SIMPLE ONE-TIME INTEREST

$I = P_0 r$

$A = P_0 + I = P_0 + P_0 r = P_0(1 + r)$

I is the ___________
A is the ________________: principal plus interest
P_0 is the ___________ (starting amount)
r is the ___________ (in ________ form.
Example: 5% = 0.05)

Example

A local business asks for a $750 loan to cover some expenses and agrees to repay it in 60 days with 5% interest. How much interest will you earn?

P_0 = $________ (the principal)
r=________ (___% rate)
I=$_____×_____=$_______.

You will earn $_______ in interest.

Question

An organization requests a $1,200 loan for a short-term project and agrees to repay it in 90 days with 6% interest. How much interest will you earn?

Simple Interest over Time

$I = P_0 \, r \, t$

$A = P_0 + I = P_0 + P_0 \, r \, t = P_0(1 + r \, t)$ I

I is the __________
A is the end amount: principal plus interest
______ is the principal (starting amount)
r is the interest rate in decimal form
t is _______
The units of measurement (years, months, etc.) for the time should match the time period for the interest rate.

Example

Imagine your state is funding a new wildlife reserve and issues bonds to raise money for the project. You purchase a $2,000 bond that pays 4% interest annually and matures in 10 years. How much interest will you earn?

Each year, you would earn ____% interest: _______×_____=$80 in interest. So over the course of ten years, you would earn a total of __×__=$800 in interest. When the bond matures, you would receive back the $2,000 you originally paid, leaving you with a total of $2,800.

Question

A nearby county is raising funds to build a new library and issues bonds to support the project. You decide to purchase a $1,500 bond that pays 3.5% interest annually and matures in 8 years. How much interest will you earn?

APR – Annual Percentage Rate

Interest rates are usually given as an ____________________________– the total interest that will be paid in the year. If the interest is paid in smaller time increments, the APR will be divided up.

For example, a 6% APR paid monthly would be divided into twelve 0.5% payments. 6 ÷ 12 = 0.5
A 4% annual rate paid quarterly would be divided into four 1% payments. 4 ÷ 4 = 1

Example

Corporate bonds are issued by companies to raise funds for their projects. Suppose you purchase a $2,000 corporate bond with a 6% annual rate, paid semi-annually, with a maturity in 3 years. How much interest will you earn?

Since interest is paid semi-annually (twice a year), the __% interest is divided into two 3% payments. ___ = $2000 (the principal)
r=______ (3% rate per half-year)
t=___ (3 years = 6 half-years)
I=2000×______×6=$360.
You will earn $360 interest in total over the three years.

Question

Municipal bonds are issued by local governments to fund public projects. Suppose you buy a $1,500 municipal bond with a 5% annual interest rate, paid semi-annually, with a maturity in 2 years. How much interest will you earn?

Question

Samira invests $5,000 into an account at an annual rate of 1.2% simple interest for 18 months.

What is the Principal in this scenario?

A 1.2%
B 0.012
C $5,000
D 1.5

What is the interest rate for this account?

A $5,000
B 1.5
C 1.2%
D 0.012

Question

Samira invests $5,000 into an account at an annual rate of 1.2% simple interest for 18 months.

What number do you use to represent the interest rate in the simple interest formula?

A $5,000
B 0.012
C 1.2%
D 1.5

What is the length of time of this investment, in years?

A 0.012
B 1.2%
C 1.5
D $5,000

Calculate the simple interest earned on this account

Example

A payday lender charges $45 in interest for a two-month loan of $600. Find the annual interest rate they are charging.

I=$45 (interest)

P_0 =$600

t=2 months

Using $I=P_0 \times r \times t$, we get __=___×__×__. Solving for r, we find r=0.0375, or 3.75%. Since the time was in months, this is the monthly interest rate. The annual rate would be 6 times this: 45% interest.

Question

A credit union charges \$20 interest for a three-month loan of \$400. Find the annual interest rate they are charging.

Compound Interest

$P_n = P_0(1+r/k)^{Nk}$

P_N is the ____________________________.
P_0 is the starting balance of the account
(also called initial deposit, or ____________)

r is the ________________________ in decimal form
k is the ____________________________________in one year

If the compounding is done annually (once a year), k = 1.
If the compounding is done quarterly, k = 4.
If the compounding is done monthly, k = 12.
If the compounding is done daily, k = 365.

Example

A new savings bond offers a fixed interest rate and compounds quarterly, providing an attractive option for long-term investments. Suppose you invest $5,000 in a savings bond with an annual interest rate of 4%, compounded quarterly. How much will your investment be worth after 15 years?

P_0 =_________ (initial deposit)

$P_n = P_0(1+r/k)^{Nk}$

r = _______ (4% annual rate)}
k= _____ (4 quarters in a year)
N=_____ (15 years)
The future value formula is:
$P_N = P_0(1+ k/r)^{N \cdot k}$
Substituting the values: $P_{\_\_}$ = _______(1+ __/_____)$^{\_\cdot\_}$= $9024.80

Question

An investment account offers an annual interest rate of 5%, compounded semiannually, to encourage long-term savings. Suppose you deposit $7,500 into the account. How much will the account balance be after 10 years?

Question

A retirement fund offers an annual interest rate of 4.5%, compounded annually, to help investors grow their savings. Suppose you contribute $10,000 to this fund. How much will the account be worth after 12 years?

Example

You know that you will need $25,000 for a down payment on a house in 15 years. If your account earns 3.5% interest compounded monthly, how much would you need to deposit now to reach your goal?

$P_n=P_0(1+r/k)^{Nk}$

$$P_0 = \frac{P_N}{(1+r/k)^{N\cdot k}}$$

$$P_0 = \frac{\_\_\_\_\_\_\_\_}{(1+\_\_\_\_\_/\_\_\_)^{\_\cdot\_}} = \$15,161.35$$

Question

You want to save $30,000 for a car purchase in 5 years. If your savings account earns 6% interest compounded semiannually, how much do you need to deposit now to meet your goal?

1. Samantha borrows \$2,400, agreeing to pay it back with 1.5% annual interest after 15 months. How much interest will she pay?

2. A retiree invests \$5,000 in a savings plan that pays 4% per year. What will the account balance be at the end of the first year?

3. Evelyn invests \$15,000 into an account at an annual rate of 0.5% simple interest for 24 months.

a. What is the principal in this scenario?

b. What is the interest rate for this account?

c. What number do you use to represent the interest rate in the simple interest formula?

d. What is the length of time of this investment, in years?

e. Calculate the simple interest earned on this account.

4. You deposit \$500 in an account earning 6% interest compounded annually. How much will you have in the account in 10 years?

Annuity Formula

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)}$$

P_N is the balance in the account after N years.
d is the ________________(the amount you deposit each year, each month, etc.)
r is the ______________ in ________ form.
k is the _________________________in one year.
If the compounding frequency is not explicitly stated, assume there are the same number of compounds in a year as there are deposits made in a year.

When do you use this?

Annuities assume that you put money in the account on a regular schedule (every month, year, quarter, etc.) and let it sit there earning interest.

Compound interest assumes that you put money in the account once and let it sit there earning interest.

Compound interest: ________________
Annuity: _________________

Example

A savings plan allows you to deposit money monthly into an account that earns interest. Suppose you deposit $150 each month into an account earning 5% annual interest, compounded monthly. How much will you have saved after 25 years?

Monthly deposit **d = $150**
Annual interest rate (r): 5% (**r = 0.05** as a decimal)
Compounding periods per year **k=12** (compounded monthly)
Number of years **N = 25**

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)} \qquad P_N = \frac{\_\_\_((1 + \_\_\_/\_\_)^{\_x\_} - 1)}{(\_\_\_\_/\_\_\_)}$$

$= \$162,561$

Question

You decide to save for a down payment on a house by depositing $200 each month into a savings account that earns 4% annual interest, compounded monthly. How much will you have saved after 15 years?

Example

A savings account pays 4% interest. If you deposit $10 a day into this account, how much will you have after 15 years? How much of that amount is from interest?

Daily deposit d= $10
Annual interest rate (r): 4% (r = 0.04 as a decimal)
Compounding periods per year k=365 (compounded daily)
Number of years N = 15

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)} \qquad P_N = \frac{\_\_((1 + \_\_\_/\_\_\_)^{\_\_x\_\_} - 1)}{(\_\_\_/\_\_\_)}$$

$= \$92,370$

Total deposits: __ × ___ × __ = $54,750
Interest earned: $_______ - $_______ = $37,620

Question

A retirement savings account offers 2.5% annual interest. If you deposit $3 per day into this account, how much will you have after 8 years? How much of that total will come from interest?

Question

You decide to invest $200 each month into an account earning 5% annual interest, compounded monthly.

a) How much will you have in the account after 25 years?

b) How much total money will you contribute to the account?

c) How much of the total balance will come from interest?

Example

You want to save $150,000 for a down payment on a house in 20 years. Your savings account earns 6% annual interest, compounded monthly. How much do you need to deposit each month to reach your goal?

Annual interest rate (r): 6% (r = 0.06 as a decimal)
Compounding periods per year k=12 (monthly deposits)
Number of years N=20
Target amount P_{20}= $150,000

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)}$$

$$d = \frac{\_\_\_(\_\_/\_\_)}{((1 + \_\_/\_\_)^{\_\_\_} - 1)} = \frac{\_\_\_\_\_\_x(\_\_\_\_/\_\_)}{((1+\_\_\_\_/\_\_)^{\_\_x\_\_} -1)}$$

$\approx \$324.68$

Question

You want to save $250,000 for your child's college education in 18 years. Your investment account earns 7% annual interest, compounded monthly. How much do you need to deposit each month to reach your goal?

Example

If you invest $50 each month into an account earning 4% annual interest, compounded monthly, how long will it take for the account to grow to $5,000?

Monthly deposit d= $50

Annual interest rate (r): 4% (r=0.04 as a decimal)

Compounding periods per year k=12 (monthly deposits)

Target amount P_N = $5,000

Unknown (N): Time in years

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)}$$

$$\frac{P_N (r/k)}{d} + 1 = (1 + r/k)^{Nk}$$

$$\log\left[\frac{P_N (r/k)}{d} + 1\right] = \log[(1 + r/k)^{Nk}] = Nk \log(1 + r/k)$$

$$\frac{\_\_\_\_ (\_\_/\_\_)}{\_\_} = ((1 + r/k)^{Nk} - 1)$$

$$N = \frac{\log\left[\frac{P_N (r/k)}{d} + 1\right]}{k \log(1 + r/k)} = \frac{\log\left[\frac{5000(0.04/12)}{50} + 1\right]}{12 \log(1 + 0.04/12)} \approx 7.2$$

Payout Annuity Formula

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

P_0 is the balance in the account at the beginning (starting amount, or __________).

d is the ______________________ (the amount you deposit each year, each month, etc.)

r is the annual interest rate in decimal form.

k is the ______________________________ in one year.

When do you use this?

Payout annuities assume that you take money from the account on a ________________________(every month, year, quarter, etc.) and let the rest sit there earning interest.

Compound interest: ____________

Annuity: _________________

Payout Annuity: ___________________v

Example

After retiring, you plan to withdraw \$1,500 every month for 25 years from your retirement account. The account earns 5% interest annually, compounded monthly. How much money will you need in your account when you retire?

d=1500: the monthly withdrawal

r=0.05: 5% annual interest rate

k=12: compounding occurs monthly

N=25: withdrawals are made for 25 years

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

$$P_0 = \frac{1500(1-(1 + 0.05/12)^{-25 \cdot 12})}{(0.05/12)} = 279{,}495$$

Question

You plan to withdraw \$2,000 every month for 15 years from your retirement account. The account earns 4% interest annually, compounded monthly. How much money will you need in your account when you retire?

Question

You want to withdraw $30,000 each year for 20 years. Your account earns 8% annual interest.

a) How much do you need in your account at the beginning?

Question

b) How much total money will you withdraw over the 20 years?

Question

c) How much of the withdrawn amount will come from interest?

Example

You know you will have $750,000 in your account when you retire. You want to take monthly withdrawals for a total of 25 years. Your retirement account earns 6% annual interest. How much will you be able to withdraw each month?

$r=0.06$: 6% annual interest rate
$k=12$: compounding monthly
$N=25$: withdrawals for 25 years
$P_0 = 750{,}000$: starting balance

$$P_0 = \frac{d(1-(1+r/k)^{-Nk})}{(r/k)}$$

$$d = \frac{P_N\,(r/k)}{(1-(1+r/k)^{-Nk})} = \frac{(\quad/\quad)}{(1-(1+\quad/\quad)^{-\ \cdot\ })} = 4827.84$$

Question

You know you will have $400,000 in your account when you retire. You want to take monthly withdrawals for a total of 20 years. Your retirement account earns 7% annual interest. How much will you be able to withdraw each month?

Question

A donor contributes $250,000 to a hospital, with instructions that it should fund annual grants for the next 25 years. If the hospital can earn 5% annual interest, how much can they allocate for grants each year?

__________ Formula

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

P_0 is the balance in the account at the __________(starting amount, or principal).

d is the loan payment (the amount you pay each year, each month, etc.)

r is the ____________________in decimal form.

k is the number of compounding periods in one year.

N is the length of the loan in years.

When do you use this?

The loan formula assumes that you make loan payments on a regular schedule (every month, year, quarter, etc.) and are paying interest on the loan.

Compound interest: ____________

Annuity: ___________

Payout Annuity: ______________

Loans: _________________

Example

You can afford \$300 per month as a car payment. If you can get an auto loan at 4% interest for 72 months (6 years), how expensive of a car can you afford? In other words, what loan amount can you pay off with \$300 per month?

d=300: the monthly loan payment
r=0.04: 4% annual interest rate
k=12: monthly compounding
N=6: payments for 6 years (72 months)

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

$$P_0 = \frac{\_\_\_\_(1-(1 + \_\_\_/\_\_)^{-6\cdot 12})}{(0.04/\_\_)} = 19{,}098$$

Question

You can afford \$250 per month as a car payment. If you secure an auto loan at 5% interest for 48 months (4 years), how expensive of a car can you afford? In other words, what loan amount can you pay off with \$250 per month?

Question

You want to take out a \$200,000 mortgage (home loan). The interest rate on the loan is 4%, and the loan is for 15 years. How much will your monthly payments be?