Tour de Distributions!

ATL-DS-0624

Goals

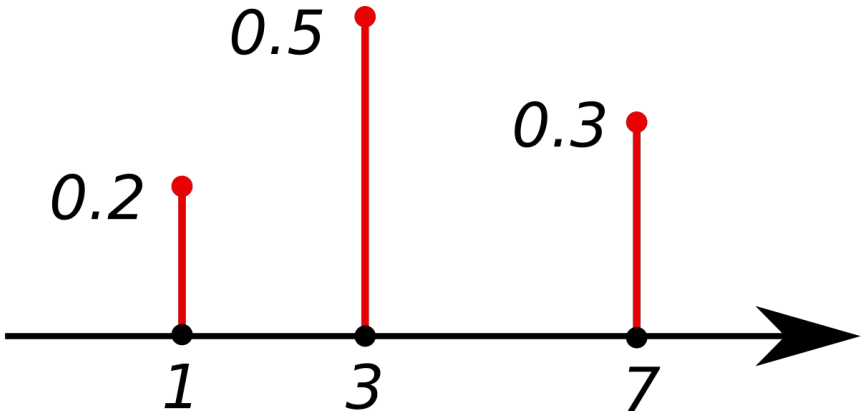
- Understand the difference between PMF and PDF.
- CDF for discrete and continuous space.
- How to Calculate and Interpret Z-Score.

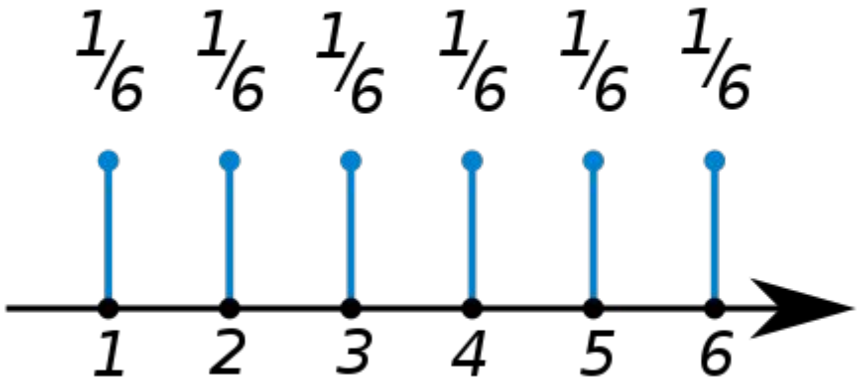
Statistical Distribution

The distribution of a variable is a description of the relative numbers of times each possible outcome will occur in a number of trials.

Probability Mass Function

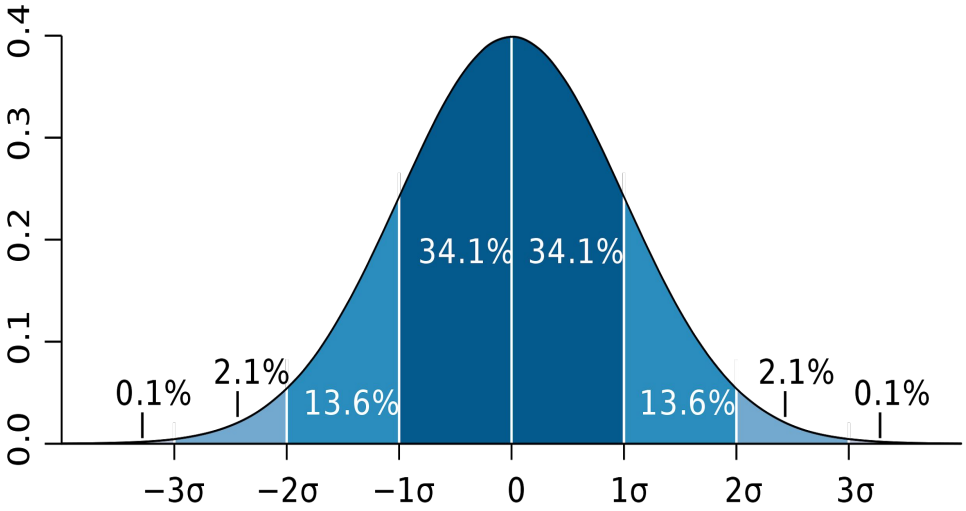
- a function that gives the probability that a **discrete random variable** is exactly equal to some value.

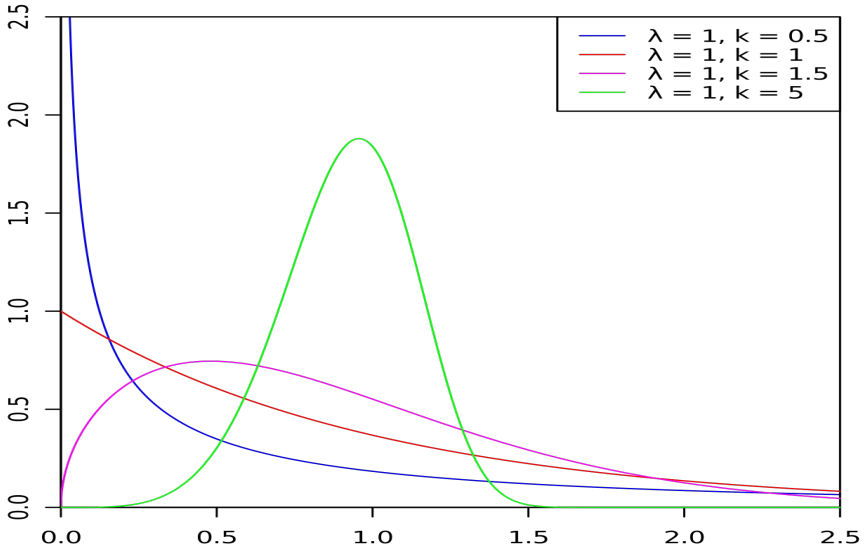




Probability Density Function

- Specifies the probability that a **continuous random variable** falling within a particular range of values, as opposed to taking on any one value.
- This probability is given by the integral of this variable's PDF over that range.

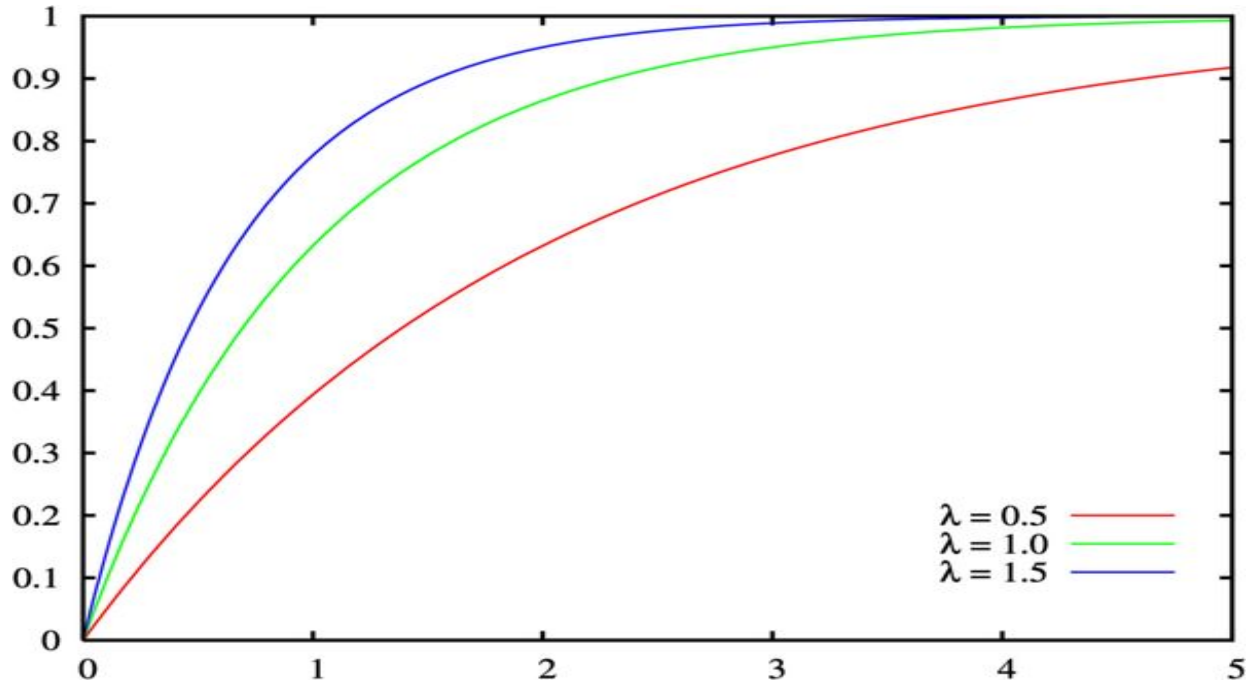




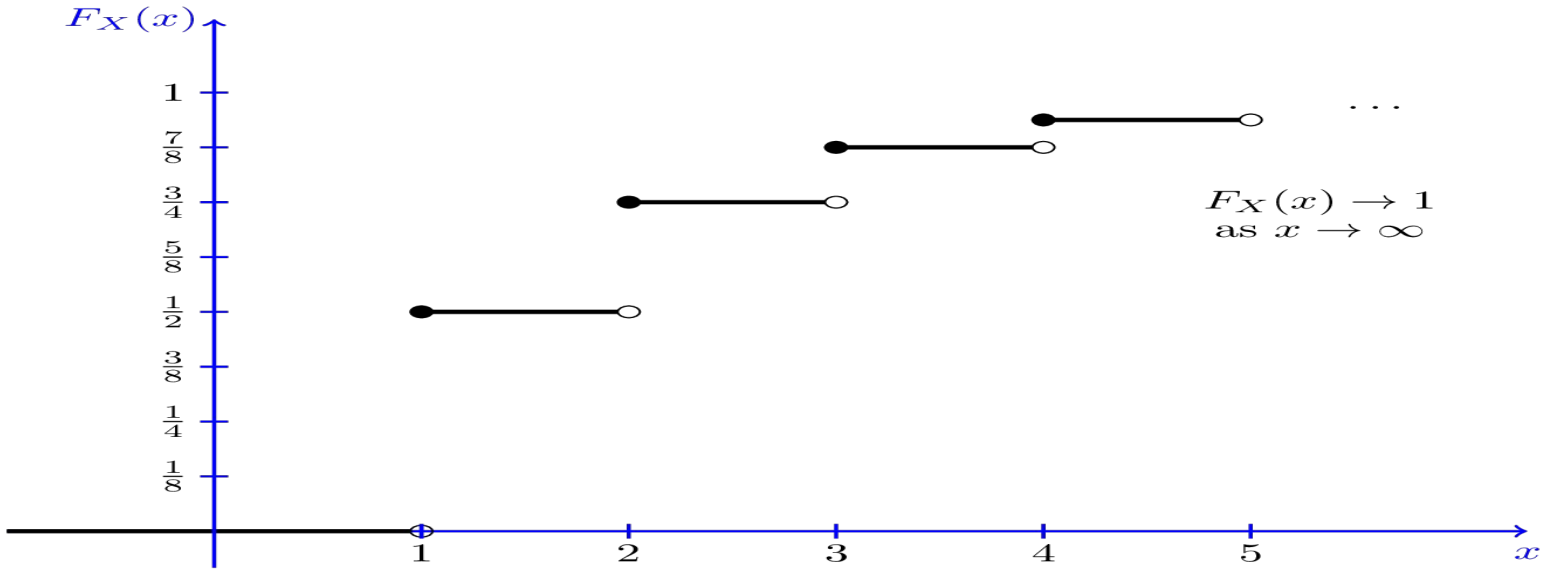
Cumulative Distribution Function

- The probability that X will take a value less than or equal to *x.*
- In the case of a continuous distribution, it gives the area under the probability density function by integrating from minus infinity to *x*.

CDF Continuous Case



CDF Discrete Case



Example Of PMF: Sum of Two Dice

Example Distribution

Set of possible values: $\mathbf{X} = \{2, 3 \ldots 12\}$

Specific value of the random variable: $x \in \mathbf{X}$

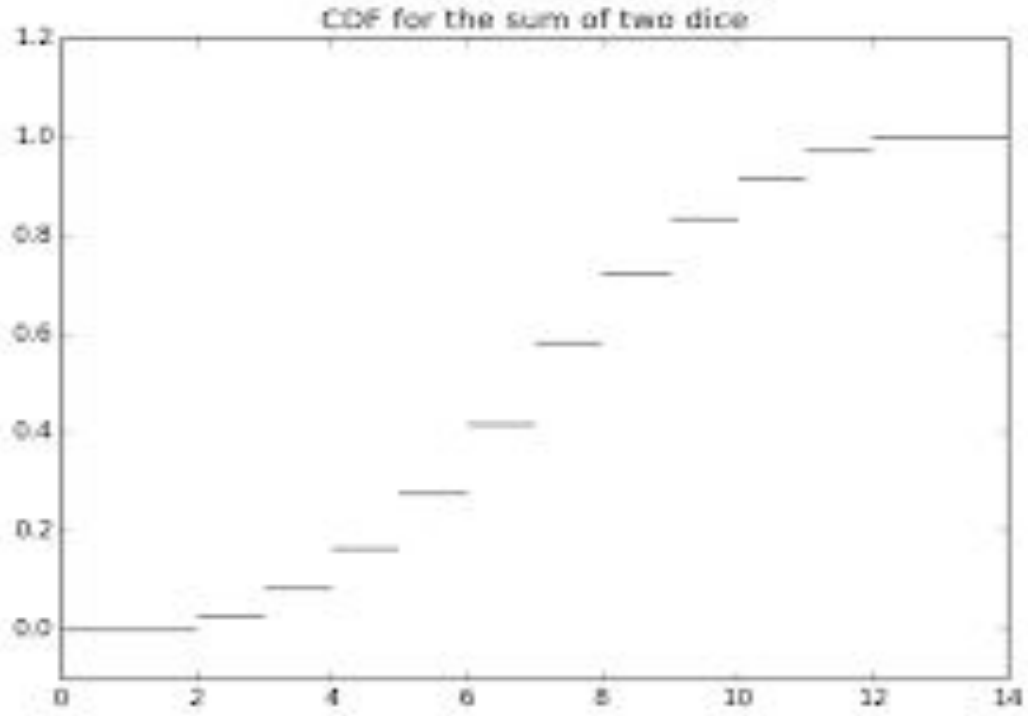
Probability of the value x: $P(x)$

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

PMF


Probability Distribution for X
probability
0.25
0.2
0.15
0.1
0.05
0
2
3
4
5
6
7
8
9
10
11
12
sum of the two dice

CDF



Gaussian Distribution (Normal Distribution)

Normal Distribution

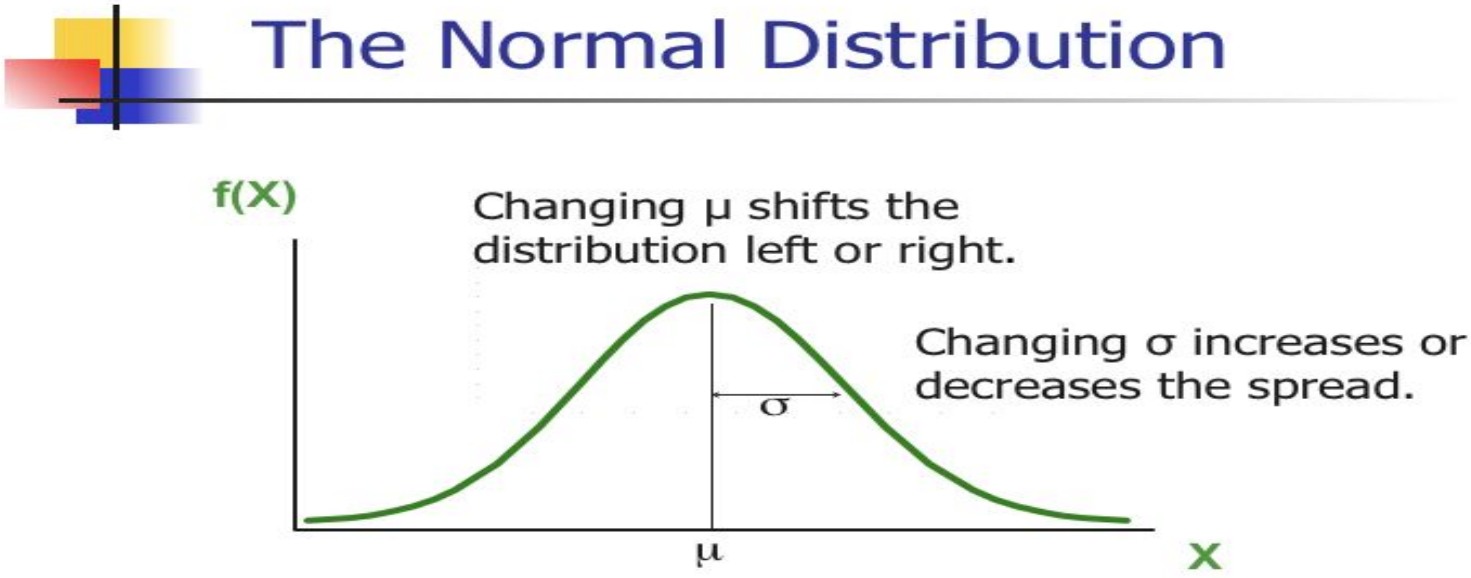
The Normal Distribution: as mathematical function (pdf)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Note constants:
π=3.14159
e=2.71828

This is a bell shaped curve with different centers and spreads depending on μ and σ

Parameters


The Normal Distribution
f(X)
Changing μ shifts the distribution left or right.
Changing σ increases or decreases the spread.
σ
μ
X

Normal distribution is defined by its mean and standard dev.

$E(X)=\mu$

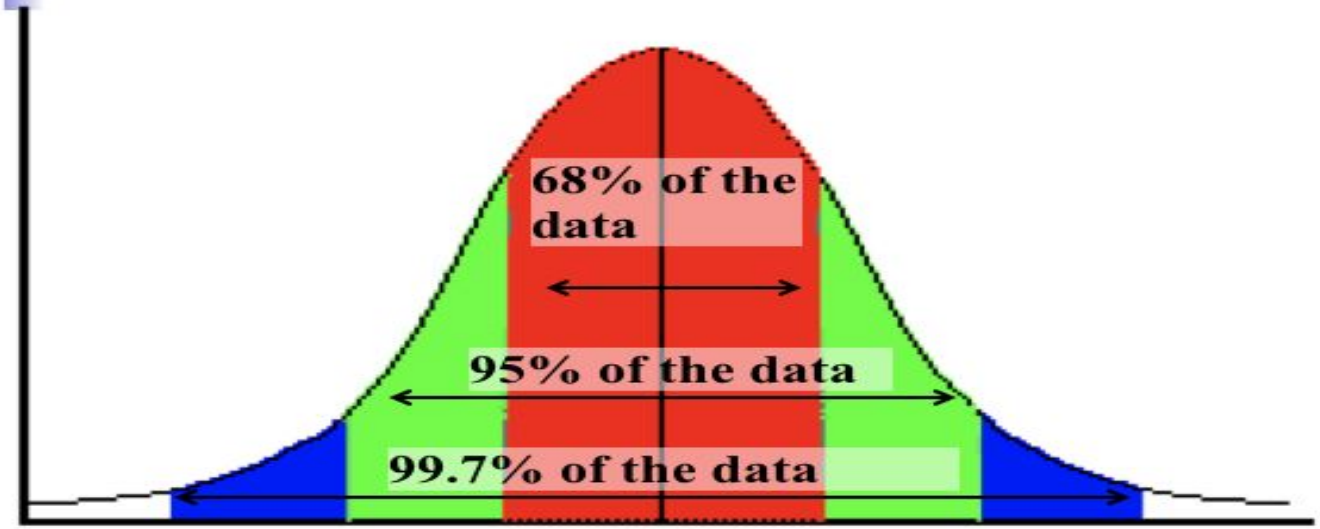
$Var(X)=\sigma^2$

Standard Deviation$(X)=\sigma$

Useful Facts

- No matter what μ and σ are,
- The area between μ-σ and μ+σ is about 68%.
- The area between μ-2σ and μ+2σ is about 95%.
- The area between μ-3σ and μ+3σ is about 99.7%. Almost all values fall within 3 standard deviations.

68-95-99.7 Rule



Standard Normal Distribution (Z)

All normal distributions can be converted into the standard normal curve by subtracting the mean and dividing by the standard deviation:

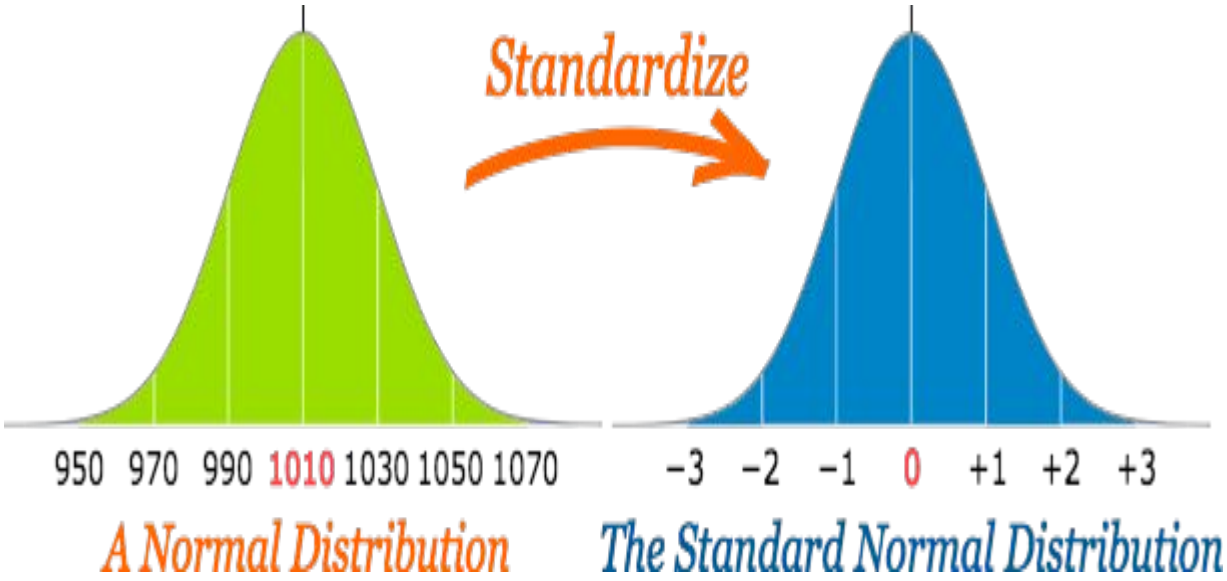
$$z = \frac{x - \mu}{\sigma}$$

μ = Mean
σ = Standard Deviation

Somebody calculated all the integrals for the standard normal and put them in a table! So we never have to integrate!

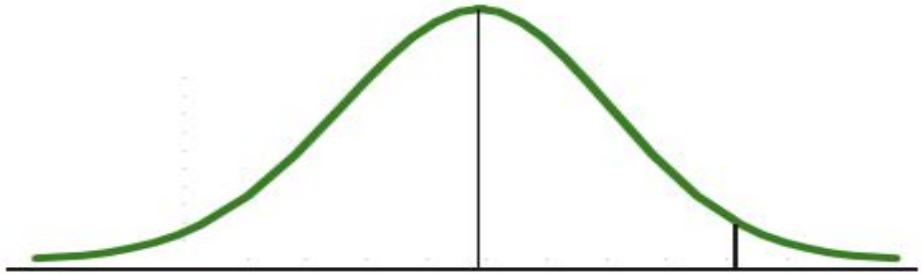
Even better, computers now do all the integration.

Standard Normal Distribution



A Standard Normal Distribution is a Normal Distribution with a mean of 0 and a standard deviation of 1

Comparing X and Z units



100 200 X ($\mu = 100, \sigma = 50$)

0 2.0 Z ($\mu = 0, \sigma = 1$)

Why Standardizing?

- Gives us a good idea the relative location of raw values
- Allows us to compare different values in a more informative way
- Scaling for features if we conduct algorithms that rely on distance metrics

Example 1

Assume snowfall follows a normal distribution over time and the mean snowfall in New York City is 25 inches with a variance of 16 inches.

What is:

1) P(X < 25) = 0.5
2) P(17 < X < 32) = 0.93
3) P(X = 25) = Not possible!!!!


```
1  z_first = (17 - 25)/4
2  z_second = (32-25)/4
3  print('z score of 17 is : ',z_first)
4  print('z score of 33 is : ',z_second)
5  stats.norm.cdf(1.75) - stats.norm.cdf(-2)
```

```
z score of 17 is :  -2.0
z score of 33 is :  1.75

0.9371907111880037
```