UF | UNIVERSITY of FLORIDA

Department of Mathematics

MAP 2302
Exam 2A
Spring 2023

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Section number

C. Under "special codes" code in the test ID numbers 2, 1.

1 ● 3 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. **1)** This test consists of 12 multiple choice questions of 5.5 points in value, plus 4 free response questions worth 40 points.

2) The time allowed is 100 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas within one day after the exam.

4) Errors in name, UFID, section, and form code will result in -5 points.

NOTE: Be sure to bubble the answers to questions 1−12 on your scantron. They are worth 5.5 points each.

1. Find the form of a particular solution to the DE. Do not solve for the coefficients.

$$y'' - 4y' + 13y = 5xe^{2x} + 3e^{2x}\sin(3x) + 7\cos x$$

a. $y = (Ax + B)e^{2x} + xe^{2x}(C\cos(3x) + D\sin(3x)) + E\cos x$.

b. $y = Axe^{2x} + xe^{2x}(C\cos(3x) + D\sin(3x)) + E\cos x + F\sin x$.

c. $y = (Ax + B)e^{2x} + xe^{2x}(C\cos(3x) + D\sin(3x)) + E\cos x + F\sin x$.

d. $y = (Ax + B)e^{2x} + (Cx + D)e^{2x}(E\cos(3x) + F\sin(3x)) + G\cos x + H\sin x$.

e. $y = (Ax + B)e^{2x} + e^{2x}(C\cos(3x) + D\sin(3x)) + E\cos x + F\sin x$.

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2. Which of the following DOES NOT exploit resonance?

a. nuclear magnetic resonance

b. AM radio

c. piano tuning

d. microwave oven

e. feedback from electric guitar

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3. To which of the following differential equations does the method of undetermined coefficients apply?

a. $y'' - 729y = t^2 - \dfrac{3}{t^2}$

b. $y'' + 18y' - y = \sqrt{t^2 + 1}$

c. $y'' + 7y = \sec^2(t) - 1$

d. $4y'' + 4y' - y = 5\sin(3t)\cos(3t)$

e. $6y'' + 8y' + 11y = \ln(e^t + 1)$

4. Which of the following equations would NOT have a particular solution of the form $y_p = P(t)e^{6t}$, where $P(t)$ is a fourth degree polynomial?

a. $y'' - 8y' + 10y = 2t^3e^{6t} - t^4e^{6t}$

b. $y'' - 8y' + 12y = 3e^{6t} + 2t^3e^{6t}$

c. $y'' - 12y' + 36y = t^2e^{6t} - 4e^{6t}$

d. $y'' + 4y' - 60y = 12t^4e^{6t} + t^2e^{6t}$

e. $y'' - y' + y = t^4e^{6t} - 4e^{6t}$

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5. Consider the differential equation

$$y'' + by' + 9y = t^2e^{-3t} + \sin(3t) + 2\cos(4t),$$

where b is a constant. For which value of b will the amplitude of a particular solution $y_p(t)$ be unbounded as $t \to \infty$?

a. 3

b. 4

c. 6

d. 10

e. 0

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6. Which of the following initial value problems would be guaranteed by the Existence and Uniqueness Theorem to have a unique solution on the interval $(-2, 3)$?

a. $x^2y'' - 3y' + y = e^x;\ y(1) = 1$

b. $4y'' + \dfrac{x^2+2}{x^2-4x+3}y' - y = 12;\ y(0) = \sqrt{3}$

c. $y'' + 8y' + 16y = 2\ln t;\ y(-1) = e$

d. $(x^2+1)y'' + xy' + y\tan(x) = 0;\ y(2) = 0$

e. None of the above

7. Which of the following differential equations has solutions that oscillate with constant amplitude as $t \to \infty$?

a. $y'' - 3y' + 5y = 0$

b. $2y'' + y = 0$

c. $4y'' + 4y' + y = 0$

d. $y'' + 2y' + 6y = 0$

e. $y'' + 4y' + 4y = 0$

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8. Solve the initial value problem.

$$y'' - 4y' + 5y = 0; \quad y(0) = 1, \quad y'(0) = 0$$

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a. $y(t) = 2e^{2t}\cos(t) - 2e^{2t}\sin(t)$

b. $y(t) = 2e^{t}\cos(2t) - e^{t}\sin(2t)$

c. $y(t) = e^{t}\cos(2t) + 2e^{t}\sin(2t)$

d. $y(t) = e^{2t}\cos(t) - 2e^{2t}\sin(t)$

e. $y(t) = e^{2t}\cos(t) + 2e^{2t}\sin(t)$

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9. Find two linearly independent solutions to the equation $t^2y'' - 9ty' + 25y = 0,\ t > 0$.

a. $y_1(t) = t,\ y_2(t) = t^{-5}$

b. $y_1(t) = t^5,\ y_2(t) = 5t^5$

c. $y_1(t) = t^{-5},\ y_2(t) = t^{-5}\ln(t)$

d. $y_1(t) = t^5\cos(\ln(t)),\ y_2(t) = t^5\sin(\ln(t))$

e. $y_1(t) = t^5,\ y_2(t) = t^5\ln(t)$

10. Which of the following pairs of functions are linearly independent? Assume intervals where both functions are continuous.

a. $y_1(t) = \ln(t^4),\ y_2(t) = 12\ln(t)$

b. $y_1(t) = 8t^6 - 2t^4 - 64t^2 + 16,\ y_2(t) = 2(4t^2 - 1)(t^4 - 8)$

c. $y_1(t) = \sin(-t),\ y_2(t) = 3\cot(t)$

d. $y_1(t) = 2\cos^2(t),\ y_2(t) = 1 - \sin^2(t)$

e. $y_1(t) = 5^{t+2},\ y_2(t) = 5^t$

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11. Find the general solution to $t^2y'' + 5ty' + 3y = 0,\ t > 0$.

a. $y(t) = C_1t + C_2t^{-1}$

b. $y(t) = C_1t^{-2} + C_2t^{-1}$

c. $y(t) = C_1t^{-3} + C_2t^{-1}$

d. $y(t) = C_1t^{-2} + C_2t$

e. $y(t) = C_1t^{-3} + C_2t$

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12. Solve the initial value problem $y'' - 2y' + y = 0$ with $y(0) = 1$ and $y'(0) = 0$.

a. $y(t) = e^t + te^t$

b. $y(t) = e^t - te^t$

c. $y(t) = e^t$

d. $y(t) = te^t + t^2e^t$

e. $y(t) = te^t - t^2e^t$

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MAP 2302 Exam 2A, Part II Free Response

Name (Last, First, legibly): ________________________________ Section #: ____________

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (9 pts) Use Reduction of Order to find a second linearly independent solution, given that $y_1 = e^x$.

$$xy'' - (2x+2)y' + (x+2)y = 0$$

2. (11 pts) Use variation of parameters to solve $y'' - 4y' + 5y = e^{2t} \sec t$ given that the two linearly independent solutions to the homogeneous DE are $y_1 = e^{2t} \cos t$ and $y_2 = e^{2t} \sin t$.

a. Find the Wronksians W, W_1, and W_2. (5 pts)

b. Find v_1. (2 pts)

c. Find v_2. (2 pts)

d. Find the general solution, combining like terms if applicable. (2 pts)

3. (10 pts) Solve the given differential equation.

$$y''' - 7y'' + 19y' - 13y = 0$$

4. (10 pts) Find a particular solution to the equation $y'' - 6y' + 8y = (t+1)e^{3t}$.

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ______________________________