SECTION 2.3

7, (8). $\frac{dy}{dx} - y - e^{3x} = 0$

Placing the equation in standard form yields

$$\frac{dy}{dx} - y = e^{3x}.$$

The integrating factor is obtained as follows:

$$\mu(x) = e^{\int -1 \, dx} = e^{-x}.$$

Hence

$$\begin{aligned}
(ye^{-x})' &= e^{3x}e^{-x} = e^{2x} \\
ye^{-x} &= \int e^{2x} \, dx + C \\
ye^{-x} &= \frac{1}{2}e^{2x} + C \\
y &= \frac{1}{2}e^{3x} + Ce^{x}.
\end{aligned}$$

9, (10). $\frac{dr}{d\theta} + r\tan\theta = \sec\theta$

Since the equation is already in standard form we obtain the integrating factor as follows:

$$\mu(\theta) = e^{\int \tan\theta \, d\theta} = e^{-\ln|\cos\theta|} = \frac{1}{|\cos\theta|}.$$

Hence

$$\begin{aligned}
\left(\frac{r}{|\cos\theta|}\right)' &= \frac{\sec\theta}{|\cos\theta|} \\
\left(\frac{r}{\cos\theta}\right)' &= \frac{\sec\theta}{\cos\theta} = \sec^2\theta \\
\frac{r}{\cos\theta} &= \int \sec^2\theta \, d\theta + C
\end{aligned}$$

$$\frac{r}{\cos\theta} = \tan\theta + C$$
$$r = \sin\theta + C\cos\theta.$$

11, (11). $(t + y + 1)dt - dy = 0$

Placing the equation in standard form yields

$$\frac{dy}{dt} - y = t + 1.$$

The integrating factor is obtained as follows:

$$\mu(t) = e^{\int -1\ dt} = e^{-t}.$$

Hence

$$(ye^{-t})' = e^{-t}(t + 1)$$
$$ye^{-t} = \int e^{-t}(t + 1)\ dt + C.$$

We integrate by parts to obtain

$$ye^{-t} = -e^{-t}(t + 1) + \int e^{-t}\ dt + C$$
$$ye^{-t} = -e^{-t}(t + 1) - e^{-t} + C$$
$$y = -t - 2 + Ce^{t}.$$

13, (13). $y\frac{dx}{dy} + 2x = 5y^3$

Placing the equation in standard form yields

$$\frac{dx}{dy} + \frac{2}{y}x = 5y^2.$$

The integrating factor is obtained as follows:

$$\mu(y) = e^{\int \frac{2}{y}\ dy} = e^{2\ln|y|} = y^2.$$

Hence

$$(xy^2)' = 5y^4$$

$$xy^2 = \int 5y^4 \, dy + C$$
$$xy^2 = y^5 + C$$
$$x = y^3 + Cy^{-2}.$$

15, (15). $(x^2+1)\frac{dy}{dx} + xy - x = 0$

Placing the equation in standard form yields

$$\frac{dy}{dx} + \frac{x}{x^2+1}y = \frac{x}{x^2+1}.$$

The integrating factor is obtained as follows:

$$\mu(x) = e^{\int \frac{x}{x^2+1} \, dx} = e^{\frac{1}{2}\ln|x^2+1|} = \sqrt{x^2+1}.$$

Hence

$$(y\sqrt{x^2+1})' = \frac{x\sqrt{x^2+1}}{x^2+1} = \frac{x}{\sqrt{x^2+1}}$$
$$y\sqrt{x^2+1} = \int \frac{x}{\sqrt{x^2+1}} \, dx + C$$
$$y\sqrt{x^2+1} = \sqrt{x^2+1} + C$$
$$y = 1 + \frac{C}{\sqrt{x^2+1}}.$$

17, (17). $\frac{dy}{dx} - \frac{y}{x} = xe^x, \; y(1) = e - 1$

Since the equation is already in standard form we obtain the integrating factor as follows:

$$\mu(x) = e^{\int \frac{-1}{x} \, dx} = e^{-\ln|x|} = \frac{1}{|x|}.$$

Hence

$$\left(y\frac{1}{|x|}\right)' = \frac{xe^x}{|x|}$$

$$
\begin{aligned}
\left(\frac{y}{x}\right)' &= \frac{xe^x}{x} = e^x \\
\frac{y}{x} &= \int\ e^x\ dx + C \\
\frac{y}{x} &= e^x + C \\
y &= xe^x + Cx.
\end{aligned}
$$

Inserting the initial conditions yields

$$
\begin{aligned}
e - 1 &= (1)e^1 + C(1) \\
C &= -1.
\end{aligned}
$$

Hence

$$
y = xe^x - x.
$$

19, (-). $t^3 \frac{dx}{dt} + 3t^2 x = t,\ \ x(2) = 0$

Placing the equation in standard form yields

$$
\frac{dx}{dt} + 3t^{-1}x = t^{-2}.
$$

The integrating factor is obtained as follows:

$$
\mu(t) = e^{\int\ \frac{3}{t}\ dt} = e^{3\ln|t|} = |t|^3.
$$

Hence

$$
\begin{aligned}
(x|t|^3)' &= |t|^3 t^{-2} \\
(xt^3)' &= t^3 t^{-2} = t \\
xt^3 &= \int\ t\ dt + C \\
xt^3 &= \frac{t^2}{2} + C \\
x &= \frac{1}{2t} + \frac{C}{t^3}.
\end{aligned}
$$

Inserting the initial conditions yields

$$
0 = \frac{1}{2 \cdot 2} + \frac{C}{2^3}
$$

$$0 = \frac{1}{4} + \frac{C}{8}$$
$$C = -2.$$

Hence

$$x = \frac{1}{2t} - \frac{2}{t^3}.$$

21, (21). $\cos x \frac{dy}{dx} + y \sin x = 2x \cos^2 x,\ y(\frac{\pi}{4}) = \frac{-15\sqrt{2}\pi^2}{32}$

Placing the equation in standard form yields

$$\frac{dy}{dx} + y \tan x = 2x \cos x.$$

The integrating factor is obtained as follows:

$$\mu(x) = e^{\int \tan x \ dt} = e^{-\ln|\cos x|} = |\sec x|.$$

Hence

$$(y|\sec x|)' = |\sec x|(2x \cos x)$$
$$(y \sec x)' = \sec x(2x \cos x) = 2x$$
$$y \sec x = \int 2x \ dx + C$$
$$y \sec x = x^2 + C$$
$$y = x^2 \cos x + C \cos x.$$

Inserting the initial conditions yields

$$\frac{-15\sqrt{2}\pi^2}{32} = \left(\frac{\pi}{4}\right)^2 \cos\left(\frac{\pi}{4}\right) + C\cos\left(\frac{\pi}{4}\right)$$
$$\frac{-15\sqrt{2}\pi^2}{32} = \left(\frac{\pi}{4}\right)^2 \frac{\sqrt{2}}{2} + C\frac{\sqrt{2}}{2}$$
$$\frac{-15\pi^2}{16} = \left(\frac{\pi}{4}\right)^2 + C$$
$$-\pi^2 = C.$$

Hence

$$y = x^2 \cos x - \pi^2 \cos x = (x^2 - \pi^2) \cos x.$$