Name:
Student ID:

CSED261: Discrete Mathematics for Computer Science
Homework 7: Induction and Recursion

Question 1. Prove that

$$\sum_{j=0}^{n} \left(-\frac{1}{2}\right)^{j} = \frac{2^{n+1} + (-1)^{n}}{3 \cdot 2^{n}}$$

whenever n is a nonnegative integer.

Solutions

Question 2. Use mathematical induction to show that $\neg(p_1 \vee p_2 \vee \cdots \vee p_n)$ is equivalent to $\neg p_1 \wedge \neg p_2 \wedge \cdots \wedge \neg p_n$ whenever $p_1, p_2, \ldots, p_n$ are propositions.

Solutions

Question 3. Let b be a fixed integer and j a fixed positive integer. Show that if $P(b), P(b+1), \ldots, P(b+j)$ are true and $[P(b) \wedge P(b+1) \wedge \cdots \wedge P(k)] \rightarrow P(k+1)$ is true for every integer $k \geq b+j$, then $P(n)$ is true for all integers n with $n \geq b$.

Solutions

Question 4. Prove that $\sum_{j=1}^{n} j(j+1)(j+2)\cdots(j+k-1) = n(n+1)\ (n+2)\cdots(n+k)/(k+1)$ for all positive integers k and n. [Hint: Use a technique from Exercise 33]

> **33.** Show that we can prove that $P(n, k)$ is true for all pairs of positive integers n and k if we show
>
> **a)** $P(1, 1)$ is true and $P(n, k) \rightarrow [P(n+1, k) \wedge P(n, k+1)]$ is true for all positive integers n and k.
>
> **b)** $P(1, k)$ is true for all positive integers k, and $P(n, k) \rightarrow P(n+1, k)$ is true for all positive integers n and k.
>
> **c)** $P(n, 1)$ is true for all positive integers n, and $P(n, k) \rightarrow P(n, k+1)$ is true for all positive integers n and k.

Solutions

Question 5. Give a recursive definition of the functions max and min so that $\max(a_1, a_2, \ldots, a_n)$ and $\min(a_1, a_2, \ldots, a_n)$ are the maximum and minimum of the n numbers $a_1, a_2, \ldots, a_n$, respectively.

Solutions

Question 6. Give a recursive definition of the set of bit strings that are palindromes.

Solutions

Question 7. Prove that Algorithm 1 for computing $n\,!$ when n is a non-negative integer is correct.

ALGORITHM 1 A Recursive Algorithm for Computing $n!$.

procedure *factorial*(n: nonnegative integer)
if $n = 0$ **then return** 1
else return $n \cdot \mathit{factorial}(n-1)$
$\{$output is $n!\}$

Solutions

Question 8. Use a merge sort to sort $4, 3, 2, 5, 1, 8, 7, 6$ into increasing order. Show all the steps used by the algorithm.

Solutions