Chap 4 Exponential and Logarithmic functions

4.1 Exponential function

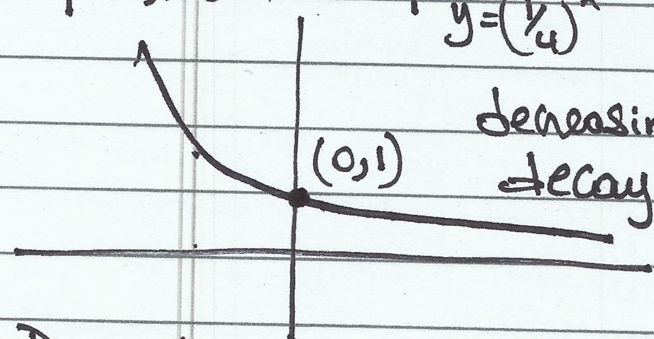
4.2

Values of a.

$$f(x) = a^x \qquad a > 0 \,;\, a \neq 1$$

$0 < a < 1$

$f(x) = a^x$ | $y = (\frac{1}{2})^x$, $y = (\frac{1}{4})^x$



$D_f = (-\infty, \infty)$.

Range: $(0, \infty)$.

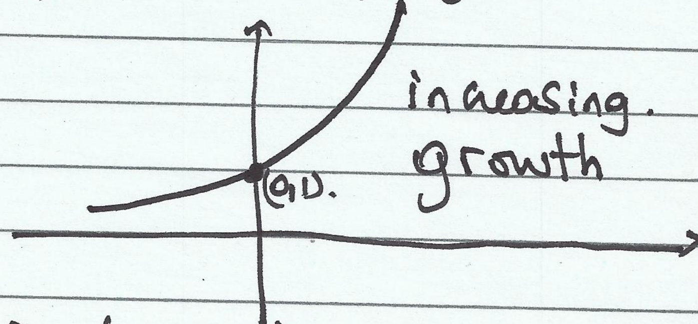
as $x \to \infty$ $y = a^x \to 0$.

$y = 0$ is Horizontal Asymptote.

as $x \to \infty$ ✓

$a > 1$

$f(x) = a^x$ | $y = 2^x$, $y = 3^x$



$D_f = (-\infty, \infty)$

Range $(0, \infty)$.

End behavior

as $x \to -\infty$ $y = a^x \to 0$

$y = 0$ is Horizontal Asymptote.

as $x \to -\infty$

$y = 3^{-x} = \frac{1}{3^x} = \left(\frac{1}{3}\right)^x$ decay.

$y = (\sqrt{2})^x$ growth.

Properties of Exponential

$a^x a^y = a^{x+y}$ | $(ab)^x = a^x b^x$

$\frac{a^x}{a^y} = a^{x-y}$ | $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

$a^{-x} = \frac{1}{a^x}$ | $a^0 = 1$

$|a^2| = |a|^2 = a^2$

The Natural base $e \sim 2.7\ldots$

$f(x) = e^x$ growth. natural Exponential function.

exponential function

$\left(1 + \frac{1}{x}\right)^x \longrightarrow e$ as $x \to \infty$

$f(x) = e^x$

$f(2) = e^2 = \ldots$

$f(0.5) = e^{0.5} = \ldots$

$f(-3) = e^{-3} = \frac{1}{e^3}$

Applications

Compound Interest.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = principal.

n=1 annually. r = rate of Interest.

n=2 Semi-annually.

n=4 Quarterly. n number of time the money is compounded.

n=12 monthly.

n=26 biweekly. t = number of years.

n=52 Weekly.

n=365 daily. future Value.

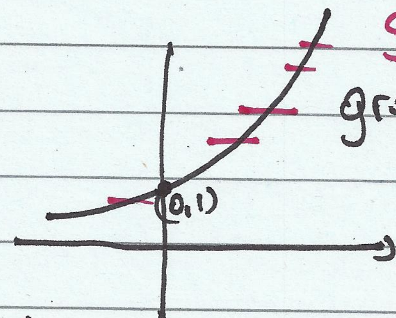
continuously $n = \infty$

$$A = Pe^{rt}$$ Continuously.

$y = a^x$ exponential

$y = x^2$ polynomial.

$y = \sqrt{x^2+1}$



growth.

Satisfy the Horizontal test

$a \neq 1$; $a > 0$

$(-\infty, \infty) \xrightarrow[\text{one-to-one.}]{a^x} (0, \infty).$

$y = \log_a x$

$(f \circ g)(x) = f(g(\underline{x})) = x$ D_g

and

$(g \circ f)(x) = g(f(\underline{x})) = x$ D_f

then $f = g^{-1}$ or $g = f^{-1}$ (inverse).

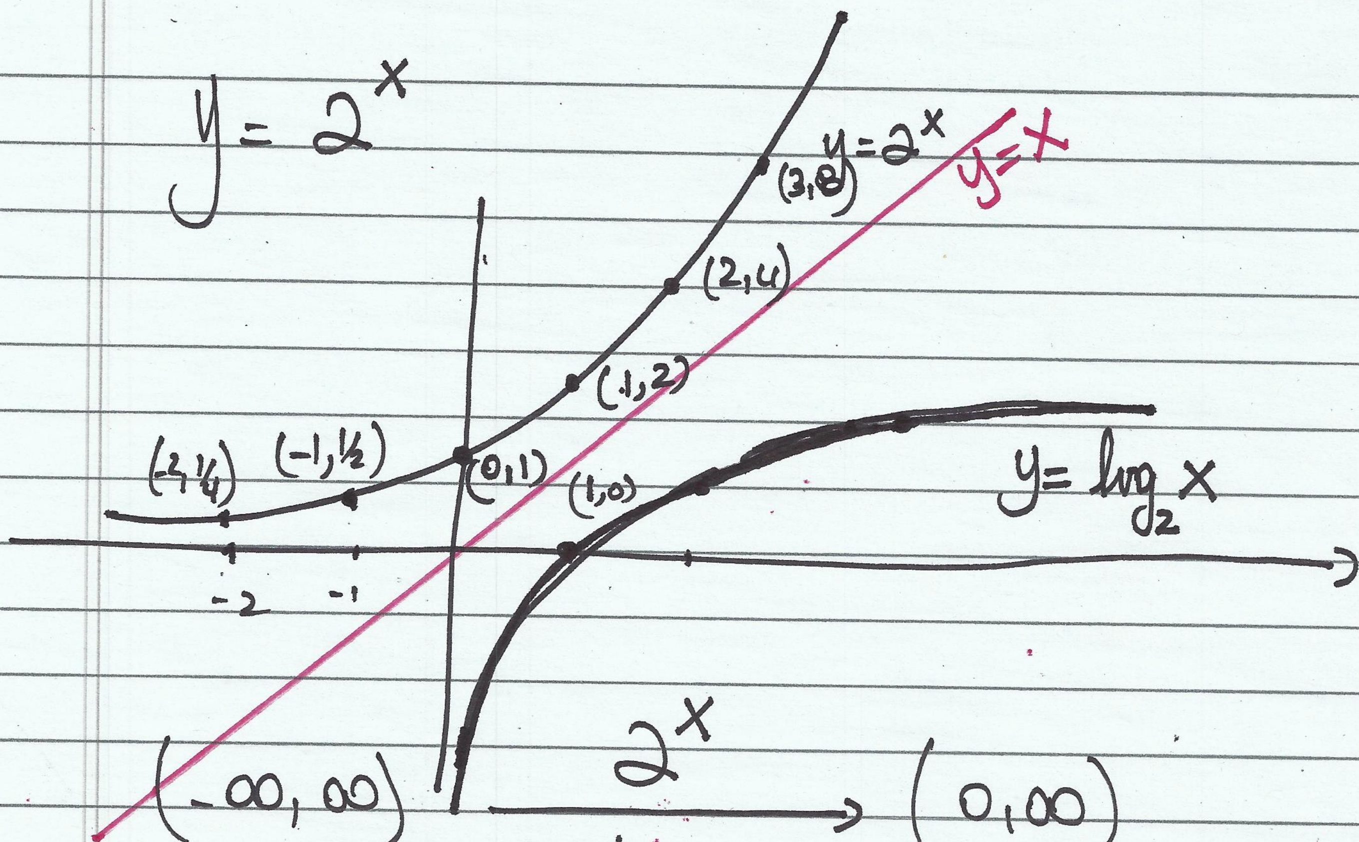
$\boxed{\log_a a^x = x}$ for any x

$\boxed{a^{\log_a x} = x}$ for $\underline{x > 0}$

$y = \log_a x$

$D = (0, \infty)$; Range $(-\infty, \infty)$.

$y = 2^x$



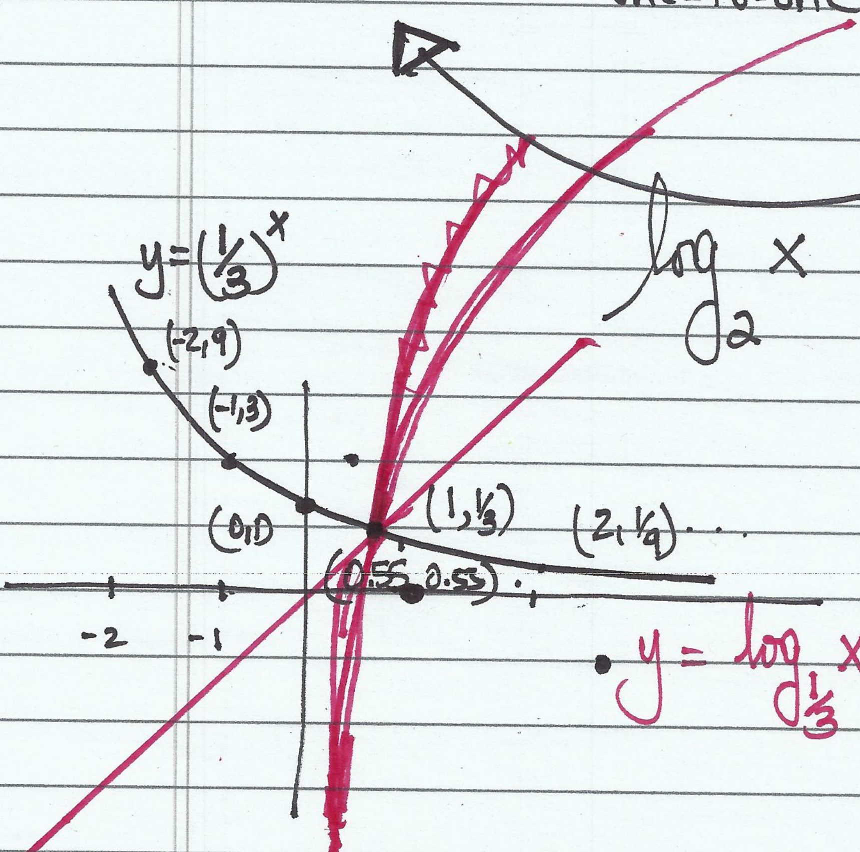
$$(-\infty, \infty) \xrightarrow{2^x} (0, \infty)$$

one-to-one.

$\log_2 x$

$\log_2 2^x = x$ for any x

$2^{\log_2 x} = x$ for $x > 0$



$y = \log_{\frac{1}{3}} x$ — $D = (0, \infty)$
$R\ (-\infty, \infty)$.

$\left(\frac{1}{3}\right)^x = x$ Solve for x

$\boxed{x = 0.55}$

$y_1 = \left(\frac{1}{3}\right)^x$
$y_2 = x$

graph them intersection

The Natural logarithmic function

$$\log_e x = \ln x$$

Natural log.

$$\ln e^x = x \quad \text{for any } x$$

$$e^{\ln x} = x \quad \text{for any } x > 0$$

$$\log_a x = y \quad \text{if and only if} \quad x = a^y.$$

$$\ln x = y \quad \text{iff} \quad x = e^y$$

~~Apply~~ $\log_a x = \log_a a^y.$

$\log_a x = y$

$x = e^y.$

Apply ln

$\ln x = \ln e^y = y$

$\ln x = y.$

Properties of $\log_a$ or Ln

1) $\log_a a^1 = 1$ $\qquad$ $\ln e = 1$

2) $\log_a 1 = 0$ $\qquad$ $\ln 1 = 0$

$\ln e^x = x$

$e^{\ln x} = x$

3) $\log_a a^x = x$ and $a^{\log_a x} = x$; $\underline{x > 0}$

any.

4) If $\log_a x = \log_a y$ then $x = y$

$\ln x = \ln y$ then $x = y$

Examples $\quad \ln\left(\frac{1}{e}\right) = \ln e^{-1} = -1$

$\ln e^x = x$

$e^{\ln 5} = 5$

$4 \ln 1 = \boxed{0}$ $\quad$ | $\quad$ $2 \ln e = \boxed{2}$

$y = \ln(x)$ domain

$\underline{x > 0}$

Domain

$y = \ln(x-1)$

$x - 1 > 0$

$\boxed{x > 1}$

$x = 1$ is a Vertical Asymptote.

$y = \ln x^2$

$x^2 > 0$

$|x| > 0$

$x < 0$ or $x > 0$

Domain $(-\infty, 0) \cup (0, \infty)$

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#50 $y = 2 + \log_2(x+1)$.

Domain

Vertical Asymptote

x-intercept.

$x + 1 > 0 \therefore \boxed{x > -1}$ $(-1, \infty)$

$\boxed{x = -1}$ VA

$0 = 2 + \log_2(x+1)$.

$\log_2 (x+1) = -2 \longleftrightarrow (x+1) = 2^{-2} = 1/4$

We Know

$\log_a A = B \longleftrightarrow A = a^B$

$\boxed{x = -3/4}$ ✓

X-int $(-3/4, 0)$.

log to Exp
$\longrightarrow$

(8) $\log_3 81 = 4 \longleftrightarrow 3^4 = 81$ ✓

$81 = 3^4$

(12) $\log_{16} 8 = 3/4 \longleftrightarrow 16^{3/4} = 8$

$(2^4)^{\frac{3}{4}} = 2^{4 \times \frac{3}{4}} = 2^3 = 8$ ✓

(16) $8^2 = 64 \longrightarrow \log_8 64 = 2.$

Apply

$\cancel{\log_8} 8^2 \quad \log_8$

$\downarrow$

$2 = \log_8 64$

(20) $10^{-3} = 0.001 \longrightarrow \log_{10}(0.001) = -3$

$\log$

(22) $\log_n n^t = \log_n 10 \longrightarrow \log_n 10 = t$

Solve the Equation If $\log_a X = \log_a Y$

$X = Y$

(34) $\log_3 3^{-5} = X \rightarrow \boxed{X = -5}$

(36) $\log_4 16 = X$; $\log_4 4^2 = X \therefore \boxed{X = 2}$