**0. So it begins (1 point).** Write your name and ID on the front page. Write the exam room. Write the IDs of your neighbors. Write the given statement and sign. Write your login in the corner of every page. Enjoy your free point ☺.

1. Tree Removal.

a) **(13 points).** Suppose we have the BST shown below. Give the tree that results from deleting "4" using the procedure from class (a.k.a. Hibbard deletion). **Use the successor, not the predecessor**. Draw your answer in the box located to the right of the tree below.
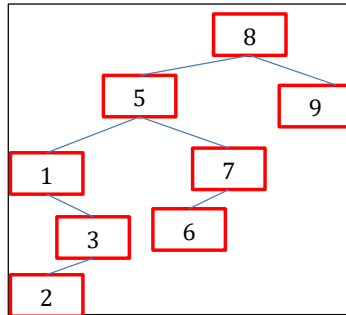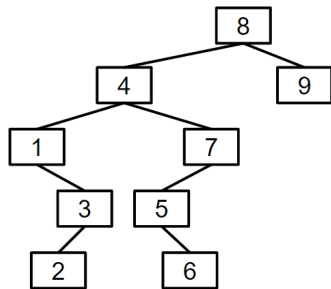
b) **(20 points).** Suppose we want to write a method that deletes the **smallest** item in a BST rooted at **TreeNode** x. Fill in the code below. The method should return the root of the BST after deletion. Assume the **TreeNode** class has three fields: item, left, and right. You may assume x is never null. If deletion results in an empty tree, you should return null.

```
private TreeNode deleteMin(TreeNode x) {
    if (x.left == null) { return x.right; }
    x.left = deleteMin(x.left);
    return x;
}
```

For reference, the **TreeNode** class is defined as shown below:

```
public class TreeNode {
    public TreeNode left;
    public TreeNode right;
    public int item;
    ...
}
```

---

2. Hash Tables. Suppose we have a correct hash table implementation of a set as seen in lecture, where:
- We store the buckets in an array, where each bucket is a linked list (i.e. separate chaining).
- We resize to 1.5x the number of buckets at the end of an add operation if the load factor is ≥ 1.
- We reduce our hashcode with floorMod(hashCode(), M). Recall that floorMod is just %, but handles negative numbers correctly.
- For parts a-c: We currently have N = 19 items and M = 41 buckets.

a) **(5 points).** What is the current load factor? It is OK to leave your answer as a fraction: 19/41

b) **(10 points).** Suppose a user calls contains. Give the minimum and maximum number of times the contains method would need to call equals. **Do not assume anything about the distribution of items in the hash table.**

Min: 0 Max: 19

c) **(10 points).** Give the minimum and maximum number of add calls that can cause the next resize.

Min: 22 Max: 22

d) **(12 points).** Suppose we define an ExamOomage class that has 4 instance variables int r, int g, int b, int a, each in the range [0, 255]. Assume that it provides a correct equals method as well as a hashCode method that returns some integer. Do not assume anything about the quality of the hashCode method. Suppose we run a timing experiment using a Problem2HashSet (as defined in the bulleted list above) as follows:

```
public static void timingTest(int N) {
    Stopwatch sw = new Stopwatch();
    Problem2HashSet<ExamOomage> hsp = new Problem2HashSet<>();
    for (int i = 0; i < N; i += 1) {
        hsp.add(ExamOomage.randomPoint()); //returns ExamOomage with
    }                                       //random r/g/b/a in [0, 255]
    System.out.println(sw.elapsedTime());
}
```

Using the code above we get the runtimes below:
```
N = 1000  : 0.026 seconds
N = 10000 : 1.00 seconds
N = 49123 : 26.96 seconds
N = 100000: 118.43 seconds
```

From the given data, which of the following best matches the **average** time **per add** operation?

Each add on average costs: ○ $\Theta(1)$ ○ $\Theta(\log N)$ ● $\Theta(N)$ ○ $\Theta(N^2)$ ○ $\Theta(2^N)$

e) **(12 points).** Give an example of a hashCode() function that would yield timing results similar to those shown in the table above.

```
return 1; // the goal here was to make a bad hash code
```

4. LLRBs.

a) **(10 points).** Suppose we have the LLRB below, where red links are given as thicker dashed lines. If we add 15, what **single** LLRB operation will we need to perform? Give your answer as `rotateLeft(X)`, `rotateRight(X)`, or `colorFlip(X)` where X is the node which we rotate or color flip. Reminder: `colorFlip(17)` would mean flipping all edges touching 17. Don't forget to fill in the value in parenthesis!

● rotateLeft(13)　　　○ rotateRight(　　)　　　○ colorFlip(　　)
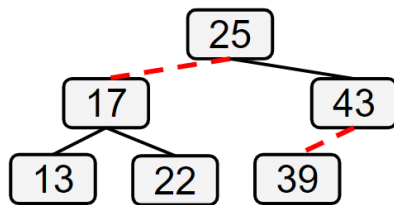
b) **(24 points).** Starting from the figure of the LLRB above (i.e. without adding 15), what LLRB operations will we need to complete if we add the value 40? By LLRB operation, we mean `rotateLeft`, `rotateRight`, and `colorFlip`. Give your answer by filling in one bubble per line and filling in the value in parenthesis, where line 1 corresponds to operation #1, etc. You may not need all five operations.

Op #1: ● rotateLeft(39)　　　○ rotateRight(　　)　　　○ colorFlip(　　)

Op #2: ○ rotateLeft(　　)　　　● rotateRight(43)　　　○ colorFlip(　　)

Op #3: ○ rotateLeft(　　)　　　○ rotateRight(　　)　　　● colorFlip(40)

Op #4: ○ rotateLeft(　　)　　　○ rotateRight(　　)　　　● colorFlip(25)

Op #5: ○ rotateLeft(　　)　　　○ rotateRight(　　)　　　○ colorFlip(　　)

c) **(12 points).** Draw the 2-3 tree corresponding to the LLRB from part a (i.e. without adding 15 or 40).
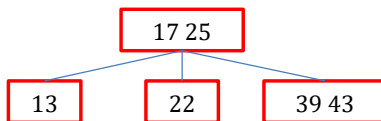

5. PNH (0 Points). What is a "desire path"?

A desire path is a path created as a consequence of erosion from human/animal traffic. It usually represents the shortest or most easily navigated route.

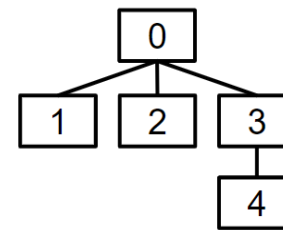## 6. WQUzaaaaaaaaaaaaaaaaaaaaaapp (24 points).

For each of the three figures below, give a sequence of 4 Weighted Quick Union `union` calls (without path compression) that could result in the figure shown. If two trees are unioned with the same weight, assume the **first argument's root is put below the second argument** (i.e. first side's new parent is the second side). If the figure is impossible to create using 4 or fewer `union` calls, fill in the circle marked "Not possible". You may not need all blanks.

Assume that before your `union` calls, all items are originally disconnected.
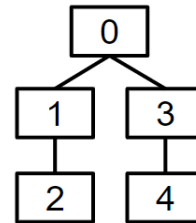
Union(_____,_____)　　　Union(__0__,__1__)　　　Union(_____,_____)
Union(_____,_____)　　　Union(__0__,__2__)　　　Union(_____,_____)
Union(_____,_____)　　　Union(__3__,__4__)　　　Union(_____,_____)
Union(_____,_____)　　　Union(__3__,__0__)　　　Union(_____,_____)
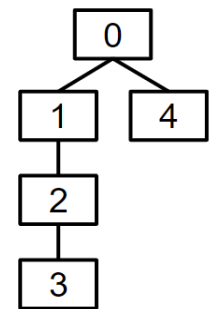
● Not possible　　　　○ Not possible　　　　● Not possible

Reminder: For each figure, either fill in the blanks for the `union` calls, or mark "Not possible".

8. Les Visiteurs. Suppose we have the tree below. Suppose it is implemented using TreeNodes.

```
    10
   /  \
  9    3
 /    / \
1    2   5
```
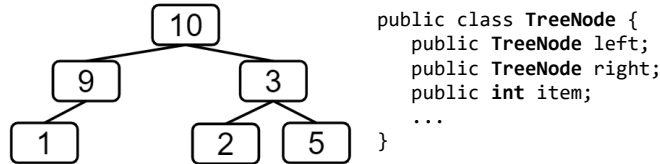
```
public class TreeNode {
    public TreeNode left;
    public TreeNode right;
    public int item;
    ...
}
```

In lecture, our implementation for the preorder traversal printed a node when it was visited, but this can be generalized so that "visiting" can be any arbitrary action. Code for this idea is given below.

```
static void preorder(TreeNode n, Visitor v) {
    if (n == null) { return; }
    v.visit(n);
    preorder(n.left, v);
    preorder(n.right, v);
}
```

```
interface Visitor {
    void visit(TreeNode x);
}
```

a. (**8 points**). Suppose we perform a **preorder** traversal on the tree above using the `Printer` visitor below. Write the output of the program in the blank provided.

```
class Printer implements Visitor {
    public void visit(TreeNode x) {
        System.out.print(x.item + " ");
    }
}
```

____10 9 1 3 2 5_____

b. (**16 points**). Suppose we perform an **inorder** traversal using the `Adder` visitor defined below. Draw the tree that results in the provided box.

```
static void inorder(TreeNode n, Visitor v) {
    if (n == null) { return; }
    inorder(n.left, v);
    v.visit(n);
    inorder(n.right, v);
}
class Adder implements Visitor {
    public void visit(TreeNode x) {
        if (x.left != null)  { x.item += x.left.item; }
        if (x.right != null) { x.item += x.right.item; }
    }
}
```
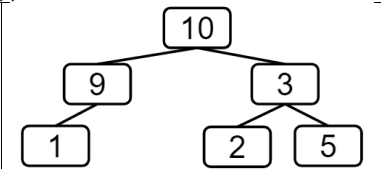
```
       23
      /  \
    10    10
   /     /  \
  1     2    5
```
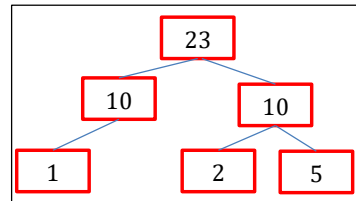
c. (**20 points**). Suppose we perform a preorder traversal using a **GlorpGlorp** on the tree to the right, i.e. we call **TreeNode**.preorder(n, new **GlorpGlorp()**), where n is the root node of the tree.

```
class GlorpGlorp implements Visitor {
    Printer printer = new Printer();

    public void visit(TreeNode x) {
        TreeNode.inorder(x, printer);
    }
}
```

```
    10
   /  \
  9    3
 /    / \
1    2   5
```

Write the output of the program in the blank provided below.

__1 9 10 2 3 5 1 9 1 2 3 5 2 5_____

d. (**12 points**). Give the DFS preorder and postorder starting from 5 if we were to treat the tree at the top of this page as a graph. If a node has multiple neighbors, traverse the smallest neighbor first.

DFS Preorder: __5 3 2 10 9 1_____

DFS Postorder: __2 1 9 10 3 5_____

9. Heaping. a) (**20 points**). In lecture and project 2A, we implemented a Heap as an array, but we could also implement as a recursive data structure, e.g. a tree of **TreeNode** objects. Write a `sink` method that sinks the value in the given **TreeNode**. You'll need to use the `min` helper method given in the text box.

```
public static void sink(TreeNode n) {
    if (n == null) { return; }
    TreeNode minN = min(n.left, n.right)
    if (minN == null) {
        return;
    }
    if (n.item > minN.item) {
        int temp = n.item;
        n.item = minN.item;
        minN.item = temp;
        sink(minN);
    }
}
```

```
TreeNode min(TreeNode a, TreeNode b):
    Returns the treenode with the smaller .item.
    If both equal, breaks ties arbitrarily.
    If both are null, returns null.
    If only one is null, returns the non-null node.

min is a static method that is part of the
TreeNode class.
```

b) (**15 points**). If we create a `SinkVisitor` whose `visit` method simply calls `sink(x)`. Give a **complete tree with 4 nodes** for which **TreeNode**.preorder(n, new SinkVisitor()) does not result in a heap if n is the root of your tree. **No credit will be given to trees that are not complete.**

```
       1
      / \
     2   3
    /
   0
```
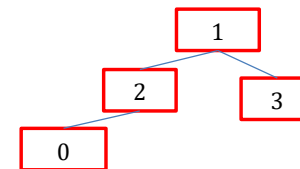
10. Asymptotics

a) **(44 points).** Give the runtime of the following functions in Θ notation. Your answer should be a function of N that is as simple as possible with no unnecessary leading constants or lower order terms. **Don't spend too much time on these**!

__Θ_N^2__
```
public static void g1(int N) {
        for (int i = 0; i < N; i += 3) {
            for (int j = 0; j < i; j += 1) {
                System.out.print("ca3");
            }
        }
    }
```

__Θ_N___
```
public static void g2(int N) {   // ASSUME N IS A POWER OF 2.
        for (int i = 0; i < N; i = i * 2) {
            g2(i);
        }
        System.out.print("power egg");
    }
```

__Θ_N^2__
```
public static void g3(int N) {
        int j = N / 2;
        for (int i = 0; i < j; i += 4) {
            for (j = 0; j < N; j += 1) {
                System.out.print("moo");
            }
        }
    }
```

__Θ_NlogN_
```
public static void g4(ArrayList<Integer> x) {
        int N = x.size(); // size takes constant time
        if (N == 1) { System.out.println(x.get(0)); return; }

        // subListCopy(P, Q) returns a copy of the list from [P, Q),
        // i.e. exclusive of Q, and takes Θ(Q – P) time.
        ArrayList<Integer> leftHalf = x.subListCopy(0, N / 2);
        ArrayList<Integer> rightHalf = x.subListCopy(N / 2, N);

        g4(leftHalf);
        g4(rightHalf);

        for (int i = 0; i < N; i += 1) {
            System.out.println(x.get(i)); // get takes constant time
        }
    }
```

b) In the first few weeks of class, when discussing `AList`, I claimed that resizing by multiplying by a constant factor works well, but resizing our array by adding a constant factor results in an unusably slow data structure. Let's now work on understanding why by examining the cost of the `resize` method. To do so, we'll use number of values copied as our cost model.

i) **(10 points).** Suppose we have an AList that starts with 1 item and doubles in size whenever an item is added that won't fit in the old array. How many values need to be copied to handle each of the first 16 add operations? The first 4 values are provided for you. For example, on the 3rd add we resize from size 2 to 4, so must copy the 2 old values.

add #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
# copied	0	1	2	0	4	0	0	0	8	0	0	0	0	0	0	0

ii) **(10 points).** Let $C_d(N)$ be the **total number of copy operations** needed to handle a sequence of N add operations using this technique where we resize by doubling. For example, $C_d(4) = 3$. Give a theta bound for $C_d(N)$.

$C_d(N) = \Theta(\_\_\_N\_\_\_)$

iii) **(10 points).** Suppose we instead have an AList that starts with 1 item and instead increases in size by 3 whenever an item is added that won't fit in the old array. How many values need to be copied to handle each of the first 16 add operations? The first 5 values are provided for you. For example, on the 5th add we resize from size 4 to 7, so must copy the 4 old values.

| add #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
# copied	0	1	0	0	4	0	0	7	0	0	10	0	0	13	0	0

iv) **(10 points).** Let $C_{a3}(N)$ be the **total number of copy operations** needed to handle a sequence of N add operations using this technique where we resize by adding 3 more array entries. For example, $C_{a3}(5) = 5$. Give a theta bound for $C_{a3}(N)$.

$C_{a3}(N) = \Theta(\_\_N^2\_\_\_)$

Note problem 10.a was a hint for this problem, which is why that problem prints out "ca3". They have the exact same pattern / runtime.

Sometime after exam, use the answers to the problems above to convince yourself that multiplicative resizing is good, and constant factor resizing is bad.

11. Yggdrasil (60 points). <u>This is a very challenging problem</u>. Write a function that takes an integer k and a min-heap h (in tree representation) and removes **the k smallest values** and returns them organized into **valid perfectly balanced BST.** For example, if we call heapToBBST(7, h) on the MinHeap in the left figure, it returns the Tree in the middle figure, and as a side-effect, h becomes the MinHeap in the right figure.
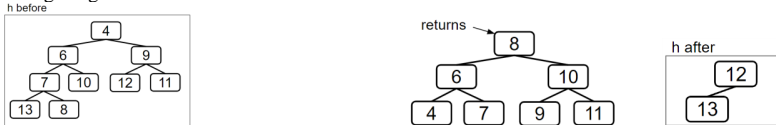
This should be done in-place, i.e., reusing the TreeNodes from the min-heap. Your function should complete in O(N log N) time, where N is the number of items in the min-heap, and use no more than O(log N) additional memory while it is running. For full credit, it must work for arbitrary values for k, but you can earn **almost full credit if your solution works for k = $2^H - 1$** (i.e. powers of 2 minus 1).

To receive 10% credit and skip this problem, fill in this box and leave the code below blank: □

```java
public class TreeNode {
    public int item;
    public TreeNode left;
    public TreeNode right;
}


public class MinHeap {
    /* Even though a MinHeap is made up of TreeNodes, the instance
     * variables are private. You can't directly access them. */

    /* removes the minimum node and returns it */
    public TreeNode removeMin() { /* ... */ }
    ...
}

public static TreeNode heapToBBST(int k, MinHeap h) {
    if (k == 0) {
        return null;
    }
    TreeNode left = heapToBBst(k/2, h);
    TreeNode middle = h.removeMin();
    TreeNode right = heapToBBst(k - (k/2) - 1, h);
    middle.left = left;
    middle.right = right;
    return middle;
}
```