UF | UNIVERSITY of FLORIDA
Department of Mathematics

MAP 2302
Exam 2A
Fall 2022

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Section number

C. Under "special codes" code in the test ID numbers 2, 1.

1 ● 3 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) This test consists of 12 multiple choice questions of five points in value, plus 4 free response questions worth 40 points.

2) The time allowed is 100 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas within one day after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted in Canvas within one week of the exam.

NOTE: Be sure to bubble the answers to questions 1−12 on your scantron. They are worth 5 points each.

1. Solve the initial value problem $y'' - 4y' + 4y = 0;\ y(0) = 4,\ y'(0) = \dfrac{43}{5}$.

a. $y(t) = -\dfrac{4}{5}e^{2t} + \dfrac{24}{5}te^{2t}$

b. $y(t) = -\dfrac{9}{5}e^{2t} + \dfrac{1}{5}te^{2t}$

c. $y(t) = -4e^{2t} + \dfrac{9}{5}te^{2t}$

d. $y(t) = 4e^{2t} + \dfrac{3}{5}te^{2t}$

e. $y(t) = \dfrac{9}{5}e^{2t} + \dfrac{1}{5}te^{2t}$

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2. Using the method of undetermined coefficients, determine the form of a particular solution for the differential equation. Do not solve for the coefficients.

$$y'' + 4y' + 5y = 3x^2e^{-2x}\cos x.$$

a. $y_p(x) = e^{-2x}[Ax^2\cos x + Bx^2\sin x]$

b. $y_p(x) = xe^{-2x}[Ax^2\cos x + Bx^2\sin x]$

c. $y_p(x) = e^{-2x}[(A_2x^2 + A_1x + A_0)\cos x + Bx^2\sin x]$

d. $y_p(x) = xe^{-2x}[(A_2x^2 + A_1x + A_0)\cos x + (B_2x^2 + B_1x + B_0)\sin x]$

e. $y_p(x) = x^2e^{-2x}[(A_2x^2 + A_1x + A_0)\cos x + (B_2x^2 + B_1x + B_0)\sin x]$

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3. Solve $y''' - y'' - 5y' - 3y = 0$.

a. $y(t) = c_1e^{3t} + c_2e^{-2t} + c_3te^{-t}$

b. $y(t) = c_1e^{-3t} + c_2e^{-2t} + c_3te^{-t}$

c. $y(t) = c_1e^{2t} + c_2e^{-2t} + c_3te^{-t}$

d. $y(t) = c_1e^{3t} + c_2e^{-t} + c_3te^{-t}$

e. $y(t) = c_1e^{-3t} + c_2e^{-2t} + c_3te^{2t}$

4. Solve the differential equation.

$$t^2\,y'' - t\,y' + 21\,y = 0,\ t < 0$$

a. $y(t) = c_1 e^t \cos[-2\sqrt{5}\ln(t)] + c_2 e^t \sin[-2\sqrt{5}\ln(t)]$

b. $y(t) = c_1 e^t \cos[2\sqrt{5}\ln(-t)] + c_2 e^t \sin[2\sqrt{5}\ln(-t)]$

c. $y(t) = c_1 t \cos[2\sqrt{5}\ln(-t)] + c_2 t \sin[2\sqrt{5}\ln(-t)]$

d. $y(t) = c_1 e^t \cos[-2\sqrt{5}t] + c_2 e^t \sin[-2\sqrt{5}t]$

e. $y(t) = c_1 t \cos[-2\sqrt{5}\ln(t)] + c_2 t \sin[-2\sqrt{5}\ln(t)]$

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5. On which interval is a solution guaranteed for the initial value problem

$$t(t-3)y'' + 4ty' - y = t^2,\ y(-4) = Y_0,\ y'(-4) = Y_1,$$

where Y_0 and Y_1 are real constants?

a. $(0, 3)$ b. $(3, \infty)$ c. $(-\infty, -3)$ d. $(-\infty, 0)$ e. $(-3, 0)$

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6. Find a particular solution to the differential equation $y'' + 4y' + 4y = 4e^{-2t}$.

a. $y_p(t) = 3e^{-2t}$

b. $y_p(t) = 2te^{-2t}$

c. $y_p(t) = 3t^2e^{-2t}$

d. $y_p(t) = 2t^2e^{-2t}$

e. $y_p(t) = 3te^{-2t}$

7. Consider the initial value problem $y'' - 2y' + y = 0,\ y(0) = 2,\ y'(0) = -1.$

I. Find the solution $y(t)$.

II. Find $\lim_{t\to\infty} y(t)$.

a. I. $y(t) = -2e^t + 3te^t$, II. $-\infty$

b. I. $y(t) = 2e^t - 3te^t$, II. $-\infty$

c. I. $y(t) = 2e^t - te^{-t}$, II. $-\infty$

d. I. $y(t) = 2e^t - 3te^t$, II. ∞

e. I. $y(t) = 2e^t - 3te^t$, II. 0

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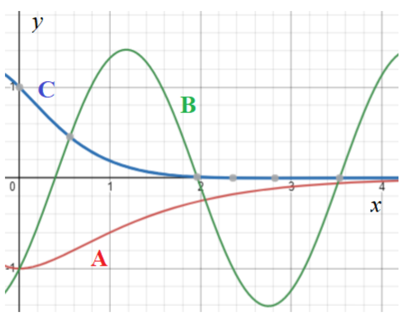
8. Which of the following could NOT be a graph of the solution to the initial value problem

$$y'' + 3y' + 2y = 0;\ y(0) = -1,\ y'(0) = 0?$$



a. A only b. B and C only c. C only d. B only e. A and C

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9. Given that $y_1(t) = \dfrac{1}{4}\sin 2t$ is a solution to $y'' + 2y' + 4y = \cos 2t$ and that $y_2(t) = \dfrac{t}{4} - \dfrac{1}{8}$ is a solution to $y'' + 2y' + 4y = t$, use the superposition principle to find a solution to the differential equation $y' + 2y' + 4y = 11t - 12\cos 2t$.

a. $y(t) = 11t/2 - 11/8 - 3\sin(2t)$

b. $y(t) = 11t/4 - 11/2 - 3\cos(2t)$

c. $y(t) = 11t/4 - 11/2 - 3\sin(2t)$

d. $y(t) = 11t/4 - 11/8 - 3\sin(2t)$

e. $y(t) = 11t/2 - 11/2 - 3\sin(2t)$

10. Solve $t^2z'' - 17tz' + 81z = 0,\ t > 0$.

a. $z = c_1t^9 + c_2e^{9t}\ln t$

b. $z = c_1e^{9t} + c_2te^{9t}$

c. $z = c_1e^{9t} + c_2t^9\ln t$

d. $z = c_1t^9 + c_2t^8\ln t$

e. $z = c_1t^9 + c_2t^9\ln t$

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11. To which of the following can the method of undetermined coefficients be applied?

I. $y'' + 18y' = (e^t + 4t)^2$

II. $y'' + y' + y = 3^t\sin(2t)$

III. $y'' + 3y' + 2y = \dfrac{\cos(2x)}{x} + \sin(2x)$

IV. $3ty'' + 2y' + 2y = t^2 + t5^t$

a. I and III only b. II and III only c. II only
d. All of them. e. I and II only

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12. Which of the following pairs of functions are linearly **independent** on the interval (0,1)?

I. $y_1 = 2\cos^2 t - 2,\ y_2 = 14\sin^2 t$

II. $y_1 = e^{5t},\ y_2 = e^{-6t}$

III. $y_1 = \tan^2 t - \sec^2 t,\ y_2 = 11$

IV. $y_1 = e^{4t},\ y_2 = te^{4t}$

a. I,IV b. II,IV c. I,III d. II,III e. all of them

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MAP 2302 Exam 2A, Part II Free Response

Name: ______________________________ Section #: ______________

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (9 pts) A differential equation and nontrivial solution $y_1(x)$ are given below. Find a second linearly independent solution using reduction of order.

$$(x-1)y'' - xy' + y = 0,\ x > 1,\ y_1(x) = e^x$$

2. (11 pts) Use variation of parameters to solve $y'' + 4y' + 4y = t^{-2}e^{-2t}, \ t > 0$.

a. Find the Wronksians $W, \ W_1,$ and W_2. (5 pts)

b. Find v_1. (2 pts)

c. Find v_2. (2 pts)

d. Find the general solution, combining like terms if applicable. (2 pts)

3. (10 pts) In the mass-spring oscillator, mass position $y(t)$ is given by $my'' + by' + ky = 0$.

3a. Solve the initial value problem representing the vibrating spring with damping if $m = 10$ kg, $b = 100$ kg/sec, $k = 290$ kg/sec^2, $y(0) = 0$ m, and $y'(0) = -0.5$ m/sec. (3 pts)

3b. Now set $b = 0$ kg/sec and solve the initial value problem using the same values for $m, k, y(0)$, and $y'(0)$ as in part 3a. (3 pts)

3c. Does damping increase or decrease the frequency of oscillation? (1 pt)

3d. Is the force of friction in the same direction or the opposite direction of the spring? (1 pt)

3e. Does the magnitude of oscillations increase or decrease with time? (1 pt)

3f. Is friction putting energy into or taking energy out of the system? (1 pt)

4. (10 pts) Solve $y'' + y = 5xe^{-3x},\ y(0) = 0,\ y'(0) = 0.$

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ____