3.3 Real Zeros of Polynomial functions

Long Division. improper fraction

$$361 \div 23 = \frac{361}{23}$$

Quotient

$$\begin{array}{r} 15 \\ 23 \overline{)361} \\ -23 \\ \hline 131 \\ -115 \\ \hline 16 \end{array}$$

Divisor

Divident.

Remainder.

$$\frac{361}{23} = 15 + \frac{16}{23} = 15\tfrac{16}{23}$$

$$361 = (15)(23) + 16$$

Long Division of Polynomials.

#10 $(5x^2 - 17x - 12) \div (x - 4)$.

$$\begin{array}{r} 5x + 3 \\ x-4 \overline{\big)\, 5x^2 - 17x - 12} \\ -\,(5x^2 - 20x) \\ \hline 3x - 12 \\ -\,(3x - 12) \\ \hline \boxed{0} \end{array}$$

$$\frac{5x^2 - 17x - 12}{x-4} = (5x+3)$$

$$5x^2 - 17x - 12 = (5x+3)(x-4)$$

18 $(1 + 3x^2 + x^4) \div (3 - 2x + x^2)$.

$$\begin{array}{r} x^2 + 2x + 4 \\ x^2 - 2x + 3 \overline{\big)\, x^4 + 3x^2 + 1} \\ -\,(x^4 - 2x^3 + 3x^2) \\ \hline 2x^3 + 1 \\ -\,(2x^3 - 4x^2 + 6x) \\ \hline 4x^2 - 6x + 1 \\ -\,(4x^2 - 8x + 12) \\ \hline \boxed{2x - 11} \end{array}$$

Remainder.

$$\frac{x^4 + 3x^2 + 1}{x^2 - 2x + 3} = x^2 + 2x + 4 + \frac{2x - 11}{x^2 - 2x + 3}$$

$$(x^4 + 3x^2 + 1) = (x^2 + 2x + 4)(x^2 - 2x + 3) + (2x - 11)$$

Synthetic Division

Use $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

When you divide $x - k$

Example(s) $(3x^3 - 2x^2 + 7x - 5) \div (x - 2)$.

$$
\begin{array}{r}
3x^2 + 4x + 15 \\
x - 2 \,\overline{)\, 3x^3 - 2x^2 + 7x - 5} \\
-\;3x^3 - 6x^2 \\
\hline
4x^2 + 7x - 5 \\
-\;4x^2 - 8x \\
\hline
15x - 5 \\
-\;15x - 30 \\
\hline
\boxed{25}
\end{array}
$$

$$
\begin{array}{r|rrrr}
2 & 3 & -2 & 7 & -5 \\
 & \downarrow & 6 & 8 & 30 \\
\hline
 & 3 & 4 & 15 & \boxed{25}
\end{array}
$$

Remai

Quotient

$3x^2 + 4x + 15$

(24) $(5x^3+18x^2+7x-6)\div(x+3)$

$$\begin{array}{r|rrrr} -3 & 5 & 18 & 7 & -6 \\ & & -15 & -9 & 6 \\ \hline & 5 & 3 & -2 & \boxed{0} \end{array}$$

Quotient $Q(x) = 5x^2+3x-2$

$R(x) = 0.$

$5x^3+18x^2+7x-6 = (5x^2+3x-2)(x+3)$

Zeros -3 ; $\frac{-3 \pm \sqrt{49}}{10}$ $= (5x-2)(x+1)(x+3)$

$-3, -1, 2/5$

$$\begin{array}{r|rrr} -1 & 5 & 3 & -2 \\ & & -5 & 2 \\ \hline & 5 & -2 & \boxed{0} \end{array}$$

#32) $\dfrac{3x^3 - 4x^2 + 5}{x - \frac{3}{2}} = \left(3x^2 - \frac{1}{2}x + \frac{3}{4}\right) + \dfrac{49/8}{(x - 3/2)}$

$\frac{3}{2}$	3	-4	0	5
		$\frac{9}{2}$	$\frac{3}{4}$	$\frac{9}{8}$
	3	$\frac{1}{2}$	$\frac{3}{4}$	$\boxed{\frac{49}{8}}$

$3x^3 - 4x^2 + 5 = \left(3x^2 - \frac{1}{2}x + \frac{3}{4}\right)\left(x - \frac{3}{2}\right) + \frac{49}{8}$

Let $f(x) = 3x^3 - 4x^2 + 5 = \left(3x^2 - \frac{1}{2}x + \frac{3}{4}\right)\left(x - \frac{3}{2}\right) + \frac{49}{8}$

$f\left(\frac{3}{2}\right) = \frac{49}{8}$

The Remainder and factor theorem.

$f(x)$ polynomial. is divided by $(X-K)$.

the remainder $r = f(K)$

$$\frac{f(x)}{(X-K)} = Q(X) + \frac{R(x)}{X-K} = \frac{Q(x)}{} + \frac{r}{X-K}$$

deg R < deg (Divisor).

$$\frac{f(x)}{(X-K)} = Q(X) + \frac{r}{(X-K)}$$

$$f(x) = Q(x)(X-K) + r$$

$$f(K) = \cancel{Q(K)}\ 0 + r$$

$$\boxed{r = f(K)}$$

(44) Given $g(x) = 2x^6 + 3x^4 - x^2 + 3$.

a) $g(2) = r$ remainder $\frac{g(x)}{x-2}$ $\boxed{175}$ d) $g(-1) = 7$

b) $g(1) = 7$

c) $g(3)$

a)

$$\begin{array}{r|rrrrrrr} 2 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 4 & 8 & 22 & 44 & 86 & 172 \\ \hline & 2 & 4 & 11 & 22 & 43 & 86 & \boxed{175} \end{array}$$

c) ~~b)~~

$$\begin{array}{r|rrrrrrr} 3 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 6 & 18 & 63 & 189 & \cancel{264} & 1692 \\ \hline & 2 & 6 & 21 & 63 & 188 & \cancel{284} & \boxed{594} \\ & & & & & & \cancel{264} & \boxed{1695} \\ & & & & & & 564 & \end{array}$$

The factor theorem

f(x) is a polynomial and has (X-K) as a factor.

$$f(x) = Q(x)(X-K)$$

$\boxed{r=0}$

(X-K) is a factor of f(x) if and only if.

$$f(K) = r = 0$$

#53 Verify that $(X-5)$ and $(X+4)$ are factor of $f(x) = X^4 - 4X^3 - 15X^2 + 58X - 40$

$f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline -4 & 1 & 1 & -10 & 8 & \boxed{0} \\ & & -4 & 12 & -8 & \\ \hline & 1 & -3 & 2 & \boxed{0} & \end{array}$$

$(X-5)$ is a factor

$f(x) = (x^3 + x^2 - 10x + 8)(X-5)$

$f(x) = (x^2 - 3x + 2)(X-5)(X+4)$.

$(x-1)(x-2)(x-5)(x+4)$

Solve $f(x) = 0$ ∴ $\boxed{X = 5, -4, 1, 2}$

56 $f(x) = 2x^3 - x^2 - 10x + 5$; $(2x-1)$

$2(x - \frac{1}{2})$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & \boxed{0} \end{array}$$

$f(x) = (2x^2 - 10)(2x-1)$

$f(x) = 0$ ∴ $x = \frac{1}{2}, \pm\sqrt{5}$

$$\begin{array}{r|rrrr} \sqrt{5} & 2 & -1 & -10 & 5 \\ & & 2\sqrt{5} & 10-\sqrt{5} & -5 \\ \hline & 2 & 2\sqrt{5}-1 & -\sqrt{5} & \boxed{0} \end{array}$$ ✓

The Rational Zero test

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

a_n leading coefficient

a_0 constant

a_i integers.

$0 \le i \le n$

Every Rational Zero of f has the form p/q

p are factors of a_0.

q are factors of a_n.

$f(x) = 2x^3 + 3x^2 - 8x + 3$.

p = factor of 3 $\quad p = \pm 1, \pm 3$

q : factor of 2 $\quad q = \pm 1, \pm 2$

possible Rational zeros of $f(x)$ are

$p/q = \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$

$1, -3, \frac{1}{2}$

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & & 2 & 5 & -3 \\ \hline & 2 & 5 & -3 & \boxed{0} \end{array} \checkmark$$

$$f(x) = (x-1)(2x^2+5x-3).$$

$$x = \frac{-5 \pm \sqrt{25+24}}{4} = \frac{-5 \pm 7}{4} \begin{cases} \boxed{-3} \\ \boxed{\tfrac{1}{2}} \end{cases}$$

$$g(x) = 8x^4 - 7x^3 + 2x^2 + 7x - 6$$

Give all the possible Rational Roots.

$p = \pm 1, \pm 2, \pm 3, \pm 6$

$q = \pm 1, \pm 2, \pm 4, \pm 8$

$$p/q = \pm 1, \pm 2, \pm 3, \pm 6, \pm \tfrac{1}{2}, \pm \tfrac{3}{2}, \pm \tfrac{1}{4}, \pm \tfrac{3}{4}, \pm \tfrac{1}{8}, \pm \tfrac{3}{8}$$

<u>TST 2. up 3.3.</u>