APPENDIX

We first prove the following proposition that will be used in the proof of Theorem 2.

Proposition 1

If M is a stable model of $Rew(\mathscr{T},O)$ containing the fact $constrainedExplanation$ and $E = F(M)$, then $\mathscr{R} \cup B \cup E \models O$.

Proof.

Let N^* be the subset of starred base atoms in M inferred by means of $Rew(R) \cup Rew(O)$ and N the corresponding set of unstarred base atoms (i.e. $N = \{p(\bar{x}) \mid p^*(\bar{x}) \in N^*\}$). By construction, $\mathscr{R} \cup N \models O$. For each $assing(\xi,c) \in M$, with ξ an arbitrary constant and c a constant, let us replace ξ with c in M, obtaining a new set L.

Observe that this step just consists in a set of syntactic changes preserving the result of all *join* operations. It follows that $\mathscr{R} \cup L \models O$. Therefore, $\mathscr{R} \cup B \cup L \models O$, because $\mathscr{R}$ is monotonic.

The set L can contain base atoms non occurring in B. These are exactly the atoms in $E = F(M)$ (see. Definition 12). Therefore $B \cup L = B \cup E$ and $\mathscr{R} \cup B \cup E \models O$. Moreover, as $constrainedExplanation \in M$, all the arbitrary constants have been assigned to actual constants and no arbitrary constant occurs in E (see. rule 20 of Definition 12). □

Theorem 2

Let $\mathscr{T} = \langle \mathscr{R} \cup B, \mathscr{A}, \emptyset \rangle$ be an abductive theory, where $\mathscr{R}$ is a non-recursive Horn program not containing any rule in whose body two dependent predicates occur and B is a finite set of facts, and O an observation. Then:

1) If $\Delta = (E,\emptyset)$ is a constrained explanation for O w.r.t. $\mathscr{T}$, then there is a stable model M of $Rew(\mathscr{T},O)$ containing the fact $constrainedExplanation$ s.t. $F(M) = E$.
2) If M is a stable model of $Rew(\mathscr{T},O)$ containing the fact $constrainedExplanation$, $E = F(M)$ and E is minimal, i.e. there is no $E' \subset E$ s.t. $\mathscr{R} \cup B \cup E' \models O$, then $\Delta = (E,\emptyset)$ is a constrained explanation for O w.r.t. $\mathscr{T}$.

Proof.

1) Let $\Delta = (E,\emptyset)$ be a constrained explanation for O w.r.t. $\mathscr{T}$. Starting from E, we define a set M s.t. M is a stable model of $Rew(\mathscr{T},O)$ and $F(M) = E$.

We first define two subsets of M, named H^* and C.

Let S be a stable model of $\mathscr{P}^\Delta = \mathscr{R} \cup B \cup E$. As $\mathscr{P}^\Delta$ is a non recursive Horn program, S exists and is unique. Moreover, by definition, $S \models O$. As $(E,\emptyset)$ is *constrained*, there is no constant c occurring in E that can be replaced with a different constant d, not occurring in $\mathscr{T}$ and O, in a minimal set of positions, obtaining a set E' s.t. $\mathscr{R} \cup B \cup E' \models O$ (Definitions 3 and 4).

Let us define the program $\mathscr{L} = \mathscr{P}^b \cup \mathscr{P}^f \cup \mathscr{P}^c$, where:

$$
\begin{aligned}
\mathscr{P}^b &= Rew(\mathscr{R}) \cup Rew(B) \cup Rew(O) \\
\mathscr{P}^f &= \mathscr{R} \cup B \cup E \cup \\
&\quad \{h_i(\overline{X}) \leftarrow \mathscr{P}_i(\overline{X},\overline{Y_i}) \;\mid\; h(\overline{X}) \leftarrow \textstyle\bigvee_{i\in[1..n]} \mathscr{P}_i(\overline{X},\overline{Y_i}) \\
&\quad\ \text{belongs to } \mathscr{R} \text{ and } i \in [1..n]\} \\
\mathscr{P}^c &= \{\overline{h}_i(x_1,\dots,x_n) \leftarrow h_i^*(x_1,\dots,x_n) \;\wedge\; h_i(y_1,\dots,y_n) \;\wedge \\
&\quad\ \textstyle\bigwedge_{i=1..n}(arbitrary(x_i) \vee x_i = y_i) \mid h_i^*(x_1,\dots,x_n) \\
&\quad\ \text{occurs in } Rew(\mathscr{R})\} \cup \\
&\quad\ \{\perp \leftarrow h_i^*(x_1,\dots,x_n), not\ \overline{h}_i(x_1,\dots,x_n) \mid\; h_i^*(x_1,\dots,x_n) \\
&\quad\ \text{occurs in } Rew(\mathscr{R})\}.
\end{aligned}
$$

$\mathscr{P}^b$ models the backward process. For each stable model of $\mathscr{P}^b$, let $\tilde{E}^*$ the set of its base starred atoms not occurring in $Rew(B)$. The corresponding set $\tilde{E}$ of unstarred base atoms is by construction s.t. $\mathscr{R} \cup B \cup \tilde{E} \models O$. $(\tilde{E},\emptyset)$ is a *template* of a set of possible explanations in the sense that it contains arbitrary constants ξ that can be replaced with actual constants obtaining other explanations.

$\mathscr{P}^b$ models the forward process allowing to derive, from the updated set of base atoms $B \cup E$ and the rules in $\mathscr{R}$, all the possible derived facts, included those in O and the auxiliary facts of the form $h_i(\overline{X})$.

$\mathscr{P}^c$ ensures that each starred atom derived with the backward process is associated to an atom derived with the forward process.

By construction, there is a stable model H of $\mathscr{L}$. Let H^* be the subset of starred atoms in H. The set H^* is a stable model of $\mathscr{P}^b$ and then its subset of base starred atoms $\tilde{E}^*$ is s.t. $(\tilde{E},\emptyset)$ is a template of possible explanations of O (in the sense explained above).

Further, H^* has an important property: For each atom $h^*(x_1,\dots,x_m) \in H^*$, there is a corresponding atom $h(c_1,\dots,c_m) \in S$ *unifying* with it (i.e. for each $i \in [1..m]$, x_i is an arbitrary constant or $x_i = c_i$). If follows that for each atom $h(x_1,\dots,x_m) \in \tilde{E}$, there is a corresponding atom $h(c_1,\dots,c_m) \in E$ *unifying* with it. This means that $(\tilde{E},\emptyset)$ is a template of $(E,\emptyset)$.

Now we set *assign* atoms. We claim that there exists a set C consisting of atoms of the form $assign(\xi,c)$, such that for each $arbitrary(\xi) \in H^*$, C contains exactly one atom $assign(\xi,c)$ (that establishes a one-to-one correspondence between arbitrary constants ξ and constants c) and, moreover, such that replacing each arbitrary constant ξ with the corresponding constant c yields a set H'^* with the property that for each atom $p^*(\bar{x}) \in H'^*$ the atom $p(\bar{x})$ is in S.

Let us consider an atom $h^*(\bar{x}) \in H^*$, where h is a derived predicate defined by means of a rule of the form (1).

By construction, for some $i \in [1..n]$, in H^* there are all the atoms belonging to a conjunction

$$\mathscr{P}_i^*(\bar{x},\ y_{i,1}(h,\bar{x}),\ \dots,\ y_{i,m_i}(h,\bar{x}))$$

such that for at least one ground conjunction $\mathscr{P}_i(\bar{x},\ c_1,\ \dots,\ c_{m_i})$, we have $S \models \mathscr{P}_i(\bar{x},\ c_1,\ \dots,\ c_{m_i})$.

Then, we include in C the atoms $assign(y_{i,j}(h,\bar{x}),c_j)$ for $j \in [1..m_i]$. We observe that:

- for each arbitrary constant ξ of the form $y_{i,j}(h,\bar{x})$, $assign(y_{i,j}(h,\bar{x}),c_j)$ is inferred by means of a pair $\mathscr{B}^* = \mathscr{P}_i^*(\bar{x},\ y_{i,1}(h,\bar{x}),\ \dots,\ y_{i,j}(h,\bar{x}),\dots,y_{i,m_i}(h,\bar{x}))$ and $\mathscr{B} = \mathscr{P}_i(\bar{x},\ c_1,\ \dots,c_j,\dots,\ c_{m_i})$ s.t. $\mathscr{R}$ contains a rule of the form (1), $\mathscr{B}$ unifies with $\mathscr{B}^*$ and $S \models \mathscr{B}$ (as we observed, such a pair always exists);
- a rule of the form (1) is used to infer assignments only for arbitrary constants ξ of the form $y_{i,j}(h,\bar{x})$, $i \in [1..n]$ and $j \in [1..m_i]$;
- given an arbitrary constant ξ of the form $y_{i,j}(h,\bar{x})$, an assignment for ξ can be inferred only by means of a rule of the form (1).

Previous conditions guarantee that a set C with the properties previously presented always exists.

Let us consider a set M containing:

(a) atoms in H^*;
(b) atoms in C;
(c) atoms in B;
(d) $constant(a)$, $term(a)$ and $assign(a,a)$, for each constant a in $\mathscr{C}$;
(e) $term(x)$, for each atom $arbitrary(x)$ included in previous steps;
(f) $candidate(x,x)$ and $compatible(x,x)$, for each atom $term(x)$ included in previous steps;

(g) $compatible(x,y)$,
for each atoms $arbitrary(x)$ and $constant(y)$ s.t. $assign(x,y)$ is included in previous steps;
(h) $incompatible(x,y)$,
for each $arbitrary(x)$ and $constant(y)$ s.t. $assign(x,y)$ is not included in previous steps;
(i) $compatible(x,y)$,
for each $assign(x,z)$ and $assign(y,z)$ included in previous steps;
(j) $compatible(y,x)$,
for each atom $compatible(x,y)$ included in previous;
(k) $candidate(x_i,y_i)$,
for each conjunction of the form $r^*(x_1,\dots,x_i,\dots,x_{n_r}), r^*(y_1,\dots,y_i,\dots,y_{n_r}), arbitrary(x_i), constant(x_i), \bigwedge_{j\in[1..n_r]\setminus\{i\}} compatible(x_j,y_j)$ which is true w.r.t. the set of atoms included in previous steps;
(l) $discard(x,z)$,
for each $assign(x,y)$ and $candidate(x,z)$ with $y \neq z$ included in previous steps;
(m) $evaluated(x)$,
for each atom $term(x)$ included in previous steps;
(n) $constrainedExplanation$;
(o) $r^*(y_1,\dots,y_n)$,
for each conjunction $r^*(x_1,\dots,x_n), assign(x_1,y_1),\dots,assign(x_n,y_n)$ which is true w.r.t. atoms included in previous steps;
(p) $r_i^*(y_1,\dots,y_n)$,
for each conjunction $r_i^*(x_1,\dots,x_n), assign(x_1,y_1),\dots,assign(x_n,y_n)$ which is true w.r.t. atoms included in previous steps;
(q) $r^+(y_1,\dots,y_n)$,
for each conjunction $r^*(x_1,\dots,x_n), assign(x_1,y_1),\dots,assign(x_n,y_n), not\ r(y_1,\dots,y_n)$ which is true w.r.t. atoms included in previous steps and $r \in \mathscr{A}$.

Now we prove that M is a stable model of $\mathscr{Q} = Rew(\mathscr{T},O)$, $F(M) = E$ and $constrainedExplanation \in M$.

Let us build the reduct $\mathscr{Q}^M$, assuming that grounding is performed with respect to constants in $\mathscr{C}$ and arbitrary constants occurring in H^*. $\mathscr{Q}^M$ is the union of:

1) $B \cup Rew(B) \cup Rew(O)$;
2) $Const(\mathscr{T})$;
3) $\bigvee_i\ h_i^*(\overline{x}) \leftarrow h^*(\overline{x})$, s.t. $h_i^*(\overline{x}) \in M$;
4) $ground(\mathscr{P}_i^*(\overline{X},\ y_{i,1}(h,\overline{X}),\ \dots,\ y_{i,m_i}(h,\overline{X})) \leftarrow h_i^*(\overline{X}))$, $\forall r \in \mathscr{R}$ of the form (1) and $\forall i \in [1..n]$;
5) $ground(arbitrary(y_{i,1}(h,\overline{X})),\ \dots,\ arbitrary(y_{i,m_i}(h,\overline{X})) \leftarrow h_i^*(\overline{X}))$, $\forall r \in \mathscr{R}$ of the form (1) and $\forall i \in [1..n]$;
6) $ground(term(X) \leftarrow constant(X))$;
7) $ground(term(X) \leftarrow arbitrary(X))$;
8) $ground(candidate(X,X) \leftarrow term(X))$;
9) $ground(candidate(X_i,Y_i) \leftarrow r^*(X_1,\dots,X_i,\dots,X_{n_r}), r^*(Y_1,\dots,Y_i,\dots,Y_{n_r}), arbitrary(X_i), constant(Y_i), \bigwedge_{j\in[1..n_r]\setminus\{i\}} compatible(X_j,Y_j))$,
for each predicate $r(X_1,\dots,X_{n_r})$ and $\forall i \in [1..n_r]$;
10) $ground(compatible(X,X) \leftarrow term(X))$;
11) $ground(compatible(X,Y) \leftarrow compatible(Y,X))$;
12) $ground(compatible(X,Y) \leftarrow assign(X,Z), assign(Y,Z))$;
13) $compatible(x,y) \leftarrow arbitrary(x), constant(y)$
s.t. $incompatible(x,y) \notin M$, $arbitrary(x) \in M$ and $y \in \mathscr{D}$
14) $ground(incompatible(X,Y) \leftarrow arbitrary(X), constant(Y), assign(X,Z), Y \neq Z)$;
15) $discard(x,y) \vee discard(x,z) \leftarrow candidate(x,y), candidate(x,z), y \neq z$
s.t. $discard(x,y), discard(x,z) \in M$;
16) $discard(x,y) \leftarrow candidate(x,y), candidate(x,z), y \neq z$
s.t. $candidate(x,z) \in M$, $discard(x,y) \in M$ and $discard(x,z) \notin M$;
17) $assign(x,y) \leftarrow candidate(x,y)$
s.t. $candidate(x,y) \in M$ and $discard(x,y) \notin M$;
18) $ground(\bot \leftarrow assign(X,Y), assign(X,Z), Y \neq Z)$;
19) $ground(r^*(Y_1,\dots,Y_n) \leftarrow r^*(X_1,\dots,X_n), assign(X_1,Y_1),\dots, assign(X_n,Y_n))$
for each predicate symbol r;
20) $evaluated(x) \leftarrow assign(x,y)$
s.t. $assign(x,y) \in M$ and $arbitrary(y) \notin M$;
21) $unevaluated(x) \leftarrow term(x)$
s.t. $term(x) \in M$ and $evaluated(x) \notin M$;
22) $ground(arbitraryExplanation \leftarrow unevaluated(X))$;
23) $constrainedExplanation$.
24) $r^+(y_1,\dots,y_n) \leftarrow r^*(x_1,\dots,x_n), assign(x_1,y_1),\dots,assign(x_n,y_n)$
s.t. $M \models r^*(x_1,\dots,x_n), assign(x_1,y_1),\dots,assign(x_n,y_n)$ and $r(y_1,\dots,y_n) \notin M$;
25) $ground(\bot \leftarrow r^+(X_1,\dots,X_n))$ for each extensional predicate r s.t. $r \notin \mathscr{A}$;

We prove that M is a minimal model of $\mathscr{Q}^M$. First, M satisfies each rule in $\mathscr{Q}^M$ by construction. In order to prove that M is minimal we show that if M is not minimal $(E,\emptyset)$ is not constrained, i.e. there is an explanation $(E',\emptyset)$ s.t. E' is obtained from E by replacing a constant c with another constant d, not occurring in $\mathscr{T}$ and O, in a minimal set of positions.

Let us assume by contradiction that M is not minimal. Then there is $W \subset M$ s.t. $W \models \mathscr{Q}^M$.

Let $a = assign(\xi_0,c_0) \in (M \setminus W)$.

By construction of M, there must be such a fact. Otherwise – if all fact in $M \setminus W$ are of types different from $assign$ – at least a rule in $\mathscr{Q}^M$ would be violated by W. If $a = assign(\xi_0,c_0)$ then $discard(\xi_0,c_0) \notin M$. Therefore, $candidate(\xi_0,c_0) \notin W$, otherwise the rule $assign(\xi_0,c_0) \leftarrow candidate(\xi_0,c_0)$ in $\mathscr{Q}^M$ (of type 19) would be violated by W.

It means that for each rule r in $\mathscr{Q}^M$ of the form (type 9):

$$candidate(\xi,c) \leftarrow r^*(x_1,\dots,\xi_0,\dots,x_{n_r}), r^*(y_1,\dots,c_0,\dots,y_{n_r}), arbitrary(\xi_0), constant(c_0), \bigwedge_{j\in[1..n_r]\setminus\{i\}} compatible(x_j,y_j) \tag{2}$$

$W \not\models Body(r)$, otherwise the rule would be violated.

As we included $assign(\xi_0,c_0)$ in C (and so in M), there is at least an atom $r^*(x_1,\dots,\xi_0,\dots,x_{n_r}) \in H^*$ (and so in M).

Now we observe that each atom of the form $r^*(x_1,\dots,\xi_0,\dots,x_{n_r}) \in M$ belongs to H^* because at least an arbitrary constant occurs in it (ξ_0), while in *starred* atoms included in M in step (o) only constants occur.

For each $\alpha = r^*(x_1,\dots,\xi_0,\dots,x_{n_r}) \in H^*$ and $\beta = r^*(y_1,\dots,c_0,\dots,y_{n_r}) \in M$ such that $M \models \bigwedge_{j\in[1..n_r]\setminus\{i\}} compatible(x_j,\ y_j)$, i.e. such that the body of the corresponding rule of the form (2) is *true* in M, $\beta \notin W$.

This holds because the body of this rule has to be *false* in W and it cannot be assumed the absence in W of *compatible* atoms because this would lead to the absence in W of some *arbitrary* or *constant* atom (see rules 13 of $\mathscr{Q}^M$), which is a contradiction. The atom β cannot belong to H^* because this would imply the absence in W of some atom in $Rew(B)$ or $Rew(O)$, which is a contradiction. Therefore, it has been included in M in step (o) in order to satisfy at least a rule in 19.

So far from the absence in W of $assign(\xi_0,c_0)$ we derived the absence in W of each starred atom derived by replacing ξ_0 with c_0.

Observe that the body of the rule:

$$r^*(y_1,\dots,c_0,\dots,y_{n_r}) \leftarrow r^*(x_1,\dots,\xi_0,\dots,x_{n_r}), assign(x_1,y_1),\dots, assign(\xi,c_0),\dots,assign(x_{n_r},y_{n_r})$$

now is false in W because we removed $assign(\xi_0,c_0)$. However, the absence of $r^*(y_1,\dots,c_0,\dots,y_{n_r})$ in W could require the absence of other atoms in it because there could be in $\mathscr{Q}^M$ another rule of the form:

$$r^*(y_1,\dots,c_0,\dots,y_{n_r}) \leftarrow r^*(x'_1,\dots,\xi_1,\dots,x'_{n_r}), assign(x'_1,y_1),\dots, assign(\xi',c_1),\dots,assign(x'_{n_r},y_{n_r}).$$

Observe that at least an arbitrary constant occurs in $r^*(x'_1,\dots,\xi_1,\dots,x'_{n_r})$. Therefore it belongs to H^* because with rules of type 15 in Definition 10 (corresponding to rules 19 of $\mathscr{Q}^M$) we derive here only atoms in which no arbitrary constant occurs. This means that $r^*(x'_1,\dots,\xi_1,\dots,x'_{n_r})$ has to belong to W (we cannot assume its absence) and that instead one of the atoms in:

$$\{assign(x'_1,y_1),\dots,assign(x'_j,c),\dots,assign(x'_{n_r},y_{n_r})\}$$

cannot belong to W. Let $assign(\xi_1,c_1)$ such an atom ($assign(\xi_1,c_1) \notin W$).

We can reiterate the process without removing any atom from H^*. This process ends because M is finite. Therefore, from the deletion of $assign(\xi_0,c_0)$ we derived the deletion of a set of *assign* atoms. Let

$$A = \{assign(\xi_0,c_0), assign(\xi_1,c_1),\dots,assign(\xi_k,c_k)\}$$

the set of *assign* atoms that we removed from M in order to obtain W (and, we recall, without removing any atom belonging to H^*). Observe that each base starred atom $b^*(x_1,\dots,x_n) \in (M \setminus W)$ has no arbitrary constants occurring in it (as observed earlier, it is an atom inserted in step (o) and in this step no atom containing arbitrary constants is included) and is s.t. $b(x_1,\dots,x_n)$ belongs to E (it cannot belong to B because $b^*(x_1,\dots,x_n)$ would be in H^* and removing atoms belonging to H^* leads to a contradiction).

In summary, from the absence in W of $assign(\xi_0,c_0)$ we derived the absence in W of a set of assign atoms and the related starred atoms derived by replacing ξ_i with c_i, with $i \in [1..k]$. Moreover we derived the absence of new base atoms in E unifying with these arbitrary constants. And still, $W \models \mathscr{Q}^M$.

What we prove, is that in this case, is that the assignment of ξ_i to c_i, with $i \in [1..k]$, was arbitrary, in the sense that c_i can be replaced with another constant d_i. For this purpose, let us consider the set:

$$A' = \{assign(\xi_0,d_0), assign(\xi_1,d_1),\dots,assign(\xi_k,d_k)\}$$

where d_i, with $i \in [0..k]$, is a new constant not occurring in $\mathscr{T}$ and O.

We define a new set M' obtained from W by:

1) inserting atoms in A'
2) inserting $r^*(y_1,\dots,d_i,\dots,y_{n_r})$ s.t. $r^*(y_1,\dots,c_i,\dots,y_{n_r})$ has been removed from M in order to obtain W and there is a rule $r \in \mathscr{Q}^M$ of the form (2) s.t. $M \models body(r)$
3) inserting $compatible(\xi_i,d_i)$ for each $compatible(\xi_i,c_i)$ removed from M in order to obtain W.
4) replacing $incompatible(\xi,d_i)$ with $incompatible(\xi,c_i)$
5) replacing $r^+(y_1,\dots,c_i,\dots,y_{n_r})$ with $r^+(y_1,\dots,d_i,\dots,y_{n_r})$, s.t. there is a rule $r \in \mathscr{Q}^M$ of the form (24) s.t. $M \models body(r)$.

By construction M' is a model of $\mathscr{Q}^{M'}$ and so it is a model of $\mathscr{Q}$. Observe that by Proposition 1, the set $E' = F(M')$ is s.t. $\mathscr{R} \cup B \cup E' \models O$, that is $(E',\emptyset)$ is an explanation for O w.r.t. $\mathscr{T}$. Moreover, by construction, E' has been obtained from E by replacing the constants c_i with the constants d_i. This means that the explanation $\Delta = (E,\emptyset)$ is not constrained. This is a contradiction.

Finally, it is easy to show that:

- $F(M) = E$. First observe that each atom $r(\bar{x}) \in F(M)$ is a base atom and only constants occur in it, because atoms of the form $r^+(\bar{x})$ are derived by means of rules 21 in Definition 12 and in each $assign(\xi,c) \in M$, c is a constant. Moreover, as $r(\bar{x}) \notin B$ it follows that that $r(\bar{x}) \in E$.
  Now, let us consider an atom $r(\bar{c}) \in E$. As Δ is constrained there must be a rule $r \in ground(\mathscr{R})$ of the form:
  $$h(\bar{x}) \leftarrow \mathscr{P}_1(\dots) \vee \cdots \vee \mathscr{B}, r(\bar{c}) \vee \cdots \vee \mathscr{P}_n(\dots)$$
  s.t. $S \models \mathscr{B}, r(\bar{c})$ (i.e. $r(\bar{c})$ is involved in the derivation of O). We recall that H^* is a *template* of S in the sense explained earlier. For the body $\mathscr{B}, r(\bar{c})$ there is a corresponding conjunction $\mathscr{B}^*, r^*(\bar{y})$ which is true in H^* and unifies with $\mathscr{B}, r(\bar{c})$. From rule 21 of Definition 12 and *assign* atoms, $r^+(\bar{c})$ is derived. Therefore, $r(\bar{c}) \in F(M)$.
- Finally, $constrainedExplanation \in M$ by construction.

2) From Proposition 1, we know that $\mathscr{R} \cup B \cup E \models O$.

Now we show that $\Delta = (E,\emptyset)$ is constrained. We prove that by contradiction assuming that Δ is not constrained.

If Δ is not constrained, there is at least a constant c occurring in E that can be replaced with a constant d, not occurring in $\mathscr{T}$ and in O, in a minimal set of positions, obtaining a set E' s.t. $\mathscr{R} \cup B \cup E' \models O$.

First, we prove that there is a model M' of $Rew(\mathscr{T},O)$, or equivalently of $ground(Rew(\mathscr{T},O))$, s.t. $F(M') = E'$. We build M', corresponding to $\Delta' = (E',\emptyset)$, following the procedure presented in first part of the proof, that is:

1) building the program $\mathscr{L}' = \mathscr{P}^b \cup \mathscr{P}'^f \cup \mathscr{P}^c$, where:
   $\mathscr{P}^b = Rew(\mathscr{R}) \cup Rew(B) \cup Rew(O)$
   $\mathscr{P}'^f = \mathscr{R} \cup B \cup E' \cup$
   $\{h_i(\overline{X}) \leftarrow \mathscr{P}_i(\overline{X},\overline{Y_i}) \mid h(\overline{X}) \leftarrow \bigvee_{i\in[1..n]} \mathscr{P}_i(\overline{X},\overline{Y_i})$ belongs to $\mathscr{R}$ and $i \in [1..n]\}$
   $\mathscr{P}^c = \{\overline{h}_i(x_1,\dots,x_n) \leftarrow h_i^*(x_1,\dots,x_n) \wedge h_i(y_1,\dots,y_n) \wedge \bigwedge_{i=1..n}(arbitrary(x_i) \vee x_i = y_i) \mid h_i^*(x_1,\dots,x_n)$ occurs in $Rew(\mathscr{R})\} \cup$
   $\{\bot \leftarrow h_i^*(x_1,\dots,x_n), not\ \overline{h}_i(x_1,\dots,x_n) \mid h_i^*(x_1,\dots,x_n)$ occurs in $Rew(\mathscr{R})\}$;
2) considering a stable model H' of $\mathscr{L}'$ and the subset H'^* of its starred atoms;
3) building a set C' of *assign* atoms as explained in the first part of the proof;
4) including in M' the atoms in H'^*, C' and the atoms defined in steps (c)-(q) of the first part of the proof.

By construction, M' is a model of the reduct $Rew(\mathscr{T},O)^{M'}$. Therefore, M' is a model of $ground(Rew(\mathscr{T},O))$ and a model of $Rew(\mathscr{T},O)$. Observe that M' cannot be a *minimal* model of $Rew(\mathscr{T},O)^{M'}$ because d does not occur in $Rew(\mathscr{T},O)$ and atoms in which d occur cannot be derived from the rewriting. Then there is a model $M'' \subset M'$ not including atoms containing d that is a model of $Rew(\mathscr{T},O)$. But M'' is also a subset of M. Then M is not a minimal model of $Rew(\mathscr{T},O)$. If follows that M is not a stable model of $Rew(\mathscr{T},O)$. This is a contradiction. $\square$