

Exam 2
CSc 75010: Theoretical Computer Science
Graduate Center of CUNY
22 November 2002
(Sample Exam)

Do five of the following six problems. Write each answer on a separate piece of paper.

1. (a) Define *computable function* and give an example of a function that is not computable.
(b) Assume that the alphabet and tape alphabet are: $\Sigma = \Gamma = \{0, 1\}$. Give an implementation level description of a Turing machine that takes as input a number in binary representation and multiplies it by 2 (that is, shifts it to the right and adds a 0).
2. (a) Show that for any finite set Σ , the set of all finite strings over Σ , Σ^* , is countable.
(b) Let $\Sigma = \{0, 1\}$. Show that the set of all infinite strings over Σ is uncountable.
3. (a) Show that the Halting Problem is undecidable, using the diagonalization method.
(b) Show that the set of all finitely defined computable functions (that is, the set of all functions that converge only for a finite number of inputs) is not decidable.
4. (a) Show that the set of decidable languages is closed under concatenation.
(b) Show that if A and \bar{A} are Turing-recognizable, then A is decidable.
5. (a) State the Post Correspondence Problem (PCP).
(b) Show that PCP is decidable over the alphabet $\Sigma = \{1\}$.
6. (a) If $A \leq_m B$ and B regular does that imply A is regular? Why or why not? Justify your answer.
(b) Show for all A, B , there exists a set J such that $A \leq_T J$ and $B \leq_T J$.