UNIVERSITY of FLORIDA

Department of Mathematics

MAP 2302
Exam 1A
Fall 2022

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Section number

C. Under "special codes" code in the test ID numbers 1, 1.

● 2 3 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) This test consists of 12 multiple choice questions of five points in value, plus 4 free response questions worth 40 points.

2) The time allowed is 100 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas shortly. Your instructor will return your tearoff sheets with your exam score in lecture. Your score will also be posted in Canvas within one week of the exam.

NOTE: Be sure to bubble the answers to questions 1−12 on your scantron.

Questions 1 − 12 are worth 5 points each.

1. Use the Euler method to find $y(0.2)$ to two decimal places for the differential equation $\dfrac{dy}{dx} = 2x + 3y$, given $y(0) = 1$ with step size $h = 0.1$.

a. $y(0.2) = 1.51$ b. $y(0.2) = 1.41$ c. $y(0.2) = 1.61$

d. $y(0.2) = 1.63$ e. $y(0.2) = 1.71$

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2. How many of the following differential equations are linear?

I. $\dfrac{dy}{dx} + 3xy = \sqrt{\sin(x)y}$ II. $\dfrac{dx}{dt} + 3t^2x = t\sin(x)$

III. $\dfrac{dx}{dt} + 3x^2t = x\sin(t)$ IV. $\dfrac{dy}{dt} + \ln|3t|y = t\sin(y)$

V. $\dfrac{dy}{dx} + e^x y = y\sin(x)$

a. 5 b. 4 c. 3 d. 2 e. 1

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3. If the differential equation below is exact, solve it.

$$(2xy + y^3)\,dx + (x^2 + 3xy^2)\,dy = 0$$

a. $x^2y + xy^2 = C$ b. $x^2y + x^2y^2 = C$ c. $x^2y + y^3 = C$

d. $x^2y + xy^3 = C$ e. The DE is not exact.

4. Solve the initial value problem.

$$\frac{dy}{dx} = \sin(x)\sec(y), \quad y(0) = 0$$

a. $y = \arcsin(1 - \cos(x))$

b. $y = \arcsin(1 + \cos(x))$

c. $y = \arcsin\left(\frac{\pi}{2} - \cos(x)\right)$

d. $y = \arccos(1 - \sin(x))$

e. $y = \arccos\left(\frac{\pi}{2} - \cos(x)\right)$

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5. Where does the Theorem of Existence and Uniqueness imply the existence of a unique solution in the $xy-$plane for the differential equation below?

$$\frac{dy}{dx} = \frac{1 + x^2}{3y - y^2}$$

a. Everywhere except for $y = 0$.

b. Everywhere except for $y = 3$.

c. The solution does not exist anywhere.

d. The solution is not unique anywhere.

e. Everywhere except for $y = 0$ and $y = 3$.

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6. Which of the following equations is separable?

a. $y' - e^y = e^{x+y}$

b. $x\dfrac{dy}{dx} - 2y = x + 2y$

c. $xy\dfrac{dy}{dx} = x^3 + y^3$

d. $(x + y)y' = x - y$

e. $y' = \ln(xy)$

7. The differential equation $2x - 2yy' = 0$ has exactly one solution curve passing through the point $(2, 1)$. Which of the following points is also on that solution curve?

a. $(-1, -1)$ b. (0,0) c. $(\sqrt{5}, \sqrt{2})$

d. $(\sqrt{2}, \sqrt{5})$ e. $(3, 2)$

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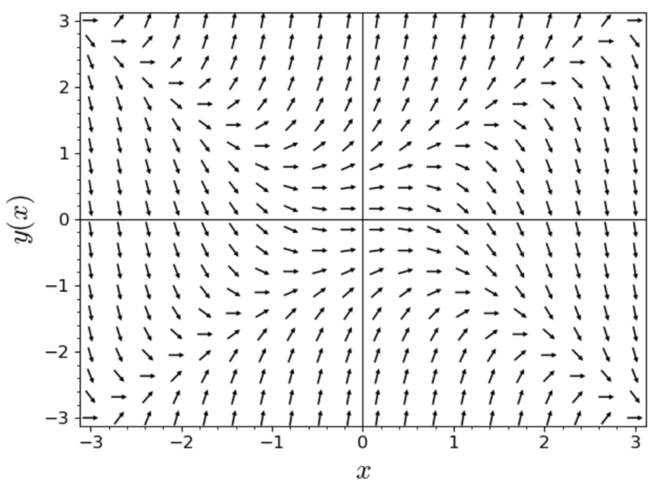
8. The direction field pictured below is described by the differential equation



a. $\dfrac{dy}{dx} = x^2 - y^2$ b. $\dfrac{dy}{dx} = y^2 - x^2$ c. $\dfrac{dy}{dx} = 2y^2 - x^2$

d. $\dfrac{dy}{dx} = y^2 + x^2$ e. $\dfrac{dy}{dx} = y^2 + 2x^2$

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9. Solve $4\dfrac{dw}{dt} + 5e^{t+w} = 0$.

a. $w = \frac{5}{4}e^t - C$ b. $w = \ln\left|\frac{5}{4}t\right| + t - C$

c. $w = -\ln\left|\frac{5e^t}{4} - C\right|$ d. $w = -\ln\left|\frac{5e^t}{4} - Ce^t\right| - t$

e. $w = -\ln\left|\frac{5e^t}{4} - Ce^t\right| + t$

10. Use an appropriate substitution to solve $\dfrac{dy}{dx} = (x+y+1)^2$.

a. $y = x - 1 + \tan(x + C)$

b. $y = x + 1 + \tan(x + C)$

c. $y = x + 1 + \sec^2(x + C)$

d. $y = -x - 1 + \tan(x + C)$

e. $y = -x - 1 + \sec^2(x + C)$

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11. Which of the following are solutions to $y'' + 4y = 0$?

I. $y = \cos(2x)$

II. $y = \sin(2x)$

III. $y = \cos(2x) + \sin(2x)$

IV. $y = \cos(x) + \sin(x)$

a. I and II only

b. II and III only

c. I, II, and III only

d. All of them.

e. I and III only

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12. Solve $\dfrac{dy}{dx} = \dfrac{y^2+1}{y(x+1)}$; $y(0) = 2$.

a. $y = \sqrt{5(x+1)^2 - 1}$

b. $y = \sqrt{6(x+1)^2 - 2}$

c. $y = \sqrt{2(x+1)^2 + 2}$

d. $y = \sqrt{4(x+1)^2}$

e. $y = \sqrt{5 - (x+1)^2}$

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MAP 2302 Exam 1A, Part II Free Response

Name: ______________________________ Section #: ______________

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (9 pts) Solve the first order linear initial value problem $ty' + (t+2)y = \dfrac{3t}{e^t}$, $y(1) = \dfrac{1}{e}$.

$P(t) =$ ______ (1 pt)

$Q(t) =$ ______ (1 pt)

Integrating factor $\mu(t) =$ ____________ (2 pt)

$y(t) =$ ________________________ $+ C$ (4 pt)

$C =$ ______ (1 pt)

2. (11 pts) Solve the Bernoulli initial value problem $\dfrac{dy}{dx} + y = e^x y^{-2},\ y(0) = 2.$

$n =$ ______ (1 pt)

$v =$ ______ (1 pt)

The transformed equation is ________________________ (1 pt)

Integrating factor $\mu(x) =$ __________ (2 pt)

$y(x) =$ ____________________ (5 pt)

$C =$ ______ (1 pt)

3. (11 pts) Solve the initial value problem $(2y^2+2y+4x^2)\,dx+(2xy+x)\,dy=0,\ y(1)=3.$

a. Show that the equation is not exact. (2 pts)

b. Find an integrating factor depending on either x or y alone. (2 pts)

c. Multiply the DE by the integrating factor and show that the new equation is exact. (2 pts)

d. Solve the DE. (3 pts)

e. Apply the initial condition to evaluate the constant. (2 pts)

4. (9 pts) Solve the homogeneous initial value problem $(x^2 + 2y^2) = xy\frac{dy}{dx}$, $y(-1) = 1$. Your answer should be in explicit form for y.

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ______________________________