



Department of Mathematics

MAP 2302

Exam 1A

Fall 2023

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

- 1) Name (last name, first initial, middle initial)
- 2) UF ID number
- 3) Section number

C. Under “special codes” code in the test ID numbers 1, 1.

- 2 3 4 5 6 7 8 9 0
- 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for “Test Form Code”, encode A.

- B C D E

E. 1) This test consists of 10 multiple choice questions of five points in value, plus 4 free response questions worth 52 points. Total is 102/100 points.

- 2) The time allowed is 90 minutes.
- 3) You may write on the test.
- 4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. 1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas shortly. Your instructor will return your tearoff sheets with your exam score in lecture. Your score will also be posted in Canvas within one week of the exam.

4) **Scantron errors in name, UF ID, section number, and form code will result in a five-point deduction.**

NOTE: Be sure to bubble the answers to questions 1–10 on your scantron.

Questions 1 – 10 are worth 5 points each.

1. Use Euler's method to approximate y_2 for the IVP $\frac{dy}{dx} = 2x - y^2$; $y(0) = 1$ using a step size of $h = 1$.

- a. 1 b. 2.2 c. -2.2 d. 2 e. 3.2
-

2. Which of the following ODEs could be made exact with an integrating factor that is either a function of x alone or y alone?

- a. $\frac{dy}{dx} = x^2 + y$ b. $\frac{dy}{dx} = e^{2x} - y$ c. $\frac{dy}{dx} = \frac{1+y}{x}$
d. $\frac{dy}{dx} = y^2 - x$ e. $(x^2 - y^2)dx + (x^2 - 2xy)dy = 0$
-

3. Determine whether the Existence and Uniqueness of Solution theorem implies that there is a unique solution for the IVP

$$\frac{dy}{dx} = y^4 + x^4, \quad y(0) = 6$$

- a. The theorem does not imply the existence of a unique solution because $f(x, y)$ is not continuous in a rectangle containing the point $(0, 6)$.

b. The theorem implies the existence of a unique solution because both $f(x, y)$ and $\frac{\partial f}{\partial y}$ are continuous in a rectangle containing the point $(0, 6)$.

c. The theorem does not imply the existence of a unique solution because $\frac{\partial f}{\partial y}$ is not continuous in a rectangle containing the point $(0, 6)$.

d. The theorem does not imply the existence of a unique solution because neither $\frac{\partial f}{\partial y}$ nor $f(x, y)$ are continuous in a rectangle containing the point $(0, 6)$.

e. The theorem implies the existence of a unique solution because only $f(x, y)$ is continuous in a rectangle containing the point $(0, 6)$.

4. For which values of n are $y(x) = x^n$ solutions to

$$2x^2 \frac{d^2y}{dx^2} - 5x \frac{dy}{dx} + 3y = 0?$$

- a. $\frac{1}{2}, -3$ b. $-\frac{1}{3}, 2$ c. $\frac{1}{3}, -2$ d. $\frac{1}{2}, 3$ e. $\frac{1}{3}, 2$
-

5. Classify the equation as an ODE, PDE, give the order, and indicate the independent and dependent variables. For ODE indicate whether the equation is linear or nonlinear.

$$e^x \frac{d^2y}{dx^2} + y^3 \sin x = \cos x.$$

- a. ODE, order 2, indep. x , dep. y , linear.
b. ODE, order 3, indep. x , dep. y , linear.
c. ODE, order 2, indep. x , dep. y , nonlinear.
d. ODE, order 3, indep. x , dep. y , nonlinear.
e. PDE, order 2, indep. x , dep. y .
-

6. Solve the initial value problem

$$\cos x(e^{2y} - y) \frac{dy}{dx} = e^y \sin 2x, \quad y(0) = 0. \text{ Note that } \sin 2x = 2 \sin x \cos x.$$

- a) $e^y + ye^{-y} + 2e^{-y} = -2 \cos x + 4$
b) $e^y + ye^{-y} + e^{-y} = 2 \cos x + 4$
c) $e^y + ye^{-y} + e^{-y} = -2 \cos x - 4$
d) $e^y + ye^{-y} + e^{-y} = -2 \cos x + 4$
e) The DE is not separable.

7. Find a particular solution to the IVP $\frac{dy}{dx} = \frac{y^2 - 9}{x^2 + 1}$, $y(0) = -9$.

a) $y = \frac{3(1 + 2e^{6\arctan(x)})}{1 - 2e^{6\arctan(x)}}$

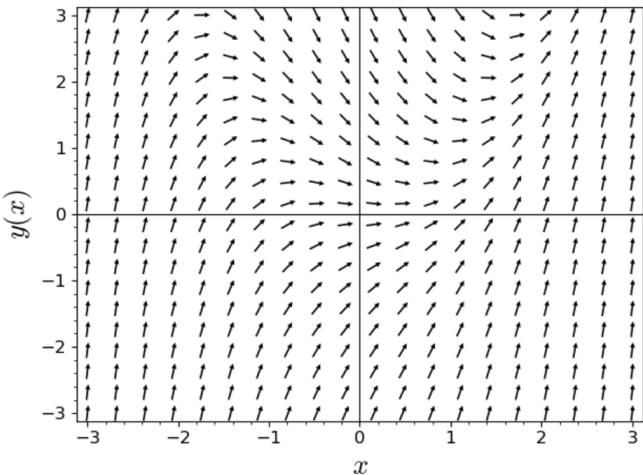
b) $y = \frac{3(1 + 2e^{6\arctan(x)})}{2e^{6\arctan(x)} - 1}$

c) $y = \frac{3(1 + 2e^{3\arctan(x)})}{1 - 2e^{3\arctan(x)}}$

d) $y = \frac{1 + 2e^{6\arctan(x)}}{1 - 2e^{6\arctan(x)}}$

e) $y = \frac{3(2 + e^{6\arctan(x)})}{2 - e^{6\arctan(x)}}$

8. The direction field pictured below is described by the differential equation



a. $\frac{dy}{dx} = x - y$

b. $\frac{dy}{dx} = y^2 - x$

c. $\frac{dy}{dx} = x^2 - y$

d. $\frac{dy}{dx} = y - x$

e. $\frac{dy}{dx} = x^2 - y^2$

9. Which of the following is a possible solution to the differential equation?

$$x \frac{dy}{dx} + (x+2)y = e^{x+1}$$

a. $y = e^{x+1} \left(\frac{1}{2x} - \frac{1}{4x^2} \right)$

b. $y = \frac{1}{4}e^{x+1}$

c. $y = \frac{1}{4}x^2 e^{2x+1}$

d. $y = e^{2x+1} \left(\frac{x}{2} - \frac{1}{4} \right)$

e. $y = e^{2x+1} \left(\frac{1}{4x} - 12 \right)$

10. Solve the DE.

$$2xydx + (x^2 - 1)dy = 0$$

a. $x^2y + y^2 = C$

b. $x^2y - y = C$

c. $x^2y + y = C$

d. $x^2y - y^2 = C$

e. No such function exists

MAP 2302 Exam 1A, Part II Free Response

Name: _____ Section #: _____

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (13 pts) Solve the initial value problem $(10 - 6y + e^{-3x})dx - 2dy = 0$, $y(0) = 2$.

2. (13 pts) Solve the Bernoulli equation $\frac{dy}{dx} + \frac{y}{2} = e^{-3x}y^{-3}$. Your answer should be in explicit form.

3. (13 pts) Solve $\frac{dy}{dx} - 10xy = x^2 + 25y^2$.

4. (13 pts) Solve the homogeneous IVP $(x^{-1}y^3 + 9y^2 + xy)dx - (9xy)dy = 0$, $y(1) = 1$.

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: _____

MAP2302 Exam 1 Fall 23 Key

MAP2302 Exam 1 Fall 23 Key

1.d

2. 3 correct possibilities

3. b

4. d

5. c

6. d

7. a

8. c

9. a

10. b

FR

$$1. \frac{10}{3}e^{3x} - 2ye^{3x} + x = -\frac{2}{3}$$

$$2. y = \pm \sqrt[4]{-4e^{-3x} + Ce^{-2x}}$$

$$3. y = \frac{1}{5\sqrt{5}} \tan(\sqrt{5}x + C) - \frac{1}{5}x$$

$$4. \frac{9\pi}{4} = 9 \arctan\left(\frac{y}{x}\right) - \ln|x|$$