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CSED261: Discrete Mathematics for Computer Science
Homework 1: Propositional Logic & Predicate Logic

Question 1. Determine whether each of these conditional statements is true or false.

1. If $1 + 1 = 3$, then unicorns exist.

2. If $1 + 1 = 3$, then dogs can fly.

3. If $1 + 1 = 2$, then dogs can fly.

4. If $2 + 2 = 4$, then $1 + 2 = 3$.

Solutions

1. **True**. When p is false, then 'if p, then q' is always true.

2. **True**. When p is false, then 'if p, then q' is always true.

3. **false**. When p is true and q is false, then 'if p, then q' is false.

4. **True**. When p and q are both true, then 'if p, then q' is true.

Question 2. Write each of these statements in the form "if p, then q" in English.

1. I will remember to send you the address only if you send me an e-mail message.

2. To be a citizen of this country, it is sufficient that you were born in the United States.

3. If you keep your textbook, it will be a useful reference in your future courses.

4. The Red Wings will win the Stanley Cup if their goalie plays well.

5. That you get the job implies that you had the best credentials.

6. The beach erodes whenever there is a storm.

7. It is necessary to have a valid password to log on to the server.

8. You will reach the summit unless you begin your climb too late.

9. You will get a free ice cream cone, provided that you are among the first 100 customers tomorrow.

Solutions

1. **If** you send me an e-mail message, **then** I will remember to send you the address.

2. **If** you were born in the United States, **then** you are a citizen of this country.

3. **If** you keep your textbook, **then** it will be a useful reference in your future courses.

4. **If** their goalies plays well, **then** the Red Wings will win the Stanley Cup.

5. **If** you had the best credentials, **then** you get the job.

6. **If** there is storm, **then** the beach erodes.

7. **If** If you have a valid password, **then** you log on to the server.

8. **If** you begin your climb too late, **then** you will not reach the summit.

9. **If** you are among the first 100 customers tomorrow, **then** you will get a free ice cream cone.

Below two questions are related to inhabitants of an island on which there are three kinds of people: knights who always tell the truth, knaves who always lie, and spies who can either lie or tell the truth. You encounter three people, A, B, and C. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the type of person each of other two is. For each of these situations, if possible, determine whether there is a unique solution and determine who the knave, knight, and spy are. When there is no unique solution, list all possible solutions or state that there are no solutions.

Question 3. A says "C is the knave," B says "A is the knight," and C says "I am the spy."

Question 4. A says "I am the knave," B says "I am the knave," and C says "I am the knave."

Solutions

Total possible case:

$$(A, B, C) = \{(knight, spy, knave), (knight, knave, spy), (spy, knight, knave),$$
$$(knave, knight, spy), (spy, knave, knight), (knave, spy, knight)\}$$

Question 3:

1. **True**. A is knight, so C should be knave. This case is True.

2. **False**. A is knight, so C should be knave but C is spy, so this case is false

3. **False**. B is knight, so A should be knight. However A is spy, so false.

4. **False**. B is knight, so A should be knight. However A is knave, so false

5. **False**. C is knight, then C is also spy. This case is false.

6. **False**. C is knight, then C is also spy. This case is false.

Possible case is,
$$(A, B, C) = (knight, spy, knave)$$

Question 4:

   All says they are knave themselves. If one of them is knight, then also he should be knave. So, all cases are false. There is no solution.

Question 5. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

Solutions

When p and q both true or both false, $p \leftrightarrow q$ is true. Otherwise, p and q are different such as 'p is true and q is false', then $p \leftrightarrow q$ is false.

We should check $(p \wedge q) \vee (\neg q \wedge \neg q)$ works logically equivalently.

1. When p and q are both true, $p \wedge q$ is true, $\neg p \wedge \neg q$ is false, so (true $\vee$ false) is true.

2. When p is true and q is false, $p \wedge q$ is false, $\neg p \wedge \neg q$ is false, so (false $\vee$ false) is false.

3. When p is false and q is true, $p \wedge q$ is false, $\neg p \wedge \neg q$ is false, so (false $\vee$ false) is false.

4. When p and q are both false, $p \wedge q$ is false, $\neg p \wedge \neg q$ is true, so (false $\vee$ true) is true.

Therefore, $(p \wedge q) \vee (\neg p \wedge \neg q)$ works equivalently with $p \leftrightarrow q$. They are logically equivalent.

Question 6. How many of the disjunctions $p \vee \neg q$, $\neg p \vee q$, $q \vee r$, $q \vee \neg r$, and $\neg q \vee \neg r$ can be made simultaneously true by an assignment of truth values to p, q, and r?

Solutions

p	q	r	$p \vee \neg q$	$\neg p \vee q$	$q \vee r$	$q \vee \neg r$	$\neg q \vee \neg r$	correct
T	T	T	T	T	T	T	F	4
T	T	F	T	T	T	T	T	5
T	F	T	T	F	T	F	T	3
T	F	F	T	F	F	T	T	3
F	T	T	F	T	T	T	F	3
F	T	F	F	T	T	T	T	4
F	F	T	T	T	T	F	T	4
F	F	F	T	T	F	T	T	4

So, when $(p, q, r) = (T, T, F)$, all of the disjunctions can be true simultaneously. Therefore, the answer is **5**.

Question 7. Determine the truth value of each of these statements if the domain consists of all real numbers.

1. $\exists x(x^3 = -1)$

2. $\exists x(x^4 < x^2)$

3. $\forall x((-x)^2 = x^2)$

4. $\forall x(2x > x)$

Solutions

1. **True**. Let $x = -1$, then $x^3 = -1$. It exists at least one x satisfiying the statement.

2. **True**. Let's pick some x bigger than 0 and smaller than 1. For example, $x = 0.1$. Then $x^4 = 0.0001$ and $x^2 = 0.01$. Therefore, it exists at least one x satisfiying the statement.

3. **True**. This is because square of any real number is always non-negative. Therefore, $(-x)^2 = x^2$ for all x.

4. **False**. Let $x = -1$, then $2x = -2$ and $x = -1$. Then $-2 > -1$, which is false. Therefore, there are counterexamples for the statement.

Question 8. Express each of these system specifications using predicates, quantifiers, and logical connectives.

1. When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.

2. No directories in the file system can be opened and no files can be closed when system errors have been detected.

3. The file system cannot be backed up if there is a user currently logged on.

4. Video on demand can be delivered when there are at least 8 megabytes of memory available and the connection speed is at least 56 kilobits per second.

Solutions

1. Let $P(x)$ be the predicates 'There is less than 30 megabytes free on the hard disk x' and $Q(y)$ be the predicates 'A warning message is sent to user y' Then,

$$\exists x P(x) \rightarrow \forall y Q(y)$$

2. Let $P(x)$ be the predicates 'System error x has been detected', $Q(y)$ be the predicates 'Directory y in the file system can be open', and $R(z)$ be the predicates 'File z can be closed'. Then,

$$\exists x P(x) \rightarrow \forall y \forall z (\neg Q(y) \land \neg R(z))$$

3. Let $P(x)$ be the predicates 'User x is currently logged on' and $Q(y)$ be the predicates 'The file system y can be backed up'. Then,

$$\exists x (P(x)) \rightarrow \forall y \neg Q(y)$$

4. Let $P(x)$ be the predicates 'There are at least 8 megabytes of memory x available', $Q(y)$ be the predicates 'The connection speed y is at least 56 kilobits per second', and $R(x)$ be the predicates 'Video on demand z can be delivered'. Then,

$$\exists x \exists y (P(x) \land Q(y)) \rightarrow \forall z R(z)$$

Question 9. Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

1. Everybody can fool Fred.

2. Evelyn can fool everybody.

3. Everybody can fool somebody.

4. There is no one who can fool everybody.

5. Everyone can be fooled by somebody.

6. No one can fool both Fred and Jerry.

7. Nancy can fool exactly two people.

8. There is exactly one person whom everybody can fool.

9. No one can fool himself or herself.

10. There is someone who can fool exactly one person besides himself or herself.

Solutions

1. $\forall x F(x, \text{Fred})$

2. $\forall x F(\text{Evelyn}, x)$

3. $\forall x \exists y F(x, y)$

4. $\neg \exists x \forall y F(x, y)$

5. $\forall x \exists y F(y, x)$

6. $\neg \exists x (F(x, \text{Fred})) \wedge (F(x, \text{Jerry}))$

7. $\exists x \exists y (\neg(x = y) \wedge F(\text{Nancy}, x) \wedge F(\text{Nancy}, y) \wedge (\forall z (F(\text{Nancy}, z) \rightarrow (z = x \vee z = y))))$

8. $\forall y (\exists x (F(x, y)) \wedge \exists z (Fz, y)) \rightarrow (x = z)$

9. $\neg \exists x F(x, x)$

10. $\exists x \exists y (\neg(x = y) \wedge F(x, y) \wedge \neg \exists z (F(x, z) \wedge \neg(z = y)))$

Question 10. Let $Q(x, y)$ be the statement "$x + y = x - y$". If the domain for both variables consists of all integers, what are the truth values?

1. $Q(1, 1)$

2. $Q(2, 0)$

3. $\forall y Q(1, y)$

4. $\exists x Q(x, 2)$

5. $\exists x \exists y Q(x, y)$

6. $\forall x \exists y Q(x, y)$

7. $\exists y \forall x Q(x, y)$

8. $\forall y \exists x Q(x, y)$

9. $\forall x \forall y Q(x, y)$

Solutions

1. **False.** $1 + 1 = 1 - 1$ is not true.

2. **True.** $2 + 0 = 2 - 0$ is true.

3. **False.** There is counterexample such as $y = 1$. (Note Q10.1)

4. **False.** There is no integer x such that $x + 2 = x - 2$, $2 = -2$ is not true.

5. **True.** For example, $x = 2, y = 0$. (Note Q10.2) So there is a pair x, y s.t. $Q(x, y)$ is true.

6. **True.** There is $y(= 0)$ for which $Q(x, y)$ is true for every x.

7. **True.** Let y is 0, then for every x, statement is true.

8. **False.** There is y s.t. $Q(x, y)$ is not true for every x.

9. **False.** There are countercases. (i.g. $x = 1, y = 1$)