Measuring the Velocity of Money

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ABSTRACT

The velocity of money is an important driver of inflation that is conventionally measured as an average for an economy as a whole. While easy to calculate from macroeconomic aggregates, such measures overlook possibly relevant heterogeneity between payment systems, across regions, and intrinsic to spending patterns. This paper proposes a new measurement methodology that leverages large-scale micro-level transaction data and modern computational techniques to measure the transfer velocity of money at the level of individual spenders, enabling comparison between subgroups. Notably, our definition of transfer velocity extends to payment systems where users are free to deposit and withdraw, even as the total balance fluctuates. This is a common feature of real-world payment systems that is not accounted for by previous approaches. We estimate the transfer velocity of Sarafu, a digital community currency in Kenya, using a newly available data set describing individual transactions over a period of time that includes the COVID-19 pandemic. Our analysis reveals distributional, temporal and geographical heterogeneity in spending patters. Some units of Sarafu are held for minutes, others for months. The system experienced dramatic changes in its total balance and in its average transfer velocity as the COVID-19 pandemic unfolded, and in response to known administrative operations. Moreover, transaction rhythms differed substantially between rural and urban areas, in particular, money moves faster in urban communities. Successful macroeconomic policies require understanding how individuals experience the economy and when those experiences diverge. The data-driven approach described in this paper improves our understanding of the heterogeneity underlying macroeconomic indicators, and represents an advance in measurement that stands to improve economic monitoring.

JEL Classification Codes: E41, E42, C63, L14

1 INTRODUCTION

The rate at which money changes hands is an important macroeconomic indicator and plays a key role in determining inflation (Wen and Arias, 2014). Previous measures of the velocity of money indicate that money moves faster between people when the economy is doing well and that money moves slower when the economy is doing poorly (Leão, 2005). These measures are typically constructed as the ratio of two large macroeconomic aggregates. For example, on the Federal Reserve Economic Data (FRED) website, a measure of the velocity of money, the Velocity of M2 (FRED ID: M2V) is calculated as the ratio of quarterly nominal gross domestic product to M2, a measure of the aggregate money supply (Federal Reserve Bank of St. Louis, 2022). However, the economics literature has long recognized that the velocity of money is likely to be different across sectors (Leontief and Brody, 1993; Brody, 2000) and between payment systems (Mbiti and Weil, 2015). Aggregate measures obscure heterogeneity in the rate at which money changes hands. It is important to capture this heterogeneity because the aggregate outcomes could be hiding signs of economic trouble in a specific sector, region, or group. Moreover, a thorough understanding of how the velocity of money differs across groups, places, and time can better inform macroeconomic policy.

This paper develops new mathematical and computational techniques that allow for a fuller empirical analysis of the velocity of money. Specifically, that within a particular payment system. The so-called *transfer velocity of money* is conventionally calculated as the ratio between the total transaction volume within a payment system and the average balance of currency in that system (Mbiti and Weil, 2013). First, we provide a new and more precise interpretation of the transfer velocity of money suitable for a world where transaction data is recorded by digital payment infrastructure at the level of individual accounts. This value is defined in terms of the durations that money is held prior to being transacted, which can be observed nearly perfectly in high-granularity transaction data. Also computed over a period of time, this new version of the indicator provides information about spending patterns that is orthogonal to the total transaction volume. Moreover, our definition remains suitable for systems far from equilibrium where spending dynamics are not static. This means that our approach can be used to study payment systems where users are free to deposit and withdraw, even as the total balance fluctuates.

While some work has been done to compute the velocity of money at a higher level of detail, this work has been largely theoretical. We build on this existing literature to expand the scope for empirical analysis. The rate at which money changes hands can be defined more precisely in terms of its inverse, that is, in terms of "holding times." Funds enter an account, are held there for some period, and then are transferred out. The inverse of a holding time has the units of a transfer velocity, so a distribution of holding times can be used to compute the average transfer velocity (Wang et al., 2003). Several papers describe theoretical agent-based models where it is possible to derive a stationary distribution of holding times, and so compute the average transfer velocity (Wang et al., 2003, 2005; Kanazawa et al., 2018). Empirical measurement is also possible following this approach (Campajola et al., 2022), but is hampered by the mathematical constraint that the distribution be stationary. We adapt the earlier theoretical work to cases where dynamics need not be stable. Our methodology produces precisely defined measurements from high granularity transaction data, and can be used to compare theoretical models of money transfer against empirical observation.

We apply this new empirical method to a large data set which describes millions of individual transactions taking place in a digital currency system in Kenya. *Sarafu* is a community currency managed by Grassroots Economics Foundation (GE), a Kenyan nonprofit organization. The "Sarafu Community Inclusion Currency 2020 - 2021" data set describes every transaction of approximately 55,000 users over a period from January 2020 to June 2021 (Ruddick, 2021). The operation of the community currency over this period and the resulting patterns of circulation have been scientifically documented (Mattsson et al., 2022a,b). One unit of Sarafu is approximately equal in value to one Kenyan shilling, and the system saw substantial use during the COVID-19 pandemic. The data describes every transaction from one account to another, allowing us to construct a network of millions of transactions. In this network, a unit of currency is held in an account for a period of time and then, when it is spent, moves to another account. From this transaction network we extract holding times of individual units of currency. Additionally, the data contains demographic and location data from the individual user accounts, allowing us to analyze how the velocity of this money varies across locations and time periods.

We find that a unit of the Sarafu community currency changes hands 0.31 times per week, on average. Furthermore, this value is composed of 44% of the issued Sarafu changing hands at 0.70 times per week. That is to say, 56% of the issued Sarafu is effectively static. The distribution of held durations that produces this average ranges from minutes to months; much of the transaction volume comes from units of Sarafu that are spent the same day they are received, sometimes in a matter of minutes, while much of the balance at any particular point in time is held for weeks or months. The Sarafu system also experienced dramatic changes in its total balance and in usage patterns over this period, making our new computational method necessary. The average transfer velocity and the circulating balance both vary strongly over time. Known administrative operations have an observable effect on these values. Finally, money moves faster in urban communities, changing hands every day or two, while in rural communities it changes hands every week or two. These results are consistent with existing work on mobile money (Mbiti and Weil, 2013), with the use of community currencies in periods of disruption (Stodder, 2009; Zeller, 2020), and with macroeconomic trends in the velocity of money (Leão, 2005).

We contribute to two threads of existing literature. The first is an empirical literature devoted to measuring the transfer velocity of money. Mbiti and Weil (2015) argue that differences between payment systems can have macroeconomic impacts, particularly in countries where payment infrastructures are developing rapidly. This has been the case with Mobile Money in Kenya, Uganda, and other countries in East Africa over more than a decade (Mawejje and Lakuma, 2017). The transfer velocity of MPesa, the leading Mobile Money service in Kenya, rose from around 2 to around 4 transactions per month between 2008 and 2010 (Mbiti and Weil, 2013). In the years since, rising interest in digital financial services, cryptocurrencies, and central bank digital currencies (CBDCs) have made the fragmentation of monetary infrastructure a relevant topic of study far beyond East Africa. Modern digital systems present new opportunities for empirical measurement, and invite us to reconsider how monetary indicators are defined. For example, is possible to know the total balance of MPesa or Bitcoin quite precisely (Mbiti and Weil, 2013; Ron and Shamir, 2013; Kondor et al., 2014; Badev and Chen, 2014; ElBahrawy et al., 2017). At the same time, heterogeneity in spending patterns comes to the fore. In estimating the velocity of M-pesa in Kenya, Mbiti and Weil (2013) note that “most e-money at any point in time is held by nonfrequent transactors, even though most transfers are done by frequent transactors”. Similarly, recent empirical work on cryptocurrencies finds dramatic differences in the transfer velocity of money across individual accounts (Campajola et al., 2022).

The second area of study to which we contribute is a growing literature that uses new data and empirical techniques to identify and analyze heterogeneity in key macroeconomic measures. Aladangady et al. (2019) use anonymized transaction data to create high-frequency consumer spending data that provides more timely insight into macroeconomic fluctuations than previous measures. There is growing evidence that macroeconomic measures, such as inflation and unemployment, and important macroeconomic channels such as monetary policy, affect individuals, groups, and locales differently (Argente and Lee, 2021; Gornemann et al., 2021; Bartscher et al., 2021). In synthesizing much of the literature on heterogeneity in unemployment, inflation, and economic growth, Goodman-Bacon (2021) argues that without understanding the ways that different groups and individuals experience the economy, macroeconomic policy-makers cannot realize the full potential for economic growth. We expand this literature to include information on the heterogeneity in the velocity of money.

In the next section, we describe the Sarafu dataset and the information contained therein. In Section 3, we describe our methods, both theoretical and empirical. Section 4 describes the results of our empirical investigation. Finally, Section 5 concludes.

2 DATA

Sarafu is a small and substantially re-transacted complementary digital currency in Kenya. The payment system operates via a mobile phone interface and one unit of Sarafu is roughly equivalent in value to a Kenyan shilling. The dataset we use includes anonymized account information for around 55,000 users and records of all Sarafu transactions conducted from January 25, 2020 to June 15, 2021 (Ruddick, 2021). Transactions totaling around 300 million Sarafu capture various economic and financial activities such as purchases, transfers, and participation in savings and lending groups. Exchange with Kenyan Shillings was facilitated only in limited instances, and otherwise Sarafu was a small, closed system relative to the national currency.

The observation period includes the first year of the COVID-19 pandemic as well as several documented pilot projects, interventions, and currency operations by the nonprofit organization managing the currency (Ussher et al., 2021; Mattsson et al., 2022a). Figure 1 plots the weekly volumes of Sarafu transfers and the balance of the system over time. The vast majority of transfers during this period were ordinary transactions among users, in the data denoted as type STANDARD. A number of transactions made by savings and lending groups are also considered transfers and are denoted in the data as type AGENT_OUT. Currency creation and dissolution in the Sarafu system took place via DISBURSEMENT and RECLAMATION type transactions, respectively.

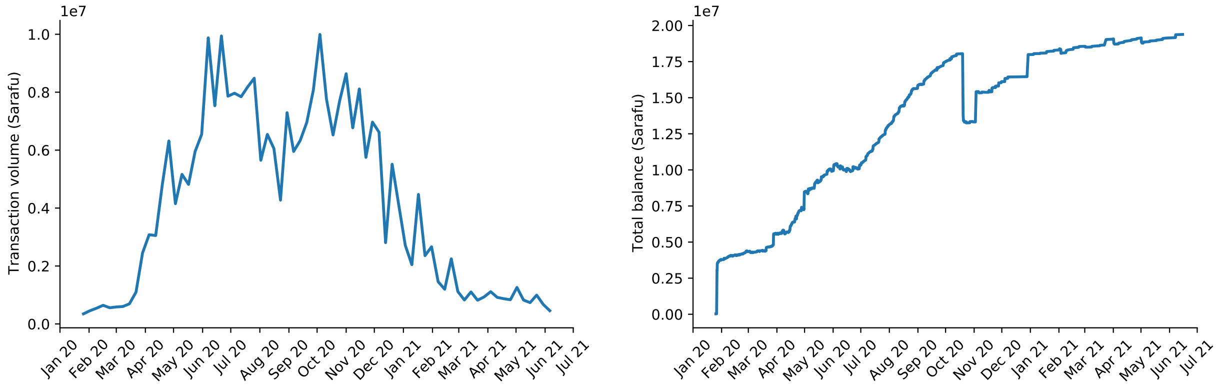


Figure 1. Weekly transaction volumes of Sarafu transfers (left) and the system balance over time (right). The reported balance does not include that of accounts used for currency management and administrative operations by staff at the nonprofit organization managing the currency.

The data also contains several contextual attributes that further describe the account holders' characteristics. The "area name" and "business type" are user-generated entries generalized into broader categories by the provider. They reflect the home location of the user and the product category of the goods or services they provide to the community. Localities are categorized into *urban*, *periurban*, and *rural* "area types." Mattsson et al. (2022a) provide precise descriptions of these data fields, together with their possible values. Network analysis shows that the structure of circulation within the Sarafu system over this period was highly modular, geographically localized, and occurring among users with diverse livelihoods (Mattsson et al., 2022b).

3 METHOD

In this section we present and reconcile two existing definitions of the transfer velocity of money, clarifying the assumptions underlying. Section 3.1 introduces an expression for the quantity theory of money as it applies to a single, individual payment system. It is noted that the transaction volume over a period of time can be decomposed into an independently observable velocity and a circulating balance, which is not necessarily equal to the total balance of the system. This extends tractability to modern digital systems where currency is routinely, and instantaneously, created and dissolved. In Section 3.2 we recount the steps taken to estimate the average transfer velocity of money using the conventional measurement methodology. We present our distributional measurement methodology in Section 3.3. Section 3.4 explains how this enables us to study heterogeneities along several dimensions. Finally, we detail our implementation in Section 3.5.

3.1 Theoretical Definition

The transfer velocity of money is conventionally defined using an identity similar to that expressing the quantity theory of money. The total flow of money, F_T, over a period of time, $T_0 < t < T_1$, is related to the amount of money in circulation, M, and its average transfer velocity, V. Equation 1 describes the familiar relation. We include the duration $T = T_1 - T_0$ in our formulation, explicitly, so that both F_T and M are amounts denoted in units of currency.

$$F_T = M \cdot T \cdot V \tag{1}$$

The transfer velocity can also be defined using the concept of a "holding time." Denoted τ, holding times are the durations between when accounts receive and re-transact particular units of money. The average transfer velocity can be found by integrating over the probability distribution of holding times for the money in the system, denoted $P(\tau)$ (Wang et al., 2003, Eqn. 8). Equation 2 expresses this in our notation.

$$V = \int_0^\infty P(\tau) \cdot \frac{1}{\tau} \cdot d\tau \text{ where } 1 = \int_0^\infty P(\tau) d\tau \tag{2}$$

These two definitions rest on remarkably similar assumptions. Formalized in the language of stochastic processes, Equation 2 holds in the case of a transaction process in a stationary state, that is, where $P(\tau)$ is independent of t (Wang et al., 2003). Equation 1 assumes M is fixed, and stationarity is the condition under which M can be readily approximated by its time-average (Mbiti and Weil, 2013). Equation 3 formulates a generalized version of the expression for F_T without this assumption, that is, where both M and $P(\tau)$ are functions also of t, cf. Wang et al. (2003, Eqn. 7). Equation 4 expresses the money flow generated by the fraction of money that participates in exchange after a period τ, cf. Wang et al. (2003, Eqn. 6).

$$F_T = \int_{T_0}^{T_1} \int_0^\infty M(t) P(\tau, t) \cdot \frac{1}{\tau} \cdot \partial\tau \partial t \tag{3}$$

$$F_T(\tau) = \int_{T_0}^{T_1} M(t) P(\tau, t) \cdot \frac{1}{\tau} \cdot dt \tag{4}$$

Using this notation, we can define the average period after which money participates in exchange. This is the average held duration over the period $T_0 < t < T_1$, denoted as $\bar{\tau}_T$ in Equation 5.

$$\bar{\tau}_T = \int_0^\infty P_T(\tau) \cdot \tau \cdot d\tau \text{ where } P_T(\tau) = F_T(\tau)/F_T \tag{5}$$

Incorporating Equation 4 into Equation 5 produces:

$$\bar{\tau}_T = \frac{1}{F_T} \int_0^{\infty} \int_{T_0}^{T_1} M(t) P(\tau, t) \partial t \partial \tau \tag{6}$$

Under the conventional assumption where M is fixed and $P(\tau)$ is independent of t, Equation 6 simplifies to Equation 7. Note the parallel with Equation 1. The average held duration is the inverse of the average transfer velocity under the same assumptions used in the conventional definition of this value.

$$F_T = M \cdot T \cdot \bar{\tau}_T^{-1} \tag{7}$$

3.2 Conventional Estimation

Conventional estimation of the velocity V uses Equation 1. In practice, F_T is a directly measurable quantity while M is not. Only rarely does the total balance of a system remain unchanged over the period $T_0 < t < T_1$ and it is common to use the time-averaged total balance (Mbiti and Weil, 2013). Equation 8 formulates an expression for V using M_{avg}, being the time-average of M. Again, $T = T_1 - T_0$.

$$V = F_T / (M_{\text{avg}} \cdot T) \text{ where } M_{\text{avg}} = \int_{T_0}^{T_1} M(t) dt / T \tag{8}$$

3.2.1 Empirical measurement

Values of F_T and $M(t)$ can be measured from large-scale micro-level transaction data. The total flow is the combined amount over transfer transactions that occur in the period $T_0 < t < T_1$. The total balance at time t is the total amount of money held across user accounts in that moment. This may be estimated by subtracting the balance of provider facing accounts from the total system balance (Mbiti and Weil, 2013).

3.3 Distributional Estimation

Distributional estimation of the average transfer velocity over the period $T_0 < t < T_1$ can be done using Equation 5. The distribution $F_T(\tau)$ can be obtained from large-scale micro-level transaction data, and so the average held duration $\bar{\tau}_T$ can be empirically measured. Equation 9 expresses the inverse relationship between the average duration held and the average transfer velocity of money under the conventional assumption that the balance is fixed and the holding time distribution is stationary. In practice, we expect this not to hold and for our estimate to diverge from the conventional estimate. As such, we denote the value we measure by V_T.

$$V_T = \bar{\tau}_T^{-1} \tag{9}$$

Equation 10 draws from Equation 6 to define the *circulating balance* of the system over the period $T_0 < t < T_1$. Denoted M_T, the circulating balance is the fixed balance that would satisfy the inverse relationship between the average held duration and the average transfer velocity of money.

$$M_T = \frac{1}{T} \int_0^{\infty} \int_{T_0}^{T_1} M(t) P(\tau, t) \partial t \partial \tau = \frac{1}{T} \cdot F_T \cdot \bar{\tau}_T \tag{10}$$

Note that distributional estimation of the velocity at a timepoint, that is $V(t)$, can be done by drawing on Equation 2. This approach has been applied to cryptocurrency systems Campajola et al. (2022). However, recall that this relies on a simplification of Equation 3 requiring a stationary distribution of holding times. In practice, the probability that money has some holding time at any particular point in time, $P(\tau, t)$, is not necessarily stable over time. And importantly, nor is this distribution directly observable.

3.3.1 Empirical measurement

This section introduces a method for obtaining the empirical $F_T(\tau)$. First, so-called "trajectory extraction" can be used to compute empirical holding times. Specifically, we measure the duration τ for which money was held prior to each of the transfer transactions that occur in the period $T_0 < t < T_1$. Kernel density estimation can then be used to smooth the empirical distribution of held durations. This makes it possible to evaluate Equation 5 numerically.

Empirical holding times can be found by tracing the flow of money through a payment system. This is done using trajectory extraction, a methodology based in the theory of walk processes on networks (Mattsson and Takes, 2021). Each transaction out of an account is allocated funds from prior transactions into that account. In this way, an amount is assigned to every possible pair of incoming to outgoing transactions. We use the so-called "well-mixed" heuristic to determine the amounts, ensuring that they reflect the proportional assignment of incoming funds to outgoing transactions without imposing arbitrary distinctions regarding how the individual spender might perceive these funds.

We consider all directly subsequent pairs of transactions, from an incoming to an outgoing transaction. Each pair corresponds to an empirical holding time. The timestamp of the outgoing transaction is the point at which the holding time is observed, and so we favor the past tense: these are "held durations."

Held durations that end in a transaction within the period $T_0 < t < T_1$ are collected into a cumulative distribution; this is $F_T(\tau)$. The observations are weighted and the total weight is used to normalize the distribution into a probability; this is $P_T(\tau)$. We obtain a smooth empirical distribution over τ using kernel density estimation (KDE) (Scott, 1992; Wand et al., 1991; Jones et al., 2018). KDE is performed in the space of $log(\tau)$, since the held durations range over several orders of magnitude—from seconds to months. Selecting a weighted gaussian kernel of suitable bandwidth in $log(\tau)$ is the equivalent of using a log-normal kernel in τ. Charpentier and Flachaire (2015) demonstrate the effectiveness of log-transformed KDE for heavy-tailed distributions of income, particularly at low values. The estimated probability density is back-transformed into the space of τ prior to numerical integration (Charpentier and Flachaire, 2015, Eqn. 9).

3.4 Heterogeneity Along Observable Dimensions

Considering different subsets of holding times makes it possible to study heterogeneity along available dimensions. There is a particular point in time at which a held duration is observed, and a particular account where this money was being held prior to this point in time. Because held durations took place at a specific account, known or derived features of the accounts can be used to study heterogeneity. Known or derived features of the transaction pair that defines the held duration can also be used. In the Sarafu data set, accounts and transactions are labeled with particular characteristics as noted in Section 2.

Held durations are grouped into categories based on the characteristics of the account at which the money was held, or of the transaction pair used to define them. An example a categorical group would be all held durations that occurred at accounts with a reported location in an "urban" area. Or, all held durations at accounts with a reported location in an "urban" area that were observed in March 2020. For each categorical group, it is possible to create a cumulative distribution of the held durations and an estimate of the average transfer velocity. These can then be compared.

From the transaction pair, it is also possible to determine what was happening to the held funds. For example, newly-created Sarafu could either enter circulation via a subsequent transfer transaction or be dissolved without ever entering circulation. Sarafu received via a transfer could remain in circulation via a subsequent transfer or be dissolved and so removed from circulation. For several analyses we consider the subset of held durations that ended in transfers, not dissolution.

3.5 Implementation

We compute the velocity of money for the Sarafu currency over the 71 full weeks from February 2nd, 2020 through June 12th, 2021. Monthly and total estimates are produced for this same period, using coarser dis-aggregation or forgoing dis-aggregation entirely. The empirical distributions are produced using the held durations observed over this period.

Transaction volumes correspond to STANDARD and AGENT_OUT transactions, collectively referred to as *transfers* throughout this work. We compute the total balance of the Sarafu system at every hour as captured by the *disbursement* and *reclamation* transactions. The balance of accounts involved in currency management and administrative operations is subtracted from the total. Specifically, these are accounts with `business_type` *system* or `held_role` VENDOR.

Empirical holding times are found using open-source software available at https://github.com/carolinamattsson/follow-the-money (Mattsson, 2020). The `--pairwise` option is used to limit trajectory extraction to pairs of sequential transactions. The system boundary is precisely defined by the *disbursement* and *reclamation* transactions. Because we are studying currency management, we select the *well-mixed* allocation heuristic (see Section 3.3.1). Holding times obtained from the recorded activity of accounts involved in currency management and administrative operations are ignored.

Note that a very small amount of Sarafu is mis-recorded in the data (Mattsson et al., 2022a). We employ the functionality provided in `follow-the-money` to infer the existence of missing funds. In extracting the empirical holding times, we do not continue to track fragments of received transactions with a size below 0.1 Sarafu. While some ambiguity arises from the cases where the entry or exit time is unobserved, we find this to be an insignificant source of noise. To quantify, we compute our estimates twice. In one formulation all unobserved funds are treated as absent from the system, and in the other, as present within the system. These give the same estimates at the precision with which we report our results. Likewise, using a size limit of 0.01 Sarafu does not change the results.

Kernel density estimation and numerical integration are implemented in Python using the `gaussian_kde` and `quad` modules from `scipy.integrate` and `scipy.stats`, respectively (Virtanen et al., 2020). The bandwidth for KDE is selected using the so-called Scott's Rule, as is the default (Scott, 1992). Analysis is performed in `pandas` (Reback et al., 2020), and figures are produced using `seaborn` (Waskom, 2021) and `matplotlib` (Caswell et al., 2019).

4 RESULTS

Over the full period, Sarafu changed hands at a rate of 0.31 transactions per week by conventional estimate. The distributional estimate is higher, as a considerable portion of the total Sarafu balance is found to be effectively static; 44% of the total Sarafu balance circulated at 0.70 transactions per week. However, this average value obscures underlying heterogeneity along several dimensions. Section 4.1 presents the distribution of held durations that produces this estimate. Section 4.2 compares the distributional estimate to the conventional estimate over time, revealing concrete effects of the COVID-19 pandemic and known interventions on the Sarafu system. Section 4.3 reveals pronounced differences in temporal spending patterns between rural and urban areas.

4.1 Distributional heterogeneity

The holding time distribution that produces the distributional estimate reveals considerable heterogeneity. Held durations vary over several orders of magnitude. Figure 2 (left) shows the probability density of observing a transaction with Sarafu that had been held, prior to the specific transaction, for a particular length of time. Sarafu is often spent the same day it is received, sometimes in a matter of minutes. It is also common to leave funds overnight to spend later in the same week, sometimes longer. Long-held Sarafu contributes little to the total spent. This wide distribution of held durations invites a different intuition than does an average velocity: much of the total balance at any point in time is slow moving—held for weeks or months—while a subset of fast-moving money produces a high volume of transactions.

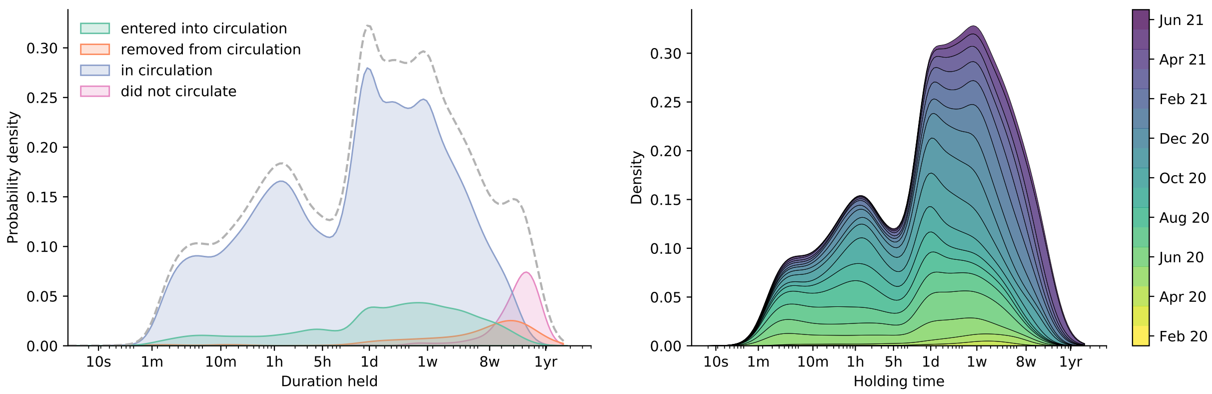


Figure 2. Probability distributions of the durations for which funds were held prior to an observed transaction (left) or an observed transfer transaction (right) between February 2020 and June 2021. Left: contributing to the overall distribution (dashed line) are funds that entered circulation by being transferred for the first time (green), remained in circulation (blue), were removed from circulation (orange), and were removed without having entered circulation (red). Right: contributing to the overall transfer distribution are durations prior to transaction activity occurring in each of the 18 months.

Obtaining the full distribution of held durations also lets us consider its constituent pieces. Currency creation and dissolution were routine features of currency management for the Sarafu system, as detailed in Section 2. In Figure 2 (left), we see that much of the newly-created Sarafu was transferred before long (green) and began circulating (blue). Still, some Sarafu was created and never transferred. A substantial portion of such funds were targeted for removal, dissolved without ever entering circulation (red). Long-held Sarafu was also targeted for removal from circulation (orange). By dis-aggregating in this way, we see that much of the distribution's long-duration tail reflects currency dissolution operations acting on inactive balances. Moreover, Figure 2 (right) confirms that even the distribution for Sarafu *remaining in or entering circulation* is not so stable over time; stationarity is not a valid assumption.

4.2 Temporal heterogeneity

The Sarafu system experienced dramatic changes over the period from January 2020 to June 2021, both due to the arrival of the COVID-19 pandemic to Kenya and due to deliberate interventions (Mattsson et al., 2022a). As a result, the average transfer velocity and the circulating balance both vary over time. Figure 3 presents monthly estimates of the average transfer velocity (left) and circulating balance (right) as computed by conventional and distributional approaches described in Section 3. Both estimates capture an increase in the rate of circulation within the Sarafu system in the spring of 2020, and its eventual slowing as economic conditions normalized. This is best explained by the counter-cyclical nature of complementary currencies, which tend to see spikes in usage levels during periods of economic disruption Stodder (2009); Zeller (2020). It is also the case that a targeted effort to introduce Sarafu to an urban slum in Nairobi began in spring 2020, with educational and outreach programs run by the Kenyan Red Cross (Mattsson et al., 2022a).

Other notable shifts in the indicators, as measured, occur at relevant points in the administrative history of Sarafu (Mattsson et al., 2022a). In July 2020, the share of static funds begins to increase and the two estimates of the average transfer velocity diverge. This corresponds to the ending of the limited cash-out functionality previously available to verified community-based savings groups; some such funds instead remained static. In October 2020, GE first announced and then conducted a large currency dissolution operation that removed long-held funds from circulation. This operation caused a sharp drop in the total Sarafu balance that mechanistically raises the conventional estimate of the average transfer velocity. The distributional estimate is affected only to the extent that this operation provoked a change in spending patterns. Indeed, the circulating balance rose considerably in October 2020 as long-held funds re-entered circulation upon threat of dissolution; the average transfer velocity was perhaps pulled somewhat below its trend. Finally, a second targeted effort to introduce Sarafu (this time to a specific area of Mombasa), which began early 2021 (Mattsson et al., 2022a), appears to have stalled the system-wide decline in the average transfer velocity.

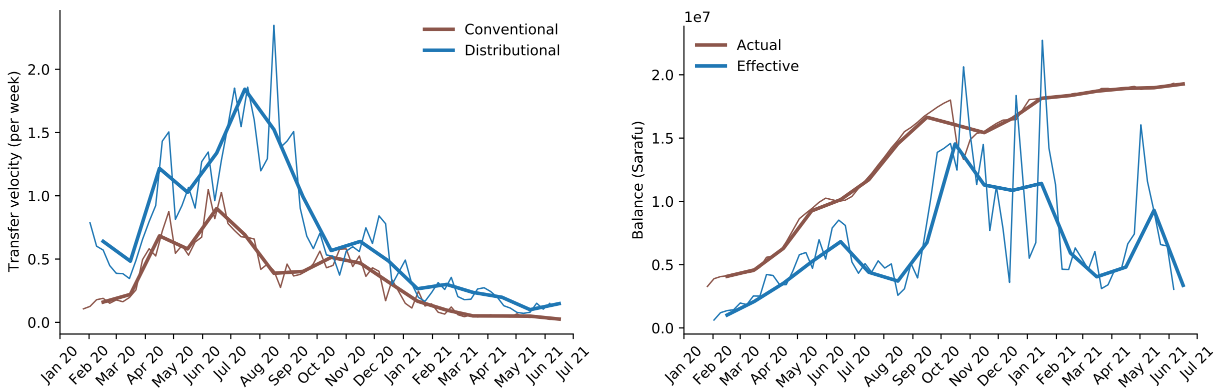
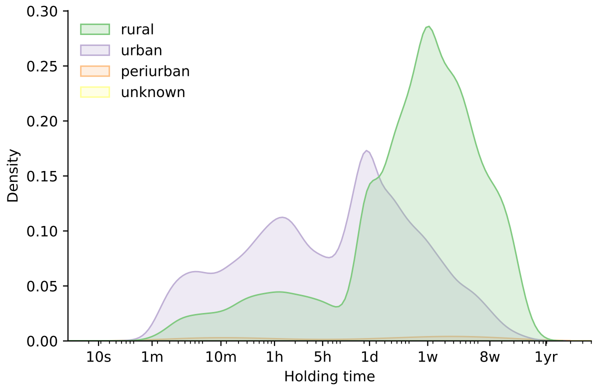


Figure 3. Conventional and distributional estimates of the average transfer velocity (left) and circulating balance (right) components of the product that makes the weekly and monthly transaction volumes.

4.3 Geographic heterogeneity

The transfer velocity of money also varies over spender characteristics. In particular, we find that transaction rhythms differ substantially between rural and urban areas. Figure 4 (left) plots the probability density of durations for which Sarafu was held by accounts registered to urban, rural, and periurban areas prior to transfers made over the observation period. Spenders in urban areas tend to hold their money for shorter periods—spend faster—than those in rural areas. Much of the transaction volume in urban areas emerges from regular, daily interaction while the rural rhythm is closer to weekly.

Sarafu is a community currency and circulation is highly localized, geographically (Mattsson et al., 2022b). For this reason, the urban and rural areas contributing to the same total transaction volume are nearly independent. Figure 4 (right) plots the average transfer velocity of Sarafu in or entering circulation, separately for urban and rural areas. These values are effectively decoupled. Considering only the average would obscure key differences. There was a dramatic increase in use centered around an urban slum in Nairobi as the COVID-19 pandemic unfolded; this area was the target of an education and outreach campaign at the time (Mattsson et al., 2022a). For some months, use in urban areas outpaced currency creation and circulation became quite rapid. Rural areas also saw a pandemic-driven increase in use, particularly in the sub-county of Kinango Kwale. However, Sarafu was more established in this area and currency management operations appear to have kept pace; the transfer velocity remained relatively steady. Eventually, in both areas, demand receded and the velocity began a gradual decrease.



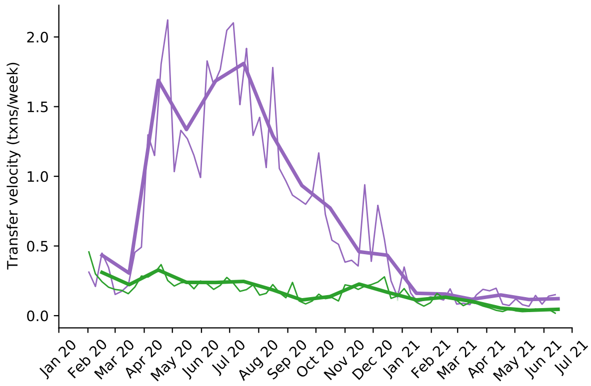


Figure 4. Contribution to the distribution of durations held by users in urban, rural, periurban, and unknown areas (left) and the weekly and monthly average across urban and rural users (right). Calculations based on funds in or entering circulation.

5 CONCLUSION

We have measured the transfer velocity of money using new mathematical and computational methods applicable to fine-grained transaction data. Our definition of velocity provides a precise interpretation of the average transfer velocity even for payment systems where spending patterns may change over time. This value is defined in terms of the durations that money is held prior to being transacted, which can be observed nearly perfectly in high-granularity transaction data. This method has allowed us to observe the empirical distribution of holding times for Sarafu, a community currency system, as well as differences across a number of dimensions including time and geographic location. We find that most of the transaction volume is driven by quick re-transactions, while a substantial portion of the balance is effectively static. Over time, we see a clear increase in the transfer velocity during the beginning of the COVID-19 pandemic. While our indicator does not change mechanistically when money is created or dissolved, spenders do appear to respond to administrative operations. Finally, we find that spenders in urban areas complete more transactions in a short period of time than do rural spenders. Generally speaking, we have uncovered a number of distributional, temporal and geographic patterns of heterogeneity in the velocity of money. Our methodology can be used in the future to conduct similar analyses on other digital payment systems, such as those operated by mobile phone providers, commercial banks, and central banks. This can lead to improved monitoring of the velocity of money to better inform macroeconomic policy.

REFERENCES

Aladangady, A., Aron-Dine, S., Dunn, W., Feiveson, L., Lengermann, P., and Sahm, C. (2019). From Transactions Data to Economic Statistics: Constructing Real-Time, High-Frequency, Geographic

Measures of Consumer Spending. *Big Data for 21st Century Economic Statistics*

Argente, D. and Lee, M. (2021). Cost of Living Inequality During the Great Recession. *Journal of the European Economic Association* 19, 913–952. doi:10.1093/jeea/jvaa018

Badev, A. I. and Chen, M. (2014). *Bitcoin: Technical Background and Data Analysis*. SSRN Scholarly Paper 2544331, Social Science Research Network, Rochester, NY. doi:10.2139/ssrn.2544331

Bartscher, A., Kuhn, M., Schularick, M., and Wachtel, P. (2021). *Monetary Policy and Racial Inequality*. Tech. Rep. 959, Federal Reserve Bank of New York, New York, NY

Brody, A. (2000). The Monetary Multiplier. *Economic Systems Research* 12, 215–219. doi:10.1080/09535310050005707

Campajola, C., D'Errico, M., and Tessone, C. J. (2022). MicroVelocity: rethinking the Velocity of Money for digital currencies. *arXiv:2201.13416 [physics, q-fin]*

Caswell, T. A., Droettboom, M., Hunter, J., Firing, E., Lee, A., Klymak, J., et al. (2019). matplotlib/matplotlib v3.1.0. doi:10.5281/zenodo.2893252

Charpentier, A. and Flachaire, E. (2015). *Log-Transform Kernel Density Estimation of Income Distribution*. Tech. Rep. 1506, Aix-Marseille School of Economics, France

ElBahrawy, A., Alessandretti, L., Kandler, A., Pastor-Satorras, R., and Baronchelli, A. (2017). Evolutionary dynamics of the cryptocurrency market. *Royal Society open science* 4, 170623

Federal Reserve Bank of St. Louis (2022). Velocity of M2 Money Stock

Goodman-Bacon, A. (2021). The myth of the rising tide | Federal Reserve Bank of Minneapolis

Gornemann, N., Kuester, K., and Nakajima, M. (2021). *Doves for the Rich, Hawks for the Poor? Distributional Consequences of Systematic Monetary Policy*. Tech. Rep. 089, University of Bonn and University of Cologne, Germany

Jones, A. T., Nguyen, H. D., and McLachlan, G. J. (2018). logKDE: log-transformed kernel density estimation. *Journal of Open Source Software* 3, 870. doi:10.21105/joss.00870

Kanazawa, K., Sueshige, T., Takayasu, H., and Takayasu, M. (2018). Kinetic theory for financial Brownian motion from microscopic dynamics. *Physical Review E* 98, 052317. doi:10.1103/PhysRevE.98.052317

Kondor, D., Pósfai, M., Csabai, I., and Vattay, G. (2014). Do the Rich Get Richer? An Empirical Analysis of the Bitcoin Transaction Network. *PLOS ONE* 9, e86197. doi:10.1371/journal.pone.0086197

Leontief, W. and Brody, A. (1993). Money-flow Computations. *Economic Systems Research* 5, 225–233. doi:10.1080/09535319300000019

Leão, P. (2005). Why does the velocity of money move pro-cyclically? *International Review of Applied Economics* 19, 119–135. doi:10.1080/0269217042000312641

Mattsson, C. (2020). carolinamattsson/follow-the-money v0.2.0

Mattsson, C. E. S., Criscione, T., and Ruddick, W. O. (2022a). Sarafu Community Inclusion Currency 2020-2021. *Scientific Data* 9. doi:10.1038/s41597-022-01539-4

Mattsson, C. E. S., Criscione, T., and Takes, F. W. (2022b). *Circulation of a digital community currency*. Tech. Rep. arXiv:2207.08941, arXiv. doi:10.48550/arXiv.2207.08941

Mattsson, C. E. S. and Takes, F. W. (2021). Trajectories through temporal networks. *Applied Network Science* 6, 1–31. doi:10.1007/s41109-021-00374-7

Mawejje, J. and Lakuma, E. C. P. (2017). *Macroeconomic Effects of Mobile Money in Uganda*. Tech. Rep. 135, Economic Policy Research Centre (EPRC), Kampala, Uganda

Mbiti, I. and Weil, D. N. (2013). The Home Economics of E-Money: Velocity, Cash Management, and Discount Rates of M-Pesa Users. *American Economic Review* 103, 369–374. doi:10.1257/aer.103.3.369

Mbiti, I. and Weil, D. N. (2015). Mobile banking: The impact of M-Pesa in Kenya. In *African Successes, Volume III: Modernization and Development* (University of Chicago Press). 247–293

Reback, J., McKinney, W., jbrockmendel, Bossche, J. V. d., Augspurger, T., Cloud, P., et al. (2020). pandas-dev/pandas: Pandas v0.24.2. doi:10.5281/zenodo.3509134

Ron, D. and Shamir, A. (2013). Quantitative Analysis of the Full Bitcoin Transaction Graph. In *Financial Cryptography and Data Security*, ed. A.-R. Sadeghi (Berlin, Heidelberg: Springer), Lecture no2tes in Computer Science, 6–24. doi:10.1007/978-3-642-39884-1_2

Ruddick, W. O. (2021). Sarafu Community Inclusion Currency, 2020-2021. *UK Data Service ReShare* https://reshare.ukdataservice.ac.uk/855142/. doi:10.5255/UKDA-SN-855142

Scott, D. W. (1992). Kernel Density Estimators. In *Multivariate Density Estimation* (New York: John Wiley & Sons, Ltd). 125–193. doi:10.1002/9780470316849.ch6

Stodder, J. (2009). Complementary credit networks and macroeconomic stability: Switzerland's Wirtschaftsring. *Journal of Economic Behavior & Organization* 72, 79–95. doi:10.1016/j.jebo.2009.06.002

Ussher, L., Ebert, L., Gómez, G. M., and Ruddick, W. O. (2021). Complementary Currencies for Humanitarian Aid. *Journal of Risk and Financial Management* 14, 557. doi:10.3390/jrfm14110557

Virtanen, P., Gommers, R., Oliphant, T. E., Haberland, M., Reddy, T., Cournapeau, D., et al. (2020). SciPy 1.0: fundamental algorithms for scientific computing in Python. *Nature Methods* 17, 261–272. doi:10.1038/s41592-019-0686-2

Wand, M. P., Marron, J. S., and Ruppert, D. (1991). Transformations in Density Estimation. *Journal of the American Statistical Association* 86, 343–353. doi:10.1080/01621459.1991.10475041

Wang, Y., Ding, N., and Xi, N. (2005). Prospects for Money Transfer Models. *arXiv:physics/0507161*

Wang, Y., Ding, N., and Zhang, L. (2003). The circulation of money and holding time distribution. *Physica A: Statistical Mechanics and its Applications* 324, 665–677. doi:10.1016/S0378-4371(03)00074-8

Waskom, M. L. (2021). seaborn: statistical data visualization. *Journal of Open Source Software* 6, 3021. doi:10.21105/joss.03021

Wen, Y. and Arias, M. (2014). What Does Money Velocity Tell Us about Low Inflation in the U.S.?

Zeller, S. (2020). Economic Advantages of Community Currencies. *Journal of Risk and Financial Management* 13, 271. doi:10.3390/jrfm13110271