UNIVERSITY of FLORIDA

Department of Mathematics

MAP 2302
Exam 4A
Spring 2022

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Section number

C. Under "special codes" code in the test ID numbers 4, 1.

1 2 3 ● 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. **1)** This test consists of 20 multiple choice questions of five points in value.

2) The time allowed is 120 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron to your exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas shortly after the exam.

NOTE: Be sure to bubble the answers to questions 1−20 on your scantron.

Questions 1 – 20 are worth 5 points each.

1. Solve the homogeneous differential equation $(3x^2 - y^2)\,dx + (xy - x^3y^{-1})\,dy = 0$.

a. $\dfrac{y^2}{4x^2} + \ln\left|\dfrac{y}{x}\right| - \dfrac{1}{4}\ln|x| = C$

b. $-\dfrac{y^2}{4x^2} + \dfrac{1}{2}\ln\left|\dfrac{y}{x}\right| - \ln|x| = C$

c. $-\dfrac{y^2}{4x^2} - \dfrac{1}{2}\ln\left|\dfrac{y}{x}\right| - \ln|x| = C$

d. $-\dfrac{y^2}{4x^2} + \dfrac{1}{2}\ln\left|\dfrac{y}{x}\right| + \ln|x| = C$

e. $-\dfrac{y^2}{4x^2} + \dfrac{1}{4}\ln\left|\dfrac{y}{x}\right| + \ln|x| = C$

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2. Solve the Bernoulli equation $y' + \dfrac{y}{x} = 7x^8y^2$, ignoring lost solutions, if any.

a. $y = \dfrac{8}{Cx - 7x^9}$

b. $y = \dfrac{2}{x - Cx^9}$

c. $y = \dfrac{4}{Cx - x^9}$

d. $y = \dfrac{2}{Cx - x^8}$

e. $y = \dfrac{4}{x - Cx^8}$

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3. Find the Laplace transform of $(t-6)^4u(t-6)$

a. $\dfrac{24e^{-6s}}{s^3}$ b. $\dfrac{12e^{6s}}{s^3}$ c. $\dfrac{12e^{6s}}{s^5}$ d. $\dfrac{12e^{-6s}}{s^3}$ e. $\dfrac{24e^{-6s}}{s^5}$

4. Use the Euler method to find y_2 to two decimal places for $y' = 2x - 3y + 1$ given $y(1) = 5$ and a step size of 0.1.

a. $y_2 = 2.88$ b. $y_2 = 2.94$ c. $y_2 = 2.76$ d. $y_2 = 2.98$ e. $y_2 = 2.92$

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5. Determine the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{11(x-6)^n}{n^2 + 8n}$.

a. $[5, \infty)$ b. $[5, 7)$ c. $(5, 7]$ d. $(5, 7)$ e. $[5, 7]$

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6. Determine the first three nonzero terms in the Taylor polynomial approximation for the given initial value problem.

$$y' = 4 \sin y + e^{2x},\ y(0) = 0.$$

a. $x + 2x^2 + 4x^3$

b. $x + 3x^2 + \frac{14}{3}x^3$

c. $x + 3x^2 + \frac{16}{3}x^3$

d. $x + 5x^2 + \frac{20}{3}x^3$

e. $x + 2x^2 + \frac{16}{3}x^3$

7. The Theorem of Existence and Uniqueness does NOT guarantee a solution to the differential equation $y' = \sqrt{y-2}$ for which value?

a. $y = 3$ b. $y = 4$ c. $y = 6$ d. $y = 2$ e. $y = 5$

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8. Use the convolution theorem to obtain a formula for the solution to the given initial value problem where $g(t)$ is piecewise continuous on $[0, \infty]$ and of exponential order.

$$y'' + 25y = g(t),\ y(0) = 0,\ y'(0) = 1$$

a. $y(t) = \frac{1}{5}\sin(5t) - \frac{1}{25}\int_0^t \sin(5(t-v))g(v)dv$

b. $y(t) = \frac{1}{5}\sin(5t) - \frac{1}{5}\int_0^t \sin(5(t-v))g(v)dv$

c. $y(t) = \frac{1}{5}\sin(5t) + \frac{2}{5}\int_0^t \sin(5(t-v))g(v)dv$

d. $y(t) = \frac{1}{5}\sin(5t) + \frac{1}{25}\int_0^t \sin(5(t-v))g(v)dv$

e. $y(t) = \frac{1}{5}\sin(5t) + \frac{1}{5}\int_0^t \sin(5(t-v))g(v)dv$

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9. Find a minimum value for the radius of convergence of a power series solution about $x_0 = 4$ for $y'' - 5xy' - 3y = 0$.

a. 4 b. $\frac{5}{2}$ c. ∞ d. $\frac{5+\sqrt{37}}{2}$ e. $\sqrt{\frac{23}{2}}$

10. Solve the initial value problem $y'' + 4y = 2\delta(t - \frac{\pi}{2}) - 6\delta(t - \pi),\ y(0) = 0,\ y'(0) = 2.$ Note: $\sin(\theta - \pi) = -\sin\theta$.

a. $y(t) = \sin(2t)\left[1 - 2u(t - \frac{\pi}{2}) + 3u(t - \pi)\right]$

b. $y(t) = \sin(2t)\left[1 - u(t - \frac{\pi}{2}) - 2u(t - \pi)\right]$

c. $y(t) = \sin(2t)\left[1 - u(t - \frac{\pi}{2}) - u(t - \pi)\right]$

d. $y(t) = \sin(2t)\left[1 - u(t - \frac{\pi}{2}) - 3u(t - \pi)\right]$

e. $y(t) = \sin(2t)\left[1 + u(t - \frac{\pi}{2}) - u(t - \pi)\right]$

11, 12. Find a general solution to the differential equation $y' - 2xy = 0$ using a power series expansion about $x_0 = -4$.

11. Find the recurrence relation.

a. $a_{n+1} = \dfrac{2a_{n-1} + 8a_n}{n+1}$

b. $a_{n+1} = \dfrac{2a_{n-1} - 4a_n}{(n+1)(n+2)}$

c. $a_{n+1} = \dfrac{2a_{n-1} - 10a_n}{n+1}$

d. $a_{n+1} = \dfrac{2a_{n-1} + 6a_n}{n+1}$

e. $a_{n+1} = \dfrac{2a_{n-1} - 8a_n}{n+1}$

12. Find the first four nonzero terms.

a. $y(x) = a_0\left[1 - 8(x+4) + 33(x+4)^2 - \dfrac{280}{3}(x+4)^3 + ...\right]$

b. $y(x) = a_0\left[1 - 8(x+4) + 33(x+4)^2 - \dfrac{250}{3}(x+4)^3 + ...\right]$

c. $y(x) = a_0\left[1 - 8(x+4) + 33(x+4)^2 - \dfrac{220}{3}(x+4)^3 + ...\right]$

d. $y(x) = a_0\left[1 - 8(x+4) - 33(x+4)^2 - 80(x+4)^3 + ...\right]$

e. $y(x) = a_0\left[1 - 8(x+4) - 33(x+4)^2 - \dfrac{275}{3}(x+4)^3 + ...\right]$

13. Determine the form of a particular solution for $y'' - 24y' + 144y = t^2e^{12t} - e^{12t}$. Do not solve for the coefficients.

a. $y_p(t) = t\left(A_2t^2 + A_1t + A_0\right)e^{12t}$

b. $y_p(t) = \left(A_2t^2 + A_1t + A_0\right)e^{12t}$

c. $y_p(t) = At^4e^{12t}$

d. $y_p(t) = t^2\left(A_2t^2 + A_1t + A_0\right)e^{12t}$

e. $y_p(t) = At^2e^{12t}$

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14. Find the first four nonzero terms in a power series expansion of the solution to the initial value problem $4y' - 2e^{2x}y = 0,\ y(0) = 3$. Note that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

a. $y(x) = 3 + \frac{3}{2}x + \frac{15}{8}x^2 + \frac{23}{16}x^3 + ...$

b. $y(x) = 3 + \frac{3}{2}x + \frac{15}{8}x^2 - \frac{27}{16}x^3 + ...$

c. $y(x) = 3 + \frac{3}{2}x + \frac{15}{8}x^2 - \frac{29}{16}x^3 + ...$

d. $y(x) = 3 + \frac{3}{2}x + \frac{15}{8}x^2 + \frac{27}{16}x^3 + ...$

e. $y(x) = 3 + \frac{3}{2}x + \frac{15}{8}x^2 + \frac{29}{16}x^3 + ...$

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15. Find a particular solution to the DE using the method of undetermined coefficients.

$$x'' + 4x' = 2t + 3$$

a. $x_p(t) = \frac{t^2}{2} - \frac{3}{8}$

b. $x_p(t) = \frac{t^2}{4} - \frac{3t}{8}$

c. $x_p(t) = \frac{t^2}{4} + \frac{5t}{8}$

d. $x_p(t) = \frac{t^2}{2} + \frac{3t}{8}$

e. $x_p(t) = \frac{t^2}{2} - 3$

16. Determine all of the singular points of $(t^2 - t - 12)x'' + (t+3)x' - (t-4)x = 0$

a. $4, -3$

b. 4 only

c. -3 only

d. $4, -3, 0$

e. $4, 3$

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17. Write $3\sum_{n=1}^{\infty} a_n x^{n+1} + \sum_{n=2}^{\infty} n b_n x^{n-1}$ as a single series with generic term x^n.

a. $2b_2 x + \sum_{n=2}^{\infty}[3a_{n-1} + (n+1)b_{n+1}]x^n$

b. $3a_1 x + \sum_{n=2}^{\infty}[3a_{n-1} + (n+1)b_{n+1}]x^n$

c. $\sum_{n=1}^{\infty}[3a_n + (n+1)b_n]x^n$

d. $\sum_{n=1}^{\infty}[3a_{n-1} + (n+1)b_{n+1}]x^n$

e. $2b_0 x + \sum_{n=1}^{\infty}[3a_{n-1} + (n+1)b_{n+1}]x^n$

18. Find a power series expansion for $\int_0^x \frac{\sin t}{t}\,dt$ given that $\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

a. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$

b. $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$

c. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)(2n+1)!}$

d. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)(2n+1)!}$

e. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)(2n)!}$

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19. Use reduction of order to find a second linearly independent solution for

$$y'' - \frac{1}{t}y' + \frac{1}{t^2}y = 0,\ t > 0,\ y_1 = t$$

a. $y_2 = te^t$

b. $y_2 = -t^2 + t + 2$

c. $y_2 = t\ln t$

d. $y_2 = t^2 \ln t$

e. $y_2 = t\ln t + e^t$

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20. Solve $y'' + y = \sec t$ using variation of parameters.

a. $y(t) = c_1 \cos t + c_2 \sin t + \sin t \ln|\cos t| + t\cos t$

b. $y(t) = c_1 \cos t + c_2 \sin t + \cos t \ln|\sin t| + t\sin t$

c. $y(t) = c_1 \cos t + c_2 \sin t + \sin t \ln|\cos t| + t\sec t$

d. $y(t) = c_1 \cos t + c_2 \sin t + \cos t \ln|\cos t| + t\sin t$

e. $y(t) = c_1 \cos t + c_2 \sin t + \cos t \ln|\cos t| + t\sec t$

Elementary Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\}$
C	$\frac{C}{s}, s > 0$
t^n	$\frac{n!}{s^{n+1}}, s > 0$
e^{at}	$\frac{1}{s-a}, \quad s > a$
$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a,\ n = 0, 1, 2, \ldots$
$\sin(bt)$	$\frac{b}{s^2+b^2}, s > 0$
$\cos(bt)$	$\frac{s}{s^2+b^2}, s > 0$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
$\sinh(bt)$	$\frac{b}{s^2-b^2}, s > \lvert b\rvert$
$\cosh(bt)$	$\frac{s}{s^2-b^2}, s > \lvert b\rvert$
$u(t-a)$	$\frac{e^{-as}}{s}, s > 0$
$\delta(t-a)$	e^{-as}

Properties of the Laplace transform

$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$

$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$ for any constant c

$\mathcal{L}\{e^{at}f(t)\}(s) = F(s-a)$ where $F(s) = \mathcal{L}\{f\}(s)$

$\mathcal{L}\{f'\}(s) = sF(s) - f(0)$

$\mathcal{L}\{f''\}(s) = s^2F(s) - sf(0) - f'(0)$

$\mathcal{L}\{f^{(n)}\}(s) = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$

$\mathcal{L}\{t^nf(t)\} = (-1)^n\frac{d^nF}{ds^n}(s)$

$\mathcal{L}\{ty'\}(s) = -sY'(s) - Y(s)$ where $Y(s) = \mathcal{L}\{y\}(s)$

$\mathcal{L}\{ty''\}(s) = -s^2Y'(s) - 2sY(s) + y(0)$