a pair of elite mathematicians[2] decided to write some notes for a subject they knew plenty about. The notes proved difficult to understand by some and could have been made easier by means of an introduction in plain simple English. Today they survive as reference documents on Rutherglen. If you can find them, and you read them, and you have this guide, then maybe you can pass MATH130.

(To be read whilst playing the introduction to the A-Team).

[1] And it might screw you anyway...

[2] Chen and Duong

Typically the syllabus is broken into two streams, calculus and algebra. This gives rise to certain problems if algebra falls behind calculus because there are prerequisites in algebra to solving some calculus problems. As such, these notes will be arranged such that the algebra material is covered first.

Contents

Chapter 2

Types of Numbers & Symbols Used

Maths is pretty much a written language used to convey what people want to do to numbers, variables, or other bits of information. There are various types of numbers, and sometimes special symbols are used to denote what conditions must be placed on those numbers.

2.1 Types of Numbers

Table 2.1 outlines the types of numbers encountered in MATH130, followed by a few additional types of numbers that are handy to bear in mind.

2.2 Types of Symbols

Table 2.2 outlines the types of symbols you are likely to encounter in MATH130. This list is partially built from Wikipedia [3].

2.3 A Brief Interlude on Language

An expression and an equation are *different* things, despite the fact that they look similar. An expression might be something like $3x - 8$, however, x has no value. By the fact that $3x - 8$ has no value *assigned* to it, it is an expression. If we were to assign a value to it, $3x - 8 = 0$ then we can call it an equation.

Symbol	Name	In MATH130	Description & Example
$\mathbb{N}$	Natural Number	Yes	Any whole number greater than zero. $1, 2, 3$
$\mathbb{R}$	Real Numbers	Yes	Any number along a continuum. $-1, 0, 1, \pi$
$\mathbb{Z}$	Integers	Yes	Any whole number. $-1, 0, 1$
$\mathbb{I}$	Irrational Numbers	Yes	Numbers which cannot be expressed as a fraction. $e, \pi, \sqrt[2]{2}$
$\mathbb{Q}$	Rational Numbers	Yes	Numbers which can be represented as a fraction. $\frac{1}{2}, 1, \frac{0}{4}$
$\mathbb{C}$	Complex Numbers	Yes [1]	Numbers which have both a real part, and an imaginary part. $\frac{2}{-1} = i$
$\mathbb{P}$	Prime Numbers	No [2]	Numbers which are divisible only by themselves and one. $1, 2, 3, 5, 7, 11, 13$

Table 2.1: Types Of Numbers

$$\begin{aligned} 3x - 8 \quad & \text{an expression} \\ 3x - 8 = 0 \quad & \text{an equation} \end{aligned} \tag{2.1}$$

Symbol	Example	Read as
$\in$	$x \in \mathbb{R}$	x is an element of $\mathbb{R}$ (real numbers)
$\notin$	$x \notin \mathbb{R}$	x is not an element of $\mathbb{R}$ (real numbers)
$\cup$	$\mathbb{I} \cup \mathbb{Q} \in \mathbb{R}$	The set $\mathbb{I}$ (irrational numbers) and $\mathbb{Q}$ (rational numbers) are in set $\mathbb{R}$ (real numbers).
$=$	$x = y$	x is equal to y
$\neq$	$x \neq y$	x is not equal to y
$\approx$	$x \approx y$	x is approximately equal to y
$\equiv$	$x \equiv y$	x is equivalent to y
$<$	$x < y$	x is less than y
$>$	$x > y$	x is greater than y
$\leq$	$x \leqslant$	x is equal to or less than y
$\geq$	$x \geqslant$	x is equal to or greater than y
$f(x)$	$f(x) = mx + b$	function of x is equal to mx + b
$\sum$	$\sum_{i=1}^{10} t_i$	sum of terms t for values 1 to 10
$\int$	$\int_0^\infty e^{-x}\,\mathrm{d}x$	integrate e^{-x} from 0 to ∞ with respect to x
$\frac{\mathrm{d}}{\mathrm{d}x} f(x)$	$\frac{\mathrm{d}}{\mathrm{d}x} f(x)$	differentiate $f(x)$ with respect to x.
$y'\,\mathrm{d}x$	$y'\,\mathrm{d}x$	differentiate y with respect to x.

Table 2.2: Types of mathematical symbols

Chapter 3

Number Systems & Factorization

There are some basic laws that need to be understood to manipulate numbers The first group of laws are called the "Distributive laws".

$$a(b+c) = ab + ac \tag{3.1}$$

$$(a+b)c = ac + bc \tag{3.2}$$

$$(a+b)(c+d) = ac + ad + bc + bd \tag{3.3}$$

Equation 3.3 gives rise to a special case called a quadratic which will be introduced in section 3.3, Introduction to Polynomials, and in further detail in chapter 6, Polynomials.
The distributive laws are all about expanding brackets, that is to say, in 3.2, first we multiply a with the first term inside the brackets (b) to give us ab, then we multply a with the second term, c to give us ac. When we add them (the $+$ symbol in the brackets) we get $ab + ac$.
Another way to think about it is a is distributed to each term inside the brackets. This also applies in 3.3 with c, and it yields the same result as in 3.2.

3.1 Introduction to Fractions

It turns out that any repeating decimal can be written as a fraction:

$$2.1 \times 100000 = \tag{3.4}$$

$$= 2.1 \times \frac{10}{10} \tag{3.5}$$

$$= \frac{21}{10} \tag{3.6}$$

$$\tag{3.7}$$

What about $1.33333\ldots$?

$$1.33333 = \frac{4}{3} \tag{3.8}$$

$$\tag{3.9}$$

Or $1.373737\ldots$??

$$let x = 1.373737\ldots \tag{3.10}$$

$$100x = 137.3737\ldots \tag{3.11}$$

$$100x - x = 137.373737\ldots - 1.373737\ldots \tag{3.12}$$

$$99x = 136 \tag{3.13}$$

$$so$$

$$x = \frac{136}{99} \tag{3.14}$$

$$\therefore 1.373737\ldots = x = \frac{136}{99} \tag{3.15}$$

For longer decimal places we need to create 2 numbers using x with the same decimal part so that when we subtract the decimal part we get a whole number.

$$let x = 36.2593593593\ldots \tag{3.16}$$

$$so 10x = 362.593593\ldots \tag{3.17}$$

$$10000x = 362593.593\ldots \tag{3.18}$$

$$10000x - 10x = 362593.593\ldots - 362.593\ldots \tag{3.19}$$

$$9990x = \ldots \tag{3.20}$$

(we have now subtracted the recurring decimal component from the fraction)

$$\therefore x = \frac{362593 - 362}{9990} \tag{3.21}$$

$$\tag{3.22}$$

As it turns out, fractions will fall into one of three categories:

(a) recurring decimals such as $\frac{99}{101} = 0.9801\ldots$.

(b) non-recurring decimals such as $\frac{1}{2} = 0.5$, these first two are called $\mathbb{Q}$ or rational numbers.

(c) non-recurring decimals such as $\pi = 3.1415\ldots$ or $\sqrt{2} = 1.4142\ldots$, these are denoted by the symbol $\mathbb{I}$, and called irrational numbers[1].

3.1.1 Fractional Operations - Adding

A good reason why we *don't* add fractions in the following way:

> add the tops (to give the numerator), add the bottoms (to give the denominator (or quotient). example: $\frac{1}{2} + \frac{1}{2} \neq \frac{2}{4} = \frac{1}{2}$
>
> The *correct* way of adding fractions is to use a common quotient then add the numerator and keep the denominator the same.[2]
>
> Consider you order 3 slices of pizza from Hot Momma's pizza at the MQ bar, and you are given 2 more for being a regular customer:

$$\frac{3}{8} = \ldots \tag{3.23}$$

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8} \tag{3.24}$$

$$\tag{3.25}$$

> You have $\frac{5}{8}$ or "five eights" of a whole pizza.[3].

[1] These numbers require some complex calculus to prove they are non-recurring, a simpler number is $0.12345678901111213 14\ldots$ which has a pattern, and is not recurring, it's just "add one" to the last number

[2] a simpler (but only partial) answer could be "you're adding like with like" – a MATH130 student from the audience of Chris Gordon's lecture at 2011-08-08 10:31AM

[3] I want a slice of that pizza if it's the supreme

What if you have different quotients? You *need* to convert to a common quotient:

$$\frac{1}{2} + \frac{7}{10} = \tag{3.26}$$
$$= \frac{5}{5} \times \frac{1}{2} + \frac{7}{10} \tag{3.27}$$
$$= \frac{5}{10} \times \frac{7}{10} \tag{3.28}$$
$$= \frac{12}{10} \tag{3.29}$$
$$= \frac{2 \times 6}{2 \times 5} \tag{3.30}$$

the two's divide out, which simplifies the fraction

$$= \frac{6}{5} \tag{3.31}$$
$$\tag{3.32}$$

Equation 3.28 is where the important heavy lifting of the operation of converting to a common quotient comes into play.

A general case:

$$\frac{a}{b} + \frac{c}{d} = \tag{3.33}$$
$$= [\frac{a}{b}\ times \frac{d}{d}] + [\frac{c}{d}\ times \frac{b}{b}] \qquad = \frac{ad + cd}{db} \tag{3.34}$$
$$\tag{3.35}$$

3.1.2 Fractional Operations - Division

Dividing fractions has a reasonably simply rule to remember: Multiply by the inverse of one fraction:

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} \times \frac{bd}{bd} \tag{3.36}$$
$$= \frac{\frac{a}{b} \times b \times d}{\frac{c}{d}\ timesb \times d} \tag{3.37}$$

divide out common terms

$$= \frac{ad}{cb} \tag{3.38}$$

3.2 Introduction to Irrational Numbers (Surds)

A *surd* is *an archaic*[4] *term for an irrational number*. This is basically a number which cannot be written as a decimal because, if you tried

(a) you'd go on forever as it has an infinite number of decimal places
and

(b) it has no repeating parts to the decimal places.

For this reason it cannot be expressed as a fraction in the form $\frac{p}{q}$ where p and q are integers. Examples of irrational numbers are $e, \pi, \sqrt[2]{2}$
A more formal[5] definition is:

$$\mathbb{I} \ni \frac{p}{q} \Bigg\} \qquad \text{p,q} \in \mathbb{Z}$$

There are some handy things we can do with irrational numbers. Consider $\sqrt[2]{8} = 2.828427124$[6]. It can be rewritten like this:

$$\sqrt[2]{8} = \sqrt[2]{2 \times 4} \tag{3.39}$$
$$= \sqrt[2]{2} \times \sqrt[2]{4} \tag{3.40}$$
$$= \sqrt[2]{2} \times 2 \tag{3.41}$$
$$= 2\sqrt[2]{2} \tag{3.42}$$

3.2.1 Rationalizing the Denominator

Often examiners will give us a fraction and say "rationalize the denominator"[7]. I don't know why - they just do. In order to get the marks in the exam, we can rationalize the denominator by multiplying that fraction by 1.
While the notion of multiplying by 1 sounds silly, consider that $1 \in \mathbb{R} = \frac{p}{q}$ where

[4] almost as old as the Maths Deptartment
[5] poorly worded, but means the same thing as above
[6] It's even longer than 2.828427124746190097603377448419396157139343750753896146353359475981464 9..., it's infinite remember!
[7] "sudo rationalize the denominator" if you want to be a troll

{p,q} can be a surd, x: $\frac{x}{x} = 1$. This gives rise to the following possibility of:

$$\frac{5}{\sqrt[2]{2}} = \tag{3.43}$$

$$= \frac{5}{\sqrt[2]{2}} \times \frac{\sqrt[2]{2}}{\sqrt[2]{2}} \tag{3.44}$$

The next part is where the useful stuff happens, if we square a square-root then they "undo" each other, and we are left over with the bit inside the square-root

$$= \frac{5 \times \sqrt[2]{2}}{(\sqrt[2]{2}) \times (\sqrt[2]{2})} \tag{3.45}$$

$$= \frac{5\sqrt[2]{2}}{2} \tag{3.46}$$

The denominator might not always be a square-root, University of North Texas' next example includes a cube-root, so we must multiply by 1 again.

$$\frac{2}{\sqrt[3]{5}} = \tag{3.47}$$

sometimes it's nicer to lay things out to see what's going on:

$$= \frac{2}{\sqrt[3]{5}} \times \frac{\sqrt[3]{5}}{\sqrt[3]{5}} \times \frac{\sqrt[3]{5}}{\sqrt[3]{5}} \tag{3.48}$$

but we still condense it into the root symbol:

$$= \frac{2}{\sqrt[3]{5}} \times \frac{\sqrt[3]{5 \times 5}}{\sqrt[3]{5 \times 5}} \tag{3.49}$$

$$= \frac{2\sqrt[3]{5 \times 5}}{(\sqrt[3]{5})(\sqrt[3]{5 \times 5})} \tag{3.50}$$

$$= \frac{2\sqrt[3]{25}}{5} \tag{3.51}$$

$$\tag{3.52}$$

The last example involves more than one term on the denominator. In this particular case, we will still multiply by 1, however we are using some trickery from an upcoming section 3.3, "Introduction to Polynomials" specifically equation 3.62.

$$\frac{2}{1 + \sqrt[2]{3}} = \tag{3.53}$$

$$= \frac{2}{1 + \sqrt[2]{3}} \times \frac{1 - \sqrt[2]{3}}{1 - \sqrt[2]{3}} \tag{3.54}$$

$$= \frac{2(1 - \sqrt[2]{3})}{1 - 3} \tag{3.55}$$

if the denominator part of above step does not make sense, then please refer to equation 3.62 in section 3.3, "Introduction to Polynomials"

$$= \frac{2(1-\sqrt[2]{3})}{-2} \tag{3.56}$$

$$= -\frac{2(1-\sqrt[2]{3})}{2} \tag{3.57}$$

$$= -\frac{1(1-\sqrt[2]{3})}{1} \tag{3.58}$$

$$= -(1-\sqrt[2]{3}) \tag{3.59}$$

The examples for rationalizing the denominator come from University of Northern Texas: http://www.math.unt.edu/mathlab/emathlab/How%20to%20Rationalize%20the%20Denominator%20of%20a%20Fraction.htm

3.3 Introduction to Polynomials

This section forms only an introduction to polynomials. More detail on polynomials is covered in chapter 6, "Polynomials".

Polynomials are a way of packing certain types of long equations into neater, more compact forms. The following equations show how the distributive laws can be applied to 3 polynomial equations. These 3 equations form the basic 3 rules of polynomials and their form should be memorised to make solving more complex problems easier down the track.

$$(a+b)^2 = (a+b)(a+b)$$

$$= a^2 + 2ab + b^2 \tag{3.60}$$

$$(a-b)^2 = (a-b)(a-b)$$

$$= a^2 - 2ab + b^2 \tag{3.61}$$

$$(a+b)(a-b) = a^3 + ab - ab - b^2$$

$$= a^2 - b^2 \tag{3.62}$$

$$(a+b)(a^2 - ab + b^2) = a^3 + b^3 \tag{3.63}$$

$$(a-b)(a^2 + ab + b^2) = a^3 - b^3 \tag{3.64}$$

Equation 3.62 is often called the difference of two squares where a^2 and b^2 represent both squares.

Equation 3.64 is often called the difference of two cubes.

3.4 Quadratics

Quadratics are an important type of the distributive law. They represent 3 coefficients and a variable. Equations 3.60 through to 3.62 are the classic 3 ways in which quadratics are introduced in textbooks. A more formal definition has been provided by the table 3.65 (from [5]).

$$ax^2 + bx + c = 0 \tag{3.65}$$

These particular quadratics are often called squares which are covered in more

Where:	
x	is the indeterminate variable
a	is the quadratic coefficient
b	is the linear coefficient
c	is the constant coefficient

Table 3.1: Components to a quadratic

detail in section 3.5, Completing the Square.

3.5 Completing the Square

Completing the square is useful for solving quadratic equations as well as graphing quadratic functions, as well as evaluating integrals in calculus. The key concept behind completing the square in MATH130 is that we want to convert a quadratic polynomial like:

$$ax^2 + bx + c$$

into the form

$$a(x - h)^2 + k$$

To do this we must find h and k. There are two ways about doing this depending on whether the value of $a = 1$.

3.5.1 General Case, when a = 1

If given an equation like:

$$x^2 + bx + c \tag{3.66}$$

we can form a square like this:

$$(x + \frac{1}{2}b)^2 = x^2 + bx + \frac{1}{4}b^2 \tag{3.67}$$

however, we have not taken into account the constant c so, we should really write the following to take it into account

$$x^2 + bx + c = (x + \frac{1}{2}b)^2 + k \tag{3.68}$$

The following examples come courtesy of Wikipedia [4] (with intermediary working provided by author):

$$x^2 + 6x + 11 = \tag{3.69}$$

First: halve b to give us 3, and then put it into the complete square form (remembering the k-value):

$$= (x + 3)^2 + k \tag{3.70}$$

Next calculate k

$$(x+3)^2 = x^2 + 6x + 9 \tag{3.71}$$

$$11 - 9 = 2 \tag{3.72}$$

$$\therefore k = 2 \tag{3.73}$$

Substitute $k = 2$ back into equation

$$x^2 + 6x + 11 = (x+3)^2 + 2 \tag{3.74}$$

Another example:

$$x^2 + 14x + 30 = \tag{3.75}$$

First: halve b to give us 3, and then put it into the complete square form (remembering the k-value):

$$= (x+7)^2 + k \tag{3.76}$$

Next calculate k

$$30 - 7^2 = -30 - 49 = -19 \tag{3.77}$$

Substitute $k = -19$ back into equation

$$x^2 + 14x + 30 = (x+7)^2 - 19 \tag{3.78}$$

Another example:

$$x^2 - 2x + 7 = \tag{3.79}$$

First: halve b to give us 3, and then put it into the complete square form (remembering the k-value):

$$= (x-1)^2 + k \tag{3.80}$$

calculate k

$$7 - (-1^2) = 7 - 1 = 6 \tag{3.81}$$

Substitute $k = 6$ back into equation

$$x^2 - 2x + 7 = (x - 1)^2 + 6 \tag{3.82}$$

From these 3 examples, the pattern should become evident as a 3 stage process:

(a) Halve b and put into the complete square form

(b) Calculate $c - h^2 = k$

(c) Substitute k back into equation and rewrite in full.

3.5.2 Non-monic Case, when a != 1

If given an equation like

$$3x^2 + 12x + 27 = \tag{3.83}$$

we can factor out the coefficient a and the complete the square as in a general case

$$3x^2 + 12x + 27 = 3(x^2 + 4x + 9) \tag{3.84}$$
$$= 3((x+2)^2 + 5) \tag{3.85}$$
$$= 3((x+2)^2) + 15 \tag{3.86}$$

this gives rise to the form:

$$a(x-h)^2 + k \tag{3.87}$$

3.5.3 Completing The Square Formulae

When $a = 1$

$$x^2 + bx + c = (x - \frac{-b}{2})^2 + k \tag{3.88}$$
$$= (x + \frac{b}{2})^2 + (c - \frac{b^2}{4}) \tag{3.89}$$

When $\neq 1$

$$ax^2 + bx + c = a(x - \frac{-b}{2})^2 + k \tag{3.90}$$
$$= a(x + \frac{b}{2a})^2 + (c - \frac{b^2}{4a}) \tag{3.91}$$

Chapter 4

Exponentials & Logarithms

Exponents, powers or indices and logarithms are ways of expressing numbers that have been multiplied or divided a number of times.

If we were to plot an exponential equation on a graph, we would notice that the graph has a constant doubling time, ie for every unit we double on the y axis, the number of units on the horizontal axis are constant.

If the graph increases at an increasing rate, it does not necessarily mean it the graph is exponential.

4.1 Powers, Exponentials and Indices

Indices are putting a power or index in the top right corner of a number, which indicates how many times we must multiply or divide that number by itself to give us a total number. With this information in mind, we need to define some parts of the language behind how logs and indices should be used.

$$b^x = y \tag{4.1}$$

When we want to perform manipulations to several numbers, there are various power

Where:	
a	is the base or *radix*
x	is the index
y	is the output

Table 4.1: Parts of an exponential function

laws we must bear in mind... They are summarised as follows:

$$b^0 = 1 \tag{4.2}$$

$$b^1 = b \tag{4.3}$$

$$b^x * b^y = b^{x+y} \tag{4.4}$$

$$b^x - b^y = b^{x-y} \tag{4.5}$$

$$(b^x)^d = b^{xd} \tag{4.6}$$

$$bx^{-y} = \frac{b}{x^y} \tag{4.7}$$

$$b^{(\frac{x}{y})} = (b^x)^{\frac{1}{y}} \tag{4.8}$$

$$= \sqrt[y]{b^x} \tag{4.9}$$

$$db^{(-\frac{x}{y})} = \frac{d}{\sqrt[y]{b^x}} \text{ combining (4.7) and (4.8)} \tag{4.10}$$

$$b^x = 0 \text{ is impossible} \tag{4.11}$$

Converting notation between the forms exhibited in 4.8 and 4.9 is often extremely useful in Calculus, in particular, differentiation.

4.2 Logarithms

Until this point, only exponentials where we want to do something to a base number have been covered. When we want to see how a base number has been affected by it's power a logarithm is the way to undo the power.

Think of a log as an undoing function to an exponential.[1]

[1] Chris Gordon, MATH130 lecturer for algebra stream, evening session on logarithms, 2011.

4.2.1 Log Laws

Many of these laws can be derived from the index laws, and have been included in a way to preserve order with those laws for easier reference.

$$\log_b(1) = 0 \quad \text{by (4.2)} \tag{4.12}$$

$$log_b(b) = 1 \quad \text{by (4.3)} \tag{4.13}$$

$$log_b(xy) = log_b(x) + log_b(y) \quad \text{by (4.4)} \tag{4.14}$$

$$log_b(\frac{x}{y}) = log_b(x) - log_b(y) \quad \text{by (4.5)} \tag{4.15}$$

$$log_b(x^d) = d * log_b(x) \quad \text{by (4.6)} \tag{4.16}$$

$$log_b(\sqrt[y]{x}) = \frac{log_b(x)}{y} \quad \text{by (4.8)} \tag{4.17}$$

$$log_b(0) = \text{undefined} \quad \text{by (4.11)} \tag{4.18}$$

(4.19)

Here's where some new stuff is introduced:

$$b^{log_b(x)} = x \text{Logs of the same base cancel as an index} \tag{4.20}$$

$$log_b(b^x) = x \text{Logs of the same base cancel as an index} \tag{4.21}$$

$$log_b(a) = \frac{1}{log_a(b)} \text{Remember back to the initial statement, that a log is an inverting function?} \tag{4.22}$$

$$log_b(x) = \frac{log_a(x)}{log_a(b)} \text{We can change the base of one log to another} \tag{4.23}$$

$$(log_a(b))(log_b(x)) = log_a(x) \tag{4.24}$$

Referring back to 4.1, but replace the variable b with a gives rise to the easy to remember translation between the logarithms and exponentials: "logs are gay" [2]

$$\log_a(y) = x \iff a^x = y \tag{4.25}$$

[2] Elizabeth Camilleri, advanced mathematics student, on my whiteboard at home. Though, this really needs her picture (included) to accompany the text for full effect.

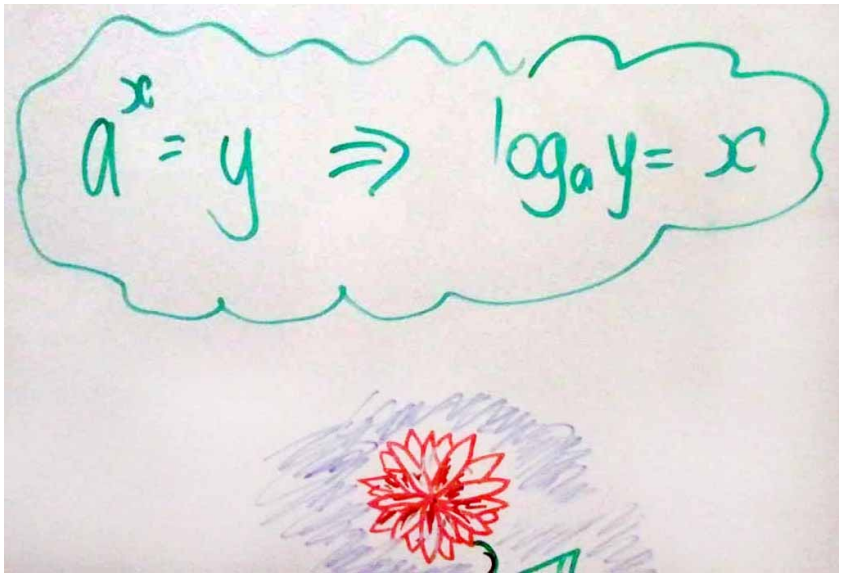

Figure 4.1: LogsAreGay

4.2.2 Converting Logarithm Bases

Suppose there are two logarithms, of different bases. It is often nice to use logarithms of the same base as it keeps the maths simpler (and sometimes things will divide or cancel out). To do this, we apply the change of base formula (4.29). Consider the following:

$$b^{log_b(x)} = x \tag{4.26}$$

$$log_a(b^{b*log_b(x)}) = log_a(x) \text{ take log base-a of both sides} \tag{4.27}$$

$$(log_a(b))(log_b(x)) = \text{by (4.6)} \tag{4.28}$$

There is an alternate way using division, and can be memorised as wrote:

$$log_b(x) = \frac{log_a(x)}{log_a(b)} \tag{4.29}$$

4.2.3 Problem solving with logs

The golden rule to remember when dealing with indices and powers is the Svensson-Cranbrookian Log Method, to be sung to "When you're happy and you know it":

> When you're having trouble with a power take a log.

Suppose we have to solve the following equation:

$$2^x = \frac{5}{3^{x+1}} \tag{4.30}$$

We use logarithms:

$$2^x = \frac{5}{3^{x+1}} \tag{4.31}$$

$$log(2^x) = log(\frac{5}{3^{x+1}}) \tag{4.32}$$

$$xlog(2) = log(5) - log(3^{x+1}) \tag{4.33}$$

$$xlog(2) = log(5) - (x+1) * log(3) \tag{4.34}$$

$$xlog(2) + xlog(3) = log - log(3) \tag{4.35}$$

$$x(log(2) + xlog(3)) = log(5) - log(3) \tag{4.36}$$

$$x = \frac{log(5) - log(3)}{log(2) + log(3)} \tag{4.37}$$

$$= \frac{log(\frac{5}{3})}{log(6)} \tag{4.38}$$

$$= \ldots \text{some decimal} \ldots \tag{4.39}$$

4.3 Euler Constant: Base *e*

Most calculations involving logarithms will be to various bases depending on the topic. In COMP it is often base 2 (binary), in other fields it is often base 10. In MATH and most of the sciences it is often base *e*, which is approximately 2.71828.[1] Numerically it is an interesting number as it crops up all over the place, sometimes in the most bizarre of locations; however we do not need to worry about that for MATH130.

The base e is used on most calculators with a button that looks like e^x. To use the logarithm with base e it is usually a button that looks like "ln", or $log_e(x)$.

An important thing to note about e, which will be raised further in calculus chapters is when you differentiate e, it is the only function where it's integral and derivative are both the same.

Chapter 5

Trigonometry

Trigonometry is about ratios of angles and sides. Specifically, trigonometry is about the ratios of angles and sides in triangles with respect to each other when drawn inside a special circle called *the unit circle*.

A brief interlude on language. Most of our trigonometric measurements are *NOT* going to be measured in degrees. Most of our measurements of angles are measured in radians. A *radian* is the ratio of angles with respect pi.
To convert between radians and degrees use the following formulae:

$$\theta^\circ = r \times \frac{180^\circ}{\pi} \tag{5.1}$$

$$r = \theta^\circ \times \frac{\pi}{180^\circ} \tag{5.2}$$

A more formal definition of the radian: *The radian measure of an angle θ is the arclength subtended by the angle in a unit circle.* This definition requires we define what a unit circle is. [1]

A full circle:

$$r = \theta^\circ \times \frac{\pi}{180^\circ} \tag{5.3}$$

$$r = 360^\circ \times \frac{\pi}{180^\circ} \tag{5.4}$$

$$r = 2 \tag{5.5}$$

[1] This works fine for *anti-clockwise* angles, however, a radian can never be negative because a length (the arclength in this case) can never be less than zero.

5.1 The Unit Circle

The unit circle is such an important part of trigonometry that it is worth looking it up on Wikipedia *AND* as many other places as possible.

A formal definition of the unit circle: $\cos(\theta)$ is the x-value of the intersection of the unit circle with the angle θ.
$\sin(\theta)$ is the y-value of the intersection of the unit circle with the angle θ[2].

UTUCDTCTV:[3]

$$\cos(\frac{-2\pi}{3}) = \dots \tag{5.6}$$

π is $\frac{1}{2}$ circle.
$\frac{\pi}{3}$ is $\frac{1}{3}$ of $\frac{1}{2}$ a circle.
At this point, there has been no trigonometry...
The rest of this is high school geometry to determine angles involving triangles (such as Pythagoras).

5.1.1 Properties of The Unit Circle

$$-1 \leq \sin(\theta) \leq \qquad 1 \tag{5.7}$$

$$\tag{5.8}$$

$\sin(\theta)$ is the y-value on the unit circle which is bound by -1 and 1.

$$\sin(-\theta) = -\sin(\theta) \tag{5.9}$$

$$\tag{5.10}$$

[2]These words are not quite complete, and would require the unit circle picture
[3]Use the Unit Circle Definition to Calculate The Value

5.1.2 SOH CAH TOA

$$\sin(\theta^\circ) = \frac{o}{h} \tag{5.11}$$

$$\cos(\theta^\circ) = \frac{a}{h} \tag{5.12}$$

$$\tan(\theta^\circ) = \frac{o}{a} \tag{5.13}$$

5.1.3 Angle Sum Formula

There are 3 angle sum formulae to learn, and while the 3rd can be deduced from the first two, it is easier to just memorise it as the derivation is beyond the scope of MATH130.

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \tag{5.14}$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\sin(\beta) \mp \cos(\beta)\sin(\alpha) \tag{5.15}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)} \tag{5.16}$$

$$\tag{5.17}$$

Chapter 6

TODO: Polynomials

Definition: Sum of natural powers of x, scaled and added.

Example:

$$6x^4 + 20x^3 + \sqrt{2}x^2 + 13x - 2 \tag{6.1}$$

$$\tag{6.2}$$

Two important terms are *coefficient*; which is a number in front of the x, and *degree* which is the highest power of x in the expression.

Polynomials are useful and common for approximating other functions. Here we approximate sin(x):

$$x - \frac{x^3}{6} \tag{6.3}$$

$$x - \frac{x^3}{6} + \frac{x^5}{120} \tag{6.4}$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} \tag{6.5}$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \tag{6.6}$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \tag{6.7}$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^11}{11!} \tag{6.8}$$

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^11}{11!} \tag{6.9}$$

$$\tag{6.10}$$

The main polynomials studied in MATH130 are:

quadratics

$$y = ax^2 + bx + c \qquad \text{General Form} \tag{6.11}$$

$$y = a(x - h)^2 + k \qquad \text{Standard Form} \tag{6.12}$$

$$y = a(x - x_1)(x - x_2) \qquad \text{Factored Form} \tag{6.13}$$

cubics

$$y = ax^3 + bx^2 + cx + d \tag{6.14}$$

6.1 Quadratics

6.1.1 General Form

$$y = ax^2 + bx + c$$

This form is perhaps the most common form of an equation - it is completely unfactored and appears the most natural (at least to me).
Key advantages of general form:

- $c = y$-intercept and is easily read straight off the equation.
- a tells us a lot about the shape of the parabola:
  - A "happy" parabola has a positive value
  - A "sad" parabola has a negative value
  - Larger values of a give steeper parabolas

6.1.2 Standard Form

$$y = a(x - h)^2 + k$$

Key advantages of standard form (sometimes called the *vertex form*):

- a tells us the same information about the shape of the parabolas as in the general form.

- h tells us the x coordinate of the minimum/maximum point, however, to get the coordinate, we must multiply h by -1.
- k tells us the y coordinate of the minimum/maximum point.
- Useful for finding the roots / zeros (where $y = 0$) of a quadratic

Consider solving:

$$8(x-7)^2 - 41 = 0 \tag{6.15}$$

Because there is only one x value it makes it easier to solve:

$$8(x-7)^2 = 41 \tag{6.16}$$

$$(x-7)^2 = \frac{41}{8} \tag{6.17}$$

$$x - 7 = \pm\sqrt{\frac{41}{8}} \tag{6.18}$$

$$x = \pm\sqrt{\frac{41}{8}} + 7 \tag{6.19}$$

It is also useful for sketching. Consider:

$$y = 2(x-1)^2 + 3 \tag{6.20}$$

(a) Start with $y = x^2$

(b) Because there is an $(x-1)$ we shift the parabola $y = x^2$ by 1 to the right.
$y = (x-1)^2$

(c) Because there is a 2 in front of the $(x-1)$, we make the parabola steeper by a factor of two.[1]
$y = 2(x-1)^2$

(d) Because there is a $+3$ on the end, it lifts the parabola up by 3.
$y = 2(x-1)^2 + 3$

6.1.3 Factored Form

$$y = a(x - x_1)(x - x_2)$$

Key advantages of standard form:

[1] called "stretching vertically"

- a tells us the same information about the shape of the parabolas as in the general form.

- x_1 and x_2 give us the x-intercepts (when multiplied by -1) of the equation.

Factored form is useful for finding the equation given a parabola. Suppose we know the x-intercepts of a parabola are -3 and -1, and it has a y-intercept of 6:

$$\begin{aligned} f(x) &= a(x - x_1)(x - x_2) && (6.21)\\ &= a(x - -3)(x - -1) && (6.22)\\ &= a(x + 3)(x + 1) && (6.23)\\ 6 &= a(0 + 3)(0 + 1) \text{substitute } x = 0 \text{ to get } y\text{-int} && (6.24)\\ 6 &= a(3) && (6.25)\\ 2 &= a && (6.26)\\ \therefore f(x) &= 2(x + 3)(x + 1) && (6.27)\\ &= 2x^2 + 8x + 6 && (6.28) \end{aligned}$$

6.1.4 Quadratic Formula

To get the roots of a quadratic (to factorize it), there is a long[2] formula we can use called the *quadratic formula* to get values of x. If given the general form of the quadratic, we can substitute the values of a, b, and c into this formula to get values for x which can then be used in the factored form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6.29}$$

[2] horrible

By applying the quadratic formula to our previous example from the factored form, we can show all 3 forms of the quadratic:

$$= 2x^2 + 8x + 6$$

where$a = 2, b = 8, c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6.30}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4(2)(6}}{2(2)} \tag{6.31}$$

$$= \frac{-8 \pm \sqrt{56 - 24}}{4} \tag{6.32}$$

$$= \frac{-8 \pm \sqrt{32}}{4} \tag{6.33}$$

$$= \frac{-8 \pm 4\sqrt{2}}{4}) \tag{6.34}$$

$$= \pm 8\sqrt{2} \tag{6.35}$$

substitute x into vertex form (6.36)

$$f(x) = a()^2 + k \tag{6.37}$$

6.2 Cubics

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Chapter 7

TODO: Absolute Values

Chapter 8

TODO:Inequalities

When we multiply or divide by a negative, change the sign of the inequality. Eg

$$-3x > 9 \tag{8.1}$$

$\div$both sides by -3

$$x > -3 \text{is NOT correct}$$

$$x < -3 \text{is correct} \tag{8.2}$$

Consider

$$\frac{3}{x-1} > 4 \tag{8.3}$$

$$3 > 4(x-1) \tag{8.4}$$

$$3 > 4x - 4 \tag{8.5}$$

$$3 + 4 > 4x \tag{8.6}$$

$$7 > 4x \tag{8.7}$$

$$\frac{7}{4} > x \tag{8.8}$$

$$\therefore x < \frac{7}{4} \tag{8.9}$$

However, this presents a problem; if we substitute $x = 1$ in, we have a "disaster", with a quotient of 0.

$$\frac{3}{1-1} > 4 \text{ cannot possibly be a solution} \tag{8.10}$$

Try $x = 0$

$$\frac{3}{0-1} > 4 \tag{8.11}$$

$$\frac{3}{-1} > 4 \tag{8.12}$$

$$-3 > 4 \text{ which we know to be false} \tag{8.13}$$

Let's ask ourselves "Why didn't this work?". What went wrong was that we don't know whether $x - 1$ is positive or negative. There are several resolutions to the problem.

(a) Take cases: consider

(i) $x - 1 > 0$

(ii) $x - 1 < 0$

And prove whether they are logically true.

(b) Multiply by a non-negative. In this case $(x-1)^2$ because a square can never be a negative number.

(c) Rearrange to a form such as $\frac{A}{B} < 0$ and consider signs.[1]

(d) Plot the inequality: $y = \frac{3}{x-1}$ and see where the y value is bigger than 4.

[1] it could also be $\frac{A}{B} > 0$

First way with cases:

$$\frac{3}{x-1} > 4 \tag{8.14}$$

$$\text{Assume } x-1 > 0 \tag{8.15}$$

$$3 > 4(x-1) \tag{8.16}$$

$$> 4x-4 \tag{8.17}$$

$$7 > 4x \tag{8.18}$$

$$\frac{7}{4} > x \tag{8.19}$$

$$x < \frac{7}{4} \tag{8.20}$$

We only keep x's where $x-1>0$, $x>1$

$$\text{Assume } x-1 < 0 \tag{8.21}$$

$$\frac{3}{x-1} < 4 \tag{8.22}$$

$$x > \frac{7}{4} \tag{8.23}$$

We only keep x's where $x-1<0$, $x<1$

So we combine our answer is when we take the answers that are logically acceptable.

Second way with squares:

$$\frac{3}{x-1} > 4 \tag{8.24}$$

$$(x-1)^2 \times \frac{3}{x-1} > 4(x-1)^2 \tag{8.25}$$

$$(x-1)3 > 4(x-1)^2 \tag{8.26}$$

$$0 > 4(x-1)^2 - 3(x-1) \tag{8.27}$$

$$> (x-1)(4(x-1)-3) \tag{8.28}$$

$$> (x-1)(4x-7) \tag{8.29}$$

$$\text{we can plot this} \tag{8.30}$$

$$\therefore 1 < x < \frac{7}{4} \tag{8.31}$$

$$\tag{8.32}$$

3rd Way:

(was skipped for tutorial exercise)

Graphical Way:

Sketch $y = \frac{3}{x-1}$

Vertical asymptote $x = 1$.

$$x = 1 + \text{a small number} \tag{8.33}$$

$$y = \text{BIG} \tag{8.34}$$

A second problem:

$$\frac{4x}{2x+3} > 2 \tag{8.35}$$

$$\tag{8.36}$$

By squares (quadratic) method:

$$\frac{4x}{2x+3} > 2 \tag{8.37}$$

$$(2x+3)^2 \times \frac{4x}{2x+3} > 2(2x+3)^2 \tag{8.38}$$

$$4x \times (2x+3) > 2(2x+3)^2 \tag{8.39}$$

$$0 > 2(2x+3)^2 - 4x(2x+3) \tag{8.40}$$

$$> (2x+3)(2(2x+3) - 4x) \tag{8.41}$$

$$> (2x+3)(4x+6-4x) \tag{8.42}$$

$$> (2x+3)(6) \tag{8.43}$$

$$> 12x + 18 \tag{8.44}$$

$$-12x > 18 \tag{8.45}$$

$$12x < -18 \tag{8.46}$$

$$x < -\frac{3}{2} \therefore x < -\frac{3}{2} \tag{8.47}$$

By signs:

$$\frac{4x}{2x+3} - 2 > 0 \tag{8.48}$$

$$\frac{4x}{2x+3} - \frac{2x+3}{2x+3} > 0 \tag{8.49}$$

$$\frac{-6}{2x+3} > 0 \tag{8.50}$$

-6always -ve (8.51)

$$2x + 3 < 0 \tag{8.52}$$

$$2x < -3 \tag{8.53}$$

$$x < \frac{-3}{2} \tag{8.54}$$

(8.55)

Chapter 9

TODO: Simultaneous Equations

Chapter 10

TODO: Matrices

Addition

$$\begin{aligned} \mathbb{Z}_{M,N} &= \mathbb{A} + \mathbb{B} \\ z_{i,j} &= a_{i,j} + b_{i,j} \end{aligned} \tag{10.1}$$

Subtraction

$$\begin{aligned} \mathbb{Z}_{M,N} &= \mathbb{A} - \mathbb{B} \\ z_{i,j} &= a_{i,j} - b_{i,j} \end{aligned} \tag{10.2}$$

Multiplication

$$\begin{aligned} \mathbb{Z}_{M,N} &= \mathbb{A} \times \mathbb{B} \\ z_{i,j} &= \sum_{k=1}^{N} a_{i,k} \times b_{k,j} \end{aligned} \tag{10.3}$$

10.1 Language of Matrices

A matrix is nothing more than a bunch of numbers arranged into a grid of M rows and N columns and is represented usually by $\mathbb{A}$. To represent a specific element inside the matrix $\mathbb{A}$, it is usually referred to as $a_{i,j}$ [1]

[1] I don't know why we change to lowercase, but it is the convention. Maybe it is to prevent confusion between referring to a whole matrix (of particular dimensions) and a specific element of a matrix.

An example of a "3 by 4" matrix called $\mathbb{A}$ is as follows:

$$\mathbb{A}_{2,3} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \end{bmatrix} \tag{10.4}$$

Or more generally:

$$\mathbb{A}_{M,N} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \dots & a_{1,j} \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots & a_{2,j} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i,1} & a_{i,2} & a_{i,3} & \dots & a_{i,j} \end{bmatrix} \tag{10.5}$$

In matrix algebra we go vertically describing the rows first then horizontally we describe the columns.[2]

There is also a special kind of matrix called the *identity matrix* often represented by $\mathbb{I}$. This is a special matrix, which must be square in shape (ie for $\mathbb{A}_{M,N}, M = N$), and consists of all zeroes except for a diagonal row of ones from the top left corner to the bottom right corner.

A 4x4 identity matrix:

$$\mathbb{I}_{4,4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10.6}$$

An important property of the identity matrix is that when a matrix is multiplied by it's identity matrix, the product is the original matrix.

$$\mathbb{A} \times \mathbb{I} = \mathbb{A} \tag{10.7}$$

Another impotant property is that a matrix can be multiplied by it's identity matrix in either order (we will see later in section 10.4, Multiplying Matrices that in matrix algebra, order is important[3].

$$\mathbb{I} \times \mathbb{A} = \mathbb{A} \tag{10.8}$$

[2] This is in contrast to the cartesian plane e.g. map reading where we go horizontally then vertically.

[3] as opposed to normal algebra where you can multiply in any which way and it doesn't matter

10.2 Adding Matrices

Two matrices, $\mathbb{A}$ and $\mathbb{B}$ can be summed if and only if the matrices have equal rows and columns. To add $\mathbb{A}$ and $\mathbb{B}$, sum the elements $a_{i,j}$ with $b_{i,j}$ such that:

$$\mathbb{Z} = \mathbb{A} + \mathbb{B} \tag{10.9}$$

where

$$z_{i,j} = a_{i,j} + b_{i,j} \tag{10.10}$$

An example:

$$\mathbb{A}_{2,2} = \begin{bmatrix} 1 & 5 \\ 3 & -4 \end{bmatrix} \tag{10.11}$$

$$\mathbb{B}_{2,2} = \begin{bmatrix} 4 & -8 \\ 2 & 10 \end{bmatrix} \tag{10.12}$$

$$\mathbb{A} + \mathbb{B} = \begin{bmatrix} 1 & 5 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 4 & -8 \\ 2 & 10 \end{bmatrix} \tag{10.13}$$

$$= \begin{bmatrix} (1+4) & (5+-8) \\ (3+2) & (-4+10) \end{bmatrix} \tag{10.14}$$

$$= \begin{bmatrix} 5 & -3 \\ 5 & 6 \end{bmatrix} \tag{10.15}$$

10.3 Subtracting Matrices

Two matrices, $\mathbb{A}$ and $\mathbb{B}$ can be subtracted if and only if the matrices have matching rows and columns. To subtract $\mathbb{B}$ from $\mathbb{A}$, subtract the elements $b_{i,j}$ from $a_{i,j}$ such that:

$$\mathbb{Z} = \mathbb{A} - \mathbb{B} \tag{10.16}$$

where

$$z_{i,j} = a_{i,j} - b_{i,j} \tag{10.17}$$

An example:

$$\mathbb{A}_{2,2} = \begin{bmatrix} 1 & 5 \\ 3 & -4 \end{bmatrix} \tag{10.18}$$

$$\mathbb{B}_{2,2} = \begin{bmatrix} 4 & -8 \\ 2 & 10 \end{bmatrix} \tag{10.19}$$

$$\mathbb{A} - \mathbb{B} = \begin{bmatrix} 1 & 5 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} 4 & -8 \\ 2 & 10 \end{bmatrix} \tag{10.20}$$

$$= \begin{bmatrix} (1-4) & (5--8) \\ (3-2) & (-4-10) \end{bmatrix} \tag{10.21}$$

$$= \begin{bmatrix} -3 & 13 \\ 1 & -14 \end{bmatrix} \tag{10.22}$$

10.4 Multiplying Matrices

Formal definitions for the multiplication of matrices like the one mentioned at the start of this chapter appear scary, but really, it is just *a lot*[4] of simple additions.

(a) The first rule for multiplying matrices is that the number of columns in $\mathbb{A}$ matches the number of rows in $\mathbb{B}$ (that is to say $\mathbb{A}_N = \mathbb{B}_M$).

(b) Remember that $\mathbb{Z}_{i,j}$ is a merely sum of several multiplication operations.

Formally, $z_{i,j} = \sum_{k=1}^{N} a_{i,k} \times b_{k,j}$

Broken down, this means for each element in a given row in $\mathbb{A}$, we must multiply that element by a matching element in it's corresponding column in $\mathbb{B}$ and sum all the results.

For the matrices $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{P}$:

$$\begin{bmatrix} p_{1,1} & p_{1,2} \\ p_{2,1} & p_{2,2} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix} \times \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{bmatrix} \tag{10.23}$$

[4] Because of this, expect to use up *a lot more* paper.

where:

$$
\begin{aligned}
p_{1,1} &= a_{1,1} \times b_{1,1} + a_{1,2} \times b_{2,1} + a_{1,3} \times b_{3,1} \\
p_{2,1} &= a_{2,1} \times b_{1,1} + a_{2,2} \times b_{2,1} + a_{2,3} \times b_{3,1} \\
p_{1,2} &= a_{1,1} \times b_{1,2} + a_{1,2} \times b_{2,2} + a_{1,3} \times b_{3,2} \\
p_{2,2} &= a_{2,1} \times b_{1,2} + a_{2,2} \times b_{2,2} + a_{2,3} \times b_{3,2}
\end{aligned}
$$

10.5 Identity Matrix

For the identity matrix:

$$\mathbb{A} \times \mathbb{I} = \mathbb{A}$$

Proof by general case:

$$
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} [(1 \times a) + (0 \times c)] & [(1 \times b) + (0 \times c)] \\ [(0 \times a) + (1 \times c)] & [(0 \times b) + (1 \times d)] \end{bmatrix} \tag{10.24}
$$

$$
= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{10.25}
$$

This is the matrix equivalent of the number 1[5]

$$1 \times a = a \tag{10.26}$$

$$\frac{1}{a} \times a = 1 \tag{10.27}$$

in matrix terms:

$$\mathbb{A}^{-1} \times \mathbb{A} = \mathbb{I} \tag{10.28}$$

This leads into the topic of calculating the inverse of a matrix.

[5]NOTE: this is only equivalent to one, it is not actually the value of one (to my knowledge).

10.6 Inverse of Matrices

For a 2x2 matrix:

$$\mathbb{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{10.29}$$

$$\mathbb{A}^{-1} = \frac{1}{ad - bc} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{10.30}$$

For a 3x3 matrix: TODO THIS...

For a 4x4 matrix, in the words of Sal Kahn, "You'll be there all day, and for a 5x5 you're almost bound to make a mistake and is best left for a computer."[6]

10.7 TODO: Dividing Matrices

10.8 Summary of Matrix Operations

[6] "In my mind, the only thing less pleasant than inverting a 3 by 3 matrix is inverting a 4 by 4 matrix." – both quotes from http://www.khanacademy.org/video/inverting-matrices–part-2?playlist=Linear%20Algebra

Chapter 11

Parabolas

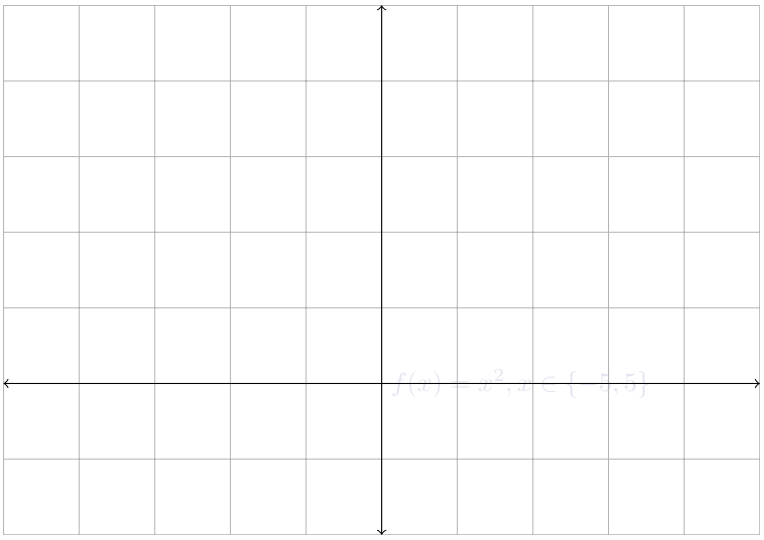

Figure 11.1: A parabolic function: $f(x) = x^2$

A parabolic function in the form $f(x) = mx^2 + c$

Where:	
m	represents a component of the gradient (covered more in section 15 “Differentiation”).
x	is the independent variable
y	is the dependent variable

Chapter 12

TODO: Counting

12.1 Progressions

Progressions are when you have a bunch of numbers in a group, and they are related to eachother in an order, such as “add one to the last one”, or “divide by one plus the last one.” They fall into two types:

Arithmetic Progressions

Geometric Progressions

12.2 Arithmetic Progressions

12.3 Geometric Progressions

Chapter 13

Introduction to Calculus

In a nutshell calculus is about measuring small changes. There are 4 major topics in calculus, with the first two of interest to MATH130 students; the latter two being more advanced topics for down the road.

Differential Calculus Studying the rates at which things change at specific points (most often, the rate at which something changes with respect to time - eg acceleration can be differentiated to determine speed of an object at a given time). This is concerned with the instantaneous rate of change. The derivative (gradient) is able to show this.

Integral Calculus The reverse of differentiation: taking a series of changes and turning this into a different metric (eg, taking a series of points of speed and turning this into the acceleration of an object). This is concerned with the accumulation of metrics. The area between a curve and the x-axis is able to show this.

Multivariable Calculus Extends the previous two types of calculus by allowing one to differentiate or integrate with respect to multiple variables.

Vector Caclulus Concerns itself with differentiating and integrating things called vector fields in 3D space. It is a subset of multivariable calculus in that a vector is a set of points in 3D that define a ray (kind of like a line, but with 2 end points) starting at 0,0,0 and ending at some x,y,z coordinate in 3D space.

There are several principles which are required for differential and integral calculus:

Limits A limit is basically a really small value that represents the difference between the two inputs of a function. We say it is *"sufficiently close"* with the result is arbitrarily close enough to be deemed the closest we can measure.

Derivatives Consider a non-linear function, $f(x)$. The derivative is the gradient of the function at a given point. The gradient changes depending on the x-value we supply; so we find a derivative at a certain point. This is done by the process of differentiation; and we then substitute the value of x in for that point to determine[1] the gradient.

Fundamental theorem States that differentiation and integration are inverse operations - that is, one will undo the other. There are two parts to integration; definite integrals and indefinite integrals (sometimes called *antiderivatives*). These will be covered in whole sections unto themselves.

[1] derive??

Chapter 14

Functions & Graphs

If you are a computer programmer, the best way to think of a function in maths is the same way you think of a function in a functional programming language. If you are not a computer programmer, perhaps the best way to think of a function is like a little machine that takes a number, it does something to that number, and it displays an output.[1] Here is an example of a function that simply "doubles" the input:

$$f(x) = 2x \tag{14.1}$$

Here is what happens when we input the number 5 into our function...

$$f(5) = 2(5) \tag{14.2}$$
$$= 10 \tag{14.3}$$

And now the number -5:

$$f(-5) = 2(-5) \tag{14.4}$$
$$= -10 \tag{14.5}$$

Functions can be (and often are) more complex, here's a quadratic function:

$$f(a) = (a+3)^2 \tag{14.6}$$
$$= (a+3)(a+3) \tag{14.7}$$
$$= a^2 + 6a + 9 \tag{14.8}$$

[1] Congratulations, you are now thinking like a programmer as well as a mathematician!

And if we substitute 5 in for a we get:

$$f(5) = 5^2 + 6(5) + 9 \tag{14.9}$$
$$= 25 + 30 + 9 \tag{14.10}$$
$$= 64 \tag{14.11}$$

For completeness, if we substitute -5 in for a we get:

$$f(-5) = -5^2 + 6(-5) + 9 \tag{14.12}$$
$$= 25 - 6(5) + 9 \tag{14.13}$$
$$= 4 \tag{14.14}$$

When it comes to graphing functions, you can rename your y axis to equal $f(x)$, so graphing your function is now the same as before graphs with one additional bonus: now you can let x be an abitrary number[2] as part of the function. Figure 14.2 shows a function, $f(x) = x$.
Types of graphs that can be encountered in MATH130:

(a) Line 14.2

(b) Parabola 11

(c) Hyperbola 14.4

(d) Cubic 14.5

(e) Absolute 14.6

A function has a *domain* which is all the acceptable values of x as inputs that the function can use. Some examples:

[2]This is called a *independent variable*

$$f(x) = x^2$$

$$domain : 0 \leq x \geq 0 \tag{14.15}$$

$$\tag{14.16}$$

$$f(x) = \frac{1}{x}$$

$$domain : x \neq 0 \tag{14.17}$$

$$\tag{14.18}$$

$$f(x) = \sqrt{x}$$

$$domain : x \geq 0 \tag{14.19}$$

$$\tag{14.20}$$

$$f(x) = \sqrt{x-2} + \sqrt{5+x}$$

$$domain : (x-2) \geq 0 \tag{14.21}$$

$$\therefore \geq 2 \tag{14.22}$$

$$\tag{14.23}$$

$$domain : 5 + x \geq 0 \tag{14.24}$$

$$\therefore \geq -5 \tag{14.25}$$

$$andso : \tag{14.26}$$

$$x : x \geq 2 \tag{14.27}$$

The last equation demonstrates that the domain is the more restrictive of the conditions of the two "sub-domains" of each square root portion of the function.

14.1 Arithmetic With Functions

Suppose we have:

$$f(x) = x^2 + 1 \tag{14.28}$$

$$g(x) = \frac{1}{1-x} \tag{14.29}$$

and

$$h = f(x) + g(x) \tag{14.30}$$

then

$$= x^2 + 1 + \frac{1}{1-x} \tag{14.31}$$

however, if

$$i = f(x) \times g(x) \tag{14.32}$$

then

$$= (x^2 + 1) \times (\frac{1}{1-x}) \tag{14.33}$$

however, if

$$j = f \circ g \tag{14.34}$$

then

$$= j(x) = f(g(x)) \tag{14.35}$$

$$= (\frac{1}{1-x})^2 + 1 \tag{14.36}$$

"Remember to "algebra" the function to minimise them and to see if they equal a simpler equation"[3]

[3] according to Gareth Richardson

14.2 Linear Functions

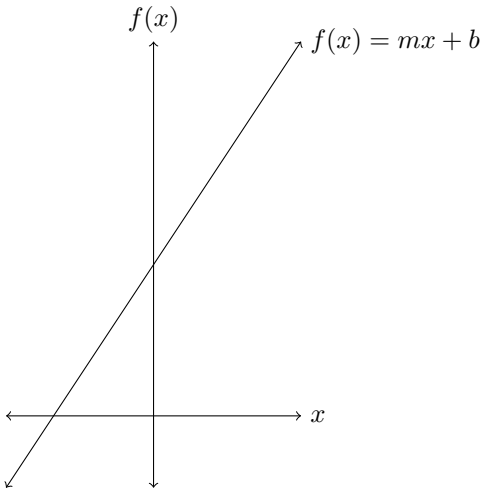

Figure 14.1: A linear function: $f(x) = mx + b$

Linear function in the *General Form* $Ax + By + C = 0$. It may also take the handy *Slope-intercept form* $f(x) = mx + b$. This is useful because the gradient of the line cane be read straight from the equation, and is just a rearrangement of the general form.

Where:	
m	represents the gradient
x	is the independent variable
y	is the dependent variable

If given only 2 points and we are to find the equation of the line:

(a) we need to determine the gradient

(b) we need to substitute the x,y values in to form the equation

Determining the gradient is done using the formula:

$$\frac{y_2 - y_1}{x_2 - x_1} = m \tag{14.37}$$

Example: we are given the points (7,1), (2,5) and need to find the equation

of the line connecting these points.

$$\frac{5-1}{2-7} = \frac{4}{-5} \tag{14.38}$$

$$= -\frac{4}{5} \tag{14.39}$$

If we want to find a parallel line passing through specific points, remember that the gradient (m) must be the same in both equations, and we must subtitute the x and y values for the specific points into the new arbitrary equation to solve for the new equation:

Example: find an equation for the line through $(3, 4)$ and parallel to the line through $(7, 1)$ and $(2, 5)$ from out previous example:

let

$$m = -\frac{4}{5}$$

substitute x,y values into slope-intercept equation

$$y = mx + b$$

$$4 = -frac45 \times 3 + b \tag{14.40}$$

and solve for b

$$4 + \frac{4}{5} \times 3 = b \tag{14.41}$$

$$4 + \frac{12}{5} = b \tag{14.42}$$

$$\frac{32}{5} = b \tag{14.43}$$

$$\therefore y = -\frac{4}{5}x + \frac{32}{5} \tag{14.44}$$

14.3 Parabolic Functions

For an introduction to parabolas, it is highly recommended to read chapter 11, “Parabolas”.

14.4 Hyperbolic Functions

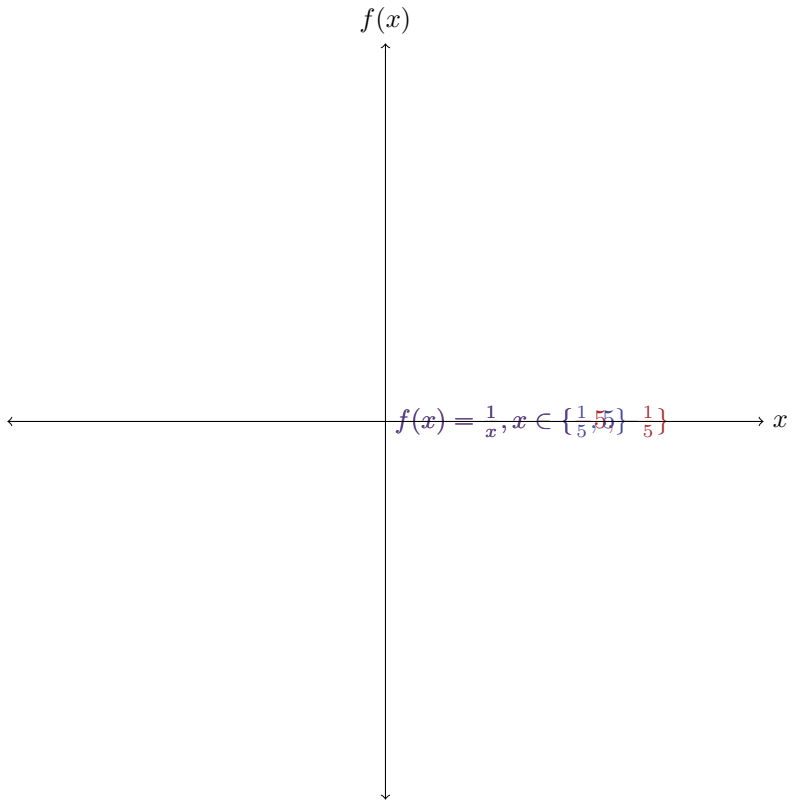

Figure 14.2: A hyperbolic function: $f(x) = \frac{1}{x}$

A hyperbolic function in the form $f(x) = \frac{1}{x}$

Where:
x is the independent variable
y is the dependent variable

14.5 Cubic Functions

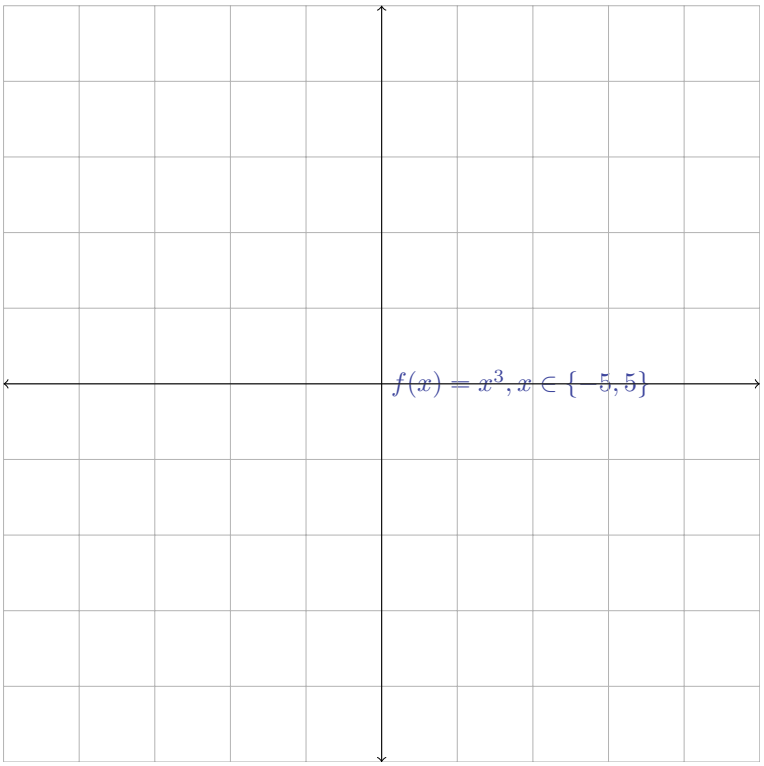

Figure 14.3: A cubic function: $f(x) = x^3$

A cubic function in the form $f(x) = mx^3 + c$

Where:	
m	represents a component of the gradient (covered more in section 15 “Differentiation”).
x	is the independent variable
y	is the dependent variable
x	is the independent variable
y	is the dependent variable

14.6 Absolute Value Functions

Figure 14.4: A function of absolute value: $f(x) = |x|$

An absolute function in the form $f(x) = |x|$

Where:	
m	represents a component of the gradient (covered more in section 15 “Differentiation”).
x	is the independent variable
y	is the dependent variable
x	is the independent variable
y	is the dependent variable

14.7 Limits

There are some types of functions you may be asked to evaluate for various values of x which are unreasonable. One such example is the hyperbolic function $f(x) = \frac{1}{x}$ where $x = 0$. In MATH130, we consider this value an illegal or "undefined" value - but there is still a way to evaluate it. Consider taking a table of values14.7: Note how as x gets smaller, y

x	5	4	3	2	1	0.8	0.6	0.4	0.2	0.1	...
y	0.20	0.25	0.33	0.50	1.00	1.25	1.67	2.50	5.00	10.00	...

Table 14.1: Table of values for hyperbolic function $f(x) = \frac{1}{x}$

gets bigger. The way this is formally worded is "as x approaches 0, y approaches infinity, and written:

$$x \to 0 \; f(x) \to \infty \tag{14.45}$$

From this point, we can see that x cannot be zero, however all other $\mathbb{R}$ are acceptable. Building on this we can define it as a set:

$$\begin{aligned} &x \to 0 \; f(x) \to \infty \\ &x \in \{\mathbb{R}, x \neq 0\} \end{aligned} \tag{14.46}$$

The key to understanding how limits work is to identify what x values are undefined or otherwise illegal. Key indicators of this phenomena are when you see x inside a squareroot sign, or as a divisor in a quotient:

Chapter 15

Differentiation

In overly simplified terms, differentiation is the process of taking a curve on a graph and finding what the gradient of that curve is. There are 4 methods which are useful for MATH130. These are:

- Power Method (well, that's what I call it)
- Product Rule
- Quotient Rule
- Chain Rule

Because the notion of calculus was a joint effort between several mathematicians[1] operating in secret, and without a project manager there are two important notations.[2]
The first notation is the "dash", "prime", or "Lagrange's" notation and appears as such:

$$f(x) = ... \tag{15.1}$$

$$f'(x) = ... \tag{15.2}$$

[1] Gottfried Leibniz woke up one day and thought "I'm going to invent a whole new branch of mathematics to annoy students for the next few hundred years." Approximately 10 years earlier, Sir Isaac Newton thought "I know what will really get Leibniz's goat... I'll get the drop on him with this idea I have." Consequently, the two never became friends.

[2] there are more, but we don't need to know about them for MATH130

Secondly is Leibniz's notation:

$$\frac{\mathrm{d}}{\mathrm{d}x}f(x) = ... \tag{15.3}$$

Euler's notation: (not so common in MATH130)

$$Df(x) = ... \tag{15.4}$$

(15.5)

Each notation has their merits and usefulness; Lagrange's form is neat and compact for simple derivatives, however Leibniz's notation describe what is being differentiated and what is in respect to, which is useful for the chain rule discussed shortly. As such, it is important to be familiar with all the above forms as they will often be used interchangeably for brevity, neatness and ease of memorising them. In terms of how you answer a question - if there is no stated style of notation, go for what "looks" like it works [3] and is clear and neat. Clear and neat usually results in the marker understanding what you are on about, so even if you are wrong, you might get partial marks.

A useful tip, when you are first getting used to differentiation, it may be handy to use Leibniz's notation and say in your mind what you are differentiating, and what it is in respect to.

15.1 Power Method

The power method is by far the easiest to understand of all 3 methods, and if possible, it may be easier to rearrange a part of an equation into index notation and differentiate that way. This method is not always possible, but by using the power laws from the series of equations starting with 4.2, sometimes a shortcut can be made, which is why Section 4 important to know very well.

Put simply, the power method can be understood as "multiply the base

[3] Munner's Law: If it looks wrong it probably is. – Cliff Munro, 1996 Cranbrook School, Design and Technology teacher. Author's corollary: If it doesn't look wrong, hopefully it's right.

by the power and subtract one from the power", and is demonstrated in equation 15.7 below.

$$f(x) = x^a + k \tag{15.6}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = ax^{a-1} \tag{15.7}$$

The value k represents a constant, often just a plain number, though it doesn't have to be. The important thing about the k-value in this example is that there is no x component. Hence it "disappears".

Functions may have more than one term, consider the following quadratic:

$$f(x) = x^2 + 2xb + b^2 \tag{15.8}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = 2x^1 + 2b \tag{15.9}$$

$$= 2(x + b) \tag{15.10}$$

Here we are asked to differentiate with respect to x (denoted by the symbol $\frac{\mathrm{d}}{\mathrm{d}x}$). To do this, we bring the power of x^2 to the front, and subtract 1 to give $2x^1$, and we do the same for the term $2xb$ by looking at the invisible power (it's there, we just don't write it out of laziness![4]): $2 * x^1 * b$

15.2 Product Rule

A function $f(x)$ is a product of two functions, $u(x) * v(x)$. For example:

$$f(x) = x^3 \sin(x) \tag{15.11}$$

$$= u(x)v(x) \tag{15.12}$$

In this case, we can see that $u(x) = x^3$ and $v(x) = \sin(x)$. While there is a mathematical proof[5] it is not necessary for MATH130. All we need to know is:

$$f(x) = u(x) * v(x) \tag{15.13}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = u'(x) * v(x) + v'(x) * u(x) \tag{15.14}$$

[4] Or convenience, neatness, brevity.

[5] refer to section 28.1 of appendix

So for 15.11, to find the derivative:

$$f(x) = x^3 \sin(x)$$
$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = \frac{\mathrm{d}}{\mathrm{d}x}(x^3) * \sin(x) + x^3 * \frac{\mathrm{d}}{\mathrm{d}x}(\sin(x)) \tag{15.15}$$
$$= 3x^2 * \sin(x) + x^3 * \cos(x) \tag{15.16}$$

This example makes use of a derivative of a trigonometric function. This will be explored in chapter 17, "Differentiation of Trig Functions" - until then, just ignore the trigonometric part.

15.3 Quotient Rule

A quotient is a division - just like from primary school: $\frac{u}{v}$. Previously we may have called the u part the numerator, nowadays it is called the *dividend*, and the v denominator previously called the denominator is now called the *divisor*, with the *quotient* being the result.
To do quotient rule differentiation, we are actually using a modified version of the product rule, however for MATH130 it is only required to think of it as a separate rule.[6] The rule takes the form of:

$$f(x) = \frac{u}{v} \tag{15.17}$$
$$f(x)\frac{\mathrm{d}}{\mathrm{d}x} = \frac{u\frac{\mathrm{d}}{\mathrm{d}x}\, v - u\, v\frac{\mathrm{d}}{\mathrm{d}x}}{v^2} \tag{15.18}$$

Although both 15.18 and 15.19 are identical, 15.19 is tidier, and may be easier to remember.

$$f'(x) = \frac{u'\, v - u\, v'}{v^2} \tag{15.19}$$

[6] Mathematical proof of this is in section 28.2 of the appendix.

Consider the following example:

$$f(x) = \frac{x^2}{4x} \tag{15.20}$$

$$f'(x) = (4x)x^2\frac{\mathrm{d}}{\mathrm{d}x} - x^2(4x\frac{\mathrm{d}}{\mathrm{d}x}) \tag{15.21}$$

$$f'(x) = (4x)2x - x^2\,(4) \tag{15.22}$$

$$f'(x) = 8x^2 - 4x^2 \tag{15.23}$$

$$f'(x) = 4x^2 \tag{15.24}$$

15.4 Chain Rule

The chain rule is useful for differentiation a function $f(x)$ where there are functions inside of functions (such as $\ln(\sin(x))$). To do this, we break the function up into it's components, give them some names [7] and apply the chain rule. The rule takes the form as follows:

$$f(x) = v(u(x)) \tag{15.25}$$

$$f(x)\frac{\mathrm{d}}{\mathrm{d}x} = f\frac{\mathrm{d}}{\mathrm{d}u}\,u\frac{\mathrm{d}}{\mathrm{d}x} \tag{15.26}$$

Here is an example of where we might use the chain rule:[8][9]

$$f(x) = (x^2)^3 \tag{15.27}$$

Let $u = x^2$ such that

$$f(x) = u^3 \tag{15.28}$$

The first step in 15.28 is to identify the chain rule, and to name the function inside u. It just makes things easier this way for most of the time.

$$f(x)\frac{\mathrm{d}}{\mathrm{d}x} = f\frac{\mathrm{d}}{\mathrm{d}u} * u\frac{\mathrm{d}}{\mathrm{d}x} \tag{15.29}$$

$$= \left[3u^2\right]\,\left[2x\right] \tag{15.30}$$

15.28 It often helps to rewrite the equation in terms of $f(x)$ and u, and then to write out the chain rule. Normally we are differentiating $f(x)$ with respect to x ($\frac{\mathrm{d}}{\mathrm{d}x}$). Here we differentiate $f(x)$ with respect to u (ie $\frac{\mathrm{d}}{\mathrm{d}u}$) *THEN* multiply by u. The rest is plain old algebra. So.. substitue values back for u.

$$= \left[3(x^2)^2\right] * 2x \tag{15.31}$$

[7] " *Let*" is possibly the most important word you will come across in mathematics. You can use it to redefine stuff if it's too complex and break it into smaller manageable pieces and put it back together again." – Chris Gordon, MATH130 lecturer, Macquarie University, 2011

[8] Note that the v function above in this example is simply u^3.

[9] While we could use the power method to solve this particular problem in 2 steps, we will demonstrate chain rule first, and then the power method.

simplify the first set of brackets with basic power laws

$$= (3x^4)(2x) \tag{15.32}$$

$$= 6x^5 \tag{15.33}$$

As a point of exercise, here's how much faster it is using the power method:

$$f(x) = (x^2)^3 \tag{15.34}$$

$$f(x) = x^6 \tag{15.35}$$

Now we use the differentiation power method, bring the power out the front and reduce the power by one

$$f(x)\frac{\mathrm{d}}{\mathrm{d}x} = 6x^5 \tag{15.36}$$

Look! Only two steps!

(15.37)

It should be noted however, that power method cannot be used for all chain rule problems (in fact, most of the time it can't - but for easy ones like this, it is far more efficient to use power method). The following example cannot use power method, and we *should* use the chain rule:

$$f(x) = \ln(\sin(x)) \tag{15.38}$$

Let

$$u(x) = \sin(x) \tag{15.39}$$

Chapter 16

TODO: Differentiation of Exponents and Logs

Chapter 17

Differentiation of Trigonometric Functions

Consider the plot of the function $f(x) = \sin(x)$ A hyperbolic function in the form $f(x) = \frac{1}{x}$ Here are some equations that are handy to remember.

Where:
x is the independent variable
y is the dependent variable

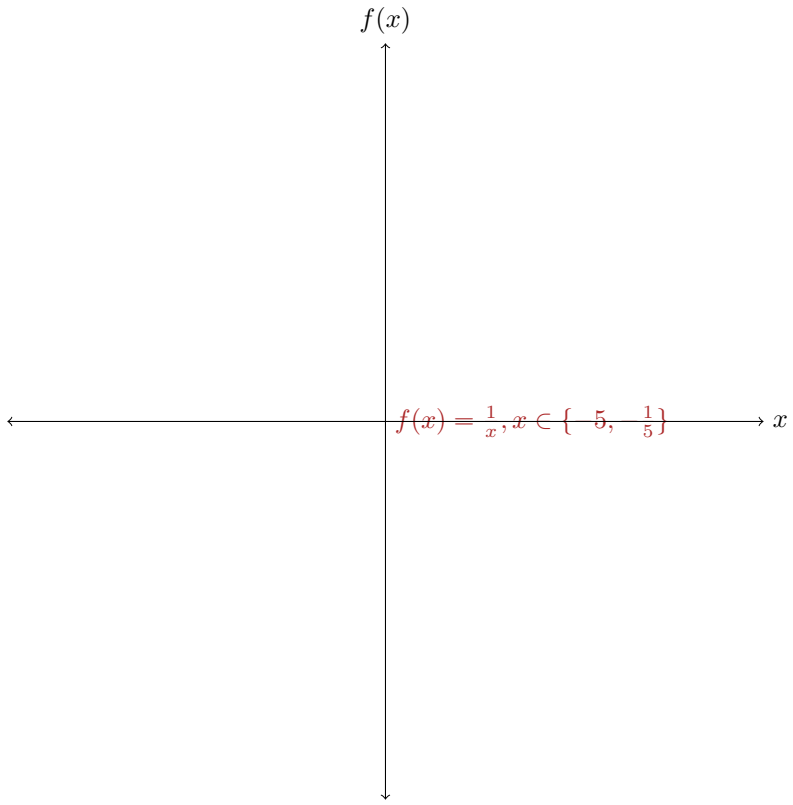

Figure 17.1: A hyperbolic function: $f(x) = \frac{1}{x}$

Source [2].

$$\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1} \tag{17.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}e^{ax} = ae^{ax} \tag{17.2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln(ax) = \frac{1}{x} \tag{17.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin(ax) = (a)\cos(ax) \tag{17.4}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos(ax) = (-a)\sin(ax) \tag{17.5}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan(ax) = (a)\sec^2(ax) \tag{17.6}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sec(ax) = (a)\sec(ax)\tan(ax) \tag{17.7}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\csc(ax) = (-a)\csc(ax)\cot(ax) \tag{17.8}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cot(ax) = (-a)\csc^2(ax) \tag{17.9}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}(\frac{x}{a}) = \frac{1}{\sqrt[2]{a^2 - x^2}} \tag{17.10}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}(\frac{x}{a}) = \frac{-1}{\sqrt[2]{a^2 - x^2}} \tag{17.11}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}(\frac{x}{a}) = \frac{a}{\sqrt[2]{a^2 - x^2}} \tag{17.12}$$

For a proof, refer to appendix section 29, “Differentiation of Trig Functions Proof”

Chapter 18

TODO: Maxima and Minima

Chapter 19

TODO: Grokking Word Problems

Chapter 20

TODO: Newton's Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \tag{20.1}$$

Newton's Method is used to find successively better approximations of the roots of a function.

We take a value of x, x_0 and subtract the function at x_0 divided by the derivative of that function at x_0. The result, x_1 is an answer close to the roots of that function.

What we can do next, is put x_1 back into the same process and get a better approximation of the function. If we repeat this over and over, we will either get an exact value of the roots of the function, or we will get pretty darn close to it.

Chapter 21

TODO: Practical Uses of Differentiation

Chapter 22

TODO: Integration

tl;dr: Integration is the reverse process of Differentiation.

If we were to delve more into what integration is, then we could describe it as finding the area "underneath a curve" (more accurately, between the curve and the x-axis).

There are many different ways of doing this, and MATH130 requires we know 2 of them:

Simpson's Rule

Trapezoidal Rule

There are two types of integration:

Anti-derivatives or *indefinite integrals*

Definite Integrals

The principles behind indefinite and definite integrals are the same, however, with definite integrals, you have a range for which you are integrating your function, where as indefinite integrals have no range (beyond the complete range of the function itself).

Chapter 23

TODO: Trapezoidal Rule

Chapter 24

TODO: Simpson's Rule

Chapter 25

TODO: Average values of a function

Chapter 26

TODO: Practical Uses of Integration

Chapter 27

Trig Identities

The following trig identities, up until eq:27.6 will be necessary to memorize. Afterwards; everything can be derived.[1]

27.1 Inverse Identities

$$\sin^{-1}\theta = \frac{1}{\sin\theta} = \csc\theta \tag{27.1}$$

$$\cos^{-1}\theta = \frac{1}{\cos\theta} = \sec\theta \tag{27.2}$$

$$\tan^{-1}\theta = \frac{1}{\tan\theta} = \cot\theta \tag{27.3}$$

27.2 Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{27.4}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \tag{27.5}$$

[1] However, if you can memorize others, it may save crucial seconds in exam situations.

27.3 Primitive Identities

$$\sin^2\theta + \cos^2\theta = 1 \tag{27.6}$$

Divide 27.6 by $\sin^2$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \tag{27.7}$$

$$1 + \cot^2\theta = \csc^2\theta \tag{27.8}$$

Divide 27.6 by $\cos^2$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \tag{27.9}$$

$$\tan^2\theta + 1 = \sec^2\theta \tag{27.10}$$

27.4 Angle Sum Identities

27.5 Double Angle Identities

Chapter 28

Proofs of Calculus

28.1 ProofofProductRule

$$x = 1 \tag{28.1}$$

28.2 Proof of Quotient Rule

$$x = 1 \tag{28.2}$$

28.3 Proof of Chain Rule

$$x = 1 \tag{28.3}$$

Chapter 29

Differentiation of Trig Functions Proof

If we have the function $f(x) = \sin(x)$, and take two points, P and Q where $P = (x, f(x))$ and $Q = (x + h, f(x + h)$ where $h \neq 0$ we can construct a line joining P and Q, and the gradient of this line is given by the "rise over run" formula:

$$\frac{\Delta Y}{\Delta X} = \frac{f(x+h) - f(x)}{(x+h) - x} \tag{29.1}$$

$$= \frac{\sin(x+h) - \sin(x)}{h} \tag{29.2}$$

Chapter 30

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[1] Giggity
[2] blunt
[3] ie, if it was "crap", you'd call it "crap"
[4] and not as one of the "ugly cousins"

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[5] Gracious Professionalism is a common law trademark of the United States Foundation for Inspiration and Recognition of Science and Technology (US FIRST).

[6] alwayz!

[7] Put a donk on it!

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