Name: _____

Exam 2
CSc 75010: Theoretical Computer Science
Graduate Center of CUNY
22 November 2002
(Yellow Version)

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Total	

Do five of the following six problems. Write each answer on a separate piece of paper.

1. (a) Define *computable function* and give an example of a function that is not computable.
   (b) Assume that the alphabet and tape alphabet are: $\Sigma = \Gamma = \{0, 1\}$. Give an implementation level description of a Turing machine that takes as input a number in binary representation and mulitplies it by 2.

2. (a) Show that the set of all positive rational numbers is countable.
   (b) Show that the set of all positive real numbers is uncountable.

3. (a) Show that the Halting Problem is undecidable, using the diagonalization method.
   (b) Show that the set of all totally undefined computable functions (that is the set of all functions that diverge for all inputs) is not decidable.

4. (a) Show that the set of Turing-recognizable languages is closed under star.
   (b) Show that if A is decidable, then A and $\bar{A}$ are Turing-recognizable.

5. (a) State the Post Correspondence Problem (PCP).
   (b) Show that PCP is decidable over the alphabet $\Sigma = \{1\}$.

6. (a) Show that for all A and B, $A \leq_m B$ implies $\bar{A} \leq_m \bar{B}$.
   (b) Show for all A, B, there exists a set J such that $A \leq_T J$ and $B \leq_T J$.