Answer Key for Quiz 0
CMP 761: Analysis of Algorithms
3 September 2002

Name: ______________________________

Student ID (Social Security Number): __________

Write your answer to each on a separate piece of paper. Staple your answer sheets to this sheet when you turn in the quiz.

1. Prove for all natural numbers n that:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

This can be done in several different ways. One way to prove this is by induction on n:

Base Case: $n = 1$.
When $n = 1$, the left hand side evaluates to 1 as does the right hand side ($\frac{1(1+1)}{2} = 1$). So, the equation holds for $n = 1$.

Inductive Step: $n > 1$.
Assume true for n, and then show true for $n + 1$. Starting with the left hand side:

$$\begin{aligned}
\textstyle\sum_{i=1}^{n+1} i &= (n+1) + \textstyle\sum_{i=1}^{n} i \\
&= (n+1) + \tfrac{n(n+1)}{2} \quad \texttt{(by IH)} \\
&= \tfrac{2(n+1)}{2} + \tfrac{n(n+1)}{2} \\
&= \tfrac{2(n+1)+n(n+1)}{2} \\
&= \tfrac{2n+2+n^2+n)}{2} \\
&= \tfrac{n^2+3n+2}{2} \\
&= \tfrac{(n+1)(n+2)}{2} \\
&= \tfrac{(n+1)((n+1)+1)}{2}
\end{aligned}$$

So, the equation holds for $n + 1$, assuming it holds for n.

Thus, by the principle of induction, for all natural numbers n,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

2. Write (in pseudo-code) an algorithm that sorts a list of n numbers.

   (More formally, design an algorithm for the following:
   Input: A sequence of n numbers $\{A[1], A[2], \ldots, A[n]\}$.
   Output: A reordering $\{A'[1], A'[2], \ldots, A'[n]\}$ of the input sequence such that $A'[1] \leq A'[2] \leq \cdots \leq A'[n]$.)

`Again, there are many different answers to this question. One possible way to sort the list would be a bubble sort:`

```
for (i = 0; i < n; i++)
{
  for (j=0; j < n-1; j++)
  {
    if (A[j] > A[j+1])
    {
       /* Swap the two elements */
      tmp = A[j];
      A[j] = A[j+1];
      A[j+1] = tmp;
    }
  }
}
```