First Exam
Theoretical Computer Science
Graduate Center
City University of New York
10 June 2003

Do three of the following four problems. Write each answer in a separate blue book.

1. Let $\Sigma = \{0, 1\}$ be a finite alphabet.

   (a) Define *regular language*.
   (b) Show the following language is regular over Σ: $L = \{w \mid w \text{ contains the substring } 1001\}$
   (c) Define a Turing machine that accepts the language L above.
   (d) Show the following language is not regular over Σ: $\{w \mid w = 0^m 1^n, m \geq n\}$

2. (a) State the Halting Problem.
   (b) Prove the Halting Problem is Turing-Recognizable (i.e. recursively enumerable).
   (c) Prove the Halting Problem is not decidable.

3. (a) State the Recursion Theorem.
   (b) Prove the Recursion Theorem.
   (c) Prove the following fixed point theorem:
   Let $t : \Sigma^* \to \Sigma^*$ be a computable function. Then there is a Turing machine F wherein $t(< F >)$ describes a Turing machine equivalent to F.

4. For the following sets, state whether the set is decidable or Turing-recognizable (or both). Justify your answer.

   (a) $E_{TM} = \{<M> \mid M \text{ is a TM and } L(M) = \emptyset\}$
   (b) A set A, where $A \leq_m B$ and B is decidable.
   (c) A set A, where $\bar{A} \leq_m A$ and $\bar{A}$ is Turing-recognizable.
   (d) $Th(\mathbf{N}, +)$
   (e) $Th(\mathbf{N}, +, \times)$