Name: Student ID:

CSED261: Discrete Mathematics for Computer Science Homework 3: Sets and Set Operations & Function and Sequences & Set Cardinality and Matrices

Question 1. Prove or disprove that if A, B and C are nonempty sets and $A \times B = A \times C$, then B = C.

Question 2. Let A, B, and C be sets. Use the the identity $A - B = A \cap \overline{B}$, which holds for any sets A and B, and the set identities to show that $(A - B) \cap (B - C) \cap (A - C) = \emptyset$.

Question 3. Show that if x is a real number, then [x] - [x] = 1 if x is not an integer and [x] - [x] = 0 if x is an integer.

Question 4. Use the identity 1/(k(k+1)) = 1/k - 1/(k+1) and telescoping to compute $\sum_{k=1}^{n} 1/(k(k+1))$.

Note. $\sum_{j=1}^{n} (a_j - a_{j-1}) = a_n - a_0$, where $a_0, a_1, ..., a_n$ is a sequence of real numbers. This type of sum is called telescoping.

Question 5. Show that the set $Z^+ \times Z^+$ is countable.

Question 6. The $n \times n$ matrix $A = [a_{ij}]$ is called a diagonal matrix if $a_{ij} = 0$ when $i \neq j$. Show that the product of two $n \times n$ diagonal matrices is again a diagonal matrix. Give a simple rule for determining this product.