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Exam 1 Spring 23

1. Solve $\dfrac{dy}{dx} = \sqrt{x+y} - 1$ using an appropriate substitution.

a. $2\sqrt{x+y+1} = x + C$ b. $2\sqrt{x-y} = y + C$ c. $2\sqrt{x-y} = x + C$

d. $2\sqrt{x+y} = x + C$ e. $\sqrt{x+y} = y + C$

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2. Which of the following satisfies the differential equation $\dfrac{d^2y}{dx^2} + y = 0$?

a. $y = \cos 2t + \sin 5t$ b. $y = 2\cos t + 5\sin t$ c. $y = 2e^t$

d. $y = x^2 + 1$ e. $y = \arctan t$

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3. Solve the initial value problem.

$$y' = \frac{y}{x} + xe^x,\ y(-1) = 0$$

a. $y = xe^x - ex$ b. $y = xe^x + ex$ c. $y = xe^x - \dfrac{x}{e}$

d. $y = e^{-x} - e^x$ e. $y = x^2e^x$

4. Which of the following statements best describes the differential equation

$$(y^3 + 5x^2y)dx + (x^3 + xy^2)dy = 0?$$

a. This equation is exact.

b. This equation can become exact by multiplying by an integrating factor $\mu(x)$ that is a function of x alone.

c. This equation can become exact by multiplying by an integrating factor $\mu(y)$ that is a function of y alone.

d. This equation is linear.

e. This equation is separable.

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5. Where does the Theorem of Existence and Uniqueness imply the existence of a unique solution in the $ty-$plane for the differential equation below?

$$\frac{dy}{dt} = \frac{1}{y} + \sqrt{y-t};\ y(t_0) = y_0$$

a. $(t_0, y_0) = (5, -1)$ b. $(t_0, y_0) = (-4, 0)$

c. $(t_0, y_0) = (7, 7)$ d. $(t_0, y_0) = (-3, -2)$

e. None of these

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6. The half-life of carbon-14 is approximately 5,730 years. A sample has a mass of 200g of carbon-14. Find the time in years it takes for the mass to be reduced to 50g.

a. $5730\frac{\ln(1/2)}{\ln(1/4)}$

b. $\frac{\ln(1/4)}{5730\ln(1/2)}$

c. $\frac{\ln(1/2)}{5730\ln(1/2)}$

d. $5730\frac{\ln(1/4)}{\ln(1/2)}$

e. $\frac{\ln(5730\cdot 1/4)}{\ln(1/2)}$

7. Which differential equation below has the highest order?

a. $u\dfrac{\partial^2 u}{\partial x^2} = t\dfrac{\partial u}{\partial t} + x\dfrac{\partial u}{\partial x}$

b. $\ln(y^5) = 4x\dfrac{d^3y}{dx^3}$

c. $t\dfrac{d^4y}{dt^4} = 4y + t^2$

d. $\dfrac{\partial^3 v}{\partial^2 x \partial t} = v^{10} + x^8 - v_{tt}$

e. $y''' - 3x^2y'' + 3y^4y' - 2y = x^7$

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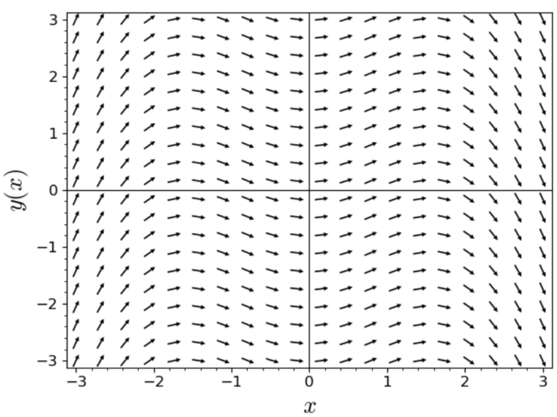
8. The direction field pictured below is described by the differential equation



a. $\dfrac{dy}{dx} = x\sin x$

b. $\dfrac{dy}{dx} = x\cos x$

c. $\dfrac{dy}{dx} = \sin x$

d. $\dfrac{dy}{dx} = y\cos x$

e. $\dfrac{dy}{dx} = y\sin x$

9. Using an appropriate substitution, find the general solution to the homogeneous differential equation. (Hint: You may need to simplify your answer to obtain one of the answer choices.)

$$\frac{dy}{dx} = \frac{y[\ln(y) - \ln(x)]}{x}.$$

a. $y = xe^{Cx+1}$

b. $y = xe^{Ce^x}$

c. $y = e^{1+Cx}$

d. $y = e^{Ce^x}$

e. $y = 0$

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10. For which value(s) of k is $\frac{dx}{dt} + x^k = t^{k+2}$ linear?

a. -2 only
b. 0 only
c. 1 only
d. 0 or -2
e. 0 or 1

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11. Find an integrating factor to make the DE exact.

$$(x + y)\sin(y)dx + (x\sin(y) + 1)dy = 0$$

a. $\cot(y)$

b. $\csc(y)$

c. $\sin(y) + \cos(y)$

d. $\tan(y)$

e. $\sec(y)$

12. Which of the following equations is homogenous?

a. $(x+3y+3)dx+(7x-3y)dy=0$

b. $x\sin(\frac{y}{x})\frac{dy}{dx}=y\sin(\frac{y}{x})+x$

c. $\frac{d^2x}{dy^2}+10\frac{dy}{dx}-3xy=\cos(x)$

d. $\frac{dy}{dx}+\sin(x)y=x^3$

e. $\frac{dy}{dx}-7y=4x$

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13. Solve the Bernoulli equation $\dfrac{dy}{dx}=\dfrac{2y}{x}-x^2y^2$.

a. $y=\dfrac{5x^2}{x^5+Cx}$

b. $y=\dfrac{x^2}{x^5+C}$

c. $y=\dfrac{5y^2}{x^5+C}$

d. $y=\dfrac{y^2}{x^5+C}$

e. $y=\dfrac{5x^2}{x^5+C}$

MAP 2302 Exam 1A, Part II Free Response

Name: ______________________________ Section #: ______________

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (13 pts)

a. Solve the first order linear initial value problem $y' - \dfrac{y}{x} = \dfrac{x}{x^2+1}$; $y(1) = 0$. (9 pts)

b. Find $\lim\limits_{x \to \infty} y(x)$ (4 pts)

2. (10 pts) Suppose we are using Euler's method to approximate the solution to the initial value problem $\frac{dy}{dx} = y^2 - 1$; $y(0) = 0$.

a. Let h be the step size used, with $h > 0$. If, after two iterations, we get that $y_2 = 0$, what is h? Use exact forms if possible. (6 pts)

b. Using the step size you found in part a., compute y_3 and y_4. Show your work. Use exact forms if possible. (4 pts)

3. (12 pts) Given the exact DE and initial condition $\dfrac{dy}{dx} = \dfrac{5x^4 - xy^2}{4y^3 + x^2y}$; $y(0) = 1$,

a. Solve the IVP. (9 pts)

b. The solution to the IVP intersects the line $y = 1$ for two values of x. One of those values is $x = 0$. What is the other one? (3 pts)

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Form A 1. d 2. b 3. c 4. b 5. d 6. d 7. c 8. b 9. a 10. e 11. b 12. b 13. e

Free response

1.$y(x) = x \arctan(x) + cx$

c=-pi/4

infinity

2.$h = \sqrt(2)$

$y_3 = -\sqrt(2)$

$y_4 = 0$

3. $x^2y^2/2 - x^5 + y^4 = C$

c=1

$x = (1/2)^(1/3)$