CHAPTER 4 HOMEWORK SOLUTIONS

Note: the first number corresponds to the homework problem from the 5th edition while the second number in parentheses corresponds to the homework problem in the 6th edition. If the problem does not occur in the 6th edition, the symbol $(-)$ is present.

SECTION 4.2

13, (13). $y'' + 2y' - 8y = 0, \ y(0) = 3, \ y'(0) = -12$

The characteristic equation is given by

$$r^2 + 2r - 8 = (r+4)(r-2) = 0$$

so there exists two real, distinct roots $r = 2, -4$.

Hence the general solution and its derivative are given by

$$y = c_1 e^{2x} + c_2 e^{-4x}$$

$$y' = 2c_1 e^{2x} - 4c_2 e^{-4x}.$$

Inserting the initial conditions yields

$$3 = c_1 e^{2\cdot 0} + c_2 e^{-4\cdot 0} = c_1 + c_2$$

$$-12 = 2c_1 e^{2\cdot 0} - 4c_2 e^{-4\cdot 0} = 2c_1 - 4c_2.$$

The linear system has the solution $c_2 = 3, \ c_1 = 0$ so that the answer is given by

$$y = 3e^{-4x}.$$

15, (15). $y'' - 4y' - 5y = 0, \ y(-1) = 3, \ y'(-1) = 9$

The characteristic equation is given by

$$r^2 - 4r - 5 = (r-5)(r+1) = 0$$

so there exists two real, distinct roots given by $r = -1, 5$. Hence the general solution and its derivative are given by

$$y = c_1 e^{5x} + c_2 e^{-x}$$

$$y' = 5c_1 e^{5x} - c_2 e^{-x}.$$

Inserting the initial conditions yields

$$3 = c_1 e^{(5)\cdot(-1)} + c_2 e^{(-1)\cdot(-1)} = c_1 e^{-5} + c_2 e$$

$$9 = 5c_1 e^{(5)\cdot(-1)} - c_2 e^{(-1)\cdot(-1)} = 5c_1 e^{-5} - c_2 e.$$

The linear system has the solution $c_1 = 2e^5,\ c_2 = e^{-1}$ so that the answer is given by

$$y = 2e^5 e^{5x} + e^{-1} e^{-x}$$

$$y = 2e^{5(x+1)} + e^{-(x+1)}.$$

17, (17). $z'' - 2z' - 2z = 0,\ z(0) = 0,\ z'(0) = 3$

The characteristic equation is given by

$$r^2 - 2r - 2 = 0$$

so there exists two real, distinct roots given by the quadratic equation

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}.$$

Hence the general solution and its derivative are given by

$$y = c_1 e^{(1+\sqrt{3})x} + c_2 e^{(1-\sqrt{3})x}$$

$$y' = c_1(1+\sqrt{3}) e^{(1+\sqrt{3})x} + c_2(1-\sqrt{3}) e^{(1-\sqrt{3})x}.$$

Inserting the initial conditions yields

$$0 = c_1 e^{(1+\sqrt{3})\cdot 0} + c_2 e^{(1-\sqrt{3})\cdot 0} = c_1 + c_2$$

$$3 = c_1(1+\sqrt{3}) e^{(1+\sqrt{3})\cdot 0} + c_2(1-\sqrt{3}) e^{(1-\sqrt{3})\cdot 0} = c_1(1+\sqrt{3}) + c_2(1-\sqrt{3}).$$

The linear system has the solution $c_1 = \sqrt{3}/2,\ c_2 = -\sqrt{3}/2$ so that the answer is given by

$$y = (\sqrt{3}/2) e^{(1+\sqrt{3})x} - (\sqrt{3}/2) e^{(1-\sqrt{3})x}$$

$$y = (\sqrt{3}/2)(e^{(1+\sqrt{3})x} - e^{(1-\sqrt{3})x}).$$

19, (19). $y'' + 2y' + y = 0,\ y(0) = 1,\ y'(0) = -3$

The characteristic equation is given by

$$r^2 + 2r + 1 = (r+1)^2 = 0$$

so there exists one real, repeated root $r = -1$. Hence the general solution and its derivative are given by

$$y = c_1 e^{-x} + c_2 x e^{-x}$$

$$y' = -c_1 e^{-x} + c_2(-x+1)e^{-x}.$$

Inserting the initial conditions yields

$$1 = c_1 e^0 + c_2(0)e^0 = c_1$$

$$-3 = -c_1 e^0 + c_2(0+1)e^0 = -c_1 + c_2.$$

The linear system has the solution $c_1 = 1,\ c_2 = -2$ so that the answer is given by

$$y = e^{-x} - 2xe^{-x}.$$