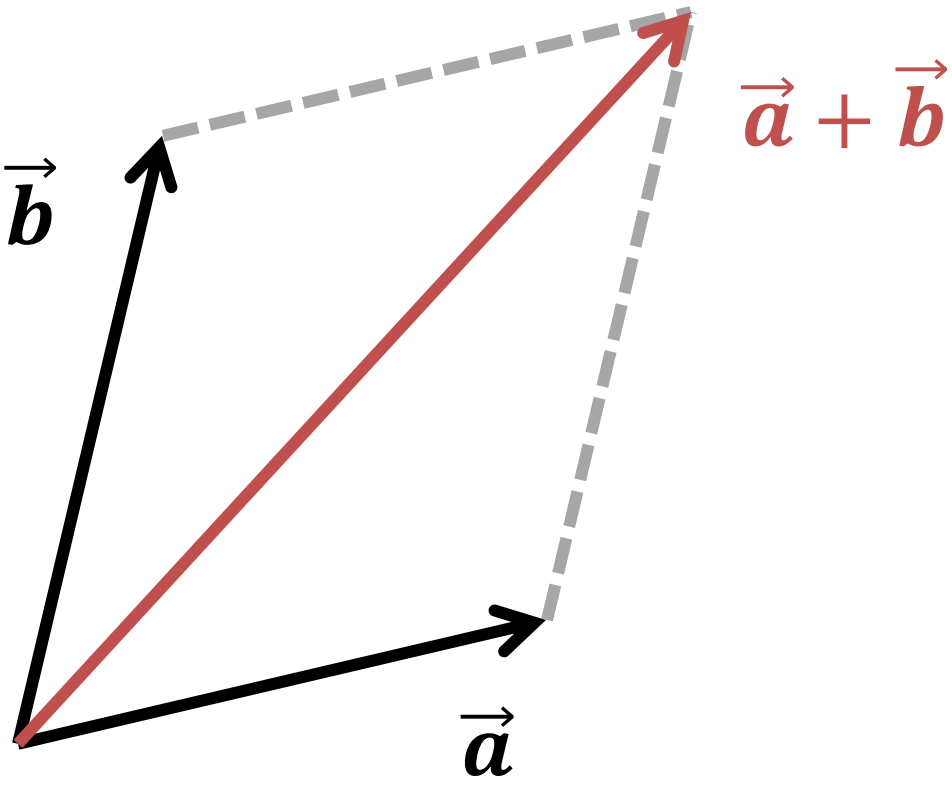
Points, Vectors, and Matrices

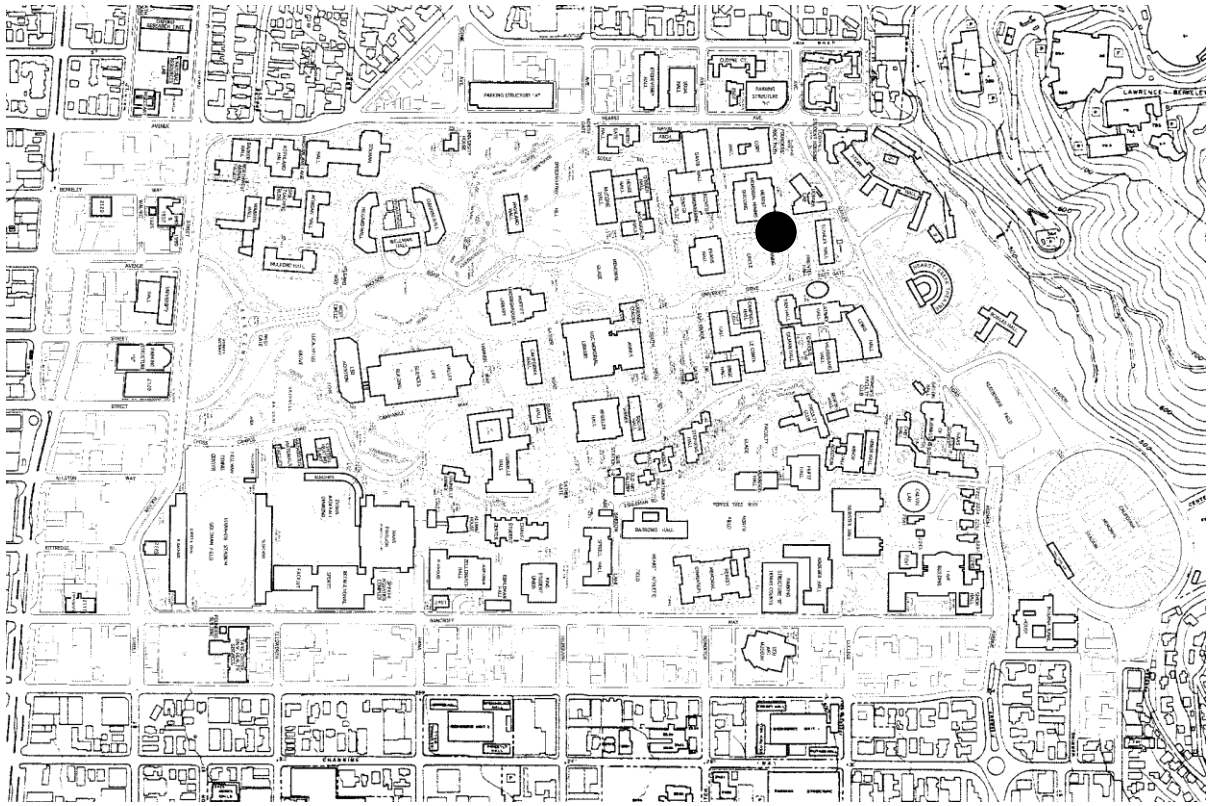


Point

- A location in space

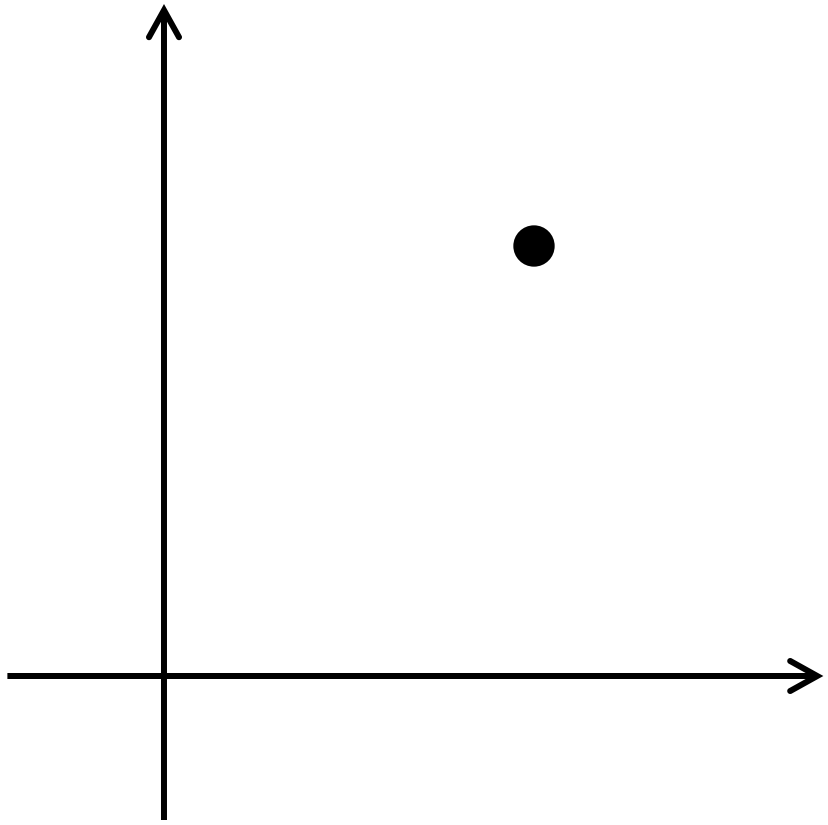
Point

- Can locate a meaningful spot



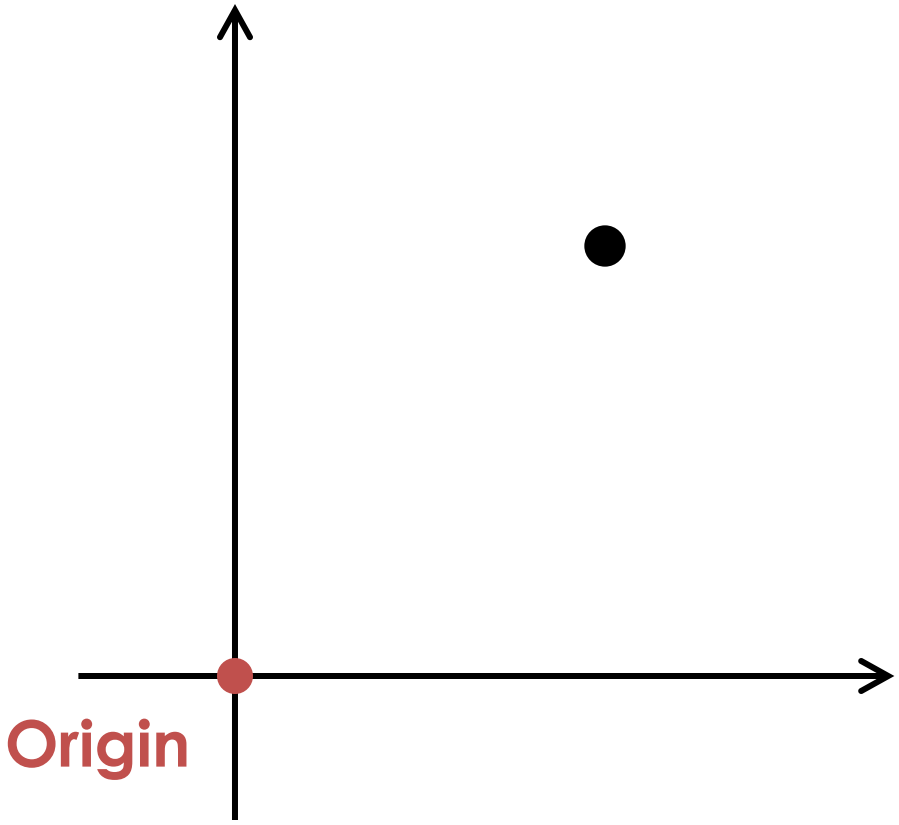
Point

- Usually embedded in a coordinate system



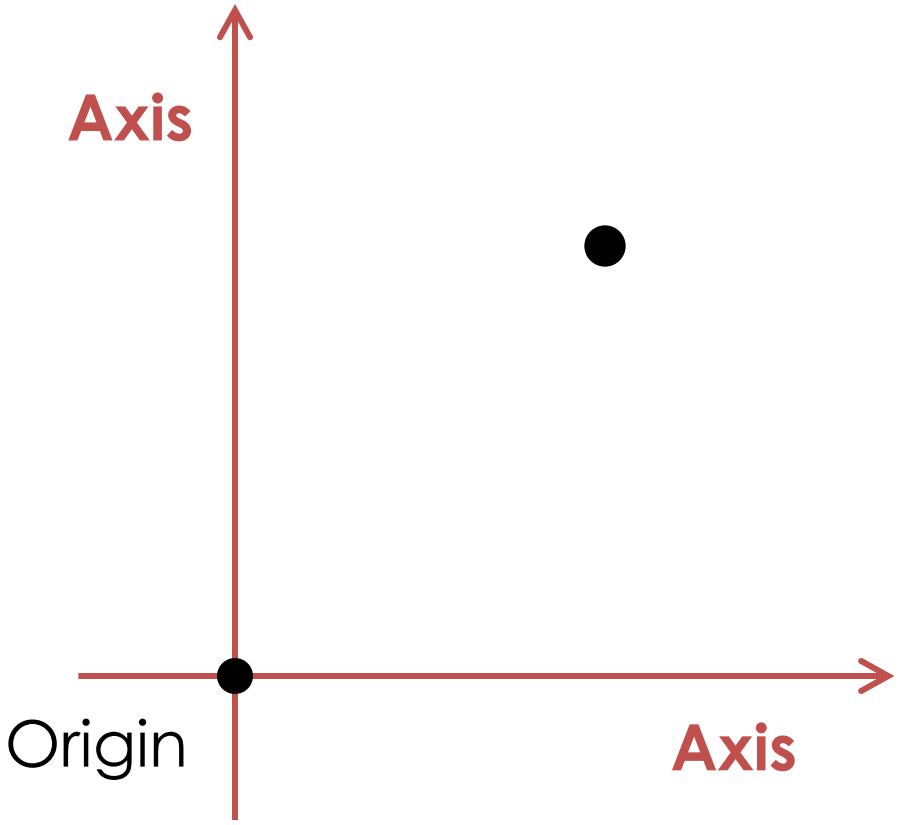
Point

- Usually embedded in a coordinate system



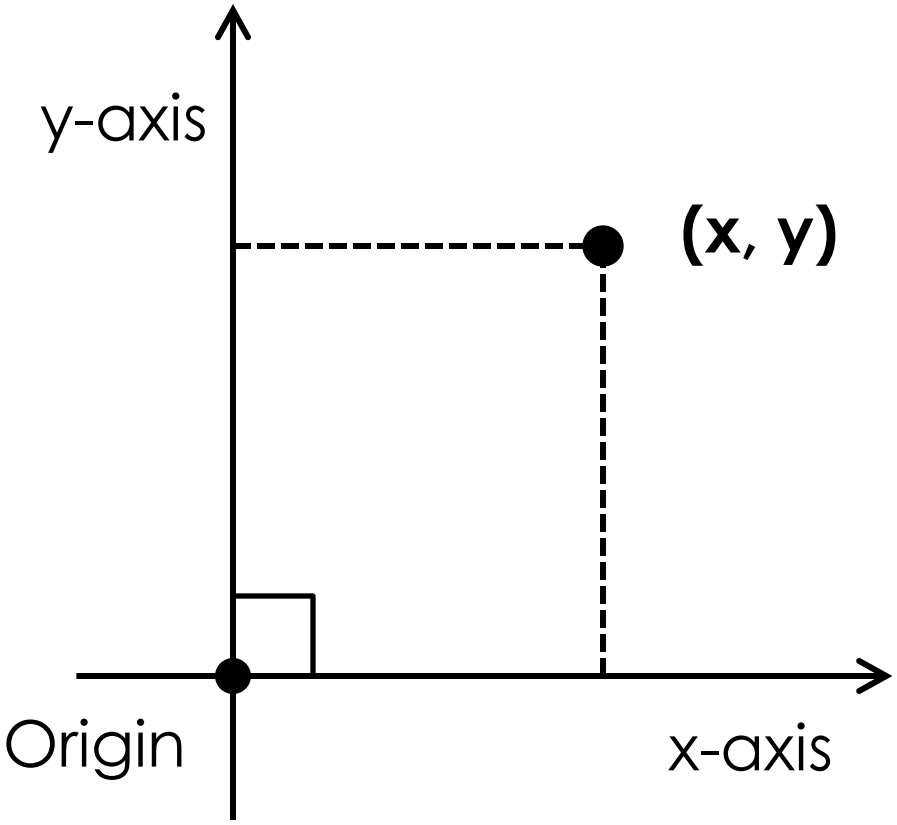
Point

- Usually embedded in a coordinate system



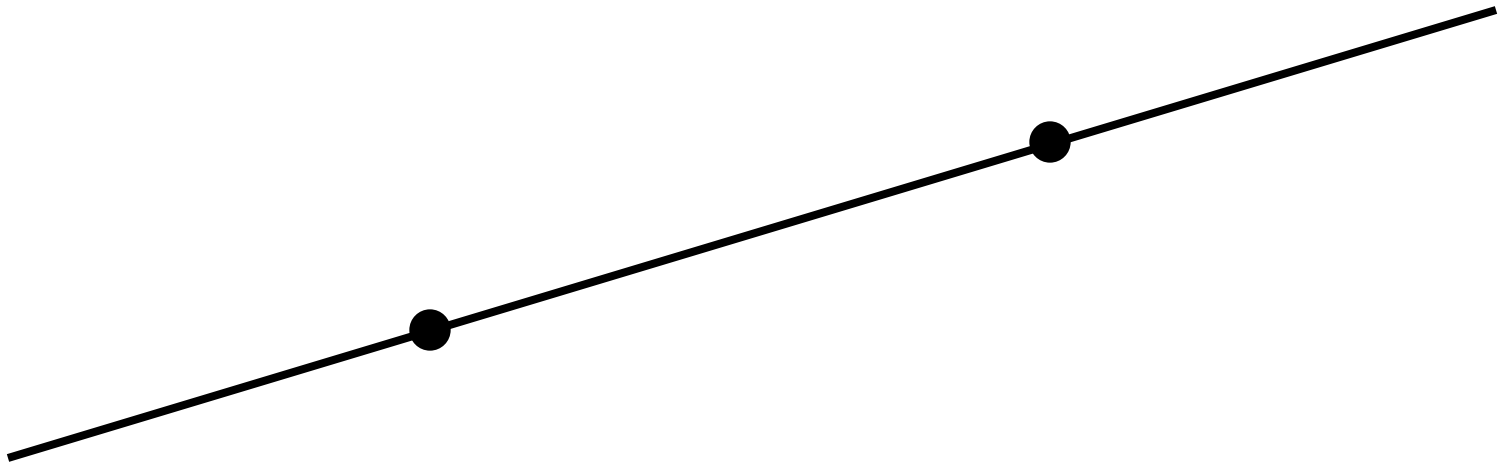
Point

- Cartesian coordinate system



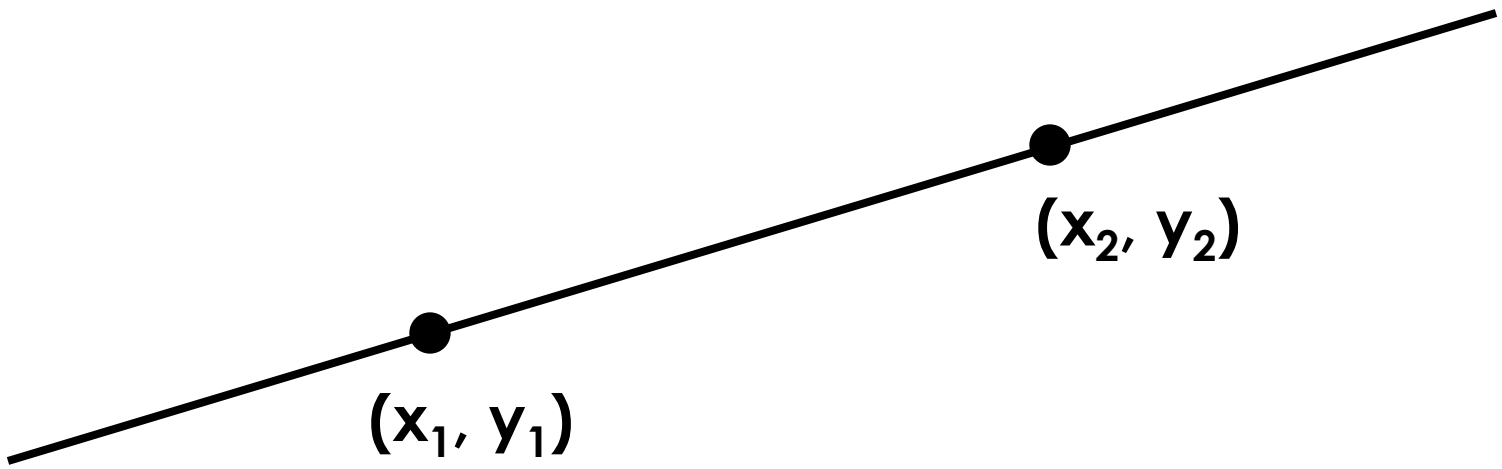
Line

- Any two distant points uniquely determine a line



Line

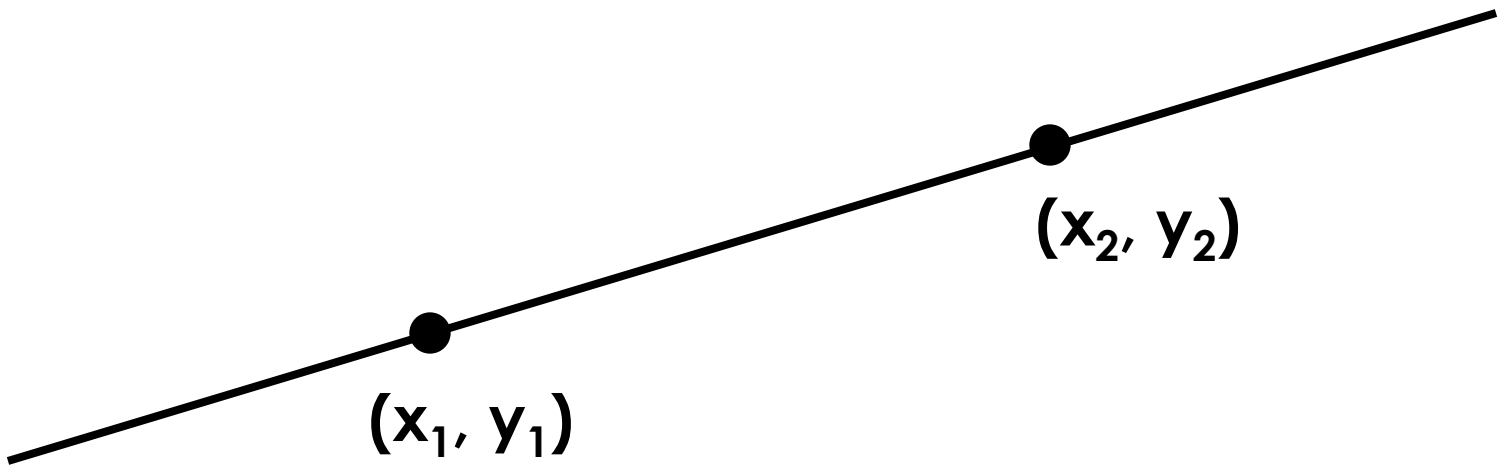
- Explicit equation: y=f(x)



Line

- Explicit equation: y=f(x)

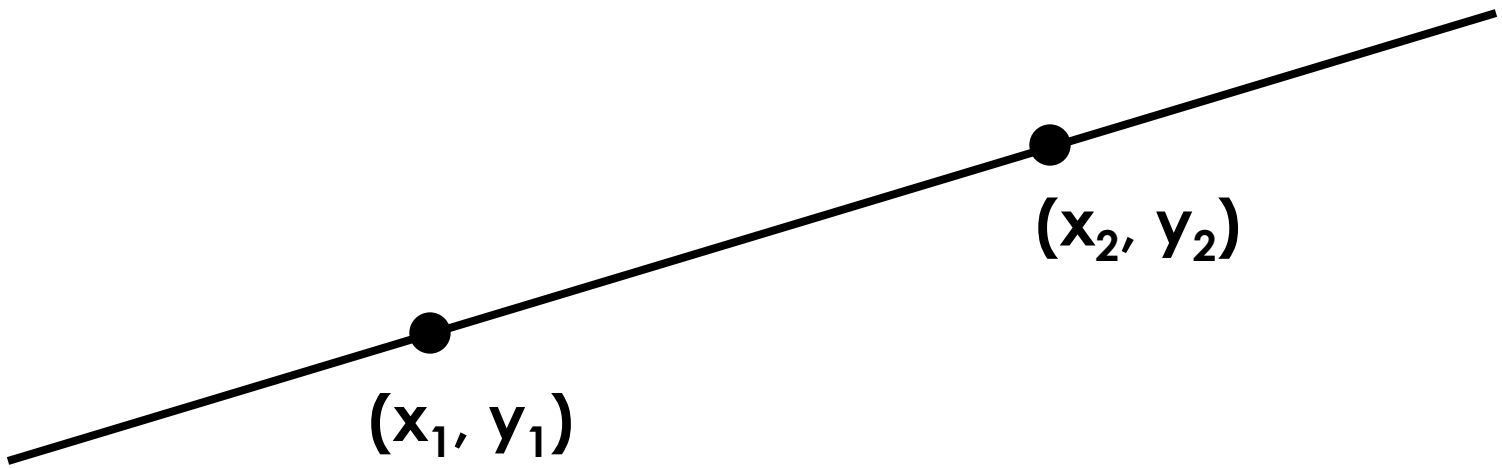
Ex) A line passing through (-2, 4) and (1, -2)



Line

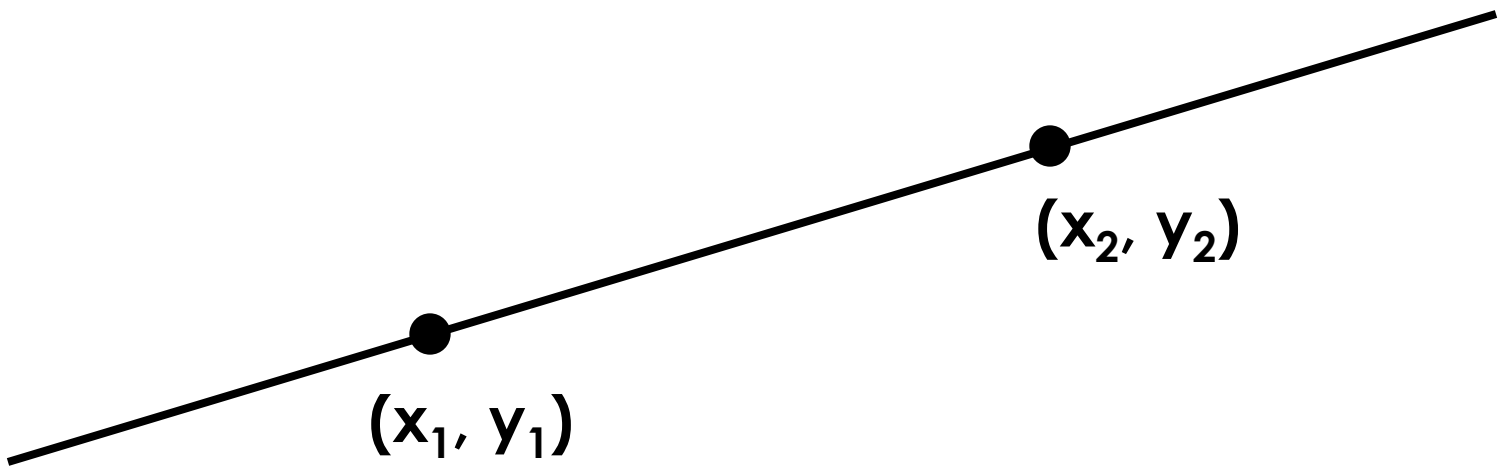
- Implicit equation: f(x,y)=0

Ex) A line passing through (-2, 4) and (1, -2)



Line

- Parametric equation: x=f(t), y=g(t)

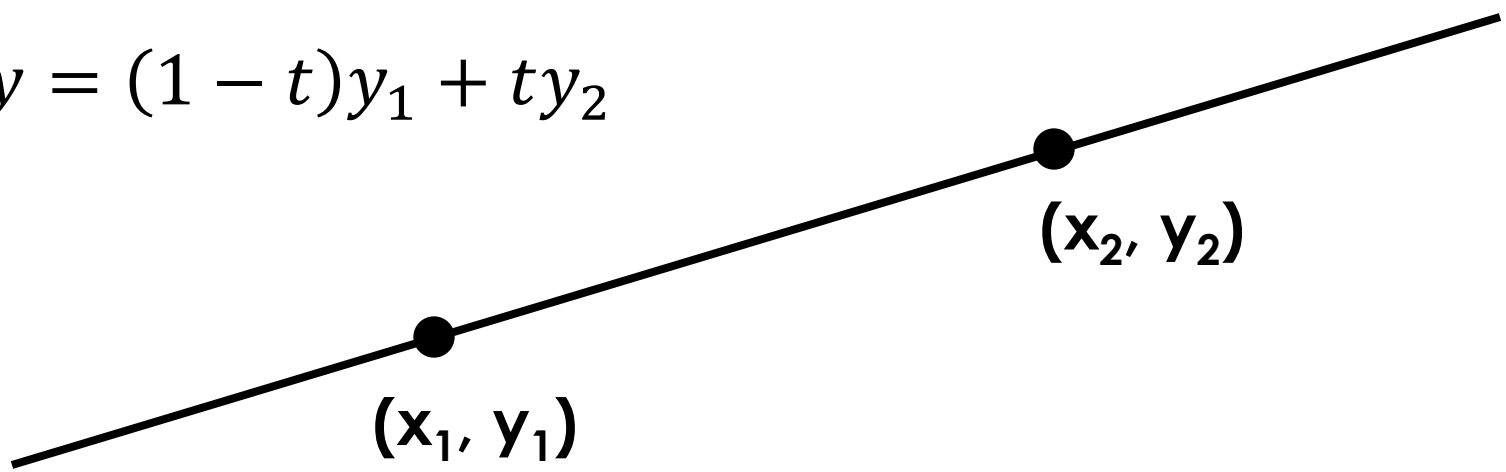


Line

- Parametric equation: x=f(t), y=g(t)

$$x = (1-t)x_1 + tx_2 \qquad t \in (-\infty, +\infty)$$

$$y = (1-t)y_1 + ty_2$$

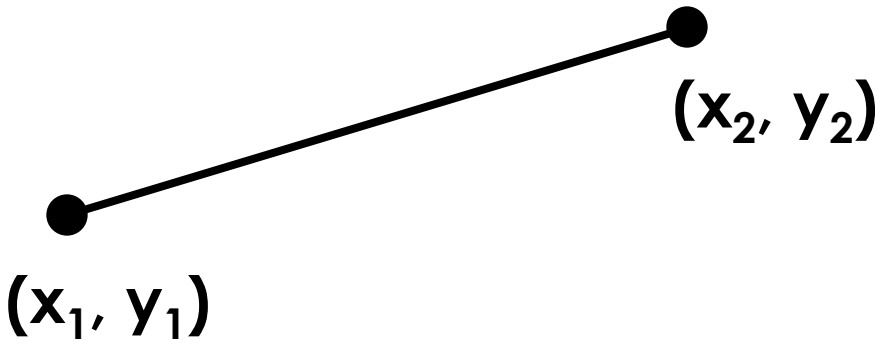


Line Segment

- Parametric equation: x=f(t), y=g(t)

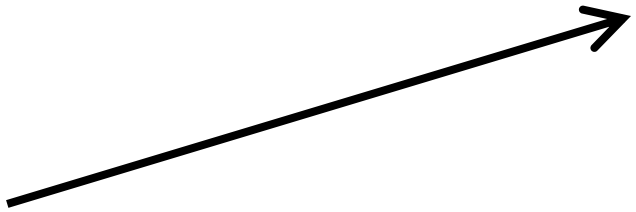
$$x = (1-t)x_1 + tx_2 \qquad t \in [0,1]$$

$$y = (1-t)y_1 + ty_2$$



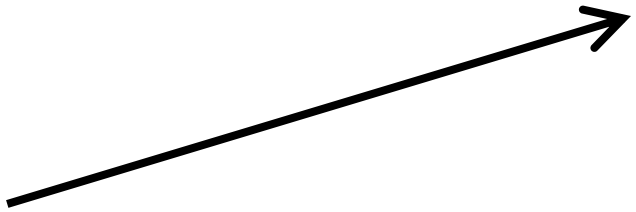
Vector

- Is similar to line segment?



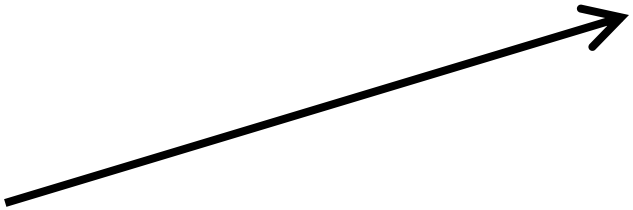
Vector

- Just a direction and a magnitude



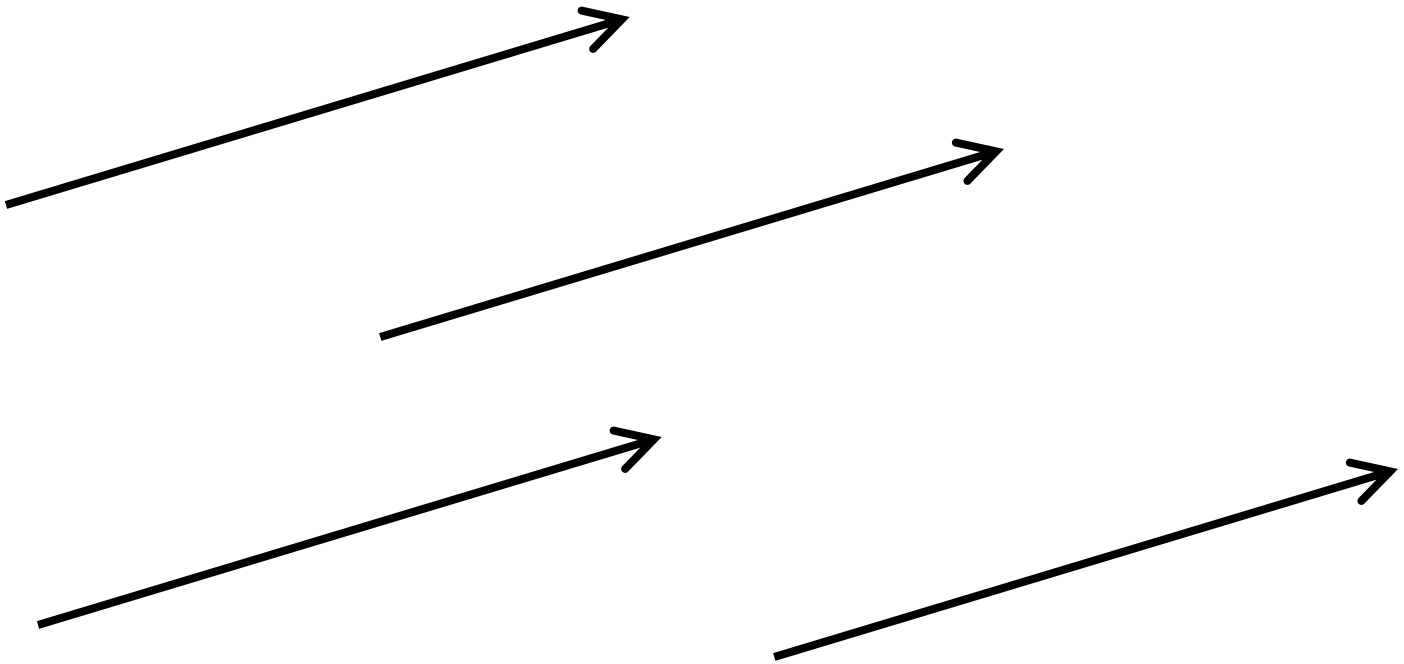
Vector

- Just a **direction** and a **magnitude**
- No information on the ~~*location*~~



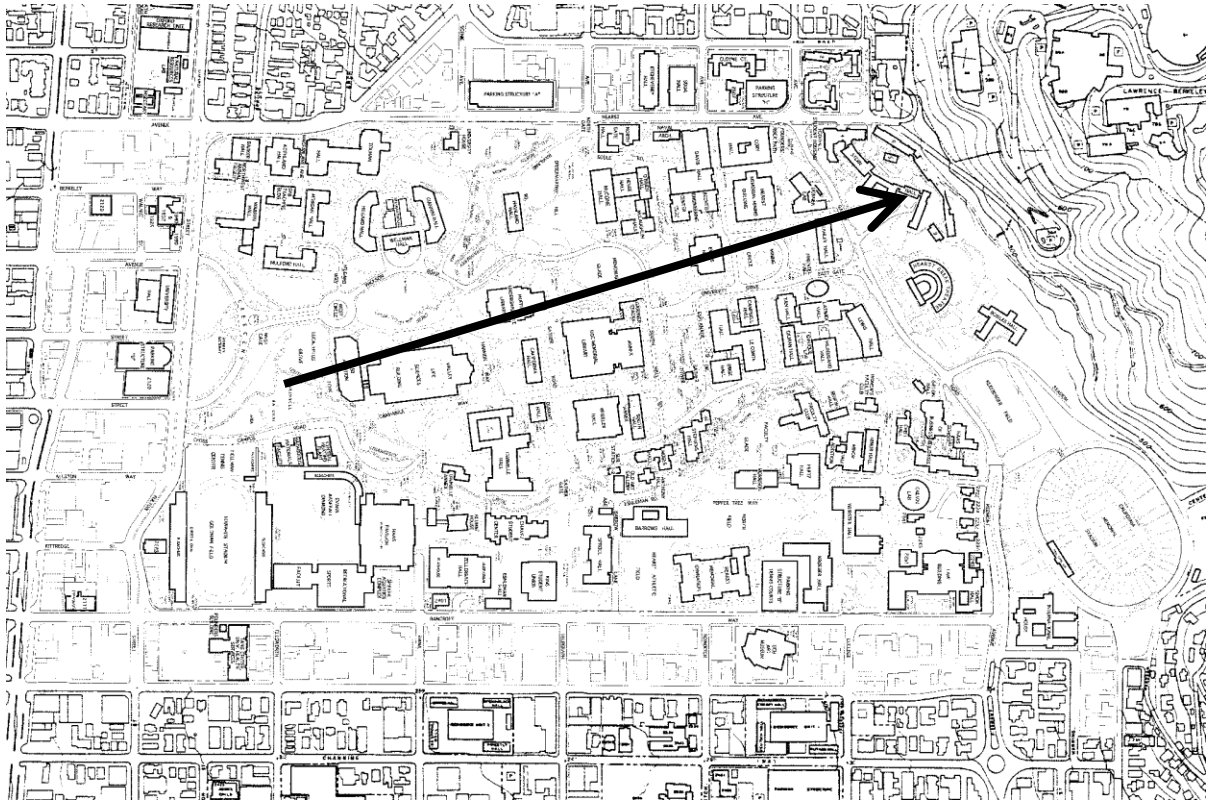
Vector

- Are these the same or different?



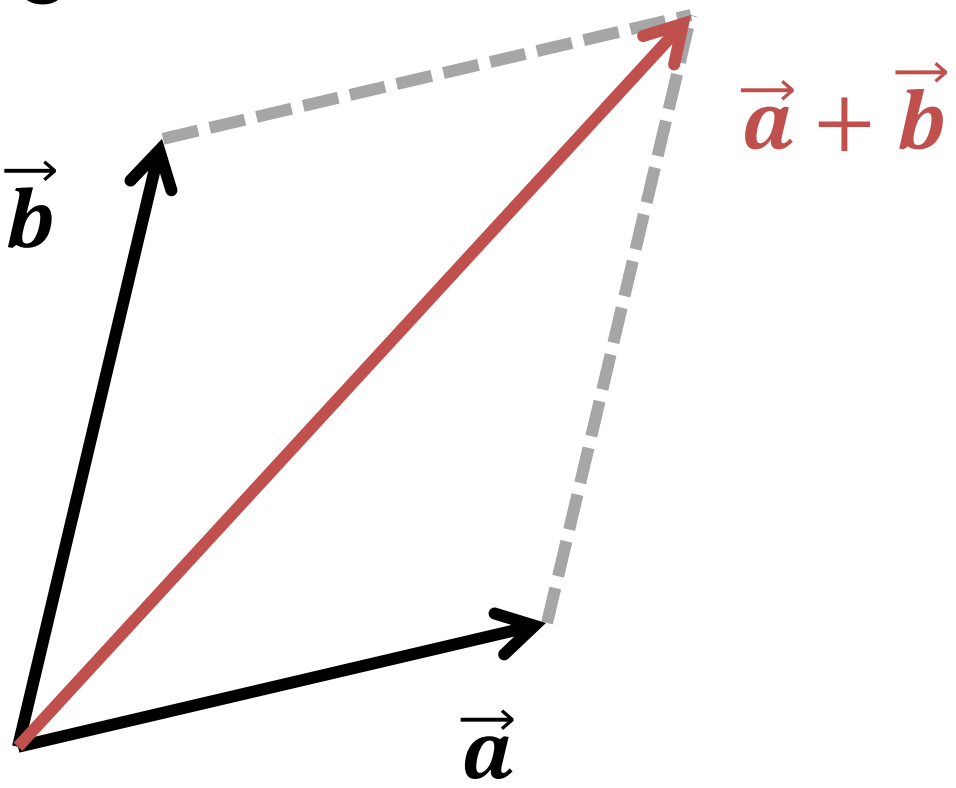
Vector

- Can represent the difference between two locations



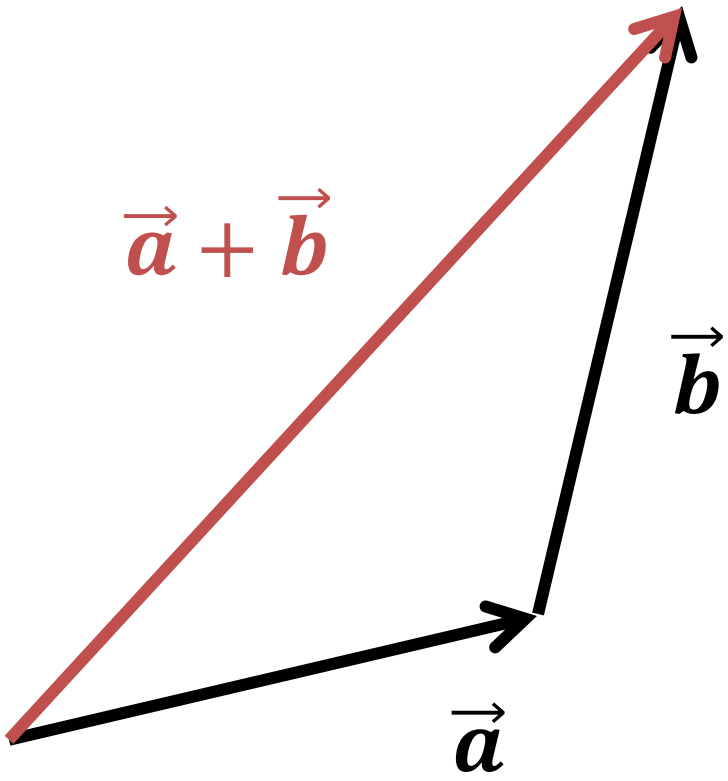
Vector Addition

- Any two vectors can be added to yield a single vector



Vector Addition

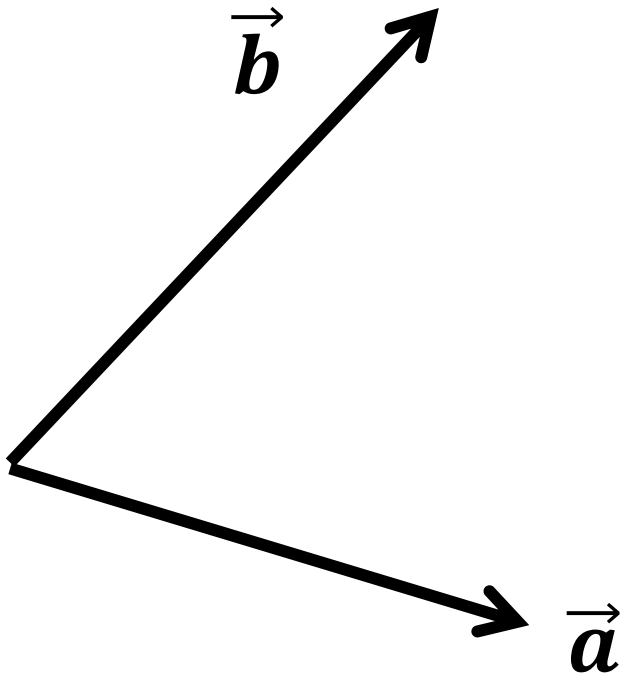
- Place two vectors head to tail, and draw a vector from free tail to free head



Vector Subtraction

- Subtraction

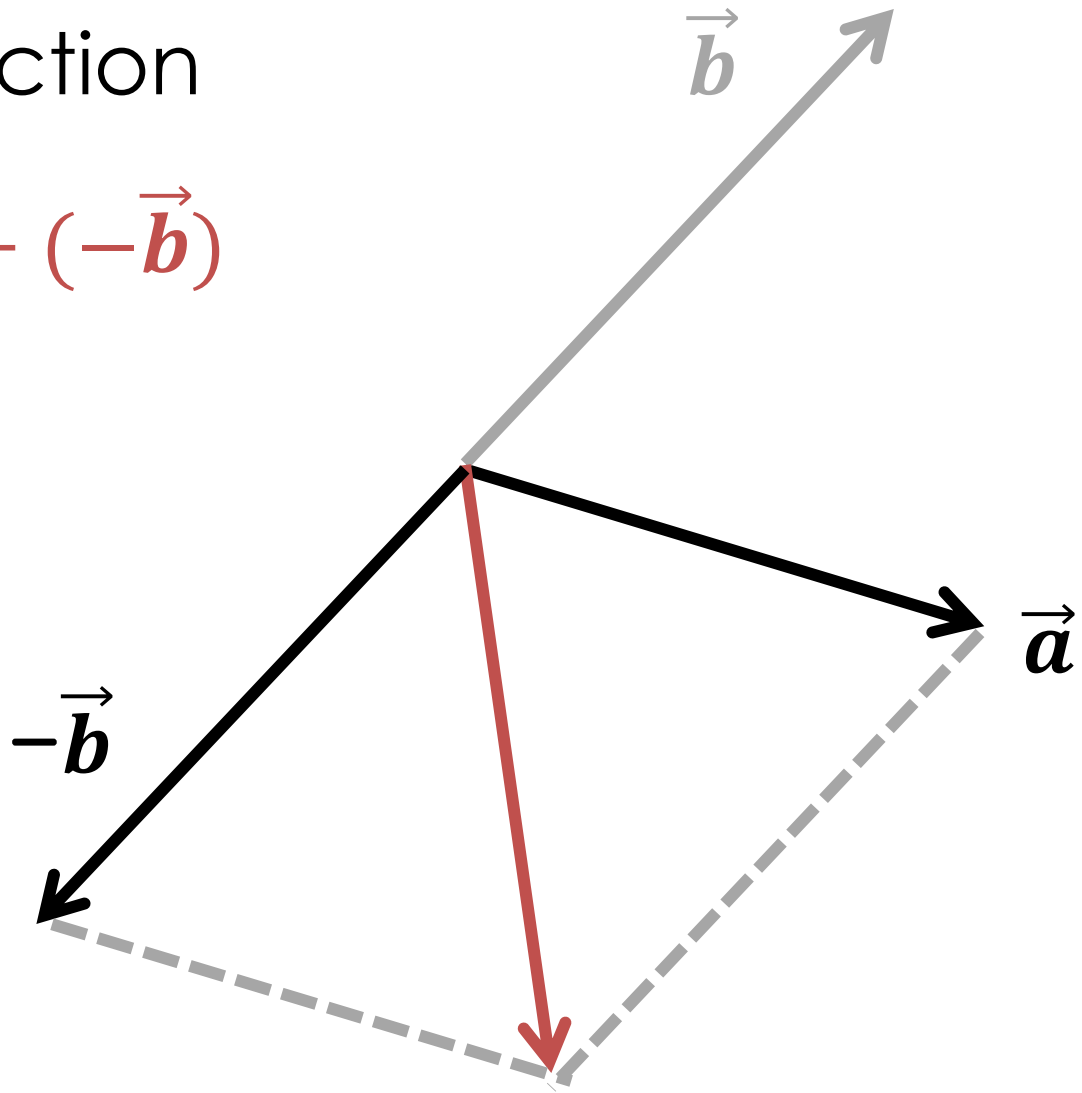
$\vec{a} - \vec{b}$



Vector Subtraction

- Subtraction

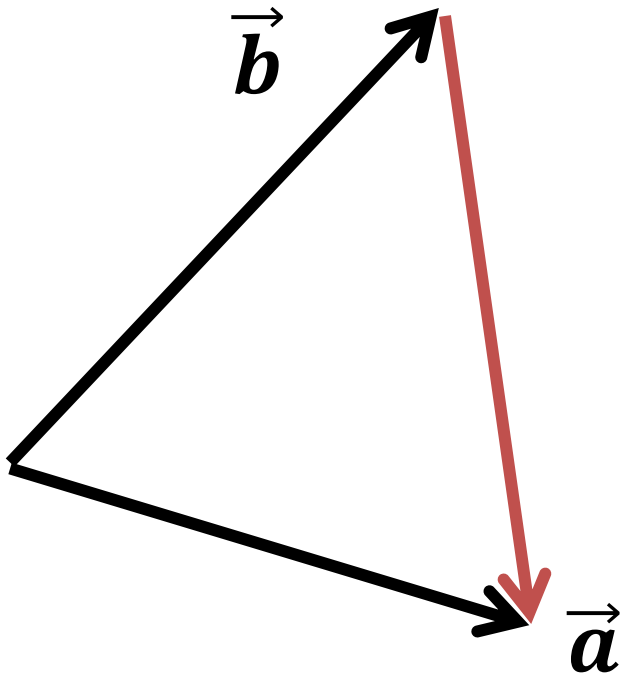
$$= \vec{a} + (-\vec{b})$$



Vector Subtraction

- Subtraction

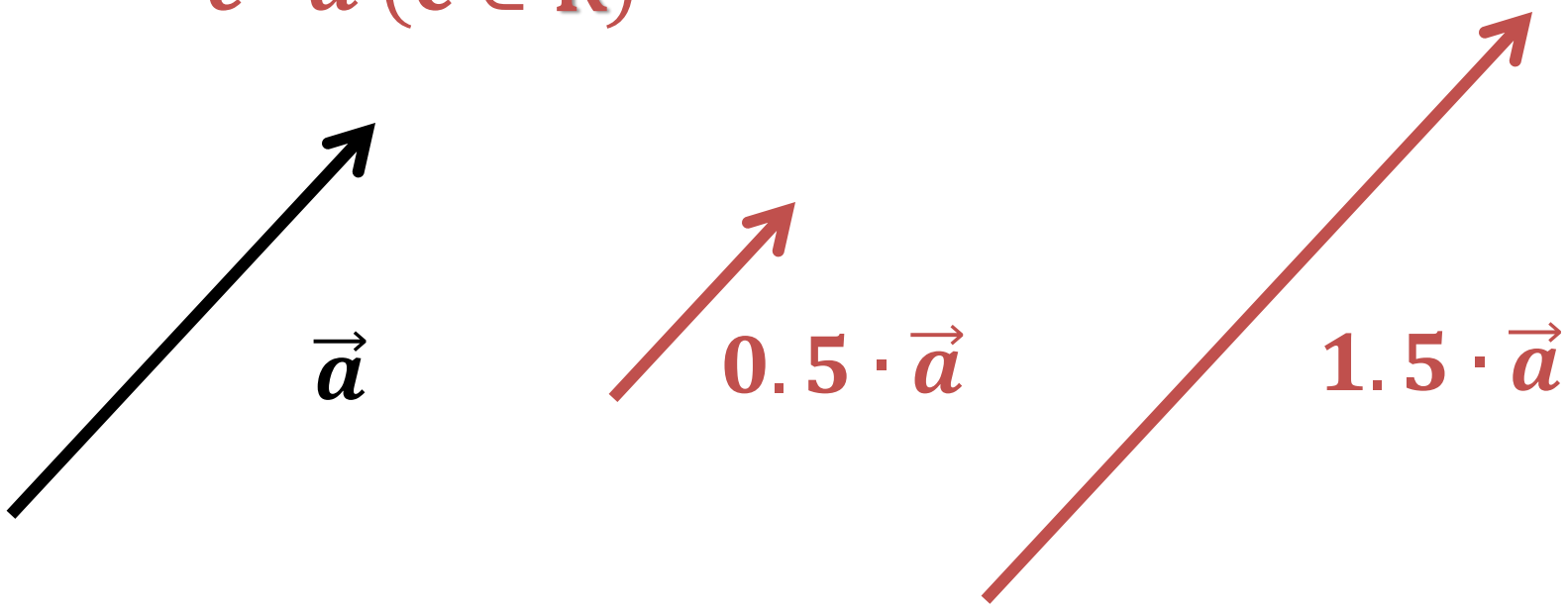
$\vec{a} - \vec{b}$



Scalar Multiplication

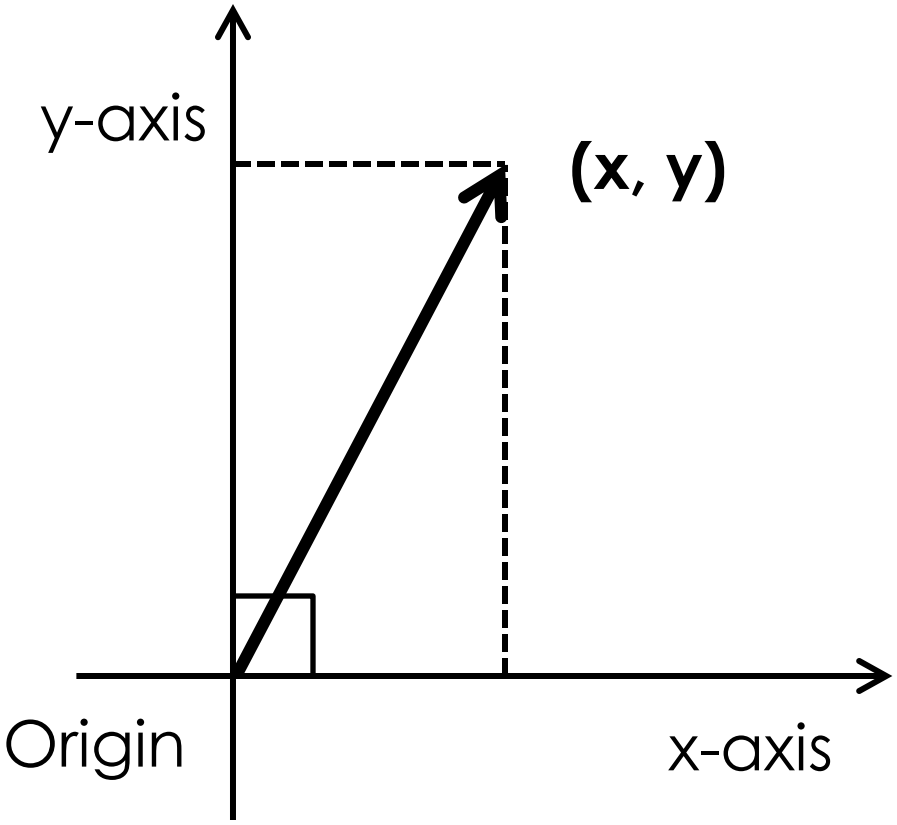
- Modifying length only, without changing direction

$c \cdot \vec{a}\ (\mathbf{c} \in \mathbf{R})$



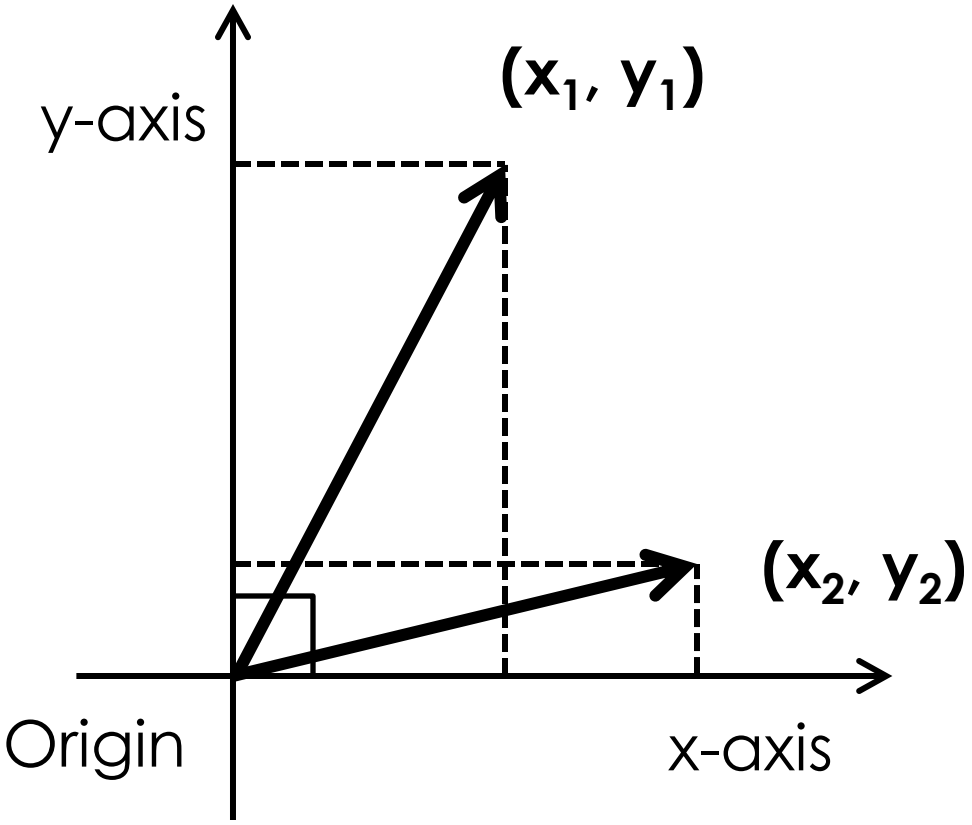
Cartesian Coordinate System

- Can be represented as coordinates



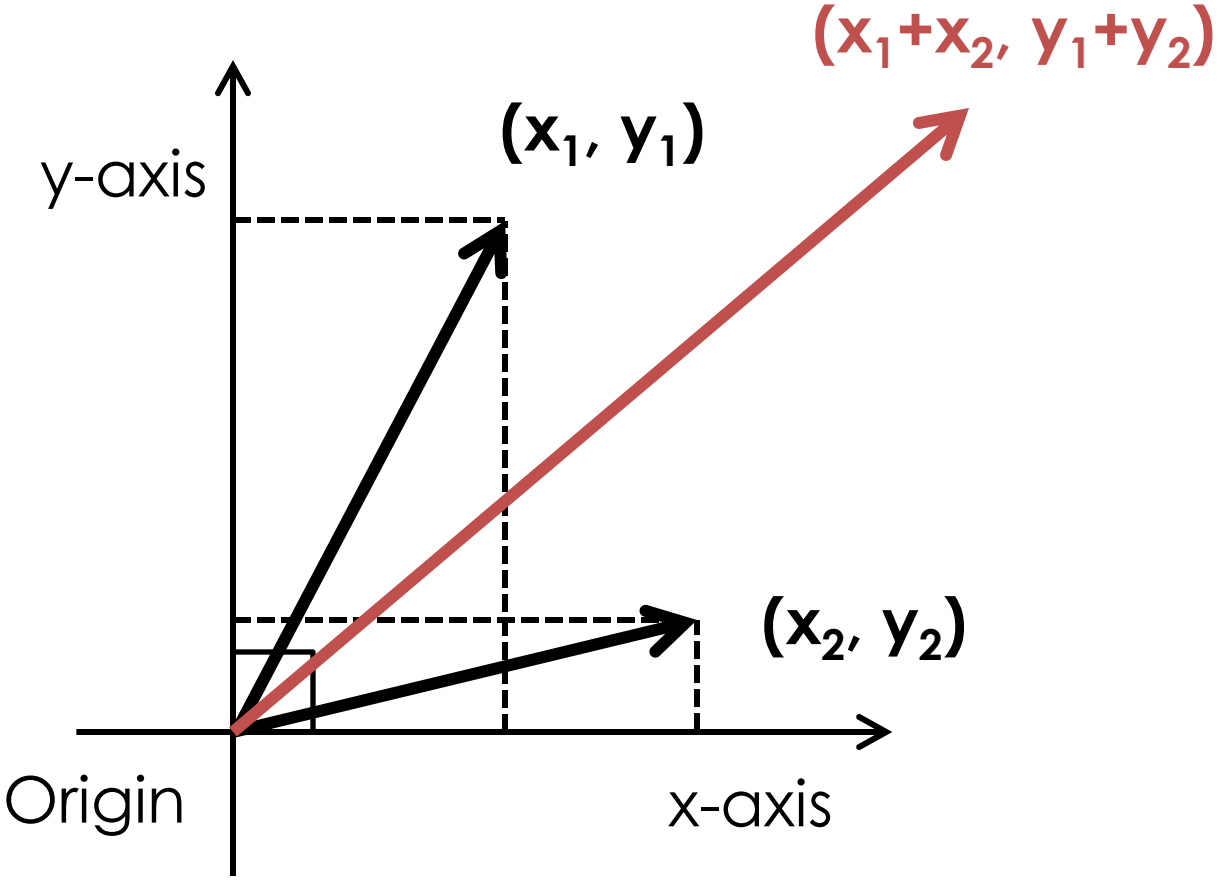
Numeric Operations

- Vector addition and subtraction?



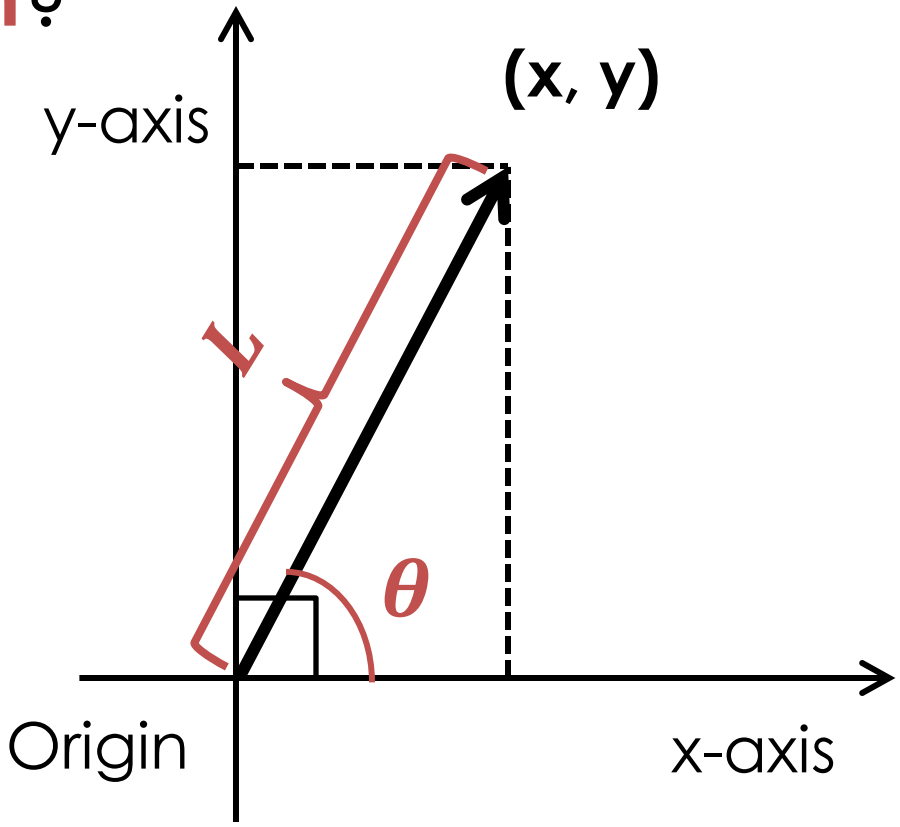
Numeric Operations

- Component-wise add/sub



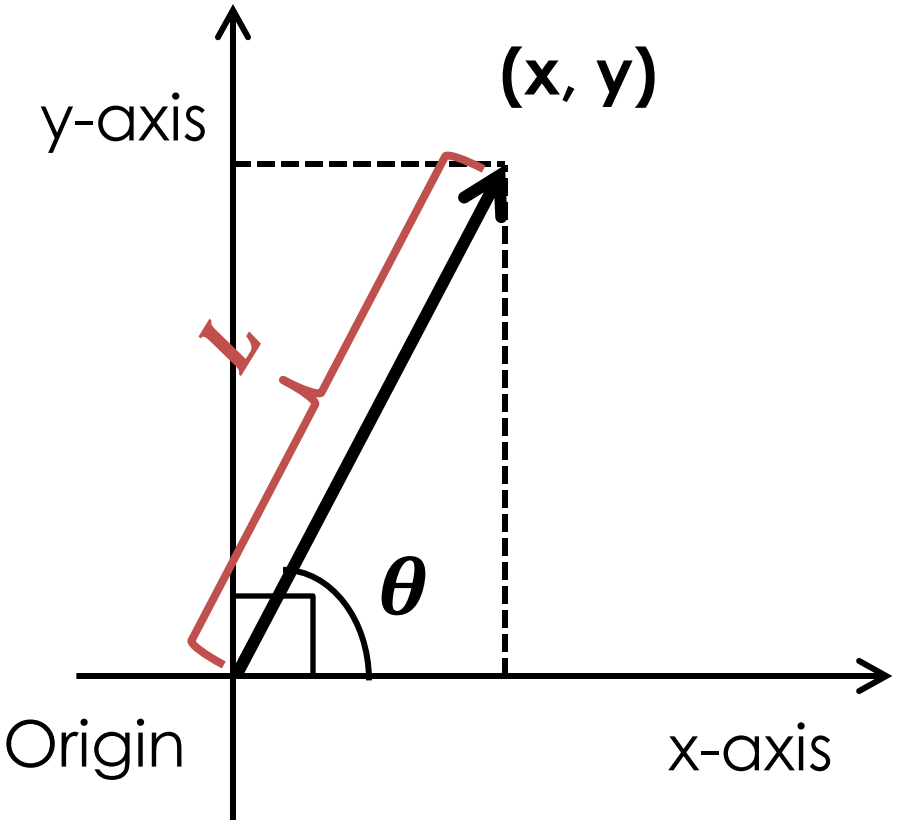
Length and Direction

- Given a vector, calc its **length** and **direction**?



Length by Pythagorean Theorem

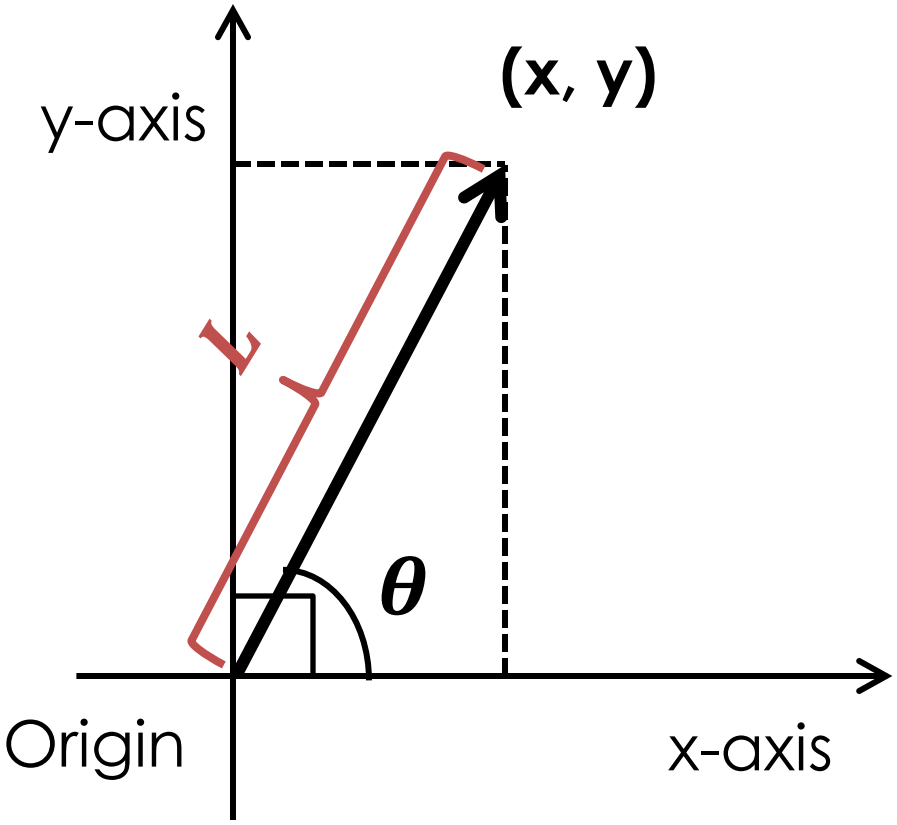
- $x^2 + y^2 = L^2$



Length by Pythagorean Theorem

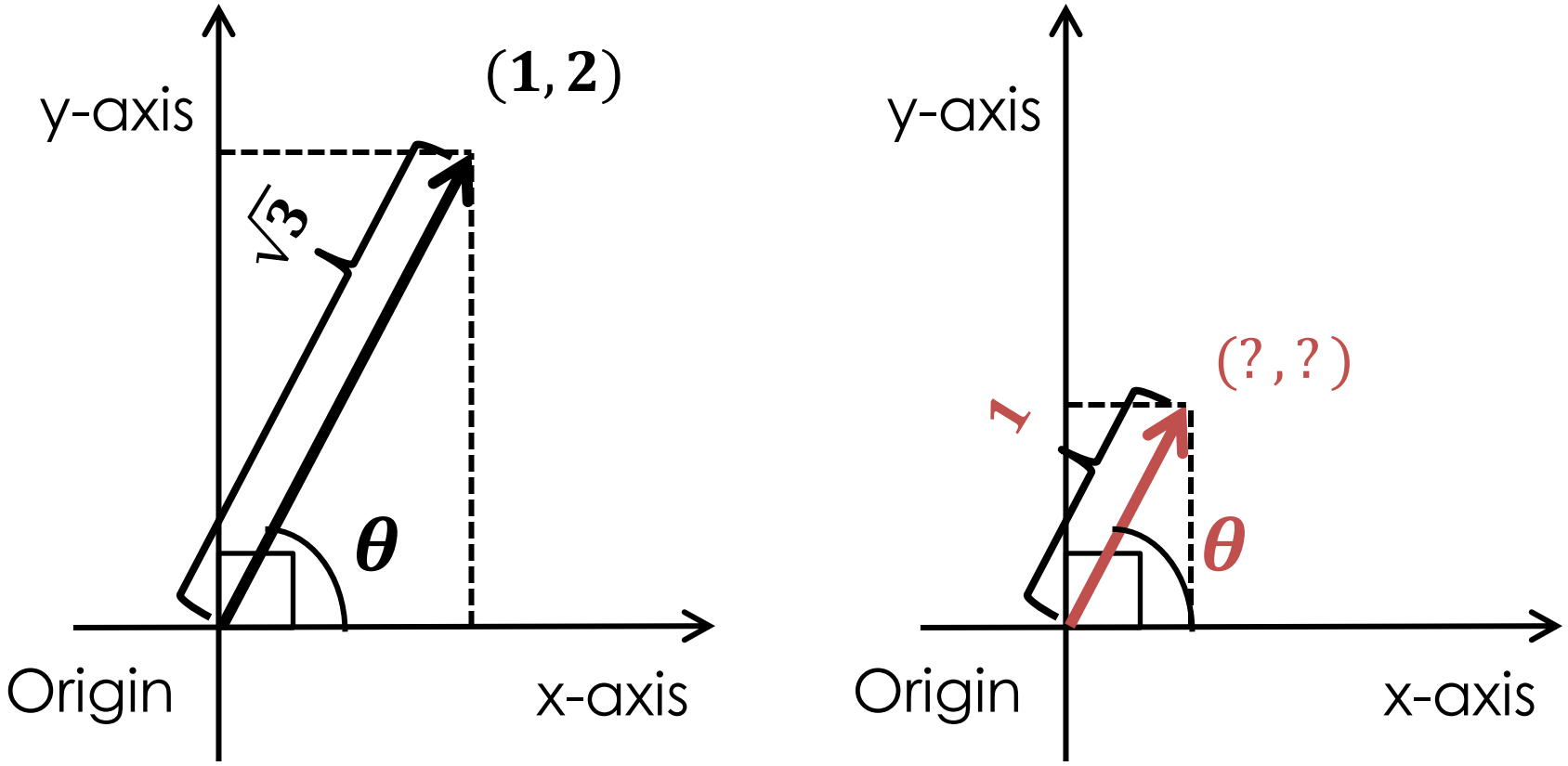
- $x^2 + y^2 = L^2$

Ex) length of $\vec{v} = (1, 2)$?



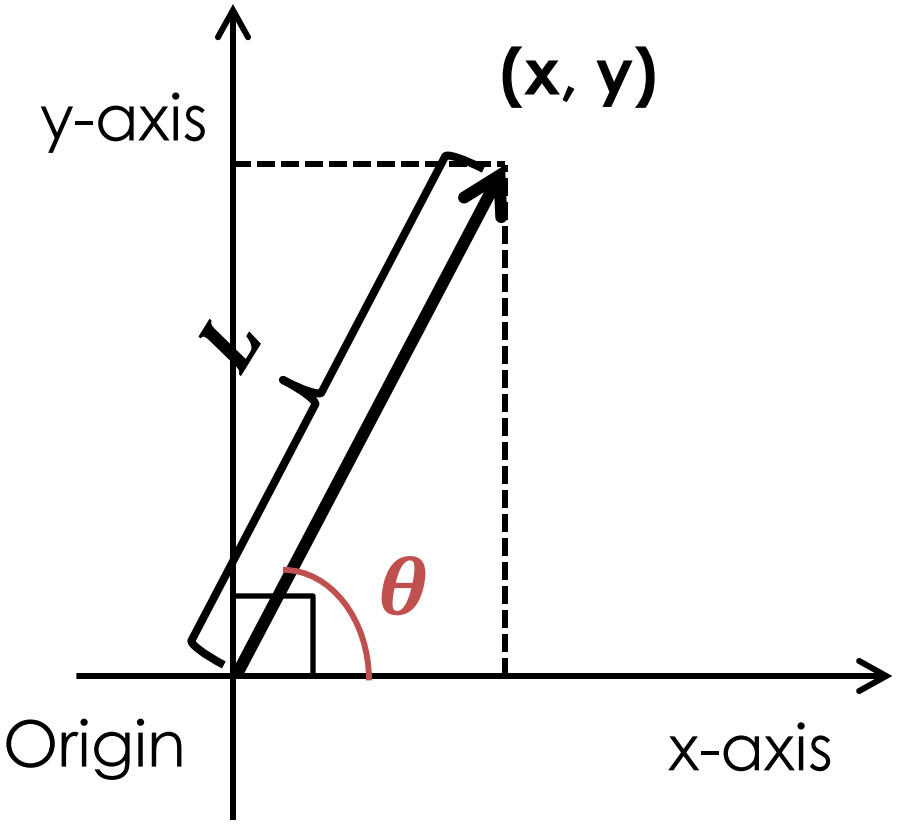
Normalization into Unit Vector

- Scale a vector such that its length becomes one while keeping its direction



Direction by Trigonometry

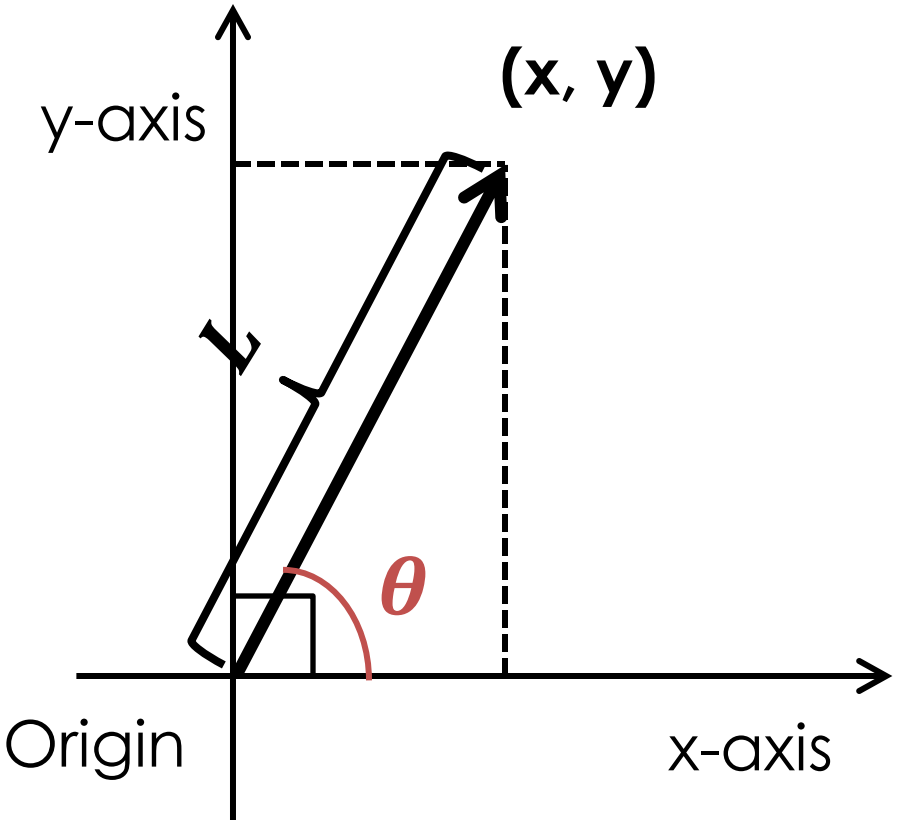
- $\tan\theta = \frac{y}{x}$



Direction by Trigonometry

- $\tan\theta = \frac{y}{x}$

Ex) length and angle of $\vec{v} = (1, \sqrt{3})$?

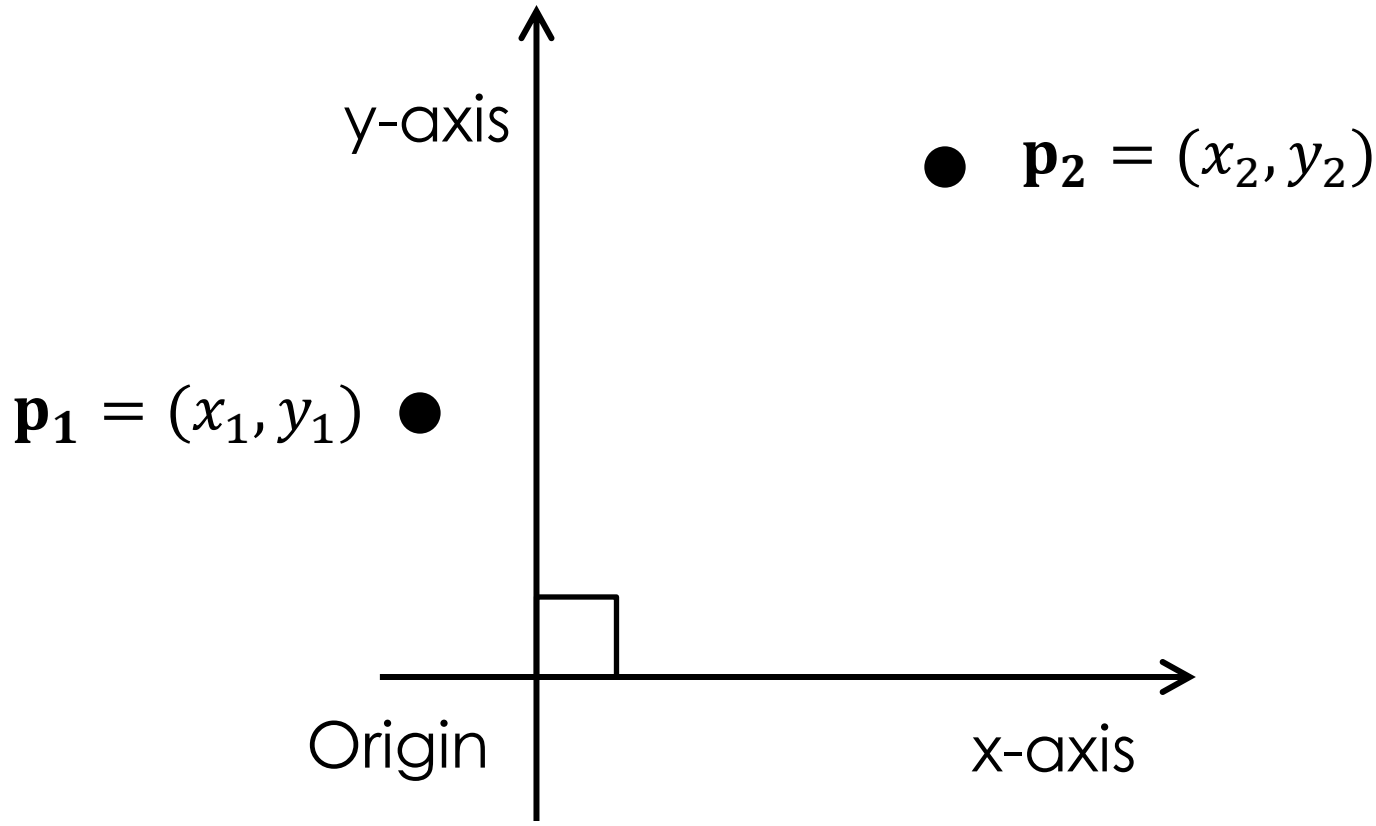


Revisiting Operations on Points

- Points can be added?
- Points can be subtracted?
- Points can be multiplied with scalars?

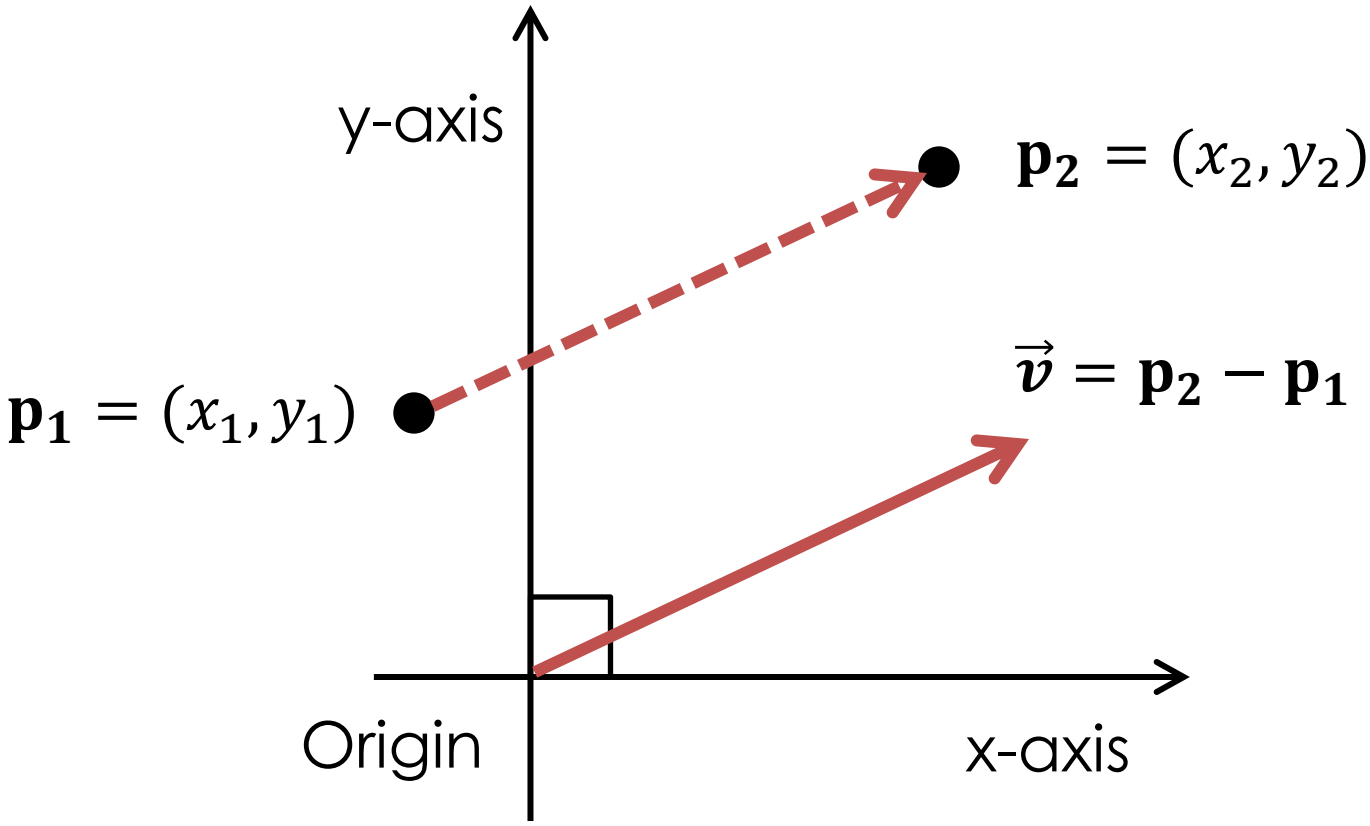
Point – Point = Vector

- **Relative location** of one point with respect to another point



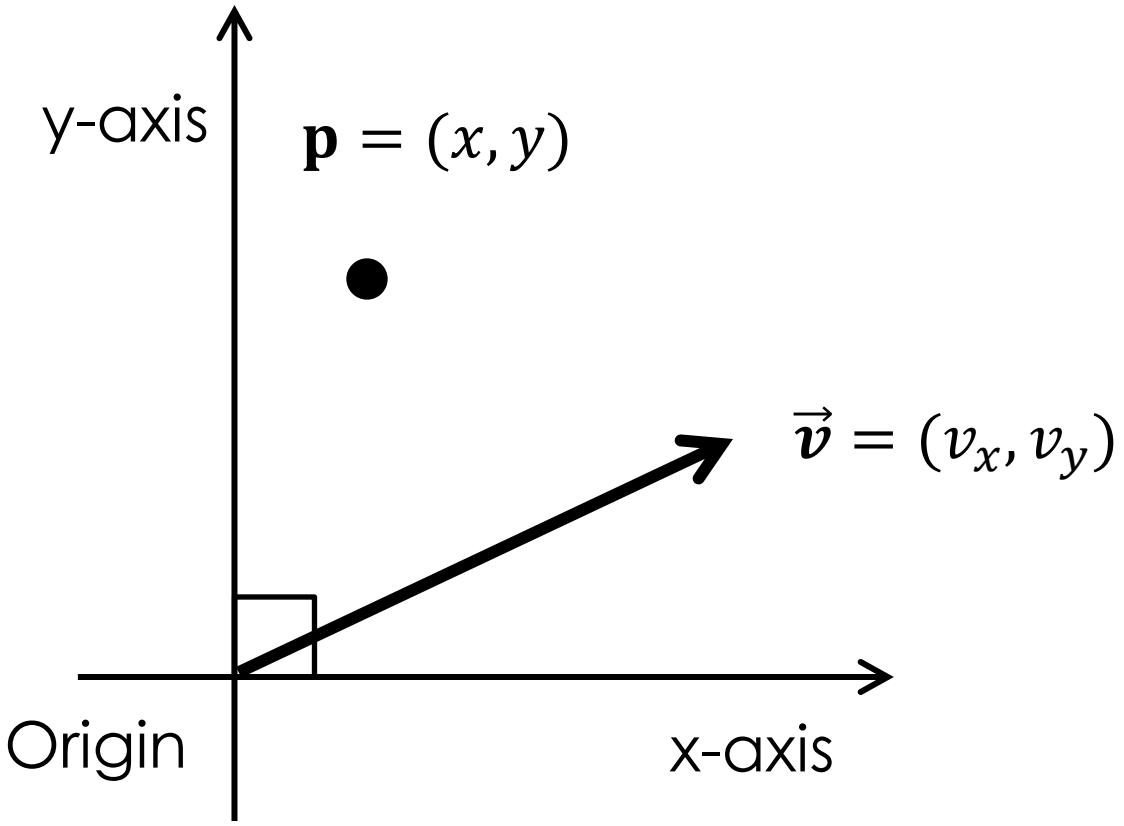
Point – Point = Vector

- **Relative location** of one point with respect to another point



Point + Vector = Point

- **Translate** a point toward a direction by a magnitude



Point + Vector = Point

- **Translate** a point toward a direction by a magnitude