Name: _____

Final Exam
CSc 75010: Theoretical Computer Science
Graduate Center of CUNY
13 December 2002

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Total	

Do five of the following six problems. Write each answer on a separate piece of paper.

1. Let $\Sigma = \{0, 1\}$ be a finite alphabet.
   (a) Define *regular language*.
   (b) Show the following language is regular over Σ: $\{w \mid w \text{ ends in a } 0\}$
   (c) Show the following language is not regular over Σ: $\{w \mid w = 0^n 1^n, n \geq 0\}$

2. For each of the following statements, state whether it is true or false. Justify your answer by providing a proof sketch or counterexample.
   (a) Every regular language is context-free.
   (b) Every regular language is Turing-recognizable.
   (c) Every decidable language is regular.

3. (a) State the Halting Problem.
   (b) Prove the Halting Problem is Turing-Recognizable.
   (c) Prove the Halting Problem is not decidable.

4. (a) State the Post Correspondence Problem (PCP).
   (b) Is the PCP undecidable for all alphabets? Why or why not? If yes, sketch a proof of the undecidability. If no, give an example of an alphabet over which PCP is decidable. Justify your answer.

5. (a) State the Recursion Theorem.
   (b) Given a Turing Machine M, the *length* of the description of $\langle M \rangle$ is the number of symbols in the string describing M. M is *minimal* if there is no Turing Machine equivalent to M with a shorter description. Show

   $$\text{MIN}_{TM} = \{\langle M \rangle \mid M \text{ is a minimal TM}\}$$

   is not Turing-recognizable.

6. For the following sets, state whether the set is decidable or Turing-recognizable (or both). Justify your answer.
   (a) $E_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$
   (b) $Th(\mathbf{N}, +)$
   (c) $Th(\mathbf{N}, +, \times)$