Lab 2

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Programming Questions

In [1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
%matplotlib inline
```

1. Correlations

Pandas

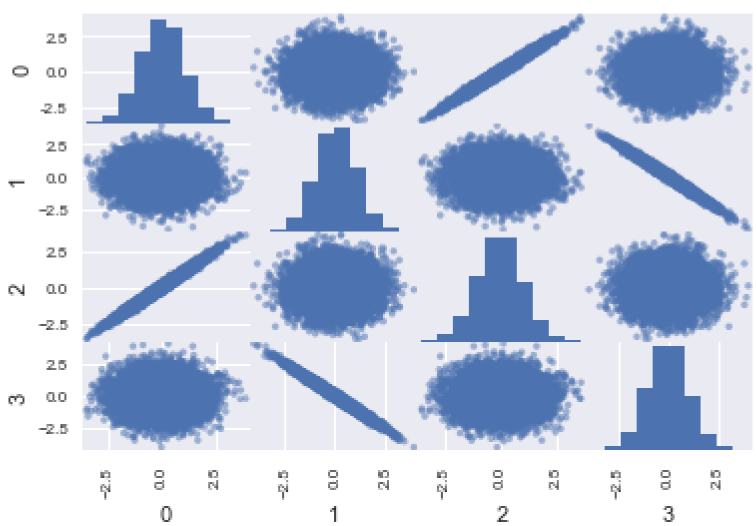
In [2]:
```
path = 'Data\DF1'
df=pd.read_csv(path,index_col=0,na_values='?')
df.head()
```

Out[2]:

	0	1	2	3
0	1.038502	0.899865	0.835053	-0.971528
1	0.320455	-0.647459	0.149079	0.352593
2	0.055480	2.234771	0.271672	-2.108739
3	-0.007260	-0.524299	-0.126550	0.670827
4	-1.237390	-1.377017	-1.049932	1.342079

In [3]:
```
from pandas.plotting import scatter_matrix
# plot each column against each other using pandas
scatter_matrix(df)
```

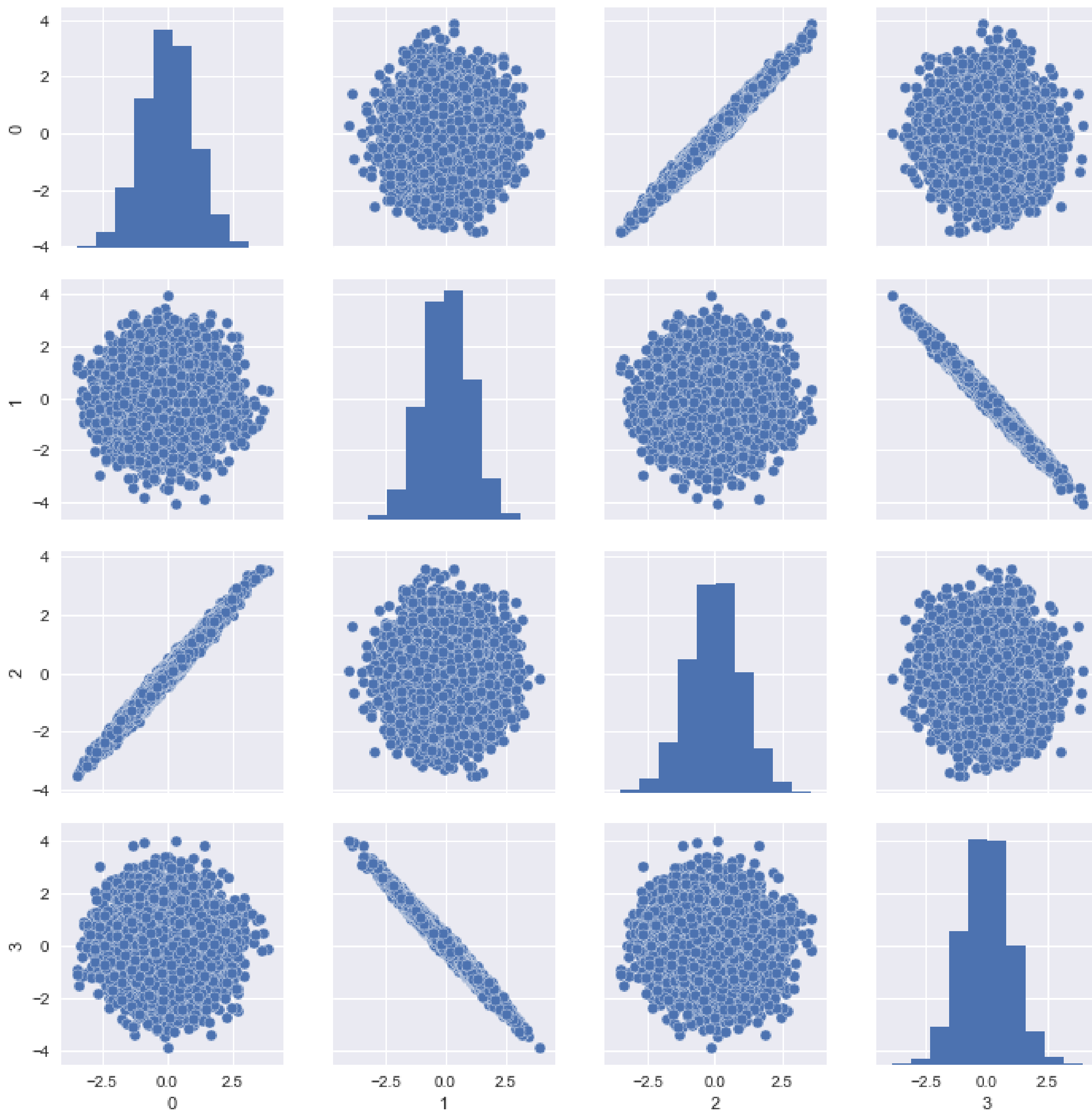
Out[3]:
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x000000000BFE87B8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000C1E9AC8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000C15E8D0>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000C3AB6D8
>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x000000000C479080>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000C51DC88>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000C5F07B8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000C6DCE80
>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x000000000C7F29B0>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000CCE3198>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000CDE7DD8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000CE90940
>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x000000000CF5C5C0>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000D01C2B0>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000D117E80>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x000000000D224828
>]], dtype=object)
```



Seaborn

In [4]:
```
# plot each column against each other using seaborn
sns.pairplot(df)
```

Out[4]: <seaborn.axisgrid.PairGrid at 0xd24ef28>



Covariance Matrix

```
In [5]: df.cov()
```

Out[5]:

	0	1	2	3
0	1.001558	-0.004012	0.991624	0.004125
1	-0.004012	1.005378	-0.004099	-0.995457
2	0.991624	-0.004099	1.001589	0.004081
3	0.004125	-0.995457	0.004081	1.005168

The covariance matrix is an n x n matrix where each element is cov(x_i,x_j)

For example: \begin{align} \sigma^2 = \begin{pmatrix} \sigma*1^2 & \sigma*{12}^2 \ \sigma_{12}^2 & \sigma_2^2 \end{pmatrix}. \end{align}

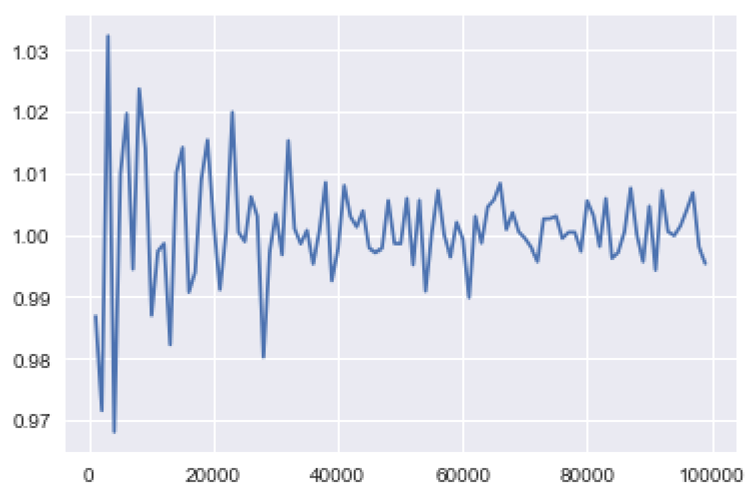
Columns 0 and 1 have a low covariance magnitude, so their plot seems to have no relation. This is the case for 0 and 3, 1 and 2, and 2 and 3 as well.

Columns 0 and 2 have a high covariance, so their plot looks like a line with a positive slope.

Columns 1 and 3 have a high negative covariance, so their plot looks like a line with a negative slope.

Covariance vs Sample Size

```
In [6]: a = {}
for i in range(1000,100000,1000):
    # take i samples and get the covariance
    samples1 = np.random.multivariate_normal((0,0,0),((1,0,0),(0,1,1),
(0,1,1)), i)
    a[i] = np.cov(samples1,rowvar=False)[1,2]
# plot the covariance against the number of samples
plt.plot(*zip(*sorted(a.items())))
plt.show()
```



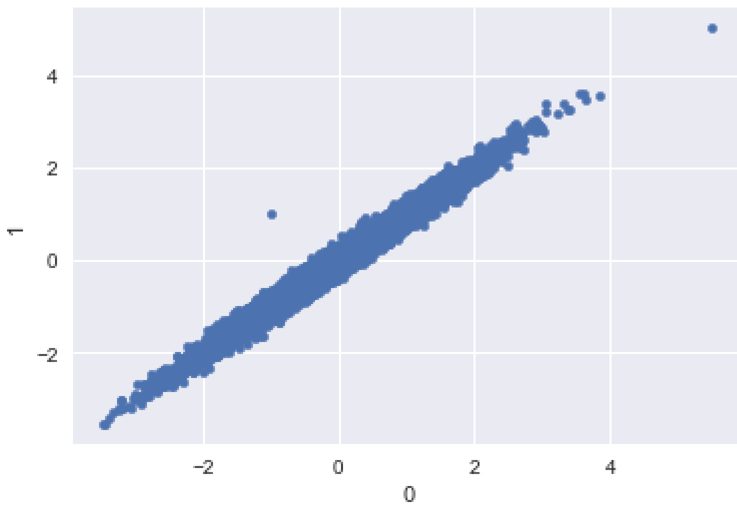
As the number of samples increases, the calculated covariance approaces the true variance.

2. Outliers

In [7]:
```
plt.clf()
path = 'Data\DF2'
df=pd.read_csv(path,index_col=0,na_values='?')
df.plot(x='0',y='1',kind='scatter')
```

Out[7]:
```
<matplotlib.axes._subplots.AxesSubplot at 0xf243c50>
```

```
<matplotlib.figure.Figure at 0xf8e8cf8>
```

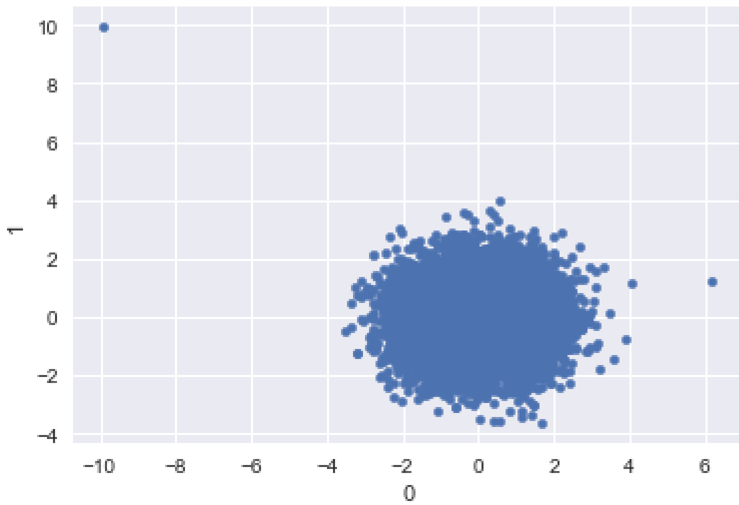


In [8]:
```
cov = df.cov()

from scipy.linalg import sqrtm
cov_inv = np.linalg.inv(sqrtm(cov))
data = pd.DataFrame(np.dot(df,cov_inv))

data.plot(x = 0, y = 1,kind='scatter')
```

Out[8]: <matplotlib.axes._subplots.AxesSubplot at 0xf79bd30>



$$
\begin{aligned}
\mathbf{C_z} &\triangleq E\{(\mathbf{z}-\bar{\mathbf{z}})(\mathbf{z}-\bar{\mathbf{z}})^T\} \\
&= E\left\{\left[(\mathbf{Qy})-(\mathbf{Q}\bar{\mathbf{y}})\right]\left[(\mathbf{Qy})-(\mathbf{Q}\bar{\mathbf{y}})\right]^T\right\} \\
&= E\left\{\left[\mathbf{Q}(\mathbf{y}-\bar{\mathbf{y}})\right]\left[\mathbf{Q}(\mathbf{y}-\bar{\mathbf{y}})\right]^T\right\} \\
&= E\left\{\mathbf{Q}(\mathbf{y}-\bar{\mathbf{y}})(\mathbf{y}-\bar{\mathbf{y}})^T\mathbf{Q}^T\right\} \\
&= \mathbf{Q}E\left\{(\mathbf{y}-\bar{\mathbf{y}})(\mathbf{y}-\bar{\mathbf{y}})^T\right\}\mathbf{Q}^T \\
&= \mathbf{QC_yQ}^T
\end{aligned}
$$

If Cy is the identity matrix and Cz is know, then Q can be determined by taking the matrix square root of Cz. We then multiply points in Z by Q^-1, and we will obtain Y. Now the point (-1,1) is more outlying than (5.5,5).

3. Even More Standard Error

Constant n

In [9]:
```python
a = []
for i in range(10000):
    # take samples of e and x
    eSamples = np.random.normal(0,1,150)
    xSamples = np.random.normal(0,1,150)
    # compute y
    ySamples = -3 + eSamples
    # calculate bHat using the equation from written problem 2
    bHat = np.dot(xSamples, ySamples)/np.dot(xSamples,xSamples)
    error = bHat - 0
    a.append(error)
#find the standard deviation of the errors
np.std(a)
```

Out[9]:
```
0.25957849857841037
```

The value -0.15 is not significant since the standard deviation is greater than |-0.15|

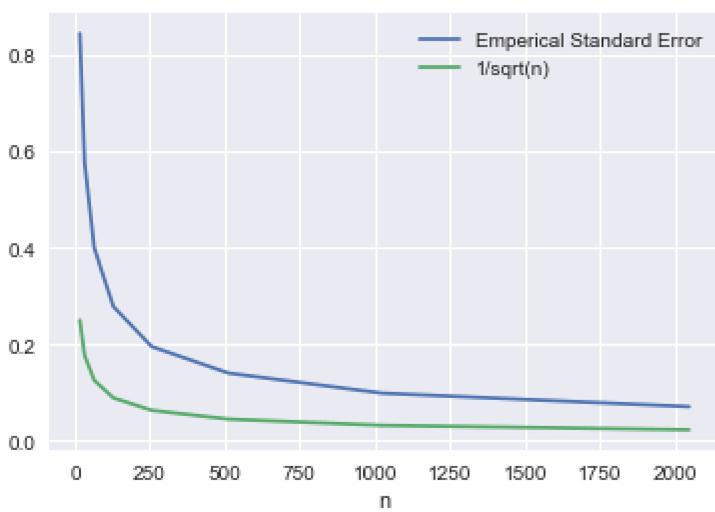
Varying n

In [11]:
```
xAxis = [2**i for i in range(4,12)]
stdError = []
invSqrt = [1/np.sqrt(i) for i in xAxis]
for n in xAxis:
    errors = []
    # perform calculation from last step with varying n
    for i in range(10000):
        eSamples = np.random.normal(0,1,n)
        xSamples = np.random.normal(0,1,n)
        ySamples = -3 + eSamples

        bHat = np.dot(xSamples, ySamples)/np.dot(xSamples,xSamples)
        error = bHat - 0
        errors.append(error)
    stdError.append(np.std(errors))

# plot emperical error and 1/sqrt(n) against n
plt.plot(xAxis,stdError, label = 'Emperical Standard Error')
plt.plot(xAxis,invSqrt, label = '1/sqrt(n)')
plt.legend()
plt.xlabel('n')
```

Out[11]:
```
<matplotlib.text.Text at 0x1048f4a8>
```



The shape of the Emperical Standard Error matches the inverse square root of the number of samples

4. Names and Frequencies

In [13]:
```
# top k names for a given year
def topNames(k, year):
    if(not year in range(1880,2016)):
        return None
    df=pd.read_csv('Names\yob{}.txt'.format(year),header=None,na_values='?')
    df.sort_values(2,ascending=False)
    return(df[0][:k])
topNames(10, 2000)
```

Out[13]:
```
0        Emily
1       Hannah
2      Madison
3       Ashley
4        Sarah
5       Alexis
6     Samantha
7      Jessica
8    Elizabeth
9       Taylor
Name: 0, dtype: object
```

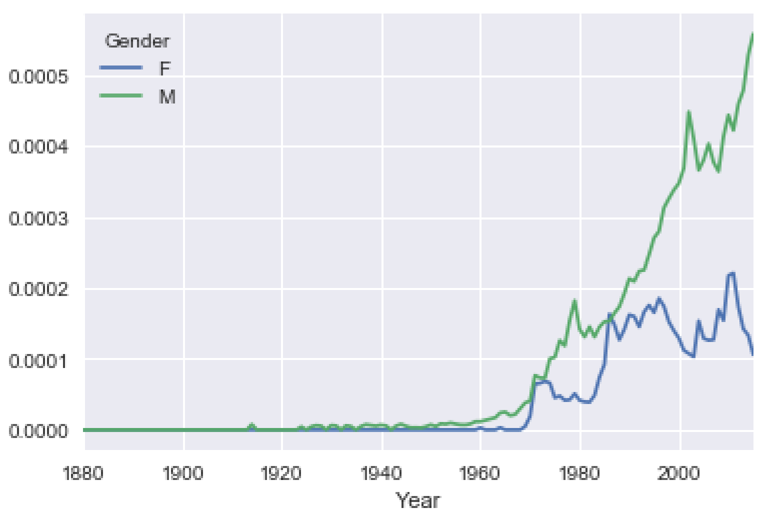
In [14]:
```
# name frequency from 1880 to 2015 based on gender
def nameFreq(name):
    names = {'M':0,'F':0}
    for year in range(1880,2016):

df=pd.read_csv('Names\yob{}.txt'.format(year),header=None,na_values='?')
        # find all the males named name, and count them
        males = df.loc[df[0] == name].loc[df[1] == 'M'][2].get_values()
        females = df.loc[df[0] == name].loc[df[1] == 'F'][2].get_values()
        if(males.size != 0):
            names['M'] += males[0]
        if(females.size != 0):
            names['F'] += females[0]
    return names
nameFreq('Ali')
```

Out[14]:
```
{'F': 9501, 'M': 23614}
```

In [201]:
```
# relative name frequency from 1880 to 2015 based on gender
def relativeNameFreq(name):
    # combine all files into one dataframe
    dataFrames = []
    for year in range(1880,2016):
        df=pd.read_csv('Names\yob{}.txt'.format(year),names=['Name', 'Gender',
 'Freq'])
        df['Year']=pd.Series(year,index=df.index)
        dataFrames.append(df)
    df = pd.concat(dataFrames)
    # get the total names in a year
    totalNames = df.pivot_table(values='Freq',index='Year',columns='Gender',ag
gfunc=np.sum)
    # get the number of people named name in a year
    totalGivenName =
df.loc[df['Name']==name].pivot_table(values='Freq',index='Year',columns='Gende
r',aggfunc=np.sum)
    # get the fraction of people named name in a year
    relativeFreq = totalGivenName.div(totalNames).fillna(0)
    # plot
    return relativeFreq
relativeNameFreq('Ali').plot()
```

Out[201]: <matplotlib.axes._subplots.AxesSubplot at 0x27b716a0>



In [202]:
```
# find all names that changed in gender popularity from 1880 to 2015
def nameGenderSwitch():
    # combine all files into one dataframe
    dataFrames = []
    for year in range(1880,2016):
        df=pd.read_csv('Names\yob{}.txt'.format(year),names=['Name', 'Gender',
 'Freq'])
        df['Year']=pd.Series(year,index=df.index)
        dataFrames.append(df)
    df = pd.concat(dataFrames)
    # get the number of people named a name in a year
    totalNames = df.pivot_table(values='Freq',index=['Name','Year'],columns='G
ender',aggfunc=np.sum).fillna(0)
    totalNames['M-F'] = np.sign(totalNames['M']-totalNames['F'])
    totalNames = totalNames['M-F']
    index = totalNames.axes[0]
    ret = {'MToF': 0, 'FToM':0}
    for a in set(index.get_level_values(0)):
        if totalNames[a].is_monotonic_increasing and
len(totalNames[a].unique()) > 1:
            ret['FToM'] += 1
            # if m < f then m > f
#             print "{} was a female name until {}, and now is a male name".fo
rmat(a,totalNames[a][totalNames[a] != -1].index[0])
        elif totalNames[a].is_monotonic_decreasing and len(totalNames[a].uniqu
e()) > 1:
            ret['MToF'] += 1
            # if f < m then m < f
#             print "{} was a male name until {}, and now is a female name".fo
rmat(a,totalNames[a][totalNames[a] != 1].index[0])
    return ret
nameGenderSwitch()
```

Out[202]: {'FToM': 756, 'MToF': 609}

Examples:

Niger was a female name until 1994, and now is a male name

Chesleigh was a male name until 2010, and now is a female name

Chade was a male name until 1985, and now is a female name

5. Visualization Tools and Missing/Hidden Values

In [209]:
```
tweets = pd.read_csv('tweets.csv')
tweets['user_location'].head()
```

Out[209]:
```
0        Wheeling WV
1                NaN
2                NaN
3             global
4    California, USA
Name: user_location, dtype: object
```

In [213]:
```
def get_state(row):
    state = []
    text = row['user_location'].lower()

    if "west virginia" in text or r"\bwv\b" in text or "Wheeling" in text:
        state.append("WV")
    if "texas" in text or r"\btx\b" in text or "austin" in text or "houston" i
n text:
        state.append("TX")
    if "new york" in text or r"\bny\b" in text or "new york city" in text:
        state.append("NY")
    return ",".join(state)

tweets['user_location'] = tweets['user_location'].astype(str)
tweets['state'] = tweets.apply(get_state,axis=1)
tweets['state'].value_counts()
```

Out[213]:
```
         226352
TX         6285
NY         4592
WV          249
TX,NY         2
Name: state, dtype: int64
```

We can find the number of states by searching for certain terms in the user_location field. We can assign more states with more strict criteria and checking for more states/cities.

1 a) $P(Z_{avg} > c) = P\left(\frac{Z_{avg}-\mu}{\sigma/\sqrt{n}} > \frac{c-\mu}{\sigma/\sqrt{n}}\right) \approx 1-\Phi\left(\frac{c-\mu}{\sigma/\sqrt{n}}\right)$

$Z_n = \frac{Z_{avg}-\mu}{\sigma/\sqrt{n}}$

$P\left(Z_n > \frac{c-\mu}{\sigma/\sqrt{n}}\right)$

$P(Z_n > .1 \cdot \sqrt{10000}) \approx 1-\Phi(10) \approx 0$
$P(Z_n > .01 \cdot 100) \approx 1-\Phi(1) \approx 0.16$
$P(Z_n > .001 \cdot 100) \approx 1-\Phi(.1) \approx 0.46$

b) $P\left(Z_n > \frac{n^{-1/3}-\mu}{\sigma/\sqrt{n}}\right) \approx 1-\Phi\left(\frac{n^{-1/3}-\mu}{\sigma/\sqrt{n}}\right)$
$P\left(Z_n > \frac{n^{-1/2}-\mu}{\sigma/\sqrt{n}}\right) \approx 1-\Phi\left(\frac{n^{-1/2}-\mu}{\sigma/\sqrt{n}}\right)$

$P\left(Z_n > \frac{n^{-2/3}-\mu}{\sigma/\sqrt{n}}\right) \approx 1-\Phi\left(\frac{n^{-2/3}-\mu}{\sigma/\sqrt{n}}\right)$

2 a) $\frac{1}{n}\sum(x_i\beta - y_i)^2 = \frac{1}{n}\sum x_i^2\beta^2 - \frac{2}{n}\sum x_i y_i\beta + \frac{1}{n}\sum y_i^2$

$A = \frac{1}{n}\sum_{i=1}^{n} x_i^2 \qquad B = -\frac{2}{n}\sum_{i=1}^{n} x_i y_i \qquad C = \frac{1}{n}\sum_{i=1}^{n} y_i^2$

1D

b) $\frac{d}{d\beta}\Big|_{\beta=\hat\beta}\frac{1}{n}\sum(x_i\beta - y_i)^2 = 0$

$\rightarrow \frac{2}{n}\sum x_i(x_i\beta - y_i) = 0$

$\rightarrow \sum x_i^2\beta - \sum x_i y_i = 0$

$\rightarrow \beta = \frac{\sum x_i y_i}{\sum x_i^2}$

c) $\hat\beta = \frac{\sum x_i(x_i\beta + e_i)}{\sum x_i^2}$

$= \frac{\sum x_i^2\beta + \sum x_i e_i}{\sum x_i^2}$

$= \beta + \frac{\sum x_i e_i}{\sum x_i^2}$

$= \beta + \frac{X^T \cdot e}{X\cdot X^T}$

$z = \frac{X^T}{X\cdot X^T}$

General

min

b) $\frac{1}{n}\sum(x_i\cdot\beta - y_i)^2$

$\rightarrow \min \frac{1}{n}\|X\vec\beta - \vec y\|_2^2$

$\rightarrow \min \frac{1}{n}(X\beta - y)^T(X\beta - y)$

$\rightarrow (\beta^T X^T - y^T)(X\beta - y)$

$\rightarrow (\beta^T X^T X\beta - 2\beta^T X^T y + y^T y)$

$\rightarrow \nabla\|X\vec\beta - \vec y\|_2^2 = 0$

$\rightarrow (2X^T X\beta - 2X^T y) = 0$

$\rightarrow \beta = (X^T X)^{-1}X^T y$

c) $\hat\beta = (X^T X)^{-1}X^T(X\vec\beta + \vec e)$

$= (X^T X)^{-1}(X^T X)\vec\beta + (X^T X)^{-1}X^T\vec e$

$= \vec\beta + (X^T X)^{-1}X^T\vec e$

$z = (X^T X)^{-1}X^T\vec e$