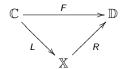
Exercise Sheet 3

MAGIC009 - Category Theory

October 25th, 2024

- 1. Find an example of a functor that is both full and faithful, one that is full but not faithful, one that is faithful but not full, and one that is neither.
- 2. Find an example of a functor $F: \mathbb{C} \to \mathbb{D}$ such that F is faithful but there exist distinct maps f_1 and f_2 in \mathbb{C} such that $F(f_1) = F(f_2)$.
- 3. Let $F: \mathbb{C} \to \mathbb{D}$ be an equivalence of categories. Show that if \mathbb{C} has a terminal object, then so does \mathbb{D} .
- 4. Prove that every functor $F \colon \mathbb{C} \to \mathbb{D}$ admits a factorisation of the form



where L is essentially surjective and R is full and faithful.

Let $\mathbb C$ be a category. A *subcategory* $\mathbb D$ of $\mathbb C$ consists of a subclass $Ob(\mathbb D)\subseteq Ob(\mathbb C)$ and a family of subclasses $\mathbb D(X,Y)\subseteq \mathbb C(X,Y)$, for $X,Y\in Ob(\mathbb D)$, which contains the identity maps and is closed under composition, so that $\mathbb D$ is itself a category.

- 5. Give an example of a subcategory of the category **Grp** of groups.
- 6. Let P be a partially ordered set and consider the associated category \underline{P} . What are the subcategories of \underline{P} ?
- 7. Let $\mathbb C$ be a category and $\mathbb D$ a subcategory of $\mathbb C$.
 - (a) Show that there is an inclusion functor $F: \mathbb{D} \to \mathbb{C}$.
 - (b) Is $F: \mathbb{D} \to \mathbb{C}$ always full?
 - (c) Is $F: \mathbb{D} \to \mathbb{C}$ always faithful?
- 8. Let $F: \mathbb{C} \to \mathbb{D}$ be a full and faithful functor. Prove that is \mathbb{C} is equivalent to a subcategory of \mathbb{D} .