Homework 4
CSc 72700: Analysis of Algoritms
CUNY Graduate Center, Fall 2001

Due Friday, 19 October

See the guidelines on the webpage for details about submitting homework. (If turning your homework in electronically, you can mail it directly to the grader at: `ivm3@columbia.edu`.)

Practice Problems

The problems in this section **are not to be submitted**. They are to help you understand the material, and some will appear on exams.

- Exercise 13.1-1 on p 246 (in the second edition: 12.1-1 on p 256).
- Exercise 13.2-1 on p 249 (in the second edition: 12.2-1 on p 259).
- Exercise 13.3-1 on p 253 (in the second edition: 12.3-1 on p 264).
- Exercise 14.1-4 on p 265 (in the second edition: 13.1-6 on p 277).
- Exercise 14.2-2 on p 267 (in the second edition: 13.2-2 on p 278).
- An **AVL** tree is a binary search tree in which the subtrees of a node differ in height by at most one. Insertions and deletions are performed as in binary search trees and then followed by rotations to preserve balance (see links on webpage for more details or 13-3, p 296 in the second edition).

  Construct a binary search tree by introducing the following keys in the given order: 2, 6, 10, 8, 3, 4, 5. Using the same sequence of keys, construct an AVL tree that uses rotations to keep balance after every insertion.

Graded Problems

These problems will be graded and should be submitted, following the guidelines on the webpage.

1. Average node depth in a randomly built binary search tree, 13-3 on p 260 (in the second edition, 12-3 on p 270).

2. Number of different binary trees, 13-4 on p 262 (in the second edition, 12-4 on p 271).
   Note, there is a typo in the first edition. In part b), the formula should read:

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

3. AVL Trees (see webpage or 13-3, p 296 second edition for defintions)

   (a) Trees of height $O(\lg n)$ are called **balanced**. Show AVL trees are balanced.

   (b) What is the most number of rotations that must be performed when inserting to maintain balance? What is the most number of rotations that must be performed, when deleting?

   (c) What is the running time of the operations: insert, delete, and search for AVL trees?