HW2

① If in REF form, one row has all zeros, but the product is non-zeros there is no solution

$$\left[\begin{array}{cc|c} x & x & x \\ 0 & 0 & 2 \end{array}\right]$$

Unique solution: no row with all-zero variables

No. If there are a row of zeros, but still the same or more equations than unknown variables, then there is still a unique solution.
If all zeros, can be ∞ solution

②a iii)

$$\begin{bmatrix} 1 & 2 \\ 2 & -4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 2 & -4 & 4 \\ 3 & -2 & 8 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 4 & 0 & 12 \\ 1 & -2 & 2 \\ 3 & -2 & 8 \end{array}\right] \quad \begin{array}{l} R_1 \leftarrow R_1 + R_3 \\ R_2 \leftarrow R_2 / 2 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 1 & -2 & 2 \\ 3 & -2 & 8 \end{array}\right] \quad R_1/4$$

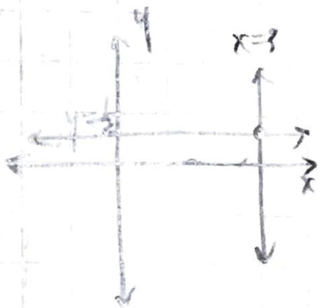
$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & -2 & -1 \\ 3 & -2 & 8 \end{array}\right] \quad R_2 \leftarrow R_2 - R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & -2 & 8 \end{array}\right]$$

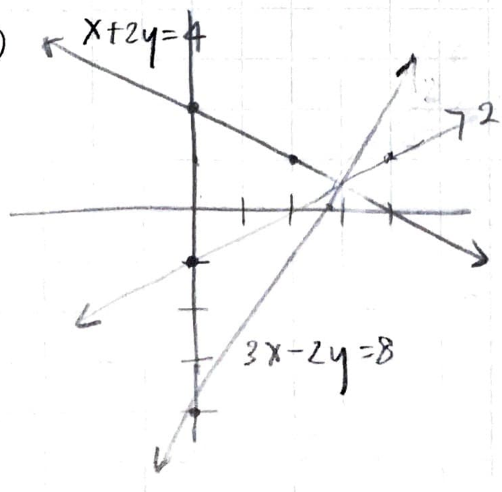
$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & 0 & 9 \end{array}\right] \quad R3 + R2$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & \frac{1}{2} \\ 1 & 0 & 3 \end{array}\right]$$

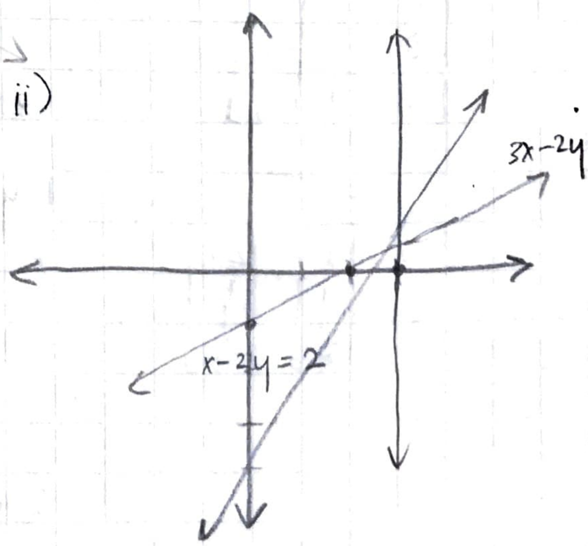
$x = 3$
$y = \frac{1}{2}$



vertical + horizontal lines

i)


$(3, \frac{1}{2})$

ii)


One of the lines is now vertical $(x=3)$

2b)

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 1 & 12 & 6 & 1 \\ 0 & 2 & 1 & 4 \\ 3 & 16 & 16 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 2 & 1 & 4 \\ 3 & 16 & 16 & 7 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1$

$0 \quad 2 \quad \frac{1}{5} \;\Big|\; -\frac{2}{5}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 0 & \frac{4}{5} & -\frac{18}{5} \\ 3 & 16 & 16 & 7 \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2/5$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 0 & 4 & -18 \\ 3 & 16 & 16 & 7 \end{array}\right]$$

$3 \quad 6 \quad 15 \;\Big|\; 9$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 0 & 1 & -\frac{9}{2} \\ \cancel{0} & \cancel{10} & \cancel{1} & \cancel{-2} \end{array}\right]$$

$R_3 \leftarrow R_3/4$

$R_4 \leftarrow R_4 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 10 & 1 & -2 \\ 0 & 0 & 1 & -\frac{9}{2} \end{array}\right]$$

$z = -\frac{9}{2}$ $\qquad$ $y = \frac{1}{4}$

$10y + z = -2$

$10y - \frac{9}{2} = -\frac{4}{2}$

$10y = \frac{5}{2}$

$y = \frac{\cancel{5}}{2} \cdot \frac{1}{\cancel{10}\,2} = \frac{1}{4}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 25 \\ \frac{1}{4} \\ -\frac{9}{2} \end{bmatrix}$$

$x + 2y + 5z = 3$

$x + \frac{1}{2} - \frac{45}{2} = \frac{6}{2}$

$x - \frac{44}{2} = \frac{6}{2}$

$x - 22 = 3$ $\qquad$ $x = 25$

2c) $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 5 \\ 3 & 9 & 6 \end{bmatrix} \begin{bmatrix} 16-11t \\ -5+3t \\ t \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} (16-11t)+2(-5+3t)+5(t) \\ 3(16-11t)+9(-5+3t)+6(t) \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 16-11t-10+6t+5t \\ 48-33t-45+27t+6t \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 6+0t \\ 3+0t \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

2d) 2 free variables: x_1, x_5

say $x_5 = t$, $x_1 = p$

$$\begin{cases} p + x_2 + 3t = 16 \\ x_3 - 3t = -17 \\ x_4 + t = 5 \end{cases}$$

$$x_2 = 16 - 3t - p$$
$$x_3 = -17 + 3t$$
$$x_4 = 5 - t$$

$$\vec{x} = \begin{bmatrix} p \\ 16-3t-p \\ -17+3t \\ 5-t \\ t \end{bmatrix}$$

3a) $\left\{ \begin{bmatrix} -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$

linear dependence:
$\{\vec{v}_1 \cdots \vec{v}_n\}$ if
scalars $\alpha_1 \ldots \alpha_n$
such that $\alpha_1 \vec{v}_1 + \ldots + \alpha_n \vec{v}_n = \vec{0}$
and at least one non-zero $\vec{v}$

not linearly dependent

$$\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 = \vec{0}$$

$$\Rightarrow \alpha_1 \begin{bmatrix} -5 \\ 2 \end{bmatrix} + \alpha_2 \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{\alpha} = A^{-1} \vec{b}$$

$$\left[\begin{array}{cc|c} -5 & 5 & 0 \\ 2 & 2 & 0 \end{array}\right]$$

3b) $\left[\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 3 & -1 & 0 \\ -1 & -2 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{array}\right]$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\left[\begin{array}{ccc|c} -1 & -2 & 0 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 3 & -1 & 0 \\ -1 & -2 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{array}\right]$ $R_1 \leftarrow -R_1$, $R_2 \leftarrow R_2 + R_1$ $\left[\begin{array}{ccc|c} 0 & -3 & 1 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

linearly dependent
no unique soln

$x_2 = \alpha$

$-x_1 - 2x_2 = 0$
$3x_2 - x_3 = 0$

$x_1 = -2\alpha$
$x_2 = \alpha$
$x_3 = 3\alpha$

$$-2\vec{v}_1 + \vec{v}_2 + 3\vec{v}_3 = \vec{0}$$

linearly dependent

3c) $\left[\begin{array}{cccc|c} 2 & 0 & 2 & 0 & 0 \\ 2 & 1 & 4 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{array}\right]$

$\left[\begin{array}{cccc|c} 2 & 0 & 2 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{array}\right]$ $R2 \leftarrow R_2 - R_1$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{array}\right]$ $R_1/2$, $R_3 - R_2$

3c cont.

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{array}\right]$$

$x_4 = \alpha$

$-3x_3 + 2\alpha = 0$
$x_3 = \frac{2}{3}\alpha$

$x_1 + x_3 = 0$
$x_1 + \frac{2}{3}\alpha = 0$
$x_1 = -\frac{2}{3}\alpha$

$x_2 + x_3 + \alpha = 0$
$x_2 + \frac{5}{3}\alpha = 0$
$x_2 = -\frac{5}{3}\alpha$

$$\vec{0} = -\frac{2}{3}v_1 - \frac{5}{3}v_2 + \frac{2}{3}v_3 + v_4$$

3d

linearly independent

cannot combine together with scalars to form $\vec{0}$.

4a

$$\begin{cases} m_1 = f_1(\alpha)\cdot a + f_1(\beta)\cdot b \\ m_2 = f_2(\alpha)\cdot a + f_2(\beta)\cdot b \end{cases}$$

$\alpha = 45°$
$\beta = -30°$

$$\begin{cases} m_1 = \cos(45°)\,a + \cos(-30°)\,b \\ m_2 = \sin(45°)\,a + \sin(-30°)\,b \end{cases}$$

$$\begin{cases} m_1 = \frac{\sqrt{2}}{2}a + \frac{\sqrt{3}}{2}b \\ m_2 = \frac{\sqrt{2}}{2}a - \frac{1}{2}b \end{cases}$$

$$\left[\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 2m_1 \\ 0 & -1-\sqrt{3} & 2m_2 - 2m_1 \end{array}\right] \quad R_2 \leftarrow R_2 - R_1$$

4b

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{\sqrt{3}} & 1 & (2m_1)\left(\frac{1}{\sqrt{3}}\right) \\ 0 & 1 & \left(\frac{-1}{1+\sqrt{3}}\right)(2m_2 - 2m_1) \end{array}\right]$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & m_1 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{3} & 0 & \left(\frac{2}{\sqrt{3}}m\right) - \left(\frac{-1}{1+\sqrt{3}}\right)(2m_2 - 2m_1) \\ 0 & 1 & \left(\frac{-1}{1+\sqrt{3}}\right)(2m_2 - 2m_1) \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 2m_1 \\ \sqrt{2} & -1 & 2m_2 \end{array}\right]$$

4b

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & m_1 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{2} & \sqrt{3} & 2m_1 \\ \sqrt{2} & -1 & 2m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{3}\cdot\sqrt{2} & 3 & 2\sqrt{3}\, m_1 \\ 3\sqrt{2} & -3 & 6m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} (\sqrt{3}\cdot\sqrt{2}) + 3\sqrt{2} & 0 & 6m_2 + 2\sqrt{3}\, m_1 \\ \sqrt{2} & -1 & 2m_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} (\sqrt{2})(\sqrt{3}+3) & 0 & 6m_2 + 2\sqrt{3}\, m_1 \\ (\sqrt{2})(\sqrt{3}+3) & -1\,(\sqrt{3}+3) & (2m_2)(\sqrt{3}+3) \end{array}\right]$$

$$\left[\begin{array}{cc|c} \sqrt{2}\cdot(\sqrt{3}+3) & 0 & 6m_2 + 2\sqrt{3}\, m_1 \\ 0 & -\sqrt{3}-3 & 2(\sqrt{3}+3)m_2 - 6m_2 + 2\sqrt{3}\, m_1 \end{array}\right]$$

$$a = \frac{(2\sqrt{3}\, m_1 + 6m_2)}{\sqrt{2}(\sqrt{3}+3)}$$

$$b = \frac{2\sqrt{3}\, m_1 + 2\sqrt{3}\, m_2}{-\sqrt{3}-3} = \frac{-2\sqrt{3}}{\sqrt{3}+3} m_1 - \frac{2\sqrt{3}}{\sqrt{3}+3} m_2$$

$$a = \underbrace{\frac{\sqrt{6}-\sqrt{2}}{2}}_{\vec{u}} m_1 + \underbrace{\left(\frac{-\sqrt{6}+3\sqrt{2}}{2}\right)}_{\vec{v}} m_2 \qquad b = \underbrace{(-1+\sqrt{3})}_{\vec{u}} m_1 + \underbrace{(-1+\sqrt{3})}_{\vec{v}} m_2$$

4c "all human beings are born free and equal in dignity and rights"

EECS16A: Homework 2

Problem 4: Filtering Out The Troll

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import wave as wv
import scipy
from scipy import io
import scipy.io.wavfile
from scipy.io.wavfile import read
from IPython.display import Audio
import warnings
warnings.filterwarnings('ignore')
sound_file_1 = 'm1.wav'
sound_file_2 = 'm2.wav'
```

Let's listen to the recording of the first microphone (it can take some time to load the sound file).

In [2]:
```
Audio(url='m1.wav', autoplay=False)
```

Out[2]:

0:00 / 0:00

And this is the recording of the second microphone (it can take some time to load the sound file).

In [3]:
```
Audio(url='m2.wav', autoplay=False)
```

Out[3]:

0:00 / 0:00

We read the first recording to the variable `corrupt1` and the second recording to `corrupt2`.

In [4]:
```
rate1,corrupt1 = scipy.io.wavfile.read('m1.wav')
rate2,corrupt2 = scipy.io.wavfile.read('m2.wav')
```

Enter the values of u and v to get the clean speech.

Note: The square root of a number a can be written as `np.sqrt(a)` in IPython.

In [5]:
```
# enter the values of u (recording 1) and v (recording 2)
u = (np.sqrt(6) - np.sqrt(2))/2
v = (-np.sqrt(6) + 3 * np.sqrt(2))/2
```

Weighted combination of the two recordings:

In [6]:
```
s1 = u*corrupt1 + v*corrupt2
```

Let's listen to the resulting sound file (make sure your speaker's volume is not very high, the sound may be loud if things go wrong).

In [7]:
```
Audio(data=s1, rate=rate1)
```

Out[7]:

0:00 / 0:10

In [ ]:

6a)

$$\vec{r} = \begin{bmatrix} \star \\ b \\ c \\ \star \\ b \\ c \end{bmatrix}$$

6b)

$$G_K \vec{m} = \vec{w}$$

$$G_K = \begin{array}{c} \begin{matrix} a & b & c \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad G_K \vec{m} = \begin{bmatrix} a \\ b \\ c \\ a \\ b \\ c \end{bmatrix}$$

6c)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 \\ \star \\ \star \\ 3 \\ 4 \\ \star \\ \star \end{bmatrix}$$

7x3

$a = 7$

$b = \star$

$c = \star$

$a + b = 3 \rightarrow 7 + b = 3$, $b = -4$

$a + c = 4 \rightarrow 7 + c = 4$, $c = 3$

$b + c = \star$

$a + b + c = \star$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \\ 3 \end{bmatrix}$$

6d)

$$\vec{r} = \begin{bmatrix} 1 \\ \star \\ 3 \\ \star \\ 4 \\ \star \\ 9 \end{bmatrix}$$

$a = 1$

$c = 3$

$a + c = 4$

$a + b + c = 9 \longrightarrow 1 + b + 3 = 9$, $b = 5$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$$

⑦ $A\vec{x} = \vec{b}$ has ∞ soln $\Rightarrow$ columns are lin dependent

i) If system has ∞ soln, must have at least 2 distinct soln

$$A\vec{x} = \vec{b}$$

$$A\vec{u} = \vec{b} \qquad A\vec{v} = \vec{b}$$

ii) $\begin{bmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_3 \\ | & | & & | \end{bmatrix}$

Columns of vector are linearly dependent if exist scalars α such that $\alpha_1 \vec{v}_1 + \dots + \alpha_n \vec{v}_n = \vec{0}$ and at least 1 nonzero α

$$\vec{b} = \vec{c}_1 u_1 + \vec{c}_2 u_2 + \dots + \vec{c}_n u_n$$

$$\vec{b} = \vec{c}_1 v_1 + \vec{c}_2 v_2 + \dots + \vec{c}_n v_n$$

iii) $A\vec{u} = \vec{b}$ $\qquad$ $A\vec{v} = \vec{b}$

$$\begin{bmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \vec{b} \qquad \begin{bmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \vec{b}$$

$$u_1 \begin{bmatrix} | \\ \vec{c}_1 \\ | \end{bmatrix} + u_2 \begin{bmatrix} | \\ \vec{c}_2 \\ | \end{bmatrix} + \dots + u_n \begin{bmatrix} | \\ \vec{c}_n \\ | \end{bmatrix} = \vec{b} \qquad v_1 \vec{c}_1 + v_2 \vec{c}_2 + \dots + v_n \vec{c}_n = \vec{b}$$

$$A\vec{u} - A\vec{v} = A(\vec{u} - \vec{v}) = \vec{b} - \vec{b} = \vec{0}$$

$$\vec{u} - \vec{v} \neq \vec{0}$$

set $\vec{x} = \vec{u} - \vec{v}$

therefore linearly dependent columns.

PS 2

7b

Known	To Show
$\{\vec{v}_1, \vec{v}_2 \cdots \vec{v}_n\}$	$\text{span}\{\vec{v}_1, \vec{v}_2 \cdots \vec{v}_n\} = \text{span}\{\vec{v}_1+\vec{v}_2, \vec{v}_2 \cdots \vec{v}_n\}$

suppose $\vec{q} \in \text{span}\{\vec{v}_1, \vec{v}_2 \cdots \vec{v}_n\}$ for some scalars a_i:

$$\vec{q} = a_1\vec{v}_1 + a_2\vec{v}_2 + \cdots a_n\vec{v}_n = a_1(\vec{v}_1+\vec{v}_2) + (-a_1+a_2)\vec{v}_2 + \cdots a_n\vec{v}_n$$

therefore $\vec{q} \in \text{span}\{\vec{v}_1+\vec{v}_2, \vec{v}_2 \cdots \vec{v}_n\}$

suppose $\vec{q} \in \text{span}\{\vec{v}_1+\vec{v}_2, \vec{v}_2 \cdots \vec{v}_n\}$ for some scalars b_i:

$$\vec{r} = b_1\vec{v}_1 + (b_1+b_2)\vec{v}_2 + b_3\vec{v}_3 + \cdots b_n\vec{v}_n = b_1\vec{v}_1 + b_2\vec{v}_2 \cdots b_n\vec{v}_n$$

thus, $\vec{r} \in \text{span}\{\vec{v}_1, \vec{v}_2 \cdots \vec{v}_n)$

Spans are same. QED

7c

Known	Show
n = pos. int	$\{A\vec{v}_1, A\vec{v}_2 \cdots A\vec{v}_k\}$
$\{\vec{v}_1, \vec{v}_2 \cdots \vec{v}_k\} \in \mathbb{R}^n$	is set of linearly dependent

By definition of linear dependence

$$a_1\vec{v}_1 + a_2\vec{v}_2 + \cdots + a_k\vec{v}_k = \vec{0}$$

such that not all "a" scalars are 0

$$A(a_1\vec{v}_1 + a_2\vec{v}_2 + \cdots + a_k\vec{v}_k) = A\vec{0}$$

$$A\vec{v}_1 + A\vec{v}_2 + \cdots + A\vec{v}_k = \vec{0}$$

Thus, $\{A\vec{v} \cdots\}$ is linearly dependent

8 I worked alone.