Recursive Partitioning for Heterogeneous Causal Effects

Motivation I: Experiments and Data-Mining

- Concerns about ex-post "data-mining"
 - In medicine, scholars required to pre-specify analysis plan
 - In economic field experiments, calls for similar protocols
- But how is researcher to predict all forms of heterogeneity in an environment with many covariates?
- ▶ Goal:
 - Allow researcher to specify set of potential covariates
 - Data-driven search for heterogeneity in causal effects with valid standard errors

Motivation II: Treatment Effect Heterogeneity for Policy

- Estimate of treatment effect heterogeneity needed for optimal decision-making
- This paper focuses on estimating treatment effect as function of attributes directly, not optimized for choosing optimal policy in a given setting
- This "structural" function can be used in future decisionmaking by policy-makers without the need for customized analysis

Preview

- Distinguish between causal effects and attributes
- Estimate treatment effect heterogeneity:
 - Introduce estimation approaches that combine ML prediction
 & causal inference tools
- Introduce and analyze new cross-validation approaches for causal inference
- Inference on estimated treatment effects in subpopulations
 - Enabling post-experiment data-mining
- ▶ NOTE: estimation versus prediction objective

"Moving the Goalpost": What is Question?

- Estimate $\tau(x) = E[\tau_i | X_i = x]$ as well as possible
 - Why? Want to hold some covariates fixed and look at the effect of others.
- Estimate $BLP[\tau_i|X_i=x]$
 - Why? "Interpretable"? The best linear predictor is a bit hard to interpret without the whole variance-covariance matrix of nonlinear functions and interactions; you have omitted variable bias on the coefficients you are explaining, relative to $\tau(x)$. My view is that simple models can be more "misinterpretable" than interpretable.
- Causal Tree: Find partition of covariate space and estimate $E[\tau_i|X_i\in S]$ for each element of partition
 - Why? Easier to interpret than BLP, but still important to report mean, median, percentiles of all covariates for each leaf to understand how leaves are different, when covariates are correlated.
- Which units have highest or lowest treatment effects?
 - Why? Helps understand who could be treated. Can be estimated directly or can draw inferences based on output of causal tree or non-parametric estimates of $\tau(x)$
 - Common practice to display differences between covariates; see Chernozhukov and Duflo (2018)
- What is the best policy mapping from X to treatments W?
 - Why? Sometimes this is the direct object of interest.
 - Fully nonparametric? See e.g. Hirano and Porter (2009)
 - With limited complexity or other constraints? See e.g. Kitagawa and Tetenov (2015), Athey and Wager (2017).
- What is the full set of covariates for which there is statistically significant heterogeneity?
 - List, Shaikh, and Xu (2016) (multiple testing)
- Tradeoffs: More personalization, reliable confidence intervals, role of assumptions, interpretability

Regression Trees for Prediction

Data

- Outcomes Y_{i} , attributes X_{i} .
- Support of X_i is \mathcal{X} .
- Have training sample with independent obs.
- Want to predict on new sample
- Ex: Predict how many clicks a link will receive if placed in the first position on a particular search query

Build a "tree":

- Partition of ${\mathcal X}$ into "leaves" ${\mathcal X}_j$
- Predict Y conditional on realization of X in each region \mathcal{X}_j using the sample mean in that region
- Go through variables and leaves and decide whether and where to split leaves (creating a finer partition) using in-sample goodness of fit criterion
- Select tree complexity using crossvalidation based on prediction quality

Regression Trees for Prediction: Components

Model and Estimation

- A. Model type: Tree structure
- B. Estimator \hat{Y}_i : sample mean of Y_i within leaf
- C. Set of candidate estimators C: correspond to different specifications of how tree is split

2. Criterion function (for fixed tuning parameter λ)

A. In-sample Goodness-of-fit function:

Qis = -MSE (Mean Squared Error)=
$$-\frac{1}{N}\sum_{i=1}^{N}(\hat{Y}_i - Y_i)^2$$

- A. Structure and use of criterion
 - i. Criterion: $Q^{crit} = Q^{is} \lambda \times \#$ leaves
 - Select member of set of candidate estimators that maximizes Q^{crit} , given λ

3. Cross-validation approach

- A. Approach: Cross-validation on grid of tuning parameters. Select tuning parameter λ with highest Out-of-sample Goodness-of-Fit Qos.
- B. Out-of-sample Goodness-of-fit function: $Q^{os} = -MSE$

Using Trees to Estimate Causal Effects

Model:

$$Y_i = Y_i(W_i) = \begin{cases} Y_i(1) & if W_i = 1, \\ Y_i(0) & otherwise. \end{cases}$$

- Suppose random assignment of W_i
- ▶ Want to predict individual i's treatment effect
 - $\tau_i = Y_i(1) Y_i(0)$
 - This is not observed for any individual
 - Not clear how to apply standard machine learning tools
- Let

$$\mu(w, x) = \mathbb{E}[Y_i | W_i = w, X_i = x]$$

 $\tau(x) = \mu(1, x) - \mu(0, x)$

Using Trees to Estimate Causal Effects

$$\mu(w, x) = \mathbb{E}[Y_i | W_i = w, X_i = x]$$

$$\tau(x) = \mu(1, x) - \mu(0, x)$$

- Approach I:Analyze two groups separately
 - Estimate $\hat{\mu}(1, x)$ using dataset where $W_i = 1$
 - Estimate $\hat{\mu}(0, x)$ using dataset where $W_i = 0$
 - Use propensity score weighting (PSW) if needed
 - Do within-group cross-validation to choose tuning parameters
 - Construct prediction using

$$\hat{\mu}(1,x) - \hat{\mu}(0,x)$$

- Approach 2: Estimate $\mu(w, x)$ using tree including both covariates
 - Include PS as attribute if needed
 - Choose tuning parameters as usual
 - Construct prediction using

$$\hat{\mu}(1,x) - \hat{\mu}(0,x)$$

Estimate is zero for x where tree does not split on w

Observations

- Estimation and cross-validation not optimized for goal
- Lots of segments in Approach 1: combining two distinct ways to partition the data

Problems with these approaches

- Approaches not tailored to the goal of estimating treatment effects
- 2. How do you evaluate goodness of fit for tree splitting and cross-validation?
 - $\tau_i = Y_i(1) Y_i(0)$ is not observed and thus you don't have ground truth for any unit

Literature

Approaches in the spirit of single tree and two trees

- Beygelzimer and Langford (2009)
 - Analogous to "two trees" approach with multiple treatments; construct optimal policy
- Dudick, Langford, and Li (2011)
 - Combine inverse propensity score method with "direct methods" (analogous to single tree approach) to estimate optimal policy
- Foster, Tailor, Ruberg, Statistics and Medicine (2011)
 - Estimate $\mu(w, x)$ using random forests, define $\hat{\tau}_i = \hat{\mu}(1, X_i) \hat{\mu}(0, X_i)$, and do trees on $\hat{\tau}_i$.
- ▶ Imai and Ratkovic (2013)
 - In context of randomized experiment, estimate $\mu(w, x)$ using lasso type methods, and then $\hat{\tau}(x) = \hat{\mu}(1, x) \hat{\mu}(0, x)$.

Estimating treatment effects directly at leaves of trees

- Su, Tsai, Wang, Nickerson, Li (2009)
 - Do regular tree, but split if the t-stat for the treatment effect difference is large, rather than when the change in prediction error is large.
- Zeileis, Hothorn, and Hornick (2005)
 - "Model-based recursive partitioning": estimate a model at the leaves of a tree. In-sample splits based on prediction error, do not focus on out of sample cross-validation for tuning.

Transformed outcomes or covariates for regressions

- ► Tibshirani et al (2014)
- Weisberg and Pontes (2015)

None of these explore cross-validation based on treatment effect.

Another Approach: Transform the Outcome

Suppose we have 50-50 randomization of treatment/control

- ▶ Then $E[Y_i^*] = 2 \cdot (\frac{1}{2}E[Y_i(1)] \frac{1}{2}E[Y_i(0)]) = E[\tau_i]$
- \triangleright Suppose treatment with probability p_i

Let
$$Y_i^* = \frac{W_i - p}{p(1 - p)} Y_i = \begin{cases} \frac{1}{p} Y_i & \text{if } W_i = 1 \\ -\frac{1}{1 - p} Y_i & \text{if } W_i = 0 \end{cases}$$

- $\qquad \text{Then } E[Y_i^*] = \left(p_{\overline{p}}^1 E[Y_i(1)] (1-p)_{\overline{1-p}}^1 E[Y_i(0)]\right) = E[\tau_i]$
- Selection on observables or stratified experiment

Let
$$Y_i^* = \frac{W_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i$$

Estimate $\hat{p}(x)$ using traditional methods

Causal Trees: (Conventional Tree, Transformed Outcome)

Model and Estimation

- A. Model type: Tree structure
- B. Estimator $\hat{\tau}_i^*$: sample mean of Y_i^* within leaf
- C. Set of candidate estimators C: correspond to different specifications of how tree is split

2. Criterion function (for fixed tuning parameter λ)

A. In-sample Goodness-of-fit function:

Qis = -MSE (Mean Squared Error) =
$$-\frac{1}{N}\sum_{i=1}^{N}(\hat{\tau}_i^* - Y_i^*)^2$$

- A. Structure and use of criterion
 - i. Criterion: $Q^{crit} = Q^{is} \lambda \times \#$ leaves
 - Select member of set of candidate estimators that maximizes Q^{crit} , given λ

3. Cross-validation approach

- A. Approach: Cross-validation on grid of tuning parameters. Select tuning parameter λ with highest Out-of-sample Goodness-of-Fit Qos.
- B. Out-of-sample Goodness-of-fit function: $Q^{os} = -MSE$

Critique of Approach: Transform the Outcome

$$Y_i^* = \frac{W_i - p}{p(1 - p)} Y_i = \begin{cases} \frac{1}{p} Y_i & \text{if } W_i = 1\\ -\frac{1}{1 - p} Y_i & \text{if } W_i = 0 \end{cases}$$

- Within a leaf, sample average of Y_i^* is not most efficient estimator of treatment effect
 - The proportion of treated units within the leaf is not the same as the overall sample proportion
- This motivates preferred approach: use sample average treatment effect in the leaf

Causal Trees:

(Causal Tree, TOT loss function)

Model and Estimation

- A. Model type: Tree structure
- B. Estimator $\hat{\tau}_i^{CT}$: sample average treatment effect within leaf (w/ PSW)
- C. Set of candidate estimators C: correspond to different specifications of how tree is split

2. Criterion function (for fixed tuning parameter λ)

A. In-sample Goodness-of-fit function:

Qis = -MSE (Mean Squared Error) =
$$-\frac{1}{N}\sum_{i=1}^{N}(\hat{\tau}_{i}^{CT}-Y_{i}^{*})^{2}$$

- A. Structure and use of criterion
 - i. Criterion: $Q^{crit} = Q^{is} \lambda \times \#$ leaves
 - Select member of set of candidate estimators that maximizes Q^{crit} , given λ

3. Cross-validation approach

- A. Approach: Cross-validation on grid of tuning parameters. Select tuning parameter λ with highest Out-of-sample Goodness-of-Fit Qos.
- B. Out-of-sample Goodness-of-fit function: $Q^{os} = -MSE$

Causal Trees

- What are you estimating? Within a leaf estimate treatment effect rather than a mean
 - Difference in average outcomes for treated and control group
 - Weight by normalized inverse propensity score in observational studies
- What is your goal? MSE of treatment effects: $-E_{S^T}\left[\sum_{i\in S^T}(\tau_i-\hat{\tau}(X_i))^2\right]$
- Problem: this is infeasible (true treatment effect unobserved)
 - We show we can estimate the criteria
- We also modify existing methods to be "honest." We decouple model selection from model estimation.
 - Split sample, one sample to build tree, second to estimate effects.
 - This changes criteria—novel idea for the literature.

$$-E_{S^T,S^E}\left[\sum_{i\in S^T}(\tau_i-\hat{\tau}(X_i;S^E))^2\right]$$

- Tradeoff:
 - COST: sample splitting means build shallower tree, less personalized predictions, and lower MSE of treatment effects.
 - ▶ BENEFIT: Valid confidence intervals with coverage rates that do not deteriorate as data generating process gets more complex or more covariates are added.

Honest Causal Trees

- Honest estimation changes expected criterion
 - Criterion anticipates that we will re-estimate effects in the leaves.
 - The bias due to "dishonest" selection of tree structure will be eliminated.
 - Eliminating the bias was the main purpose of cross-validation in standard method.
 - We face uncertainty in what honest sample will estimate
 - Small leaves will create noise.
 - > Splitting on variables that don't affect treatment effect can reduce variance
- Criterion for splitting and cross-validation changes
 - ▶ Given set of leaves, MSE on test set taking into account re-estimation.
 - Uncertainty over estimation set and test set at time of evaluation.

Standard Prediction Trees

$$MSE_{\mu}(\mathcal{S}^{\text{te}}, \mathcal{S}^{\text{est}}, \Pi) \equiv \frac{1}{\#(\mathcal{S}^{\text{te}})} \sum_{i \in \mathcal{S}^{\text{te}}} \left\{ \left(Y_i - \hat{\mu}(X_i; \mathcal{S}^{\text{est}}, \Pi) \right)^2 - Y_i^2 \right\}$$
$$EMSE_{\mu}(\Pi) \equiv \mathbb{E}_{\mathcal{S}^{\text{te}}, \mathcal{S}^{\text{est}}} \left[MSE_{\mu}(\mathcal{S}^{\text{te}}, \mathcal{S}^{\text{est}}, \Pi) \right]$$

Conventional CART uses for training and CV, respectively:

$$-\mathrm{MSE}_{\mu}(\mathcal{S}^{\mathrm{tr}}, \mathcal{S}^{\mathrm{tr}}, \Pi) = \frac{1}{N^{\mathrm{tr}}} \sum_{i \in \mathcal{S}^{\mathrm{tr}}} \hat{\mu}^{2}(X_{i}; \mathcal{S}^{\mathrm{tr}}, \Pi)$$

$$-\text{MSE}_{\mu}(\mathcal{S}^{\text{tr,cv}}, \mathcal{S}^{\text{tr,tr}}, \Pi)$$

$$= \frac{1}{N^{tr,cv}} \sum_{i \in \mathcal{S}^{tr,cv}} ((\widehat{\mu}(X_i; \mathcal{S}^{tr,tr}))^2 - 2\widehat{\mu}(X_i; \mathcal{S}^{tr,cv}) \widehat{\mu}(X_i; \mathcal{S}^{tr,tr}))$$

Honest Prediction Trees

$$-\text{EMSE}_{\mu}(\Pi) = -\mathbb{E}_{(Y_i, X_i), \mathcal{S}^{\text{est}}} [(Y_i - \mu(X_i; \Pi))^2 - Y_i^2]$$

$$-\mathbb{E}_{X_i, \mathcal{S}^{\text{est}}} [(\hat{\mu}(X_i; \mathcal{S}^{\text{est}}, \Pi) - \mu(X_i; \Pi))^2] =$$

$$\mathbb{E}_{X_i} [\mu^2(X_i; \Pi)] - \mathbb{E}_{\mathcal{S}^{\text{est}}, X_i} [\mathbb{V}(\hat{\mu}^2(X_i; \mathcal{S}^{\text{est}}, \Pi))],$$

This uses fact that estimator on independent sample is unbiased

$$\widehat{\mathbb{V}}(\widehat{\mu}(x; \mathcal{S}^{\text{est}}, \Pi)) \equiv \frac{S_{\mathcal{S}^{\text{tr}}}^{2}(\ell(x; \Pi))}{N^{\text{est}}(\ell(x; \Pi))}$$

$$\widehat{\mathbb{E}}\left[\mathbb{V}(\widehat{\mu}^{2}(X_{i}; \mathcal{S}^{\text{est}}, \Pi)|i \in \mathcal{S}^{\text{te}}\right] \equiv \frac{1}{N^{\text{est}}} \cdot \sum S_{\mathcal{S}^{\text{tr}}}^{2}(\ell)$$

$$-\widehat{\mathrm{EMSE}}_{\mu}(\mathcal{S}^{\mathrm{tr}}, N^{\mathrm{est}}, \Pi) \equiv \frac{1}{N^{\mathrm{tr}}} \sum_{i \in \mathcal{S}^{\mathrm{tr}}} \hat{\mu}^{2}(X_{i}; \mathcal{S}^{\mathrm{tr}}, \Pi) - \left(\frac{1}{N^{\mathrm{tr}}} + \frac{1}{N^{\mathrm{est}}}\right) \cdot \sum_{\ell \in \Pi} S_{\mathcal{S}^{\mathrm{tr}}}^{2}(\ell(x; \Pi))$$

Standard Causal Trees

$$MSE_{\tau}(\mathcal{S}^{\text{te}}, \mathcal{S}^{\text{est}}, \Pi) \equiv \frac{1}{\#(\mathcal{S}^{\text{te}})} \sum_{i \in \mathcal{S}^{\text{te}}} \left\{ \left(\tau_{i} - \hat{\tau}(X_{i}; \mathcal{S}^{\text{est}}, \Pi) \right)^{2} - \tau_{i}^{2} \right\}$$

$$EMSE_{\tau}(\Pi) \equiv \mathbb{E}_{\mathcal{S}^{\text{te}}, \mathcal{S}^{\text{est}}} \left[MSE_{\tau}(\mathcal{S}^{\text{te}}, \mathcal{S}^{\text{est}}, \Pi) \right]$$

$$\widehat{MSE}_{\tau}(\mathcal{S}^{\text{te}}, \mathcal{S}^{\text{tr}}, \Pi) \equiv -\frac{2}{N^{\text{tr}}} \sum_{i \in \mathcal{S}^{\text{te}}} \hat{\tau}(X_{i}; \mathcal{S}^{\text{te}}, \Pi) \cdot \hat{\tau}(X_{i}; \mathcal{S}^{\text{tr}}, \Pi)$$

$$+\frac{1}{N^{\mathrm{tr}}}\sum_{i\in\mathcal{S}^{\mathrm{te}}}\hat{\tau}^{2}(X_{i};\mathcal{S}^{\mathrm{tr}},\Pi).$$

For training and CV, respectively:

$$-\widehat{\mathrm{MSE}}_{\tau}(\mathcal{S}^{\mathrm{tr}}, \mathcal{S}^{\mathrm{tr}}, \Pi) = \frac{1}{N^{\mathrm{tr}}} \sum_{i \in \mathcal{S}^{\mathrm{tr}}} \hat{\tau}^{2}(X_{i}; \mathcal{S}^{\mathrm{tr}}, \Pi)$$
$$-\widehat{\mathrm{MSE}}_{\tau}(\mathcal{S}^{\mathrm{tr,cv}}, \mathcal{S}^{\mathrm{tr,tr}}, \Pi)$$

Honest Causal Trees

$$-\mathrm{EMSE}_{\tau}(\Pi) = \mathbb{E}_{X_i}[\tau^2(X_i; \Pi)] - \mathbb{E}_{\mathcal{S}^{\mathrm{est}}, X_i}\left[\mathbb{V}(\hat{\tau}^2(X_i; \mathcal{S}^{\mathrm{est}}, \Pi))\right]$$

This uses fact that estimator on independent sample is unbiased

For training and CV, respectively:

$$-\widehat{\mathrm{EMSE}}_{\tau}(\mathcal{S}^{\mathrm{tr}}, N^{\mathrm{est}}, \Pi) \equiv \frac{1}{N^{\mathrm{tr}}} \sum_{i \in \mathcal{S}^{\mathrm{tr}}} \hat{\tau}^{2}(X_{i}; \mathcal{S}^{\mathrm{tr}}, \Pi)$$

$$-\left(\frac{1}{N^{\text{tr}}} + \frac{1}{N^{\text{est}}}\right) \cdot \sum_{\ell \in \Pi} \left(\frac{S_{\mathcal{S}_{\text{treat}}}^{2}(\ell)}{p} + \frac{S_{\mathcal{S}_{\text{control}}}^{2}(\ell)}{1-p}\right)$$

$$-\widehat{\mathrm{EMSE}}_{\tau}(\mathcal{S}^{\mathrm{tr,cv}}, N^{\mathrm{est}}, \Pi)$$

Inference

Attractive feature of trees:

- Can easily separate tree construction from treatment effect estimation
- Tree constructed on training sample is indep. of sampling variation in test sample
- Holding tree from training sample fixed, can use standard methods to conduct inference within each leaf of the tree on test sample
 - Use any valid method for treatment effect estimation, not just method used in training. Asymptotic theory as usual within a leaf.
- Once you have the partition, just run a regression on second sample interacting leaf dummies with treatment indicator. Everything is as usual.
- We do not require ANY assumptions about sparsity of true data-generating process. Coverage does not deteriorate at all as you increase number of covariates.
 - But we do not attempt to make fully personalized estimates

Comparing Alternative Approaches to Preferred Honest Causal Tree

Dishonest with double the sample

- Does worse if true model is sparse (also the case where bias is less severe)
- Has similar or better MSE in many cases, but poor coverage of confidence intervals

Splitting on statistical criteria of model fit

- Paper shows formally how these methods differ (proposed in a small related literature, one that doesn't consider honesty and cross-validation issues)
- Splitting on T-statistic on treatment effect ignores variance reduction from reducing imbalance on covariates
- Splitting on overall model fit prioritizes level heterogeneity above treatment effects

Application: Treatment Effect Heterogeneity in Estimating Position Effects in Search

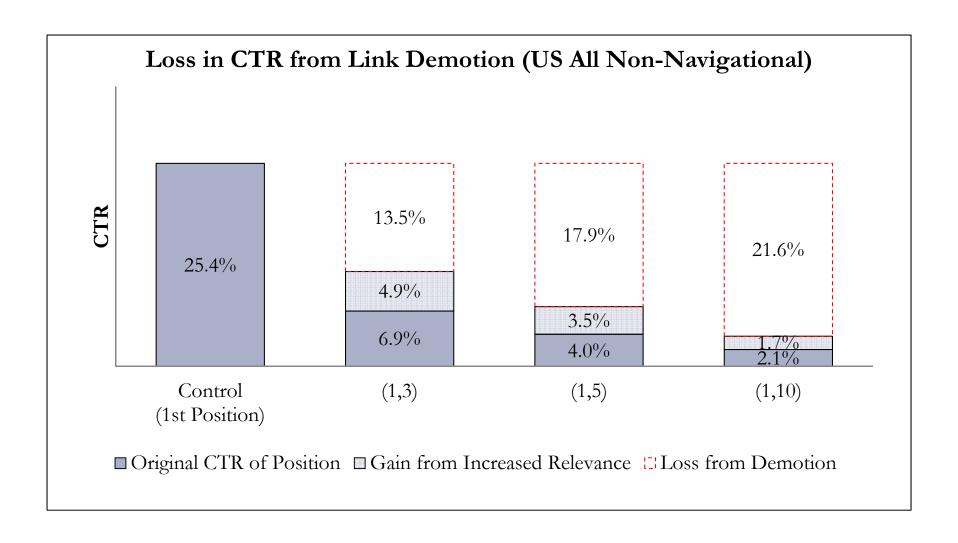
Queries highly heterogeneous

- Tens of millions of unique search phrases each month
- Query mix changes month to month for a variety of reasons
- Behavior conditional on query is fairly stable

Desire for segments.

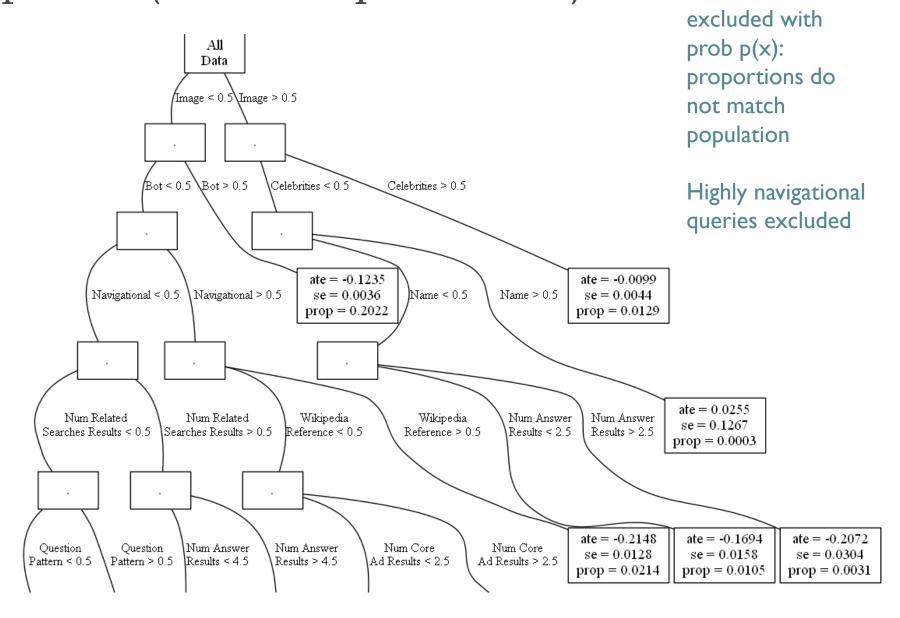
- Want to understand heterogeneity and make decisions based on it
- "Tune" algorithms separately by segment
- Want to predict outcomes if query mix changes
 - For example, bring on new syndication partner with more queries of a certain type

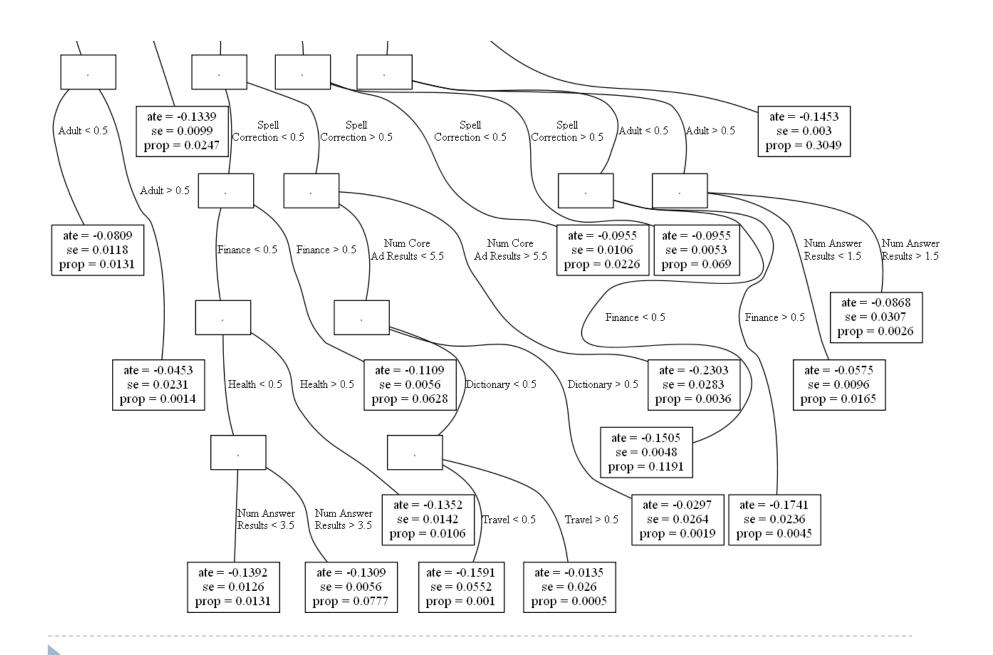
Relevance v. Position



Search Experiment Tree: Effect of Demoting Top Link (Test Sample Effects)

Some data





	Hoi	nest Estima	ites	Adaptive Estimates			
	Treatment	Standard		Treatment	Standard		
	Effect	Error	Proportion	Effect	Error	Proportion	
Use Test Sample	-0.124	0.004	0.202	-0.124	0.004	0.202	
	-0.134	0.010	0.025	-0.135	0.010	0.024	
-	-0.010	0.004	0.013	-0.007	0.004	0.013	
for Segment	-0.215	0.013	0.021	-0.247	0.013	0.022	
Means & Std	-0.145	0.003	0.305	-0.148	0.003	0.304	
	-0.111	0.006	0.063	-0.110	0.006	0.064	
Errors to Avoid	-0.230	0.028	0.004	-0.268	0.028	0.004	
Bias	-0.058	0.010	0.017	-0.032	0.010	0.017	
	-0.087	0.031	0.003	-0.056	0.029	0.003	
	-0.151	0.005	0.119	-0.169	0.005	0.119	
Variance of	-0.174 0.026	0.024 0.127	0.005 0.000	-0.168 0.286	0.024 0.124	0.005 0.000	
estimated	-0.030	0.127	0.000	-0.009	0.124	0.000	
	0.425	0.014	0.011	-0.114	0.015	0.010	
treatment effects	-0.159	0.055	0.001	-0.143	0.053	0.001	
in training	-0.014	0.026	0.001	0.008	0.050	0.000	
sample 2.5 times	-0.081	0.012	0.013	-0.050	0.012	0.013	
	-0.045	0.023	0.001	-0.045	0.021	0.001	
that in test	-0.169	0.016	0.011	-0.200	0.016	0.011	
sample (adaptive	-0.207	0.030	0.003	-0.279	0.031	0.003	
	-0.096	0.011	0.023	-0.083	0.011	0.022	
estimates	-0.096	0.005	0.069	-0.096	0.005	0.070	
biased)	-0.139	0.013	0.013	-0.159	0.013	0.013	
,	-0.131	0.006	0.078	-0.128	0.006	0.078	

Analyzing Field Experiments: Revisit Karlan and List (AER)

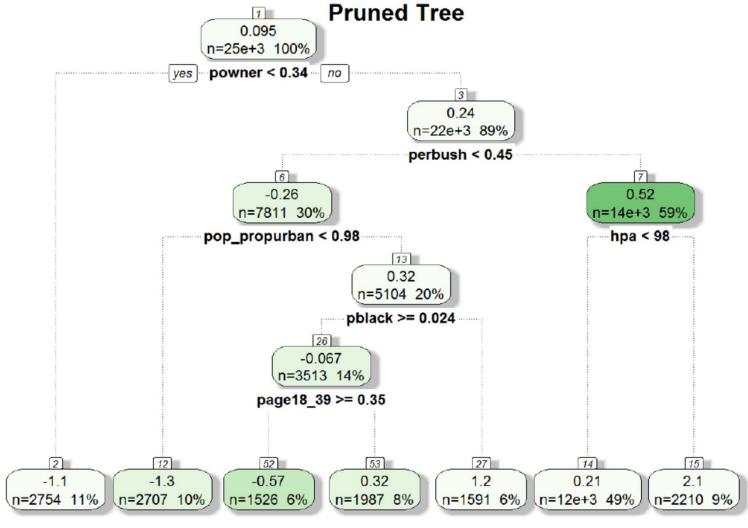
Field experiment on charitable giving

- Solicitations: treatment groups are offered match of various sizes, example gifts, and limits
- Finding: match increases gift amount, but larger sizes do not have incremental effect
- Some exploration of heterogeneity

Apply causal trees:

- Explore heterogeneity more systematically
- Highlight the risks of data mining
- Unlike the search application, it appears there isn't a lot of treatment effect heterogeneity

Effect of All Treatments (Pooled) Training Data



Rattle 2015-Aug-20 23:20:55 athey

Training Sample

```
: Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
leavesr-1.342
                                 0.2768 7.006 2.51e-12 ***
                        1.9395
leavesr-1.083
                        1.6189
                                 0.2845 5.691 1.28e-08 ***
leavesr-0.573
                        0.9498
                                 0.3696 2.570 0.01018 *
leavesr0.213
                        0.3260
                                 0.1350 2.415 0.01574 *
leavesr0.321
                        0.6566
                                 0.3257 2.016 0.04379 *
leavesr1.171
                        0.2839
                                 0.3669 0.774 0.43907
leavesr2.086
                        1.6176
                                 0.3288 4.921 8.68e-07 ***
leavesr-1.342:treatment -1.3417
                                 0.3445 -3.895 9.86e-05 ***
: leavesr-1.083:treatment -1.0826
                                  0.3475 -3.116 0.00184 **
                                  0.4593 -1.248 0.21201
: leavesr-0.573:treatment -0.5733
leavesr0.213:treatment 0.2131
                                  0.1648 1.294 0.19581
: leavesr0.321:treatment 0.3210
                                  0.4035 0.796 0.42632
leavesr1.171:treatment
                       1.1707
                                 0.4527 2.586 0.00972 **
leavesr2.086:treatment 2.0856
                                 0.3951 5.279 1.31e-07 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Estimation Sample

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
leavesr-1.342	0.62900	0.28579	2.201	0.02775	*
leavesr-1.083	0.51463	0.28810	1.786	0.07407	
leavesr-0.573	0.75687	0.40245	1.881	0.06003	
leavesr0.213	0.34395	0.13808	2.491	0.01275	*
leavesr0.321	1.07704	0.34707	3.103	0.00192	**
leavesr1.171	1.09966	0.36281	3.031	0.00244	**
leavesr2.086	3.39262	0.31770	10.679	< 2e-16	***
leavesr-1.342:treatment	0.37260	0.34983	1.065	0.28685	
leavesr-1.083:treatment	0.29186	0.35475	0.823	0.41067	
leavesr-0.573:treatment	0.10772	0.48868	0.220	0.82553	
leavesr0.213:treatment	0.17869	0.16888	1.058	0.29002	
leavesr0.321:treatment	-0.28147	0.42071	-0.669	0.50347	
leavesr1.171:treatment	-0.08332	0.44850	-0.186	0.85262	
leavesr2.086:treatment	0.89775	0.39055	2.299	0.02153	*

Interpretation: Sample Splitting is Key

- Training set makes it look like there is lots of heterogeneity
- Estimation sample accurately shows most of that is spurious
- You get the right, if disappointing, answer out of the estimation sample

Summary

- Key to approach
 - Distinguish between causal and predictive parts of model
- Combining two literatures
 - Combining very well established tools from different literatures
 - Systematic model selection with many covariates
 - Optimized for problem of causal effects
 - In terms of tradeoff between granular prediction and overfitting
 - With valid inference
 - While sacrificing fully personalized predictions, but gaining...
 - ▶ Easy to communicate method and interpret results
 - Dutput is a partition of sample, treatment effects and standard errors