## 0.1 Final exam equation sheet

Layer energy equation: $$\frac{dE}{dt} = I_{\downarrow} + I_{\uparrow} \tag{1}$$

Solar constant: $$S = \frac{S_0}{4}(1-\alpha) \tag{2}$$

Total grey body flux $$I = \varepsilon\sigma T^4 \tag{3}$$

where $\sigma = 5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$

transmissivity tr: $$I_{\text{transmitted}} = \text{tr} I_0 \tag{4}$$

reflectity $\alpha$ $$I_{\text{reflected}} = \alpha I_0 \tag{5}$$

absorbtivity abs $$I_{\text{absorbed}} = \text{abs} I_0 \tag{6}$$

Kirchoff's law $$\varepsilon = \text{abs} \tag{7}$$

$CO_2$ radiative forcing $$\Delta F = \left(3.8\text{Wm}^{-2}\right)\frac{\ln(\text{newp CO }2/\text{ origp CO2})}{\ln(2)} \tag{8}$$

Conservation of Energy: $$\alpha \text{I}_0 + abs\text{I}_0 + \text{trI}_0 = \text{I}_0 \tag{9}$$

moist static energy: $$h_m = c_p T + l_v w_v + gz \tag{10}$$

moist adiabatic lapse rate: $$\Gamma = \frac{dT}{dz} = \frac{-g}{c_p + l_v \frac{dw_v}{dT}} \tag{11}$$

hydrostatic balance: $$dp = -\rho g dz \tag{12}$$

mass in a layer in $kg/m^2$: $$M = \int_{z_1}^{z_2} \rho(z) dz \tag{13}$$

energy in an ocean layer: $$\Delta E = \rho_w D c_w \Delta T \tag{14}$$

Conservation of energy for layer: $$\frac{d\Delta E}{dt} = \Delta F \tag{15}$$

change of temperature for an ocean layer: $$\frac{d\Delta T}{dt} = \frac{\Delta F}{\rho_w c_w D} \tag{16}$$

Planck feedback: $$\frac{dI_G}{dT} = \frac{d\left(-\sigma T^4\right)}{dT} = f_{planck} = -4\sigma T^3 = -1/\lambda \tag{17}$$

Conservation of energy with feedback: $$\frac{\Delta E}{dt} = \Delta F - 4\sigma T^3 \Delta T \tag{18}$$

Climate adjustment to abrupt forcing: $$\Delta T(t) = \lambda \Delta F\left(1 - e^{-t/\tau}\right) \tag{19}$$

Climate adjustment timescale: $$\tau = \rho_w c_w D \lambda \tag{20}$$

Climate sensitivity: $$\Delta T = \lambda \Delta F \tag{21}$$

Climate mean temperature budget: $$\rho_w c_w D \frac{dT}{dt} = \Delta F + \sum f_n \Delta T \tag{22}$$

Climate feedback factor: $$f_n = \frac{\Delta R}{\Delta T} = \left(\frac{\Delta R}{\Delta\text{ climate}}\right)\left(\frac{\Delta\text{ climate}}{\Delta T}\right) \tag{23}$$

Climate sensitivity with feedbacks: $$\lambda = -\frac{1}{\sum f_n} \tag{24}$$

oxygen isotopes: $$\delta^{18}\text{O} = \left[ \frac{\left({}^{18}\text{O}/{}^{16}\text{O}\right)_{\text{sample}}}{\left({}^{18}\text{O}/{}^{16}\text{O}\right)_{\text{standard}}} - 1 \right] \times 1000 \tag{25}$$

Carbon Isotope Mass Balance: $$\begin{aligned}&(M_{final})(\delta_{13}C_{final}) \\ &= (M_{initial})(\delta_{13}C_{initial}) \\ &+ (M_{added})(\delta_{13}C_{added})\end{aligned} \tag{26}$$

Carbon Isotopic Excursion CIE: $$\delta_{13}C_{final} - \delta_{13}C_{initial} \tag{27}$$

$$M_{added} = -CIE \times \frac{M_{initial}}{(\delta_{13}C_{final} - \delta_{13}C_{added})} \tag{28}$$

Kaya indentity: $$I = P \times A \times EI \times CI \tag{29}$$

## 0.2 Final exam constants

$$\sigma = 5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4} \tag{30}$$

$$c_p = 1004\, J kg^{-1}\, K^{-1} \tag{31}$$

$$c_w = 4186\, J kg^{-1}\, K^{-1} \tag{32}$$

$$\rho_w = 1000\, kg\, m^{-3} \tag{33}$$

$$l_v = 2.5 \times 10^6\, J kg^{-1} \tag{34}$$

$$1 \text{ ppm } CO_2 = 2.1 \text{ Gtonnes C} \tag{35}$$

$$M_{CO2} = 44\, kg\, kmole^{-1} \tag{36}$$

$$M_{^{12}C} = 12\, kg\, kmole^{-1} \tag{37}$$