$$\text{Let } x_0 \text{ be the state vector for the car} = \begin{bmatrix} x_{\text{world}} \\ y_{\text{world}} \\ \varsigma_{\text{world}} \\ \dot{x}_{\text{car}} \\ \dot{y}_{\text{car}} \\ \dot{\varsigma} \\ M_{\text{pitch}} \\ F_z \\ \omega_F \\ \omega_R \end{bmatrix}$$

$$\text{Let } y_0 \text{ be the control vector for the car} = \begin{bmatrix} \varsigma_{\text{steering}} \\ \tau_F \\ \tau_R \end{bmatrix}$$

These are all in standard metric units with a angles being in degrees.

ASSUMPTIONS

- Assumes ς_{world} has 0 having in positive x
- Assumes yaw is positive in the clockwise direction
- Assumes pitch moment is positive in the counter clockwise direction
- We have a tire model that outputs forces caused by the tires $F_{Rx}, F_{Ry}, F_{Fx}, F_{FY}$

We will now solve for the state dot vector.

$\dot{x_w}$ and $\dot{y_w}$ can be found by rotating $\dot{x_{car}}$ and $\dot{y_{car}}, \varsigma_w$ degrees. From there
$\dot{x_w} = \dot{x_c} \cos \varsigma_w + \dot{y_c} \sin \varsigma_w$
$\dot{y_w} = -\dot{x_c} \sin \varsigma_w + \dot{y_c} \cos \varsigma_w$

To find the the $\dot{\varsigma_w}$ it is trivially $\dot{\varsigma}$ from the state vector.

To find the accelerations of the car we can appeal to F=ma. For the x direction there are 3 forces: F_{Rx}, F_{Fx}, drag. The same is true for the y direction. Therefore:
$\ddot{x} = \frac{F_{Rx}+F_{Fx}-A_x\dot{x}^2}{m}$
$\ddot{y} = \frac{F_{Ry}+F_{Fy}-A_y\dot{y}^2}{m}$

where A_x and A_y are the aero constants for longitudinal and lateral movement respectively.

For $\ddot{\varsigma}$ we simply need to find the α of the car taken about the axis parallel to the Z axis and going through the center of mass of the car. While this assumes the center of mass is in a static location I think this i s a reasonable assumption. To calculate the moments we can use the y components from each tire. Based on our assumptions positive yaw is CW, so the front tire induces a positive moment if its force is positive. Therefore:
$\ddot{\varsigma} = \frac{aF_{Fx} - bF_{Rx}}{I_{22}}$.
Where a and b are the distances from the CG for the front and back tires receptively and I_{22} is the rotational inertia taken about the axis described above.

For the change pitch moment there is no easy way to calculate this. We are hoping to leave this as 0 and hope that the other variables can account for it and act as a feedback loop.

For the change in down force we will first compute the down force and then compute its derivative with respect to time. The down force is equal to $F_z = N_c A_x \dot{x}^2$ where A_x is the longitudinal aero constant and N_c is the nose cone constant (how much of drag gets converted to down force). To take the time derivative of this gives us. By implicit differentiation we get that
$\dot{F}_z = 2N_c A_x \dot{x}\ddot{x}$

For the angular acceleration of the wheels we can imagine the wheel as having 2 torques applied to it. First is that of the motor. The second it the frictional force. The frictional force is that same as the tire force that is parallel to the direction of the tire. For the rear tire that is just the F_{Rx}. For the front tire this is $F_{Fx} \cos \varsigma_{\text{steering}} + F_{Fy} \sin \varsigma_{\text{steering}}$. To get the torque we just multiply by the radius of the tire r. Therefore:
$\alpha_F = \frac{\tau_F - r(F_{Fx} \cos \varsigma_{\text{steering}} + F_{Fy} \sin \varsigma_{\text{steering}})}{I_w}$
$\alpha_R = \frac{\tau_R - r(F_{Rx})}{I_w}$