① If in RREF form, one row has all zeros but the product is non-zeros there is no solution

$$\left[\begin{array}{cc|c} x & x & x \\ 0 & 0 & 2 \end{array}\right]$$

Unique solution: no row with all-zero variables

∞ sol: <n non-zero rows & any row w all-zero variable coeffs.

No. If there are a row of zeros, but still the same or more equations than unknown variables, then there is still a unique solution.

If all zeros, can be ∞ solutions

(2a) iii)

$$\begin{bmatrix} 1 & 2 \\ 2 & -4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 2 & -4 & 4 \\ 3 & -2 & 8 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 8 & 4 \\ 3 & -2 & 8 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 4 & 0 & 12 \\ 1 & -2 & 2 \\ 3 & -2 & 8 \end{array}\right] \quad R_1 \leftarrow R_1 + R_2 \quad R_2 \leftarrow R_2/2$$

$8y = 4$

$y = \frac{1}{2}$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 1 & -2 & 2 \\ 3 & -2 & 8 \end{array}\right] \quad R_1/4$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & -2 & -1 \\ 3 & -2 & 8 \end{array}\right] \quad R_2 \leftarrow R_2 - R_1$$

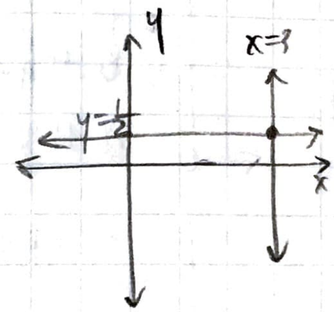
$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & -2 & 8 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & 0 & 9 \end{array}\right] \quad R_3 + R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & \frac{1}{2} \\ 1 & 0 & 3 \end{array}\right]$$

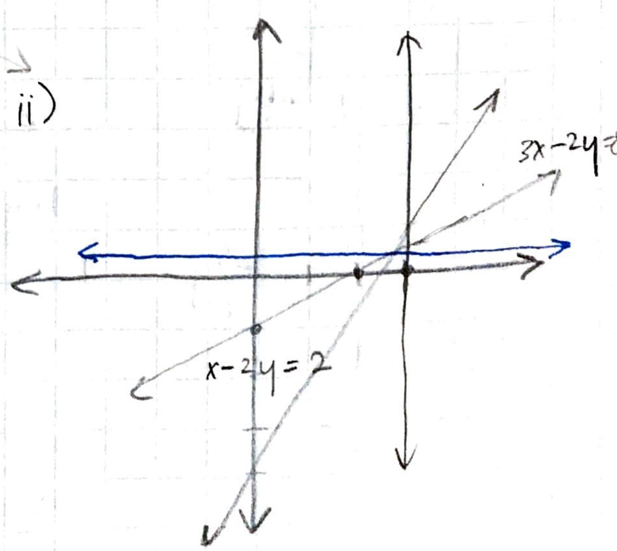
$x = 3$

$y = \frac{1}{2}$



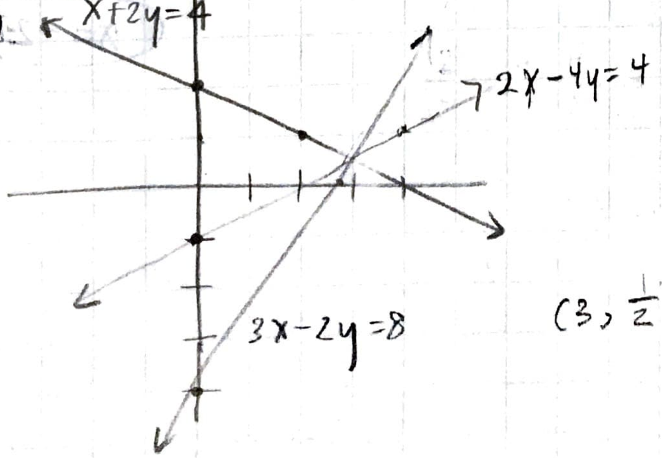

vertical + horizontal lines

ii)




one of the lines is now vertical (x=3)

i)




$(3, \frac{1}{2})$

2b

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 1 & 12 & 6 & 1 \\ 0 & 2 & 1 & 4 \\ 3 & 16 & 16 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 2 & 1 & 4 \\ 3 & 16 & 16 & 7 \end{array}\right] \quad R_2 \leftarrow R_2 - R_1$$

$$0 \quad 2 \quad \frac{1}{5} \;\Big|\; -\frac{2}{5}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 0 & \frac{4}{5} & \cancel{-\frac{18}{5}}\ \frac{22}{5} \\ 3 & 16 & 16 & 7 \end{array}\right] \quad R_3 \leftarrow R_3 - R_2/5$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 0 & 4 & \cancel{-18}\ 22 \\ 3 & 16 & 16 & 7 \end{array}\right] \quad 3 \quad 6 \quad 15 \;\Big|\; 9$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 0 & 1 & \cancel{-\frac{9}{2}}\ \frac{11}{2} \\ \cancel{0} & \cancel{10} & \cancel{1} & \cancel{-2} \end{array}\right]$$

$R_3 \leftarrow R_3 / 4$

$R_4 \leftarrow R_4 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 1 & 0.1 & -0.2 \\ 0 & 0 & 1 & \frac{11}{2} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$z = \frac{11}{2}$

$y + \frac{1}{10} z = -0.2$

$y + \frac{11}{20} = -\frac{4}{20}$

$y = -\frac{15}{20} = -\frac{3}{4}$

$y = -\frac{3}{4}$

$x + (2)(-\frac{3}{4}) + 5(\frac{11}{2}) = 3$

$x - \frac{3}{2} + \frac{55}{2} = \frac{6}{2}$

$x + \frac{52}{2} = \frac{6}{2}$

$x + 26 = 3$

$x = -23$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 0 & 1 & \frac{11}{2} \end{array}\right]$$

$z = -\frac{9}{2}$ $\qquad y = \frac{1}{4}$

$10y + z = -2$

$10y - \frac{9}{2} = -\frac{4}{2}$

$10y = \frac{5}{2}$

$y = \frac{5}{2} \cdot \frac{1}{10} = \frac{1}{4}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 25 \\ \frac{1}{4} \\ -\frac{9}{2} \end{bmatrix}$$

$x + 2y + 5z = 3$

$x + \frac{1}{2} - \frac{45}{2} = \frac{6}{2}$

$x - \frac{44}{2} = \frac{6}{2}$

$x - 22 = 3$ $\qquad x = 25$

2c

$$\begin{bmatrix} 1 & 2 & 5 \\ 3 & 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 3 & 9 & 6 \end{bmatrix} \begin{bmatrix} 16-11t \\ -5+3t \\ t \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} (16-11t)+2(-5+3t)+5(t) \\ 3(16-11t)+9(-5+3t)+6(t) \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 16-11t-10+6t+5t \\ 48-33t-45+27t+6t \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 6+0t \\ 3+0t \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

~~2d~~

2 free variables: $x_2, x_5$

say $x_5 = t$, $x_1 = p$

$$\begin{cases} p + x_2 + 3t = 16 \\ x_3 - 3t = -17 \\ x_4 + t = 5 \end{cases}$$

$x_2 = p$

$x_1 = 16 - 3t - p$

$x_3 = -17 + 3t$

$x_4 = 5 - t$

$$\vec{x} = \begin{bmatrix} p \\ 16-3t-p \\ -17+3t \\ 5-t \\ t \end{bmatrix}$$

RREF

col 2. and col 5 do not have 1 in pivot position ⇒ tf, free variables

(3a) $\left\{\begin{bmatrix}-5\\2\end{bmatrix},\begin{bmatrix}5\\2\end{bmatrix}\right\}$

linear dependence:
$\{\vec{v}_1 \dots \vec{v}_n\}$ if
scalars $\alpha_1 \dots \alpha_n$
such that $\alpha_1\vec{v}_1 + \dots + \alpha_n\vec{v}_n = \vec{0}$
and at least one non-zero $\vec{v}$

not linearly dependent

$$\alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 = \vec{0}$$

$$\Rightarrow \alpha_1\begin{bmatrix}-5\\2\end{bmatrix} + \alpha_2\begin{bmatrix}5\\2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\vec{\alpha} = A^{-1}\vec{b}$$

$$\left[\begin{array}{cc|c}-5 & 5 & 0\\ 2 & 2 & 0\end{array}\right]$$

(3b) $\left[\begin{array}{ccc|c}-1 & 1 & -1 & 0\\ 1 & 2 & 0 & 0\\ 0 & 3 & -1 & 0\\ -1 & -2 & 0 & 0\\ 1 & -1 & 1 & 0\end{array}\right]$ $\left[\begin{array}{ccc|c}1 & -1 & 1 & 0\\ 0 & 3 & -1 & 0\\ 0 & 0 & 0 & 0\\ -1 & -2 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right]$ $\left[\begin{array}{ccc|c}-1 & -2 & 0 & 0\\ 0 & 3 & -1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right]$

$\left[\begin{array}{ccc|c}1 & -1 & 1 & 0\\ 0 & 3 & -1 & 0\\ 0 & 3 & -1 & 0\\ -1 & -2 & 0 & 0\\ 1 & -1 & 1 & 0\end{array}\right]$ $R_1 \leftarrow -R_1$, $R_2 \leftarrow R_2 + R_1$ $\left[\begin{array}{ccc|c}0 & -3 & 1 & 0\\ 0 & 3 & -1 & 0\\ 0 & 0 & 0 & 0\\ -1 & -2 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right]$

linearly dependent
no unique soln

$x_2 = \alpha$
free variable

$-x_1 - 2x_2 = 0$
$3x_2 - x_3 = 0$

$x_1 = -2\alpha$
$x_2 = \alpha$
$x_3 = 3\alpha$

$$-2\vec{v}_1 + \vec{v}_2 + 3\vec{v}_3 = \vec{0}$$

linearly dependent

(3c) $\left[\begin{array}{cccc|c}2 & 0 & 2 & 0 & 0\\ 2 & 1 & 4 & -1 & 0\\ 0 & 1 & -1 & 1 & 0\end{array}\right]$

$\left[\begin{array}{cccc|c}2 & 0 & 2 & 0 & 0\\ 0 & 1 & 2 & -1 & 0\\ 0 & 1 & -1 & 1 & 0\end{array}\right]$ $R2 \leftarrow R_2 - R_1$

$\left[\begin{array}{cccc|c}1 & 0 & 1 & 0 & 0\\ 0 & 1 & 2 & -1 & 0\\ 0 & 0 & -3 & 2 & 0\end{array}\right]$ $R_1/2$, $R_3 - R_2$

3c) cont.

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 1 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \end{array}\right]$$

free variable: $3\alpha$

$x_4 = \alpha$

$-3x_3 + 2\alpha = 0$
$x_3 = \frac{2}{3}\alpha$

$x_1 + x_3 = 0$
$x_1 + \frac{2}{3}\alpha = 0$
$x_1 = -\frac{2}{3}\alpha$

$x_2 + x_3 + \alpha = 0$
$x_2 + \frac{5}{3}\alpha = 0$
$x_2 = -\frac{5}{3}\alpha$

$\vec{0} = -\frac{2}{3}v_1 - \frac{5}{3}v_2 + \frac{2}{3}v_3 + v_4$

$v_1 + \frac{2}{3}(3\alpha) = 0$
$v_1 = -2\alpha$

$\vec{0} = -2v_1 - v_2 + 2v_3 + v_4$

$v_2 + \alpha = 0$
$v_2 = -\alpha$

$v_3 - 2\alpha = \alpha$
$v_3 = 2\alpha$

$v_4 = \alpha$

3d) linearly ~~in~~dependent

can~~not~~ combine together with scalars to form $\vec{0}$.

can take linear combination $\alpha \cdot \vec{o}$ where $\alpha \neq 0$ to get $\vec{0}$

4a)
$$\begin{cases} m_1 = f_1(\alpha)\cdot a + f_1(\beta)\cdot b \\ m_2 = f_2(\alpha)\cdot a + f_2(\beta)\cdot b \end{cases}$$

$\alpha = 45°$
$\beta = -30°$

$$\begin{cases} m_1 = \cos(45°)a + \cos(-30°)b \\ m_2 = \sin(45°)a + \sin(-30°)b \end{cases}$$

$$\begin{cases} m_1 = \frac{\sqrt{2}}{2}a + \frac{\sqrt{3}}{2}b \\ m_2 = \frac{\sqrt{2}}{2}a - \frac{1}{2}b \end{cases}$$

4b)
$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & m_1 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 2m_1 \\ \sqrt{2} & -1 & 2m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 2m_1 \\ 0 & -1-\sqrt{3} & 2m_2 - 2m_1 \end{array}\right] \quad R_2 \leftarrow R_2 - R_1$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{\sqrt{3}} & 1 & (2m_1)\left(\frac{1}{\sqrt{3}}\right) \\ 0 & 1 & \left(\frac{-1}{1+\sqrt{3}}\right)(2m_2 - 2m_1) \end{array}\right]$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \left(\frac{2}{\sqrt{3}}m_1\right) - \left(\frac{-1}{1+\sqrt{3}}\right)(2m_2 - 2m_1) \\ 0 & 1 & \left(\frac{-1}{1+\sqrt{3}}\right)(2m_2 - 2m_1) \end{array}\right]$$

4b

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & m_1 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 2m_1 \\ \sqrt{2} & -1 & 2m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{3}\cdot\sqrt{2} & 3 & 2\sqrt{3}m_1 \\ 3\sqrt{2} & -3 & 6m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} (\sqrt{3}\cdot\sqrt{2}) + 3\sqrt{2} & 0 & 6m_2 + 2\sqrt{3}m_1 \\ \sqrt{2} & -1 & 2m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} (\sqrt{2})(\sqrt{3}+3) & 0 & 6m_2 + 2\sqrt{3}m_1 \\ (\sqrt{2})(\sqrt{3}+3) & -1(\sqrt{3}+3) & (2m_2)(\sqrt{3}+3) \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{2}\cdot(\sqrt{3}+3) & 0 & 6m_2 + 2\sqrt{3}m_1 \\ 0 & -\sqrt{3}-3 & 2(\sqrt{3}+3)m_2 - 6m_2 + 2\sqrt{3}m_1 \end{array}\right]$$

$$a = \frac{(2\sqrt{3}m_1 + 6m_2)}{\sqrt{2}(\sqrt{3}+3)}$$

$$b = \frac{2\sqrt{3}m_1 + 2\sqrt{3}m_2}{-\sqrt{3}-3} = \frac{-2\sqrt{3}}{\sqrt{3}+3}m_1 - \frac{2\sqrt{3}}{\sqrt{3}+3}m_2$$

$$a = \underbrace{\frac{\sqrt{6}-\sqrt{2}}{2}m_1}_{\vec{u}} + \underbrace{\left(\frac{-\sqrt{6}+3\sqrt{2}}{2}\right)m_2}_{\vec{v}} \qquad b = \underbrace{(-1+\sqrt{3})m_1}_{\vec{u}} + \underbrace{(-1+\sqrt{3})m_2}_{\vec{v}}$$

4c "all human beings are born free and equal in dignity and rights"

6a)

$$\vec{r} = \begin{bmatrix} \star \\ b \\ c \\ \star \\ b \\ c \end{bmatrix}$$

6b)

$$G_R \vec{m} = \vec{w}$$

$$G_R = \begin{array}{c} \begin{matrix} a & b & c \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad G_R \vec{m} = \begin{bmatrix} a \\ b \\ c \\ a \\ b \\ c \end{bmatrix}$$

6c)

$7 \times 3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 \\ \star \\ \star \\ 3 \\ 4 \\ \star \\ \star \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 4 \end{bmatrix}$$

$a = 7$
$b = \star$
$c = \star$
$a + b = 3 \rightarrow 7 + b = 3$, $b = -4$
$a + c = 4 \rightarrow 7 + c = 4$, $c = -3$
$b + c = \star$
$a + b + c = \star$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \\ -3 \end{bmatrix}$$

6d)

$$\vec{r} = \begin{bmatrix} 1 \\ \star \\ 3 \\ \star \\ 4 \\ \star \\ 9 \end{bmatrix}$$

$a = 1$
$c = 3$
$a + c = 4$
$a + b + c = 9 \longrightarrow 1 + b + 3 = 9$
$b = 5$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$$

⑦ . $A\vec{x} = \vec{b}$ has $\infty$ soln $\Rightarrow$ columns are lin dependent

i) If system has $\infty$ soln, must have at least 2 distinct soln

$$A\vec{x} = \vec{b}$$

$$A\vec{u} = \vec{b} \qquad A\vec{v} = \vec{b} \qquad \vec{u} \neq \vec{v}$$

ii)

$$\begin{bmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_3 \\ | & | & & | \end{bmatrix}$$

Columns of vector are linearly dependent if exist scalars $\alpha$ such that $\alpha_1 \vec{v}_1 + \ldots + \alpha_n \vec{v}_n = \vec{0}$ and at least 1 nonzero $\alpha$

$$\vec{b} = \vec{c}_1 u_1 + \vec{c}_2 u_2 + \cdots + \vec{c}_n u_n$$

$$\vec{b} = \vec{c}_1 v_1 + \vec{c}_2 v_2 + \cdots + \vec{c}_n v_n$$

iii) $A\vec{u} = \vec{b}$ $\qquad$ $A\vec{v} = \vec{b}$

$$\begin{bmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \vec{b} \qquad \begin{bmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \vec{b}$$

$$u_1 \begin{bmatrix} | \\ \vec{c}_1 \\ | \end{bmatrix} + u_2 \begin{bmatrix} | \\ \vec{c}_2 \\ | \end{bmatrix} + \cdots + u_n \begin{bmatrix} | \\ \vec{c}_n \\ | \end{bmatrix} = \vec{b} \qquad v_1 \vec{c}_1 + v_2 \vec{c}_2 + \cdots + v_n \vec{c}_n = \vec{b}$$

$$A\vec{u} - A\vec{v} = A(\vec{u} - \vec{v}) = \vec{b} - \vec{b} = \vec{0}$$

$$\vec{u} \neq \vec{v}$$

$$\vec{u} - \vec{v} \neq \vec{0}$$

set $\vec{x} = \vec{u} - \vec{v}$

therefore linearly dependent columns.

PS 2

for lin. dependent vectors: at least one scalar needs to be non-zero

7b

Known

$\{\vec{v}_1, \vec{v}_2 \dots \vec{v}_n\}$

To Show

$\text{span}\{\vec{v}_1, \vec{v}_2 \dots \vec{v}_n\} = \text{span}\{\vec{v}_1+\vec{v}_2, \vec{v}_2 \dots \vec{v}_n\}$

Suppose $\vec{q} \in \text{span}\{\vec{v}_1, \vec{v}_2 \dots \vec{v}_n\}$ for some scalars $a_i$:

$$\vec{q} = a_1\vec{v}_1 + a_2\vec{v}_2 + \dots a_n\vec{v}_n = a_1(\vec{v}_1+\vec{v}_2) + \underbrace{(-a_1+a_2)}_{\to a_2}\vec{v}_2 + \dots a_n\vec{v}_n$$

therefore $\vec{q} \in \text{span}\{\vec{v}_1+\vec{v}_2, \vec{v}_2 \dots \vec{v}_n\}$

Suppose $\vec{q} \in \text{span}\{\vec{v}_1+\vec{v}_2, \vec{v}_2 \dots \vec{v}_n\}$ for some scalars $b_i$:

$$\vec{r} = b_1\vec{v}_1 + (b_1+b_2)\vec{v}_2 + b_3\vec{v}_3 + \dots b_n\vec{v}_n = b_1\vec{v}_1 + b_2\vec{v}_2 \dots b_n\vec{v}_n$$

thus, $\vec{r} \in \text{span}\{\vec{v}_1, \vec{v}_2 \dots \vec{v}_n)$

Spans are same. QED

7c

Known

$n$ = pos. int

$\{\vec{v}_1, v_2 \dots v_k\} \in \mathbb{R}^n$

Show

$\{A\vec{v}_1, A\vec{v}_2 \dots A\vec{v}_k\}$ is set of linearly dependent

By definition of linear dependence

$$a_1\vec{v}_1 + a_2\vec{v}_2 + \dots + a_k\vec{v}_k = \vec{0}$$

such that not all "a" scalars are 0 ✓

$$A(a_1\vec{v}_1 + a_2\vec{v}_2 + \dots + a_k\vec{v}_k) = A\vec{0}$$

$$A\vec{v}_1 + A\vec{v}_2 + \dots + A\vec{v}_k = \vec{0}$$

Thus, $\{A\vec{v} \dots\}$ is linearly dependent

8 I worked alone.