UF | UNIVERSITY of FLORIDA

Department of Mathematics

MAP 2302
Exam 3A
Fall 2025

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated (-5 for errors):

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Section number

C. Under "special codes" code in the test ID numbers 3, 1.

1 2 ● 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A. (-5 for errors)

● B C D E

E. 1) This test consists of 11 multiple-choice questions worth 5 points each, plus 4 free-response questions worth 45 points.

2) The time allowed is 90 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay. Blank or missing scantrons or free-response sections will count as zero and are not grounds for a retest.

2) You must turn in your scantron and tear-off sheets to your exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas shortly after the exam. Your instructor will return your tear-off sheet in class. Your score will also be posted in Canvas within one week of the exam.

4) Scantron errors in name, section, form code, or UFID result in a non-negotiable 5-point deduction.

NOTE: Be sure to bubble the answers to questions 1–11 on your scantron. They are worth 5 points each.

1. Determine the Laplace transform of $t^2(t-1)^2$.

   (a) $\frac{2}{s^3} - \frac{8}{s^4} + \frac{24}{s^5}$
   (b) $\frac{2}{s^3} - \frac{12}{s^4} + \frac{12}{s^5}$
   (c) $\frac{2}{s^3} - \frac{12}{s^4} - \frac{12}{s^5}$
   (d) $\frac{2}{s^3} - \frac{12}{s^4} + \frac{24}{s^5}$
   (e) $\frac{1}{s^2} - \frac{12}{s^4} - \frac{24}{s^5}$

2. What is the Laplace transform of $f(t) = te^{2t}\sin t$?

   (a) $\frac{(s-2)}{[(s-2)^2+1]^2}$
   (b) $\frac{2}{[(s-2)^2+1]^2}$
   (c) $\frac{2(s-2)}{[(s-2)^2+1]^2}$
   (d) $\frac{2s}{[(s-2)^2+1]}$
   (e) $\frac{2(s-2)}{[(s-2)^2+1]}$

3. Find the inverse Laplace transform of $\frac{s}{(s^2+4s+5)}$.

   (a) $e^{-t}[\cos 2t - 2\sin 2t]$
   (b) $e^{-2t}[\cos 2t - 2\sin 2t]$
   (c) $e^{-t}[\cos 2t - 2\sin t]$
   (d) $e^{-2t}[2\cos t - \sin t]$
   (e) $e^{-2t}[\cos t - 2\sin t]$

4. Solve the initial value problem using Laplace transforms:

$$y'' + y = \delta(t-\pi), \quad y(0) = 0, \quad y'(0) = 0$$

   What is $y(t)$?

   (a) $u(t-\pi)\cos(t-\pi)$
   (b) $\sin t$
   (c) $\delta(t-\pi)\sin t$
   (d) $u(t-\pi)\sin(t-\pi)$
   (e) $u(t-\pi)$

5. What is the inverse Laplace transform of $\frac{e^{-2s}}{s^2+1}$?

   (a) $\sin(t-2)$
   (b) $u(t-2)\cos(t-2)$
   (c) $\delta(t-2)\sin t$
   (d) $u(t-2)t\sin(t-2)$
   (e) $u(t-2)\sin(t-2)$

6. Find the Laplace transform of

$$y(t) = \int_0^t (t-v)e^{3v}\,dv$$

   (a) $\frac{1}{s^2(s-3)^2}$
   (b) $\frac{1}{s^2(s-3)}$
   (c) $\frac{1}{s(s-3)^2}$
   (d) $\frac{1}{s} + \frac{1}{s-3}$
   (e) $\frac{1}{(s-3)^2(s-2)}$

7.

$$\int_{-\infty}^{\infty} \sin(3t)\delta\left(t - \frac{\pi}{2}\right) dt =$$

   (a) -1
   (b) 0
   (c) 1
   (d) ∞
   (e) $-\infty$

8.

$$\mathcal{L}\{t^3\delta(t-2)\} =$$

   (a) e^{-2s}
   (b) $e^{2s}(s^3 - 6s^2 + 12s - 8)$
   (c) $8e^{-2s}$
   (d) $e^{2s}(s^3 + 6s^2 + 12s + 8)$
   (e) $8e^{s-2}$

9. Which Laplace transform corresponds to the differential equation solution:

$$y'' + 2y' + 2y = 0, \quad y(0) = 0,\ y'(0) = 1$$

   (a) $\frac{s+1}{(s+1)^2+1}$
   (b) $\frac{s}{s^2+2s+2}$
   (c) $\frac{1}{(s+1)^2+1}$
   (d) $\frac{s+2}{s^2+2s+2}$
   (e) $\frac{s^2+1}{s^2+2s+2}$

10. If $f(t) = t^2$, what is $\mathcal{L}\{f(t)u(t-1)\}$?

   (a) $e^{-s} \cdot \frac{2}{s^3}$
   (b) $e^{-s} \cdot \left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right)$
   (c) $e^{-s}\left(\frac{2}{s^3} - \frac{2}{s^2} + \frac{1}{s}\right)$
   (d) $e^{-s}\left(\frac{2}{s^3} - \frac{2}{s^2} - \frac{1}{s}\right)$
   (e) $e^{-s} \cdot \frac{1}{s^2}$

11. Use the convolution theorem to obtain a formula for the solution to the IVP where $g(t)$ is piecewise-continuous on $[0, \infty)$ and of exponential order.

$$y'' - 2y' + y = g(t),\ y(0) = -1,\ y'(0) = 1$$

   (a) $y(t) = te^t - e^t + \int_0^t e^{t-v}(t-v)g(v)\,dv$
   (b) $y(t) = 2te^t - 2e^t + \int_0^t e^{t-v}(t-v)g(v)\,dv$
   (c) $y(t) = 2te^t - e^t + 2\int_0^t e^{t-v}(t-v)g(v)\,dv$
   (d) $y(t) = te^t - e^t + \int_0^t e^{t}(t-v)g(v)\,dv$
   (e) $y(t) = 2te^t - e^t + \int_0^t e^{t-v}(t-v)g(v)\,dv$

MAP 2302 Exam 3A, Part II Free Response

Name (LAST, FIRST, legibly): ______________________________ Section #: ________

(-5 for errors in name or section number)

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. Solve the IVP $y'' - 4y' + 8y = 3\delta(t-3);\ y(0) = 0,\ y'(0) = 0.$ (11 pts)

2. (11 pts) Determine the Laplace Transform of $f(t) = (t-2)e^t$ using the definition of the Laplace Transform. **No credit will be given for any other method or use of tables**.

3. (11 pts) Solve $y(t) + \int_0^t e^{t-v} y(v)\, dv = \cos t$.

4. (12 pts) Let $f(t) = \begin{cases} 0 & t < 5 \\ t - 3 & 5 \leq t < 10 \\ 2 & t \geq 10 \end{cases}$

a. Express f in terms of unit step functions.(5 pts)

b. Find $\mathscr{L}\{f(t)\}$. (3 pts)

c. Find $\mathscr{L}\{e^t f(t)\}$. (3 pts)

d. Is $f(t)$ continuous, piecewise-continuous, or neither? (1 pt)

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid during this exam.

Signature: ____________________

$f(t)$	$\mathcal{L}\{f(t)\}$
C	$\frac{C}{s}, \quad s>0$
t^n	$\frac{n!}{s^{n+1}}, \quad s>0$
e^{at}	$\frac{1}{s-a}, \quad s>a$
$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}, \quad s>a, \quad n=0,1,2,\dots$
$\sin(bt)$	$\frac{b}{s^2+b^2}, \quad s>0$
$\cos(bt)$	$\frac{s}{s^2+b^2}, \quad s>0$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}, \quad s>a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}, \quad s>a$
$u(t-a)$	$\frac{e^{-as}}{s}, \quad s>0$
$\delta(t-a)$	e^{-as}

Table 1: Elementary Laplace Transforms

Property	**Expression**
Linearity	$\mathscr{L}\{f+g\} = \mathscr{L}\{f\} + \mathscr{L}\{g\}$
Constant Scaling	$\mathscr{L}\{cf\} = c\mathscr{L}\{f\}$, for any constant c
Exponential Shift	$\mathscr{L}\{e^{at}f(t)\} = F(s-a)$, where $F(s) = \mathscr{L}\{f(t)\}$
First Derivative	$\mathscr{L}\{f'(t)\} = sF(s) - f(0)$
Second Derivative	$\mathscr{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$
n^{th} Derivative	$\mathscr{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$t^n f(t)$	$\mathscr{L}\{t^n f(t)\} = (-1)^n \frac{d^nF(s)}{ds^n}$
Integral of $f(t)$	$\mathscr{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$
Derivative of $y(t)$	$\mathscr{L}\{y'(t)\} = sY(s) - y(0)$
Second Derivative of $y(t)$	$\mathscr{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$

Table 2: Properties of the Laplace Transform