taller Carolina Cepeda Valencia

1. a NXOR b

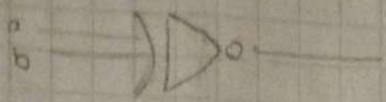



tabla de Verdad.

| entrada | salida |
|---|---|
| 00 | 1 |
| 01 | 0 |
| 10 | 0 |
| 11 | 1 |

matriz

$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array}$$

Derivando la matriz NXOR = NOT XOR

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \underbrace{\overset{\begin{array}{cccc}00 & 01 & 10 & 11\end{array}}{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}}}_{XOR} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

2. (NOT P) AND (NOT Q) = NOT (P OR Q)

1. (NOT P) AND (NOT Q)

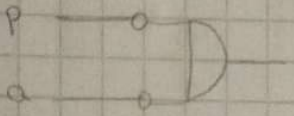



tabla de verdad

| entrada (pq) | salida |
|---|---|
| 00 | 1 |
| 01 | 0 |
| 10 | 0 |
| 11 | 0 |

matriz

$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

2.2 NOT (P OR Q)

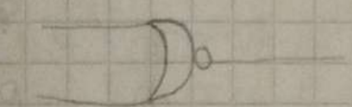


tabla de verdad

| entrada | salida |
|---|---|
| 00 | 1 |
| 01 | 0 |
| 10 | 0 |
| 11 | 0 |

matriz

$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

Como se puede observar.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Scribe

Derivando la matriz ①

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

multiplicado por operador AND

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{AND} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Derivando la matriz ②

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \overset{OR}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

③ P AND (Q OR R) = (P AND Q) OR (P AND R)

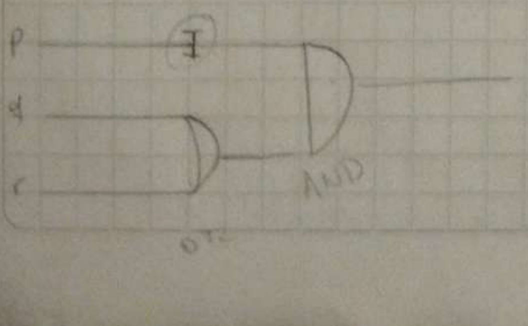
3.1 P AND (Q OR R)

tabla de verdad

| p | q | r | p and (q or r) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

matriz ①

$$\begin{array}{c|cccccccc} & 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & \end{array}$$

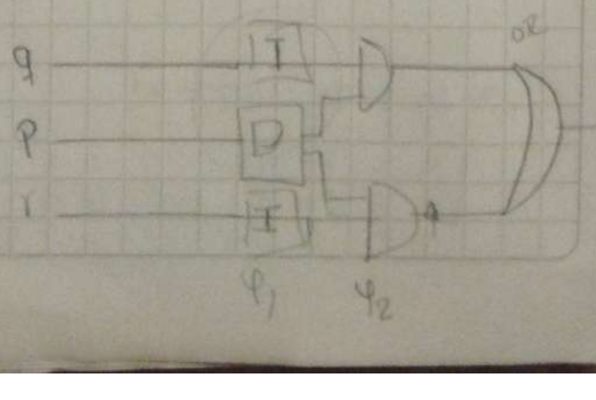



3.2 (P AND Q) OR (P AND R)

| p | q | r | operación |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

matriz ②

$$\begin{array}{c|cccccccc} & 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{array}$$



Al igualar las matrices nos damos cuenta que es la misma:

Derivando la matriz 1

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Derivando la matriz 2:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\varphi_1 = B = I \otimes A$$

$$\varphi_2 = C = AND \otimes AND \cdot \varphi_1$$

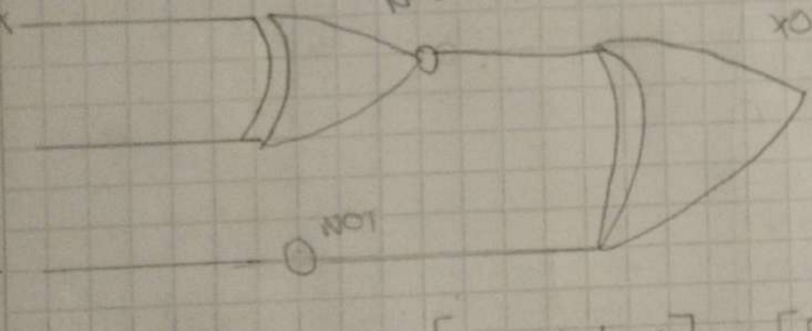
④ (NOT (x XOR y)) XOR (NOT z)

tabla de Verdad

| x | y | z | NOT x XOR y | NOT z | op |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |

matriz

$$\begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cccccccc} 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$




multiplicando de abajo hacia arriba.

$$NXOR \otimes NOT = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \cdot (NXOR \otimes NOT) = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

⑤ Adicionador de un bit con acarreo (3 entradas, dos salidas)
Intente derivar la matriz como composición de circuitos.
Intente modelar solo el circuito de la suma (sin acarreo).

Circuito

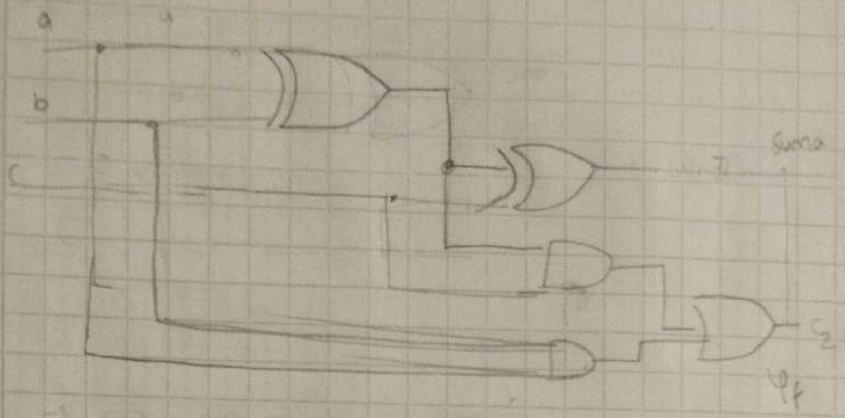



tabla de verdad

| a | b | c | suma | $c_2$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Matriz

$$\begin{array}{c}\\ 00\\ 01\\ 10\\ 11\end{array}\begin{array}{cccccccc}000 & 001 & 010 & 011 & 100 & 101 & 110 & 111\\ \end{array}$$

$$\begin{array}{c|cccccccc} & 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111\\ \hline 00 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 01 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0\\ 10 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0\\ 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{array}$$

$\varphi_f = I \otimes AND \cdot ($

circuito suma = circuito XOR

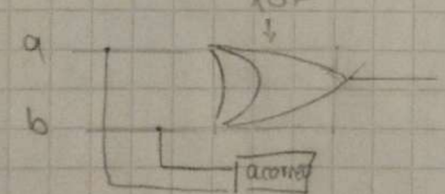



tabla de verdad

| a | b | suma |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Matriz (la misma que XOR)

$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11\\ \hline 0 & 1 & 0 & 0 & 0\\ 1 & 0 & 1 & 1 & 0\end{array}$$

Al comparar con XOR, observamos la misma derivación.