Name: `Your name here`
Student ID: `Your student ID here`

CSED261: Discrete Mathematics for Computer Science
Homework 1: Propositional Logic & Predicate Logic

Question 1. Determine whether each of these conditional statements is true or false.

1. If $1 + 1 = 3$, then unicorns exist.
2. If $1 + 1 = 3$, then dogs can fly.
3. If $1 + 1 = 2$, then dogs can fly.
4. If $2 + 2 = 4$, then $1 + 2 = 3$.

Solutions

Question 2. Write each of these statements in the form "if p, then q" in English.

1. I will remember to send you the address only if you send me an e-mail message.
2. To be a citizen of this country, it is sufficient that you were born in the United States.
3. If you keep your textbook, it will be a useful reference in your future courses.
4. The Red Wings will win the Stanley Cup if their goalie plays well.
5. That you get the job implies that you had the best credentials.
6. The beach erodes whenever there is a storm.
7. It is necessary to have a valid password to log on to the server.
8. You will reach the summit unless you begin your climb too late.
9. You will get a free ice cream cone, provided that you are among the first 100 customers tomorrow.

Solutions

Below two questions are related to inhabitants of an island on which there are three kinds of people: knights who always tell the truth, knaves who always lie, and spies who can either lie or tell the truth. You encounter three people, A, B, and C. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the type of person each of other two is. For each of these situations, if possible, determine whether there is a unique solution and determine who the knave, knight, and spy are. When there is no unique solution, list all possible solutions or state that there are no solutions.

Question 3. A says "C is the knave," B says "A is the knight," and C says "I am the spy."

Question 4. A says "I am the knave," B says "I am the knave," and C says "I am the knave."

Solutions

Question 5. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

Solutions

Question 6. How many of the disjunctions $p \vee \neg q$, $\neg p \vee q$, $q \vee r$, $q \vee \neg r$, and $\neg q \vee \neg r$ can be made simultaneously true by an assignment of truth values to p, q, and r?

Solutions

Question 7. Determine the truth value of each of these statements if the domain consists of all real numbers.

1. $\exists x(x^3 = -1)$
2. $\exists x(x^4 < x^2)$
3. $\forall x((-x)^2 = x^2)$
4. $\forall x(2x > x)$

Solutions

Question 8. Express each of these system specifications using predicates, quantifiers, and logical connectives.

1. When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.
2. No directories in the file system can be opened and no files can be closed when system errors have been detected.
3. The file system cannot be backed up if there is a user currently logged on.
4. Video on demand can be delivered when there are at least 8 megabytes of memory available and the connection speed is at least 56 kilobits per second.

Solutions

Question 9. Let F(x, y) be the statement “x can fool y,” where the domain consists of all people in the world. Use quantifiers to express each of these statements.

1. Everybody can fool Fred.
2. Evelyn can fool everybody.
3. Everybody can fool somebody.
4. There is no one who can fool everybody.
5. Everyone can be fooled by somebody.
6. No one can fool both Fred and Jerry.
7. Nancy can fool exactly two people.
8. There is exactly one person whom everybody can fool.
9. No one can fool himself or herself.
10. There is someone who can fool exactly one person besides himself or herself.

Solutions

Question 10. Let $Q(x, y)$ be the statement "$x + y = x - y$". If the domain for both variables consists of all integers, what are the truth values?

1. $Q(1, 1)$
2. $Q(2, 0)$
3. $\forall y Q(1, y)$
4. $\exists x Q(x, 2)$
5. $\exists x \exists y Q(x, y)$
6. $\forall x \exists y Q(x, y)$
7. $\exists y \forall x Q(x, y)$
8. $\forall y \exists x Q(x, y)$
9. $\forall x \forall y Q(x, y)$

Solutions