Week 07 review worksheet — exercises for §7

E7.1 **(warm-up)** Is $S^2 = \mathrm{id}$ on $U(\mathfrak{sl}_2)$? Is $S^2 = \mathrm{id}$ on all Hopf algebras?

E7.2 **(exponentiate a primitive element)** (a) Let H be a Hopf algebra and let $x \in H$ be primitive. Suppose that we can formally write elements $e^{\hbar x} = \sum_{n=0}^{\infty} \hbar^n \frac{x^n}{n!}$ (where $\hbar$ is a formal parameter) and that we can manipulate infinite series according to the same rules as finite linear combinations.

Show that $e^{\hbar x}$ is grouplike. (In other words, show that, exponentiating a primitive element, we obtain a grouplike element.)

(b) Discuss $\hbar$-adic Hopf algebras where formal exponentials described in (a) actually make sense.