Optimization model of a Compressed Air Energy Storage (CAES)

Autor: Fahim Sadat M.Sc.

Contents

List of Abbreviations

Greek Symbols		**Units**
α	quality factor	-
η	efficiency	%
Π	pressure ratio	-
ϑ	temperature ratio	-

Roman Symbols		**Units**
C	costs	Euro
c	specific costs	Euro/W
$\dot{\mathrm{E}}$	exergy flow rate	W
h	enthalpy	J/kg
$\mathrm{H_u}$	heat value of the fuel	J/kg
m	mass	kg
$\dot{\mathrm{m}}$	mass flow rate	kg/s
$\mathrm{m_{rel}}$	mass ratio	-
n	polytropic exponent	-
p	pressure	bar
P	electrical power	W
$\dot{\mathrm{Q}}$	heat flow rate	W
$\mathrm{R_s}$	specific gas constant	J/kgK
R	revenue	Euro
T	temperature	K
V	volume	m^3
y	binary variable	-
Z	compressibility factor	-
z	depth	m
z^a	auxiliary variable	-

Abbreviations

CAES	Compressend Air Energy Storage

1 Introduction

Figure 1.1 provides an overview of the implemented Compressed Air Energy Storage (CAES) system. First the implemented mathematical model shall be elucidated

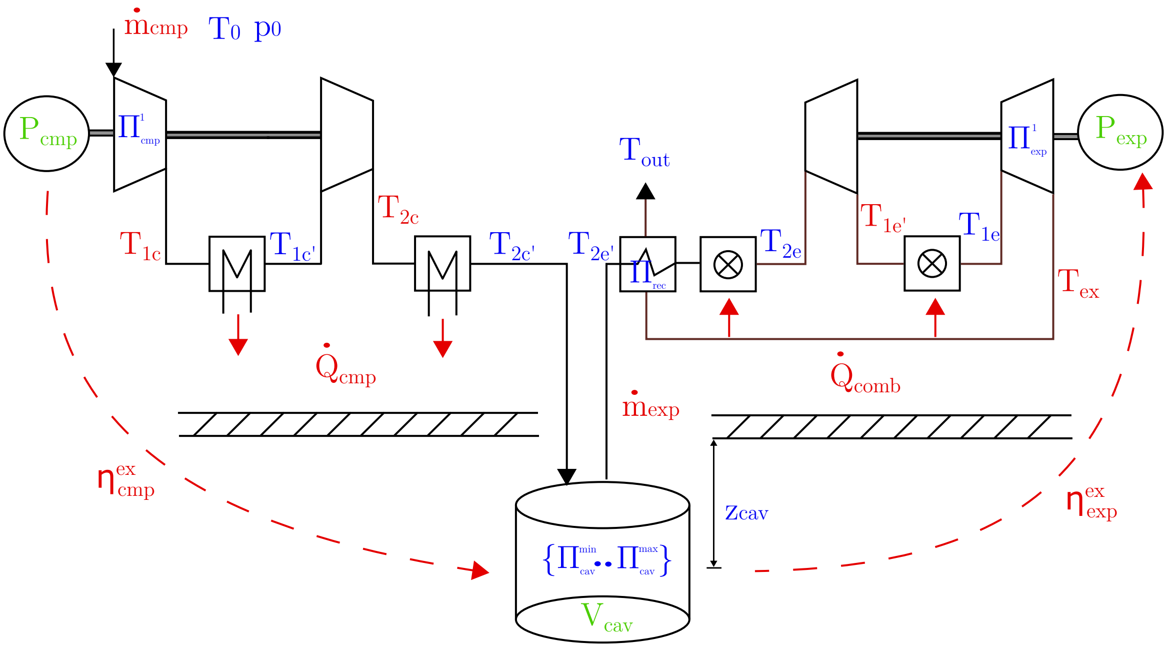

Figure 1.1: Model of the Compressed air energy storage system

before deriving a formulation of mixed-integer linear program for optimization purposes. The system is supposed to operate under variable pressure ratio as it was object of investigation in [1] to provide a better overall efficiency. The green coloured attributes can be set to desired values while creating an instance of the class "Diabatic". All blue attributes are set to default values which are summarized in Table 3.1 at the end of this document. The red marked variables are provided as results by the implemented algorithm. To what each of these expression is referring to will be highlighted in Chapter 2.

2 Mathematical model

The base of operation to describe gas behaviour shall be the General Gas Law according to Equation 2.1 with the system pressure p, temperature T and volume V. The total mass of air is described by m whereas Z and R_s represent the compressibility factor and the specific gas constant of dry air.

$$p \cdot V = Z(p, T) \cdot m \cdot R_s \cdot T \tag{2.1}$$

More detailed information about the accurate equation of state of air as a real gas can be found in [2].

2.1 Compression train

The electrical power demand P_{cmp} for compressing air up to the cavern pressure can be determined according to Equation 2.2.

$$P_{cmp} = \frac{\dot{m}^{air}_{cmp} \cdot (\Delta h^{is}_1 + \Delta h^{is}_2)}{\eta_{cmp} \cdot \eta^N_{mech} \cdot \eta^N_{mo}} \tag{2.2}$$

Δh^{is} represents the isentropic enthalpy difference between the inlet and outlet of the compressor, η_{cmp} the compressor efficiency, η^N_{mech} and η^N_{mo} the nominal mechanical and motor efficiencies. The compressor operates isentropically and the efficiency reduction in part load is approximated by the following linear approach.

$$\eta_{cmp} = \left(\frac{P_{cmp}}{P^N_{cmp}} \cdot (1 - \alpha_{cmp}) + \alpha_{cmp} \right) \cdot \eta^{is}_{cmp} \tag{2.3}$$

P^N_{cmp} represents the nominal compressor power and η^{is}_{cmp} the nominal isentropic compressor efficiency. The quality parameter α_{cmp} is introduced to provide different compressor characteristics. While $\alpha_{cmp} = 1$ represents an ideal compressor operating at a constant efficiency does $\alpha_{cmp} = 0$ represent a linear efficiency decrease at part load. Inserting Equation 2.3 into Equation 2.2 and rearranging delivers for the air mass flow rate $\dot{m}^{air}_{cmp}$:

$$\dot{m}^{air}_{cmp} = \frac{\left(\frac{{P_{cmp}}^2}{P^N_{cmp}} \cdot (1 - \alpha_{cmp}) + \alpha_{cmp} \cdot P_{cmp} \right) \cdot \eta^{is}_{cmp} \cdot \eta^N_{mo} \cdot \eta^N_{mech}}{\Delta h^{is}_{1c} + \Delta h^{is}_{2c}} \tag{2.4}$$

At this point it shall be noted that $\Delta h_1^{is} = h(s_{c1}, p_{c1}) - h(s_0, p_0)$ is a function of the entropy s and the pressure p. These values are obtained from the fluid library "CoolProp" which also applies for Δh_2^{is}. For the share of the compression heat $\dot{Q}_{cmp}$ we obtain

$$\dot{Q}_{cmp} = \eta_{mo}^{N} \cdot \eta_{mech}^{N} \cdot P_{cmp} - \dot{m}_{cmp}^{air} \cdot (h_{2c'} - h_0) \tag{2.5}$$

The exergetical efficiency η_{ex}^{cmp} of the cavern as a storage device can be calculated according to Equation 2.6 where $\dot{E}_{caes,i}^{in}$ represents the exergy flow into the cavern.

$$\eta_{ex}^{cmp} = \frac{\dot{E}_{caes}^{in}}{P_{cmp}} \tag{2.6}$$

2.2 Expansion train

The electrical expansion power is calculated according to Equation 2.7. The total mass flow rate varies slightly after each combustion chamber as we additionally need a gas supply for the combustion.

$$P_{exp} = (\dot{m}_{2e} \cdot \Delta h_{2e}^{is} + \dot{m}_{ex} \cdot \Delta h_{1e}^{is}) \cdot \eta_{tur} \cdot \eta_{gen}^{N} \cdot \eta_{mech}^{N} \tag{2.7}$$

η_{tur} and η_{gen}^{N} represent the efficiencies of the turbine and the generator. The required mass flow rate of the fuel can be estimated according to Equation 2.8. H_u represents the heat value of the fuel, h_{rec} describes the enthalpy of the air behind the recuparator and η_{comb}^{N} considers the nominal energy losses during the combustion.

$$\dot{m}_{fuel,1} \approx \frac{\dot{m}_{exp}^{air} \cdot (h_{2e} - h_{rec})}{\eta_{comb}^{N} \cdot H_u} \tag{2.8}$$

Consequently we obtain for the total flow rate behind the first combustion chamber according to Equation 2.9

$$\dot{m}_{2e} \approx \dot{m}_{exp}^{air} \cdot \underbrace{\left[1 + \frac{h_{2e} - h_{rec}}{\eta_{comb}^{N} \cdot H_u}\right]}_{M_2} \tag{2.9}$$

and respectively behind the second combustion chamber according to Equation 2.10. It shall be noted that the quadratic term of Equation 2.10 has been neglected since its impact is marginal.

$$\dot{m}_{ex} \approx \dot{m}_{exp}^{air} \cdot \underbrace{\left[1 + \frac{h_{2e} - h_{rec}}{\eta_{comb}^{N} \cdot H_u} + \frac{h_{1e} - h_{1e'}}{\eta_{comb}^{N} \cdot H_u}\right]}_{M_1} \tag{2.10}$$

In the aforementioned calculations it is assumed that the impact of the exhaust gas on the fluid properties is negligible as it is imperative that $\dot{m}_{exp}^{air} \gg \dot{m}_{fuel}$. Equation 2.11 describes the approach of the efficiency decrease in the part load analogous to the compression train.

$$\eta_{tur} = \left[\sqrt{\frac{P_{exp}}{P_{exp}^{N}}} \cdot (1 - \alpha_{exp}) + \alpha_{exp}\right] \cdot \eta_{exp}^{is} \tag{2.11}$$

P_{exp}^{N} represents the nominal turbine power and η_{exp}^{is} the corresponding isentropic efficiency. The parameter α_{exp} can take values between 1 and 0. Inserting Equations 2.9-2.11 into Equation 2.7 and rearranging delivers following expression for the required air mass flow rate in order to provide a certain expansion power.

$$\dot{m}_{exp}^{air} = \frac{P_{exp}}{\eta_{total} \cdot \left[\sqrt{\frac{P_{exp}}{P_{exp}^{N}}} (1 - \alpha_{exp}) + \alpha_{exp} \right] \cdot \left(M_2 \cdot \Delta h_{2e}^{is} + M_1 \cdot \Delta h_{1e}^{is} \right)} \tag{2.12}$$

The product of the nominal efficiencies considering the turbine, motor and the mechanics are merged to η_{total}. The necessary combustion heat can be approximately determined according to Equation 2.13.

$$\dot{Q}_{comb} = \dot{m}_{exp}^{air} \cdot \left[\frac{(h_{2e} - h_{rec}) + (h_{1e} - h_{1e'})}{\eta_{comb}^{N}} \right] \tag{2.13}$$

The expansion efficiency of the cavern can be determined according to Equation 2.14 where $\dot{E}_{caes}^{out}$ describes the exergy flow of the air leaving the cavern and $\dot{E}_{fuel}$ the exergy flow of the fuel.

$$\eta_{ex}^{exp} = \frac{P_{exp}}{\dot{E}_{caes}^{out} + \dot{E}_{fuel}} \tag{2.14}$$

2.3 Cavern

The polytropic change of state of a real gas can be determined according to Equation 2.15 [3].

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\left(\frac{n \cdot [1 + Z_T]}{n \cdot [1 - Z_p] - 1}\right)_{1,2}} \tag{2.15}$$

where n stands for the polytropic exponent and Z_T and Z_p for the gradients of the compressibility factors. The index $_{1,2}$ indicates the average values between two stages. In order to describe the compression process in the cavern the ideal gas law is applied with $Z = 1$ and therefore $Z_T = Z_p = 0$. The justification of this assumption will be shown in Section 2.4. Incorporating the following definitions

$$m_{rel} = \frac{m_{cav}}{m_{cav,0}} \quad ; \quad \Pi_{cav} = \frac{p_{cav}}{p_{cav,0}} \quad ; \quad \vartheta_{cav} = \frac{T_{cav}}{T_{cav,0}}$$

into the Equation 2.1 and differentiating delivers the differential Equation 2.16 of mass change in the cavern over time. The index $_0$ indicates a reference state.

$$\frac{dm_{rel}}{dt} = \frac{d\Pi_{cav}}{dt} \cdot \frac{1}{\vartheta_{cav}} - \frac{\Pi_{cav}}{(\vartheta_{cav})^2} \cdot \frac{d\vartheta_{cav}}{dt} \tag{2.16}$$

From Equation 2.15 we obtain for the derivation of the temperature ratio with respect to the pressure ratio of an ideal gas as follows.

$$\frac{d\vartheta_{cav}}{d\Pi_{cav}} = \left(\frac{n-1}{n}\right)_{1,2} \cdot \Pi_{cav}^{-\left(\frac{1}{n}\right)_{1,2}} \tag{2.17}$$

Inserting Equation 2.17 into Equation 2.16 and integrating delivers

$$\Pi_{cav} = \left(\frac{\dot{m}_{cmp} - \dot{m}_{exp}}{m_{cav,0}} \cdot t + \sqrt[n_{1,2}]{\Pi_{cav,0}}\right)^{n_{1,2}} \tag{2.18}$$

where the integration constant $\Pi_{cav,0}$ specifies the initiative pressure ratio of the cavern for $t = 0$. In the present model the cavern is supposed to operate isothermal which means that $n_{1,2} = n = 1$

2.4 Model assumptions

- The dry air has been treated as a real gas. The applied fluid library Coolprop bases on large measurement data and provides fairly accurate fluid properties.
- The assumption of an isentropic compression holds for a good isolated compressor and an almost reversible process.
- The cavern is treated to be isothermal as the temperature difference of +-8K are observed in Huntorf [citation missing]. Figure 2.1 shows the compressibility factor of dry air over the operating pressure range (red dashed lines). The assumption of an ideal gas in this case would lead to an error below 1 %.

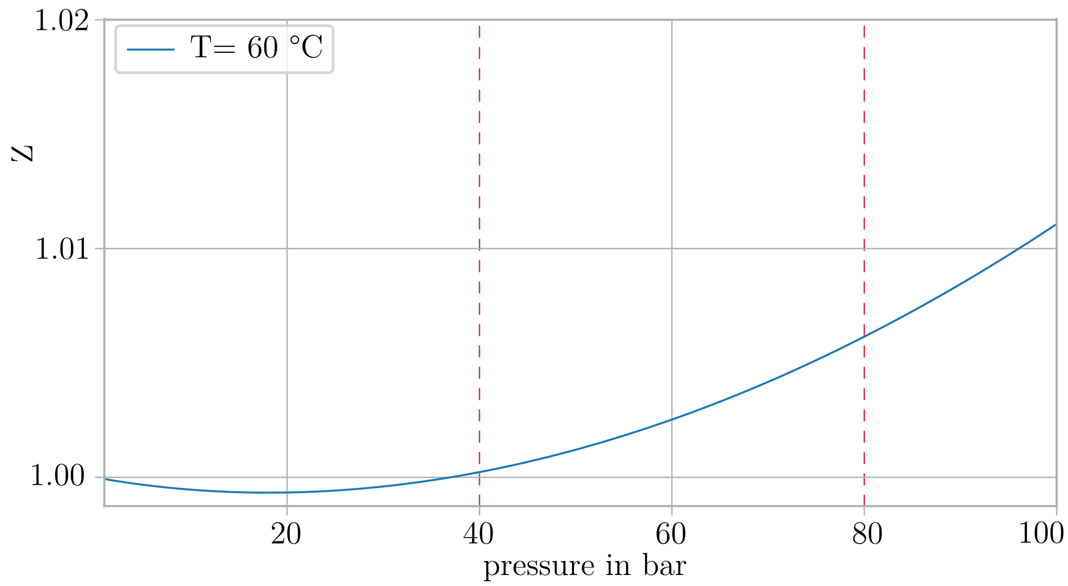

Figure 2.1: The Compressibility Factor Z for constant temperature and varying cavern pressure

- Implying a constant efficiency for motor/generator is fairly sufficient as most of the modern machines operate quite constant [4]. The accuracy doesn't hold for the heat exchanger as the heat transfer in part load falls of in quality. Also the simple approach of the combustion process causes inaccuracies.
- The operation characteristics of the compressor and turbine is more complex considering the surge and choke lines. Thus, a validation of the model is indispensable in order to quantify the provided results.

3 Mixed-integer linear program

A mixed-integer linear program has been built to simulate an operation period of the afore presented CAES system. The program basically provides the optimal operation of a plant for a given objective function and the corresponding constraints. The equations provided in Chapter 2 form a non-linear equation system but it is not mandatory to declare them as a simplified linear formulation. The advantage of a linear program lies in the faster computing time at the cost of accuracy. Especially for computationally very intensive models the mixed-integer linear program is preferred whereas computationally less sophisticated models can be targeted by a non-linear approach.

3.1 Objective function

Equation 3.1 represents the objective function which intends to minimize the sum of the costs reduced by the revenues for n time steps.

$$\min \overset{!}{=} \left[\sum_{i=1}^{n} (C_i - R_i) \cdot \Delta t_i \right] \tag{3.1}$$
$$C_i = P_{cmp,i} \cdot (c_{spot} + c_{cmp}^{chg} + c_{cmp}^{var})_i + \dot{Q}_{comb,i} \cdot (c_{fuel} + c_{emi})_i$$
$$R_i = P_{exp,i} \cdot (c_{spot} - c_{exp}^{var})_i$$

The costs contain the specific expenses of electricity at the spot market c_{spot}, all electricity charges c_{chg}, the specific variable cost of the compressor c_{cmp}^{var}, the specific fuel c_{fuel} and emission cost c_{emi}.
The revenues consider beside the spot market prices the specific variable cost of the turbine c_{exp}^{var}.

3.2 Constraints

The constraints of the compressor are described in Equation 3.2 ensuring that the compressor power is either zero or operating between the minimum and maximum load range. The relation between compression power, mass flow and storage pressure is linearised and portrayed in Equation 3.3. The binary variable y_{cmp} ensures that the mass flow is equal zero while the compressor is not operating and $a_{cmp}, b_{cmp}, c_{cmp}$

represent the polynomial coefficients of the linear fit. Equations 3.4 to 3.7 serve to resolve the product of the continuous variable describing the storage overpressure $\Pi_{cav,ov}$ and the binary variable according to the 'Big-M" type method which is commonly used for continuous variables bounded by zero. Π_{cav}^{min} and $\Pi_{cav,ov}^{max}$ represent defined parameters while z_{cmp}^{a} is an auxiliary continuous variable. The compression heat is outlined in Equation 3.8 with the parameters d_{cmp} and e_{cmp} resulting from Equation 2.5.

$$y_{cmp} \cdot P_{cmp}^{min} \leq P_{cmp} \leq y_{cmp} \cdot P_{cmp}^{N} \tag{3.2}$$

$$\dot{m}_{cmp} = a_{cmp} \cdot y_{cmp} + b_{cmp} \cdot P_{cmp} + c_{cmp} \cdot (\Pi_{cav}^{min} \cdot y_{cmp} + z_{cmp}) \tag{3.3}$$

$$z_{cmp}^{a} \leq \Pi_{cav,ov}^{max} \cdot y_{cmp} \tag{3.4}$$

$$z_{cmp}^{a} \leq \Pi_{cav,ov} \tag{3.5}$$

$$z_{cmp}^{a} \geq \Pi_{cav,ov} - (1 - y_{cmp}) \cdot \Pi_{cav,ov}^{max} \tag{3.6}$$

$$z_{cmp}^{a} \geq 0 \tag{3.7}$$

$$\dot{Q}_{cmp} = d_{cmp} \cdot P_{cmp} - e_{cmp} \cdot \dot{m}_{cmp} \tag{3.8}$$

The expansion train can be described analogically according to Equations 3.2 to 3.7 by replacing the index. The compression heat shall be replaced with the combustion heat which can be calculated according to Equation 3.9.

$$\dot{Q}_{comb} = d_{exp} \cdot y_{exp} + e_{exp} \cdot P_{exp} + f_{exp} \cdot (\Pi_{cav}^{min} \cdot y_{exp} + z_{exp}^{a}) \tag{3.9}$$

Finally, the inequality according to Equation 3.10 ensures that only one status variable is active in each time step, which means that the compressor and the turbine can't operate at the same time.

$$y_{cmp} + y_{exp} \leq 1 \tag{3.10}$$

Equation 3.11 defines the upper and lower limit of the cavern pressure. In each time step the cavern pressure changes according to Equation 3.12. η_{cav}^{loss} describes the pressure loss and $\Pi_{cav,ov}^{t-1}$ declares the overpressure of the previous time step. Equation 3.13 defines the overpressure in the first and last time step.

$$\Pi_{cav,ov}^{min} \leq \Pi_{cav,ov} \leq \Pi_{cav,ov}^{max} \tag{3.11}$$

$$\Pi_{cav,ov}^{t} = (1 - \eta_{cav}^{loss}) \cdot \Pi_{cav,ov}^{t-1} + \frac{\dot{m}_{cmp} - \dot{m}_{exp}}{m_{cav,0}} \cdot \Delta t \tag{3.12}$$

$$\Pi_{cav,ov}^{init} = \Pi_{cav,ov}^{0} \tag{3.13}$$

3.3 Link to the mathematical model

Figure 3.1 demonstrates at the example of the compression train the link between the thermodynamic model and the mixed-integer linear program. The blue dots represents the calculated values according to Section 2. The polynomial coefficients of the fitted surface are applied in the linear program to represent the characteristic of the plant. The same applies for the expansion train to represent the mass flow and the combustion heat as a function of the power and cavern pressure.

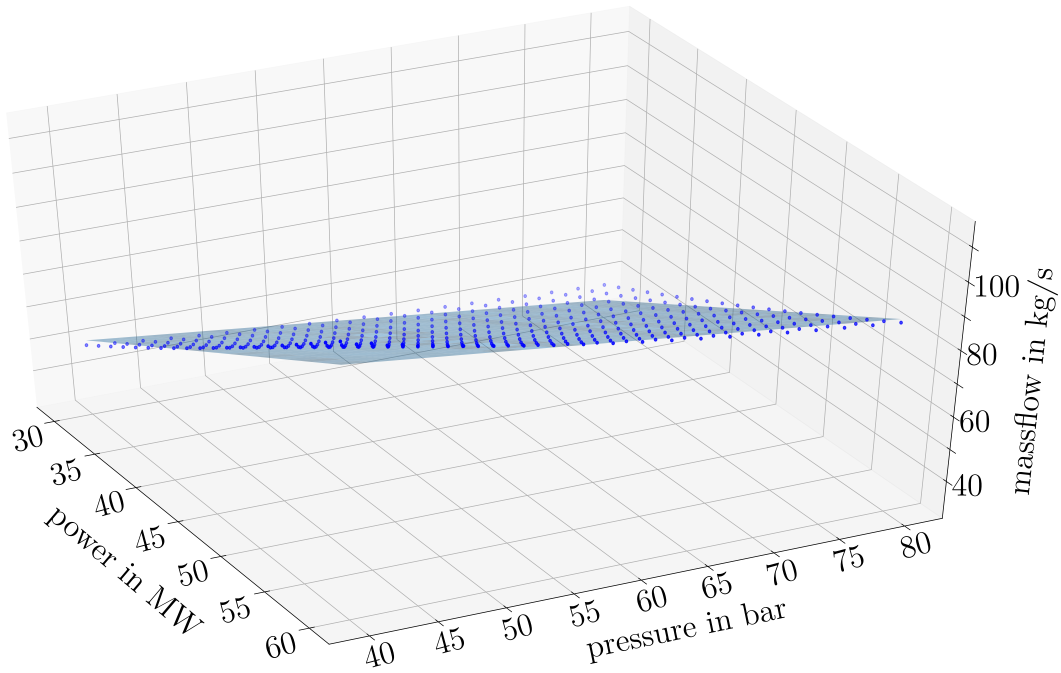

Figure 3.1: Example - Linear fit of the compression train

Table 3.1: Default values as an attribute of an instance

Ambient		**Value**	**Unit**
Pressure	p_0	1.013	bar
Temperature	T_0	288.15	K
Compression train			
Nominal compressor efficiency	η_{cmp}^{is}	85	%
Nominal motor efficiency	η_{mo}^{N}	97	%
Nominal mechanical efficiency	η_{mech}^{N}	99	%
Quality factor	α_{cmp}	0.8	-
Pressure ratio 1	Π_{cmp}^{1}	6	-
Pressure ratio 2	Π_{cmp}^{2}	11.8 - 21.38	-
Temperature	$T_{1c'}$	338.15	K
Temperature	$T_{2c'}$	338.15	K
Expansion train			
Nominal turbine efficiency	η_{tur}^{N}	90	%
Nominal generator efficiency	η_{gen}^{N}	98	%
Nominal mechanical efficiency	η_{mech}^{N}	99	%
Nominal combustion efficiency	η_{comb}^{N}	98	%
Nominal heat transfer efficiency	η_{ht}^{N}	95	%
Quality factor	α_{tur}	0.7	-
Pressure ratio recuperation	Π_{rec}	1.1	-
Pressure ratio 1	Π_{exp}^{1}	11.63	-
Pressure ratio 2	Π_{exp}^{2}	6.94 - 10.03	-
Temperature	$T_{2e'}$	338.15	K
Temperature	T_{2e}	763.15	K
Temperature	T_{1e}	1218.15	K
Temperature behind recuperator	T_{out}	372.15	K
Cavern			
Pressure range	Π_{cav}	40 - 80	bar
Pressure loss	Π_{cav}^{loss}	0.1	%/h
Depth	z	600	m
Optimization			
Power range for part load		$P^N - 0.5 \cdot P^N$	MW

Bibliography

[1] HE, Qing ; LI, Guoqing ; LU, Chang ; DU, Dongmei ; LIU, Wenyi: A compressed air energy storage system with variable pressure ratio and its operation control. In: *Energy* (2018)

[2] LEMMON, Eric W. ; JACOBSEN, Richard T. ; PENONCELLO, Steven G. ; FRIEND, Daniel G.: Thermodynamic Properties of Air and Mixtures of Nitrogen, Argon and Oxygen From 60 to 2000 K at Pressures to 2000 MPa. (2000)

[3] RIST, Dieter: *Dynamik realer Gase*. Springer-Verlag Berlin Heidelberg, 1996. – ISBN 3–540–58638–5

[4] VOLZ, Dipel.-Ing. G.: Ratgeber; Elektrische Motoren in Industrie und Gewerbe: Energieeffizienz und Ökodesign-Richtlinie.