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CSED261: Discrete Mathematics for Computer Science
Homework 8: Counting & Discrete Probability

Question 1. Suppose that p and q are prime numbers and that $n = pq$. Use the principle of inclusion–exclusion to find the number of positive integers not exceeding n that are relatively prime to n.

Question 2. Let $n_1, n_2, ..., n_t$ be positive integers. Show that if $n_1 + n_2 + \cdots + n_t - t + 1$ objects are placed into t boxes, then for some $i, i = 1, 2, ..., t$, the ith box contains at least n_i objects.

Question 3. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

Question 4. Give a combinatorial proof that if n is a positive integer then $\sum_{k=0}^{n} k^2\binom{n}{k} = n(n+1)2^{n-2}$. [**Hint:** Show that both sides count the ways to select a subset of a set of n elements together with two not necessarily distinct elements from this subset. Furthermore, express the righthand side as $n(n-1)2^{n-2} + n2^{n-1}$.]

Question 5. How many different strings can be made from the letters in *MISSISSIPPI*, using all the letters?

Question 6. Two events E_1 and E_2 are called independent if $p(E_1 \cap E_2) = p(E_1)p(E_2)$. For each of the following pairs of events, which are subsets of the set of all possible outcomes when a coin is tossed three times, determine whether or not they are independent.

- E_1: tails comes up with the coin is tossed the first time; E_2: heads comes up when the coin is tossed the second time.

Question 7. Show that if E and F are independent events, then $\overline{E}$ and $\overline{F}$ are also independent events.

Question 8. Suppose that a Bayesian spam filter is trained on a set of 500 spam messages and 200 messages that are not spam. The word "exciting" appears in 40 spam messages and in 25 messages that are not spam. Would an incoming message be rejected as spam if it contains the word "exciting" and the threshold for rejecting spam is 0.9?

Question 9. Prove the law of total expectations.
