UF UNIVERSITY of FLORIDA
Department of Mathematics

MAP 2302
Exam 4A
Fall 2022

A. Sign your bubble sheet on the back at the bottom <u>in ink</u>.

B. In pencil, write and encode in the spaces indicated:

- **1)** Name (last name, first initial, middle initial) (-5 if not coded or coded incorrectly)
- **2)** UF ID number (-5 if not coded or coded incorrectly)
- **3)** Section number (-5 if not coded or coded incorrectly)

C. Under “special codes” code in the test ID numbers 4, 1.

1 2 3 ● 5 6 7 8 9 0
● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for “Test Form Code”, encode A.

● B C D E (-5 if not coded or coded incorrectly)

E.
- **1)** This test consists of 20 multiple choice questions of five points in value.
- **2)** The time allowed is 120 minutes.
- **3)** You may write on the test.
- **4)** Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

- **1)** Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.
- **2)** You must turn in your scantron to your exam proctor. Be prepared to show your picture <u>I.D. with a legible signature</u>.
- **3)** The answers will be posted in Canvas shortly after the exam.

NOTE: Be sure to bubble the answers to questions 1−20 on your scantron.

Questions 1 – 20 are worth 5 points each.

1. Using the method of undetermined coefficients, determine the form of a particular solution for the differential equation

$$y'' + 64y = 9t^4 \sin(8t) - 2\cos(8t)$$

a. $y_p(t) = e^t[(A_4t^4 + A_3t^3 + A_2t^2 + A_1t + A_0)\cos(8t) + (B_4t^4 + B_3t^3 + B_2t^2 + B_1t + B_0)\sin(8t)]$

b. $y_p(t) = te^t[A\cos(8t) + ((B_4t^4 + B_3t^3 + B_2t^2 + B_1t + B_0)\sin(8t)]$

c. $y_p(t) = t^2[(A_4t^4 + A_3t^3 + A_2t^2 + A_1t + A_0)\cos(8t) + (B_4t^4 + B_3t^3 + B_2t^2 + B_1t + B_0)\sin(8t)]$

d. $y_p(t) = t[(A_4t^4 + A_3t^3 + A_2t^2 + A_1t + A_0)\cos(8t) + (B_4t^4 + B_3t^3 + B_2t^2 + B_1t + B_0)\sin(8t)]$

e. $y_p(t) = t^2[A\cos(8t) + ((B_4t^4 + B_3t^3 + B_2t^2 + B_1t + B_0)\sin(8t)]$

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2. Solve the initial value problem

$$y'' + 6y' + 25y = 0, \ y(0) = 2, \ y'(0) = -13$$

a. $y(t) = e^{3t}\left[4\cos(4t) + \frac{9}{4}\sin(4t)\right]$

b. $y(t) = e^{-3t}\left[2\cos(4t) + \frac{1}{2}\sin(4t)\right]$

c. $y(t) = e^{-3t}\left[4\cos(4t) - \frac{9}{4}\sin(4t)\right]$

d. $y(t) = e^{-3t}\left[2\cos(4t) - \frac{9}{4}\sin(4t)\right]$

e. $y(t) = e^{-3t}\left[2\cos(4t) - \frac{7}{4}\sin(4t)\right]$

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3. A solution to the initial value problem below, where Y_0 and Y_1 are real constants, is guaranteed on which interval?

$$(7 + t^2)\, y'' + t\, y' - y = \sec^2 t, \ y(-3) = Y_0, \ y'(-3) = Y_1,$$

a. $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$ b. $\left(0, \frac{\pi}{2}\right)$ c. $\left(0, \frac{3\pi}{2}\right)$ d. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

e. There is no guarantee on any interval.

4. The power series $1 + \dfrac{x}{2} + \dfrac{x^2}{4} + \dfrac{x^3}{8} + ...$ represents which of the following functions?

a. $\cos(\frac{x}{2})$ b. $\sin(\frac{x}{2})$ c. $\frac{2}{2-x}$ d. $\frac{2}{1-x}$ e. $\sqrt{1+x}$

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5. Suppose the recurrence relation for the coefficients of an infinite series representing the solution to a differential equation is $\sum_{n=0}^{\infty} c_n x^n$ where $c_{n+1} = \dfrac{n+2}{3(n+1)} c_n,\ n \geq 0$. What is the radius of convergence?

a. 1 b. 2 c. $\frac{1}{3}$ d. 3 e. 9

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6. Consider the following power series.

$$\text{I.} \sum_{n=0}^{\infty} c_n x^n \quad \text{II.} \sum_{n=3}^{\infty} c_n n(n-1)(n-2)x^{n-3} \quad \text{III.} \sum_{n=1}^{\infty} c_n x^n \quad \text{IV.} \sum_{n=1}^{\infty} c_{n+2}(n+2)(n+1)(n)x^{n-1}$$

Which of these represent the same function?

a. I and IV only.

b. II and III only.

c. I and III only.

d. II and IV only.

e. They all represent different functions.

7. Find the first five nonzero terms in a Taylor series solution to the IVP

$$y'' = x + y - y^2,\ y(0) = -1,\ y'(0) = 1.$$

a. $y = -1 + x - x^2 + \frac{2x^3}{3} - \frac{x^4}{3}$ b. $y = -1 + x - x^2 - \frac{2x^3}{3} - \frac{x^4}{3}$

c. $y = -1 + x - x^2 + \frac{2x^3}{3} - \frac{x^4}{2}$ d. $y = -1 + x - x^2 + \frac{2x^3}{3} + \frac{x^4}{3}$

e. $y = -1 + x - x^2 + \frac{2x^3}{3} + \frac{x^4}{2}$

Questions 8-9 form a group.

8. Find the recurrence relation for a power series solution about $x = 0$ for $y'' + xy = 0$.

a. $a_{n+1} = -\dfrac{a_{n-1}}{(n+1)(n+2)},\ n = 1, 2, 3, ...$ b. $a_{n+2} = -\dfrac{a_{n-1}}{(n+1)(n+2)},\ n = 1, 2, 3, ...$

c. $a_{n+2} = -\dfrac{a_n}{(n+1)(n+2)},\ n = 1, 2, 3, ...$ d. $a_{n+1} = -\dfrac{a_n}{(n+1)(n+2)},\ n = 1, 2, 3, ...$

e. $a_{n+2} = \dfrac{a_{n-1}}{(n+1)(n+2)},\ n = 1, 2, 3, ...$

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9. (Continued from above) Find the first four nonzero terms.

a. $y = a_0[1 - \frac{x^3}{6} + ...\,] + a_1[x - \frac{x^4}{12} + ...\,]$ b. $y = a_0[1 - \frac{x^3}{2} + ...\,] + a_1[x - \frac{x^4}{12} + ...\,]$

c. $y = a_0[1 - \frac{x^3}{6} + ...\,] + a_1[x - \frac{x^4}{6} + ...\,]$ d. $y = a_0[1 - \frac{x^3}{4} + ...\,] + a_1[x - \frac{x^4}{12} + ...\,]$

e. $y = a_0[1 - \frac{x^3}{6} + ...\,] + a_1[x - \frac{x^4}{8} + ...\,]$

10. Find the first four nonzero terms in a power series solution about $x = 0$ to the differential equation $(\cos x)y'' + y = 0$.

a. $y = a_0[1 + \frac{x^2}{2} + \dots] + a_1[x - \frac{x^3}{6} + \dots]$

b. $y = a_0[1 - \frac{x^2}{2} + \dots] + a_1[x - \frac{x^3}{6} + \dots]$

c. $y = a_0[1 - \frac{x^2}{3} + \dots] + a_1[x - \frac{x^3}{6} + \dots]$

d. $y = a_0[1 - \frac{x^2}{2} + \dots] + a_1[x - \frac{x^3}{3} + \dots]$

e. $y = a_0[1 - \frac{x^2}{4} + \dots] + a_1[x - \frac{x^3}{6} + \dots]$

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11. Find a minimum value for the radius of convergence of a power series solution about the ordinary point $x = 1$ for the differential equation $(x^2 - 2x + 10)y'' + xy' - 4y = 0$.

a. 2 b. 3 c. $\sqrt{10}$ d. 4 e. $2\sqrt{10}$

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12. The singular points of the DE $(x^2 - 4)y'' + 3(x - 2)y' + 5y = 0$ are

a. There are no singular points.

b. -2, 0, and 2.

c. 2 only.

d. -2 only.

e. -2 and 2 only.

Questions 13-14 form a group.

13. Using the method of variation of parameters, the DE $y'' - 4y' + 4y = (x+1)e^{2x}$ has

a. $W = e^{4x},\ W_1 = -x^2e^{4x},\ W_2 = xe^{4x}$

b. $W = e^{4x},\ W_1 = -(x+1)e^{4x},\ W_2 = xe^{4x}$

c. $W = e^{4x},\ W_1 = -(x+1)xe^{4x},\ W_2 = (x+1)e^{4x}$

d. $W = e^{4x},\ W_1 = (x+1)xe^{4x},\ W_2 = (x+1)e^{4x}$

e. $W = e^{4x},\ W_1 = -(x+1)xe^{4x},\ W_2 = -(x+1)e^{4x}$

14. (Continued from 13.) The particular solution $y_p(x) =$

a. $\frac{1}{6}x^3e^{2x} + \frac{1}{2}x^2e^{4x}$ b. $\frac{1}{6}x^3e^{2x} + \frac{1}{2}x^2e^{2x}$

c. $\frac{1}{6}x^3e^{2x} + \frac{3}{2}x^2e^{2x}$ d. $\frac{1}{3}x^3e^{4x} + \frac{1}{2}x^2e^{2x}$

e. $\frac{1}{3}x^3e^{2x} + \frac{1}{2}x^2e^{2x}$

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15. Solve $4x^2y'' + 17y = 0,\ y(1) = -1,\ y'(1) = -\frac{1}{2}$.

a. $y(x) = -\frac{1}{2}\sqrt{x}\cos(2\ln x)$

b. $y(x) = -2\sqrt{x}\sin(2\ln x)$

c. $y(x) = -\sqrt{x}\cos(2\ln x)$

d. $y(x) = -2\sqrt{x}\cos(2\ln x)$

e. $y(x) = -\sqrt{x}\sin(2\ln x)$

16. Solve $\dfrac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$.

a. $2\sqrt{y-2x+3} = x + c$

b. $2\sqrt{y-2x+3} = x^2 + c$

c. $2\sqrt{y-2x+3} = \ln x + c$

d. $\frac{1}{2}\sqrt{y-2x+3} = x + c$

e. $\sqrt{y-2x+3} = x + c$

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17. Find $Y(s)$ for the initial value problem $y'' + 4y' + 6y = 1 + e^{-t},\ y(0) = y'(0) = 0$.

a. $Y(s) = \dfrac{2s+1}{s(s+1)(s^2+4s+4)}$

b. $Y(s) = \dfrac{2s+1}{s(s+1)(s^2+4s+6)}$

c. $Y(s) = \dfrac{2s+1}{s(s+1)(s^2+4s+3)}$

d. $Y(s) = \dfrac{s+1}{s(s+1)(s^2+4s+6)}$

e. $Y(s) = \dfrac{s+2}{s(s+1)(s^2+4s+6)}$

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18. Find the inverse Laplace transform of $\dfrac{s}{s^2+9}e^{-\pi s/2}$.

a. $\cos[3(t-\frac{\pi}{2})]u(t-\frac{\pi}{2})$ b. $\sin[3(t-\frac{\pi}{2})]u(t-\frac{\pi}{2})$

c. $\cos[2(t-\frac{\pi}{2})]u(t-\frac{\pi}{2})$ d. $\sin[2(t-\frac{\pi}{2})]u(t-\frac{\pi}{2})$

e. $2\cos(t-\pi)u(t-\frac{\pi}{2})$

19. Which of the following statements is **FALSE**?

a. The Laplace transform of $\delta(t)$ is 1.

b. The function $f(t) = \cos(kt)$ is of exponential order for any real value of k.

c. If $f(t)$ is piecewise continuous on $[0, \infty)$ and of exponential order α, then the Laplace transform of $f(t)$ exists for some $s > \alpha$.

d. $f(t) = \dfrac{1}{t-3}$ is piecewise continuous on $[0, \infty)$.

e. If $f(t)$ is piecewise continuous on $[0, \infty)$ and of exponential order and $F(s) = \mathscr{L}\{f(t)\}(s)$, $\lim_{s\to\infty} F(s) = 0$

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20. Let $y = kx$ where k is a constant. The orthognoal trajectories are

a. lines b. parabolas c. circles d. ellipses e. hyperbolic paraboloids

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$f(x)$	Maclaurin Series	I.O.C.
$\dfrac{1}{1-x}$	$= \sum_{n=0}^{\infty} x^n,$	$(-1, 1)$
$\ln(1-x)$	$= -\sum_{n=1}^{\infty} \dfrac{x^n}{n},$	$[-1, 1)$
$\ln(1+x)$	$= \sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{x^n}{n},$	$(-1, 1]$
$\arctan x$	$= \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)},$	$[-1, 1]$
e^x	$= \sum_{n=0}^{\infty} \dfrac{x^n}{n!},$	$(-\infty, \infty)$
$\cos x$	$= \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!},$	$(-\infty, \infty)$
$\sin x$	$= \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!},$	$(-\infty, \infty)$

Elementary Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\}$
C	$\frac{C}{s}, s > 0$
t^n	$\frac{n!}{s^{n+1}}, s > 0$
e^{at}	$\frac{1}{s-a}, \quad s > a$
$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a,\ n = 0, 1, 2, \ldots$
$\sin(bt)$	$\frac{b}{s^2+b^2}, s > 0$
$\cos(bt)$	$\frac{s}{s^2+b^2}, s > 0$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
$\sinh(bt)$	$\frac{b}{s^2-b^2}, s > \lvert b \rvert$
$\cosh(bt)$	$\frac{s}{s^2-b^2}, s > \lvert b \rvert$
$u(t-a)$	$\frac{e^{-as}}{s}, s > 0$
$\delta(t-a)$	e^{-as}

Properties of the Laplace transform

$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$

$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$ for any constant c

$\mathcal{L}\{e^{at}f(t)\}(s) = F(s-a)$ where $F(s) = \mathcal{L}\{f\}(s)$

$\mathcal{L}\{f'\}(s) = sF(s) - f(0)$

$\mathcal{L}\{f''\}(s) = s^2F(s) - sf(0) - f'(0)$

$\mathcal{L}\{f^{(n)}\}(s) = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$

$\mathcal{L}\{t^nf(t)\} = (-1)^n\frac{d^nF}{ds^n}(s)$

$\mathcal{L}\{ty'\}(s) = -sY'(s) - Y(s)$ where $Y(s) = \mathcal{L}\{y\}(s)$

$\mathcal{L}\{ty''\}(s) = -s^2Y'(s) - 2sY(s) + y(0)$