

MODELLI PER LA MECCANICA DEL VOLO

→ MODELLI DEL VELIVOLO

→ MODELLO DELL' AMBIENTE

- TERRA
- ATMOSFERA

MODELLI DEL VELIVOLO

① PUNTO MATERIALE "ORIENTATO"

3 g.d.L. (gradi di libertà)

libertà



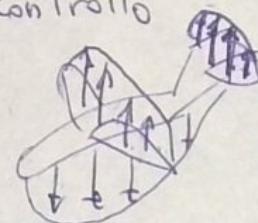
PUNTO
+
SISTEMA
DI RIF.

- Forze aerodinamiche dipendono dall'orientazione del velivolo stesse
- Perdono significato le distribuzioni di forze e masse.

② CORPO RIGIDO

6 g.d.L. { LIN(3)
ROT(3)

- equilibrio
- stabilità
- controllo



- PORTANTZA ALI
- PORTANTZA CODA

3 g.d.L.

③

FLESSIBILE

• AERODELASTICITÀ

2 LINEAR

• VERTICALE

• CONGUD.

1 ROTAZIONALE

• PECHEGGIO

1 SI TRASCURA

• PESO, IL LATERODINAMICO

MODELLO DELLA TERRA

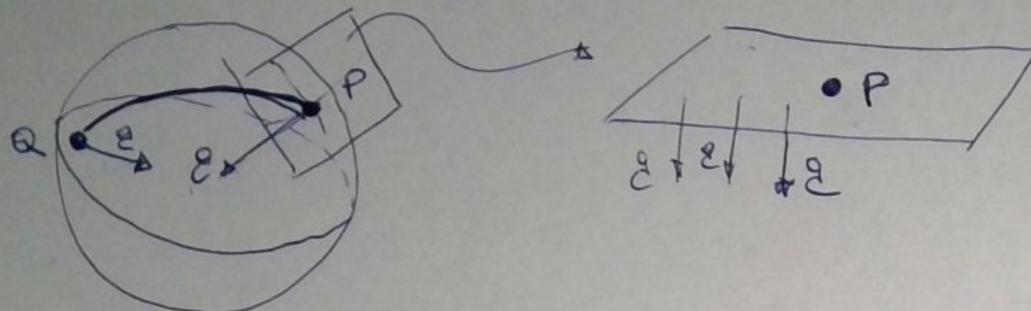
→ PIATTA

→ NON ROTANTE

① CAMPO GRAVITAZIONALE COSTANTE E UNIFORME

② TRAIETTORIE AD QUOTA COSTANTE SONO RETTILINEE

③ TERRA È UN RIFERIMENTO INERZIALE



MODELLO ATMOSFERA

θ TEMPERATURA

p PRESSIONE

g DENSITÀ

IN FUNZIONE
DELLA QUOTA
 h

(POTESI)

- GAS PERFETTO
- ARIA È IN QUIETE
- PROFILI DI TEMPERATURA IN FUNZIONE DELLA QUOTA.

SI VEDA PRESENTAZIONE
PER DETTAGLI.

+ CONDIZIONI DI RIFERIMENTO ($h=0$)

- $\theta_0 = 288.15 \text{ K}$
- $P_0 = 101325 \text{ Pa}$
- $\rho_0 = 1.225 \text{ kg/m}^3$



POLITECNICO
MILANO 1863



DIPARTIMENTO DI
SCIENZE E TECNOLOGIE
AEROSPAZIALI



POLI-Wind

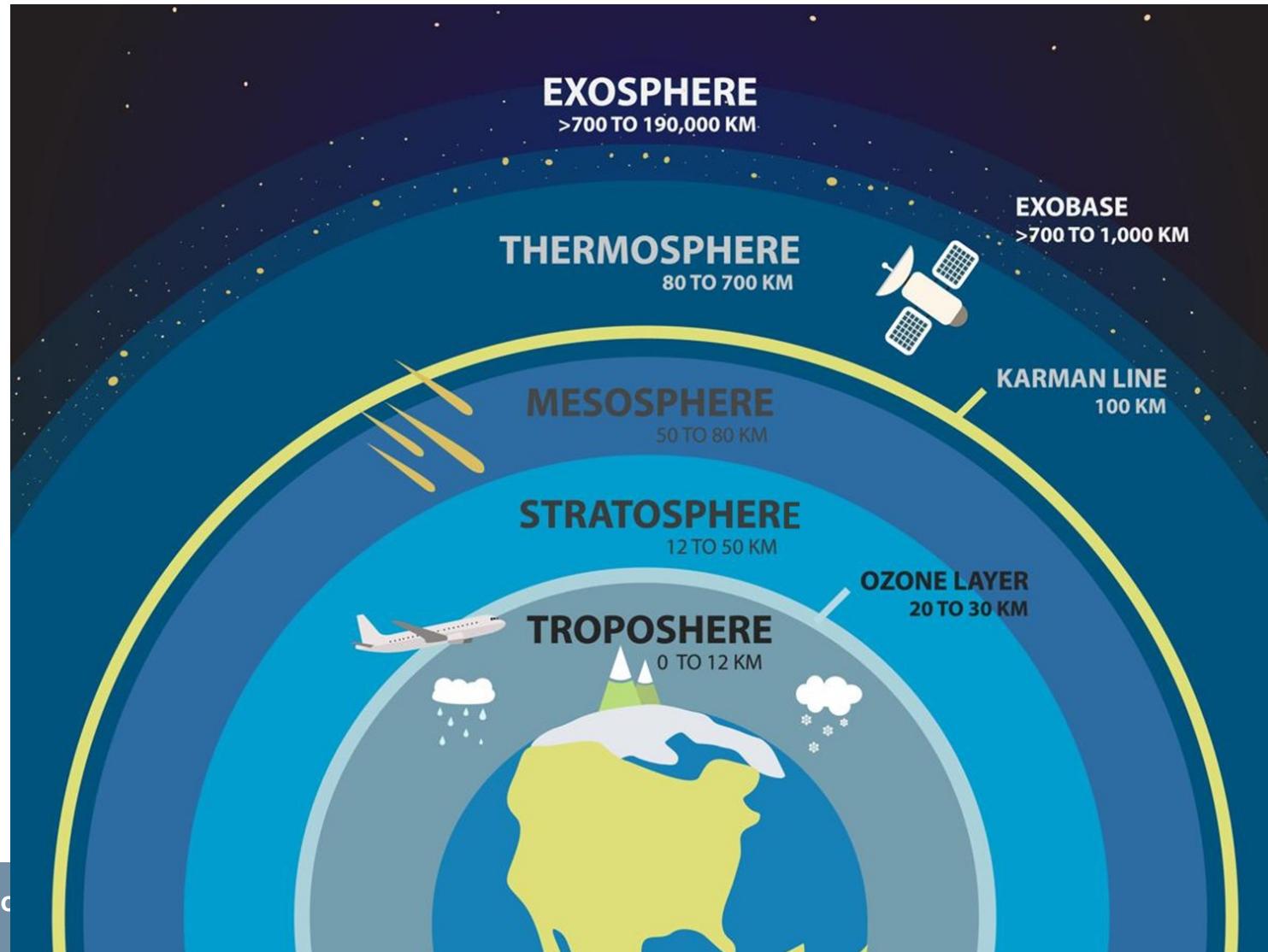


FMS
Flight Mechanics
& Flight Systems
Laboratory

International Standard Atmosphere

Fondamenti di Meccanica del Volo Atmosferico

Earth's Atmosphere



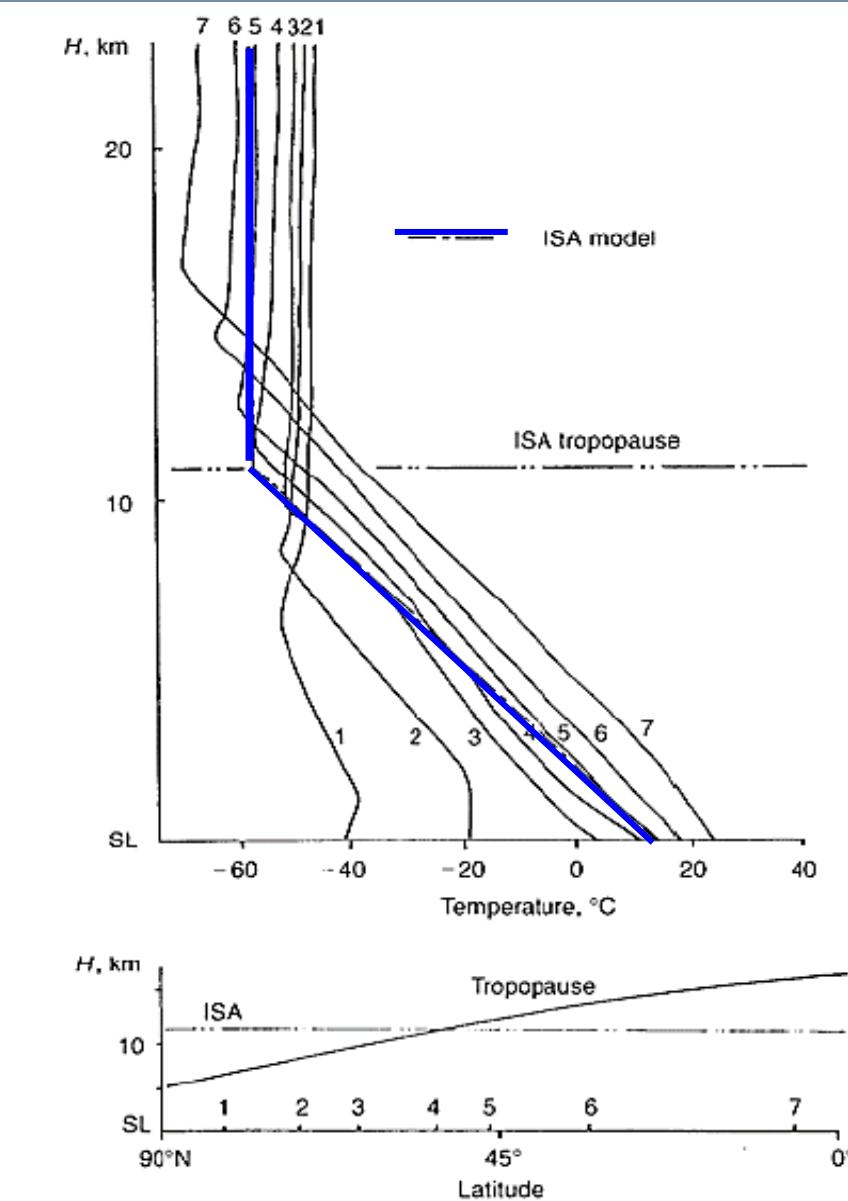
Troposphere

- From the **ground up to** an altitude between **8 km** at the poles to **17 km** at the equator
- It is characterized by a transfer of energy from the surface of the Earth
- Contains about **80% of the mass** of the atmosphere

Stratosphere

- From the top of the troposphere up to **50÷55 km**
- It is composed of two vertical sub-layers:
 - the **bottom layer** features a **constant temperature profile**
 - the temperature of the **top layer** is **increasing with altitude**
- The top layer of the (real) stratosphere is rich in **ozone**

Definition of the ISA



The International Standard Atmosphere

(ISA) is a **mathematical model** for the lower atmosphere, defined in 1964 by the International Civil Aviation Organization (ICAO).

Hypotheses:

- 1) Ideal gas
- 2) Still (static equilibrium) & dry
- 3) Constant chemical composition
- 4) Piecewise linear behavior of temperature with altitude
→ from measurements

The ISA model defines the vertical behavior of **temperature, pressure, density, viscosity, speed of sound** with **altitude**.

Derivation of the Model from Hypotheses (I)

- **Equilibrium** in the vertical direction

$$p \, dA - (p + dp) \, dA - \rho g \, dA \, dh = 0$$

By simplification we get the **hydrostatic equation**

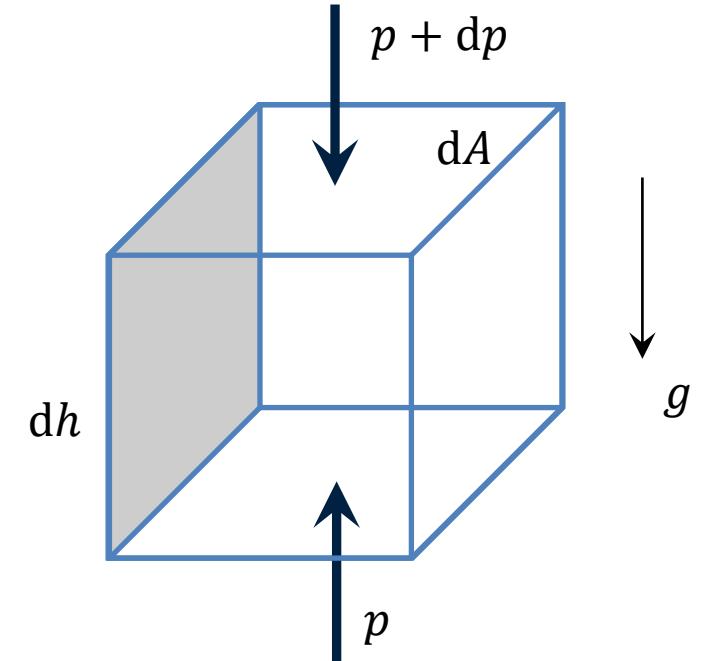
$$dp = -\rho g \, dh$$

- For an **ideal** gas

$$\rho = \frac{p}{R\vartheta}$$

Upon substitution in the equation of equilibrium we get

$$\frac{dp}{p} = -\frac{g}{R\vartheta} \, dh$$



Derivation of the Model from Hypotheses (II)

- By **integration** between an initial and a generic **altitude**,

$$\int_{p_{\text{in}}}^p \frac{dp}{p} = - \int_{h_{\text{in}}}^h \frac{g}{R\vartheta} dh \quad \xrightarrow{\text{yields}} \quad \ln \frac{p}{p_{\text{in}}} = - \frac{g}{R} \int_{h_{\text{in}}}^h \frac{dh}{\vartheta}$$

- We define the **temperature gradient** $\lambda \equiv d\vartheta/dh$ so that if

$$\lambda \equiv \text{const.} \quad \xrightarrow{\text{yields}} \quad \vartheta = \vartheta_0 + \lambda(h - h_0)$$

- From practical measurements, the ISA model **defines**

$$\lambda_{\text{ISA}} = \begin{cases} -0.0065 \text{ K/m}, & \text{in the troposphere} \\ 0 \text{ K/m}, & \text{in the stratosphere} \end{cases}$$

Derivation of the Model from Hypotheses (III)

Troposphere, constant $\lambda = -6.5 \frac{K}{km}$ ($d\vartheta = \lambda dh$):

- Integrating the pressure profile in the troposphere – temperature linear with altitude between the ground ‘0’ an the top of this layer - yields

$$\ln \frac{p}{p_0} = -\frac{g}{R\lambda} \int_{\vartheta_0}^{\vartheta} \frac{d\vartheta}{\vartheta} \quad \Rightarrow \quad \ln \frac{p}{p_0} = -\frac{g}{R\lambda} \ln \frac{\vartheta}{\vartheta_0} \quad \Rightarrow \quad \frac{p}{p_0} = \left(\frac{\vartheta}{\vartheta_0} \right)^{-\frac{g}{R\lambda}}$$

Stratosphere, constant $\lambda = 0$

- Integrating in the stratosphere – constant temperature between the base of the layer ‘ s ’ and the top altitude where the model is applicable - yields

$$\ln \frac{p}{p_s} = -\frac{g}{R} \int_{h_s}^h \frac{dh}{\vartheta} \quad \Rightarrow \quad \ln \frac{p}{p_s} = -\frac{g}{R\vartheta_s} (h - h_s) \quad \Rightarrow \quad \frac{p}{p_s} = \exp \left(-\frac{g}{R\vartheta_s} (h - h_s) \right)$$

Derivation of the Model from Hypotheses (IV)

Density is obtained from temperature and pressure for an ideal gas

$$\frac{\rho}{\rho_0} = \left(\frac{\vartheta}{\vartheta_0} \right)^{-\left(1 + \frac{g}{R\lambda}\right)}$$

troposphere

$$\frac{\rho}{\rho_s} = \exp\left(-\frac{g}{R\vartheta_s}(h - h_s)\right)$$

stratosphere

Reference Values

Pressure at the **bottom** of the troposphere and stratosphere

Unit	Troposphere	Stratosphere
Pa	101325	22632
atm	1	0.22
mb	1013	226
mm _{Hg}	760	169.75
in _{Hg}	29.92	6.68

Temperature at the **bottom** of the stratosphere – 11'000 m

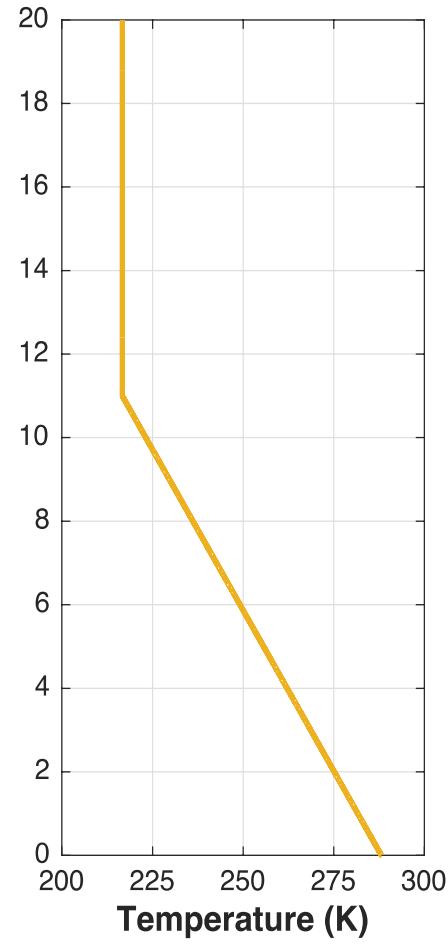
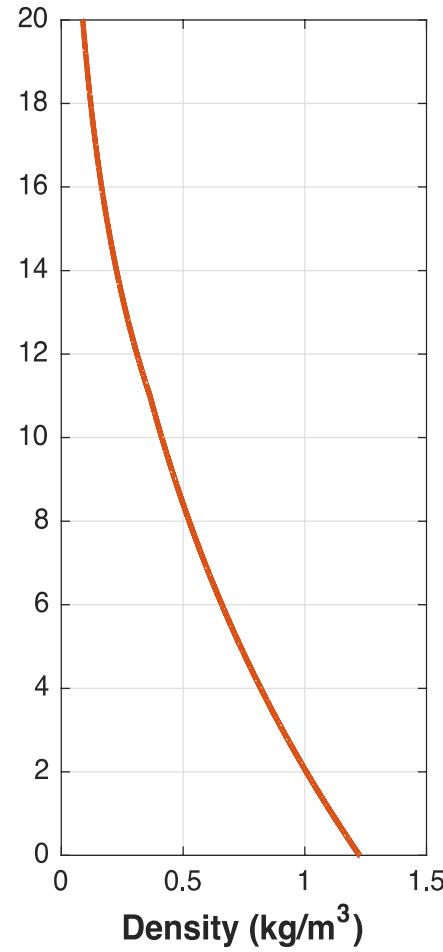
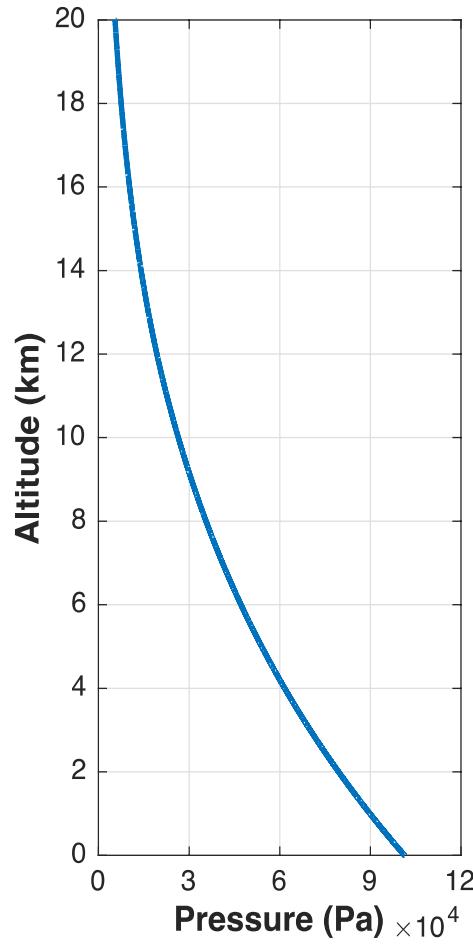
Unit	Troposphere	Stratosphere
K	288.15	216.65

From **measurements**

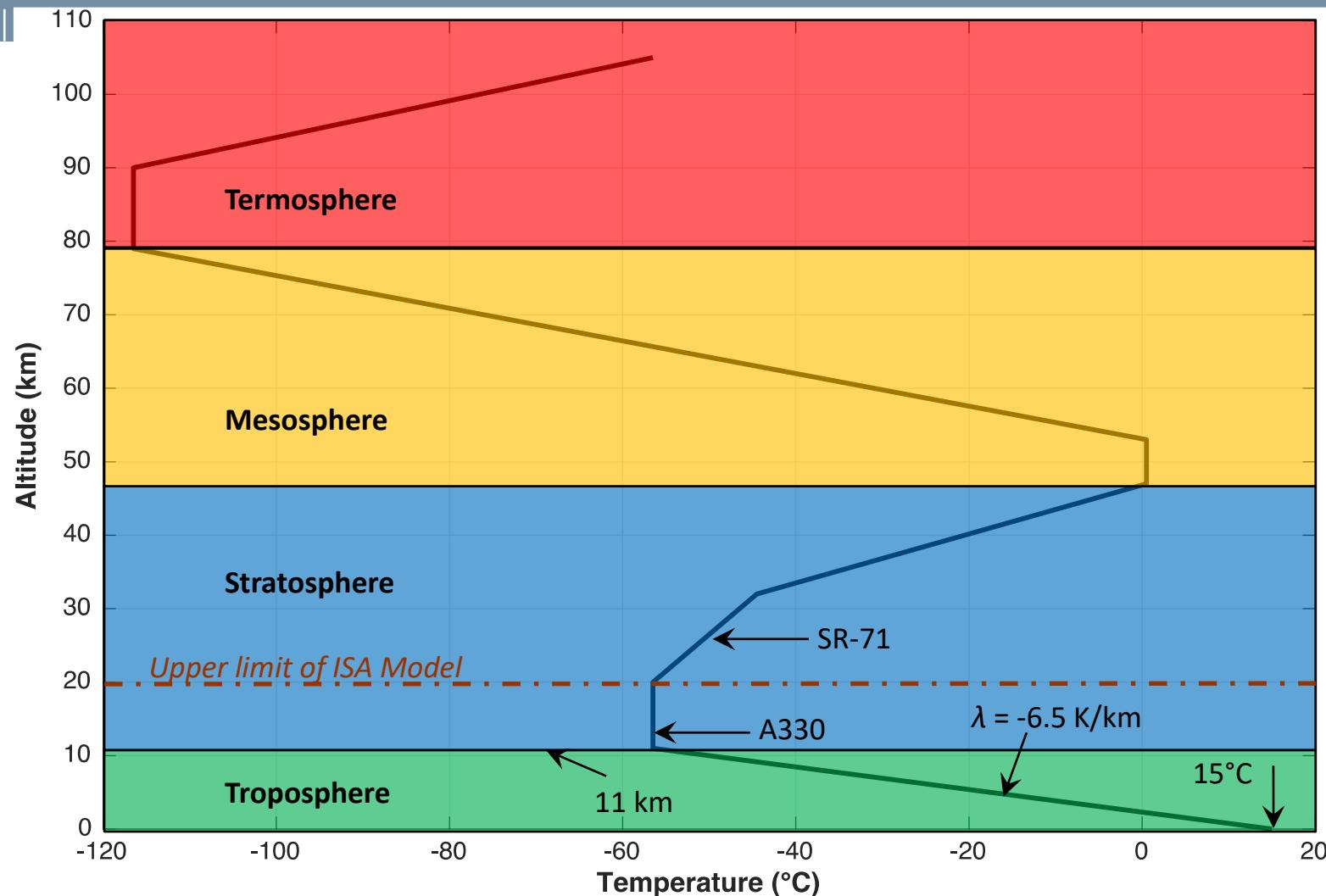


To ensure continuity in the **model**

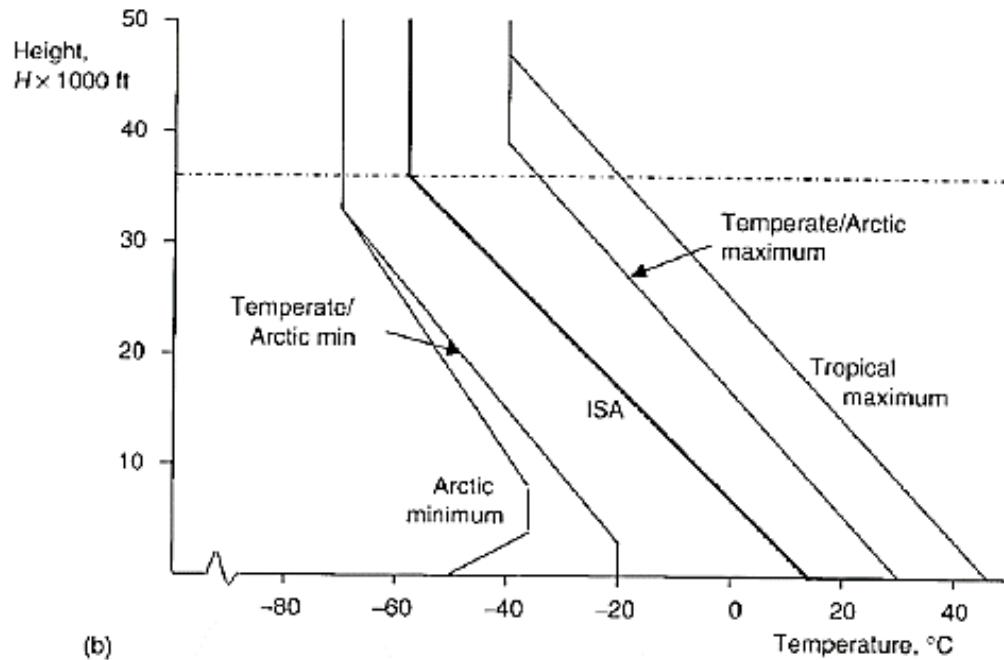
ISA Pressure, Density and Temperature



Vertical layers of the Atmosphere



Dealing with Non-Standard Atmosphere



Aircraft performance is expressed in standard atmosphere conditions

Any non-standard condition is represented as a shift from the reference temperature

$$\vartheta = \vartheta_{\text{ISA}} + \Delta\vartheta$$

Writing $\vartheta_{\text{ISA}} - 10^\circ\text{C}$ means to have a temperature that is 10°C less than the one expected from the ISA at that altitude.



POLITECNICO
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AEROSPAZIALI

AER



POLI-Wind



Flight Mechanics
& Flight Systems
Laboratory

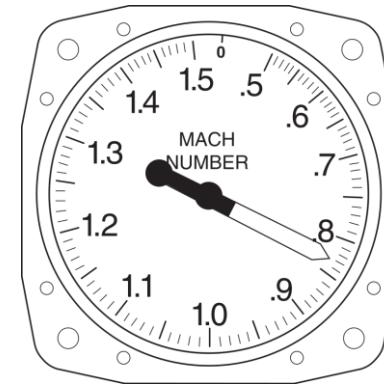
Flight instruments

Pressure Based Instruments

Much instrumentation on an airplane is based on a measurement of air pressure.

Some pressure-based flight instruments:

- **Altimeter** → altitude
- **Airspeed indicator** → flight speed
- **Machmeter** → Mach number
- **Vertical speed indicator** → vertical speed



The barometric altimeter is the basic instrument for measuring altitude.

It is based on the principle of the barometer, and therefore it needs a measurement of the static pressure.

Given a reference pressure, the altimeter provides an estimate of the pressure altitude, according to the expression $h_{PA} = h_{ISA}(p; p_{ref})$.

$$h_{PA}(p; p_{ref}) = \frac{\vartheta_0}{\lambda} \left(\left(\frac{p}{p_{ref}} \right)^{-\frac{R\lambda}{g}} - 1 \right) \quad h_{PA}(p; p_{ref}) = h_s - \frac{R\vartheta_s}{g} \ln \frac{p}{p_{ref}}$$

Troposphere

Stratosphere

Regulation of the Altimeter

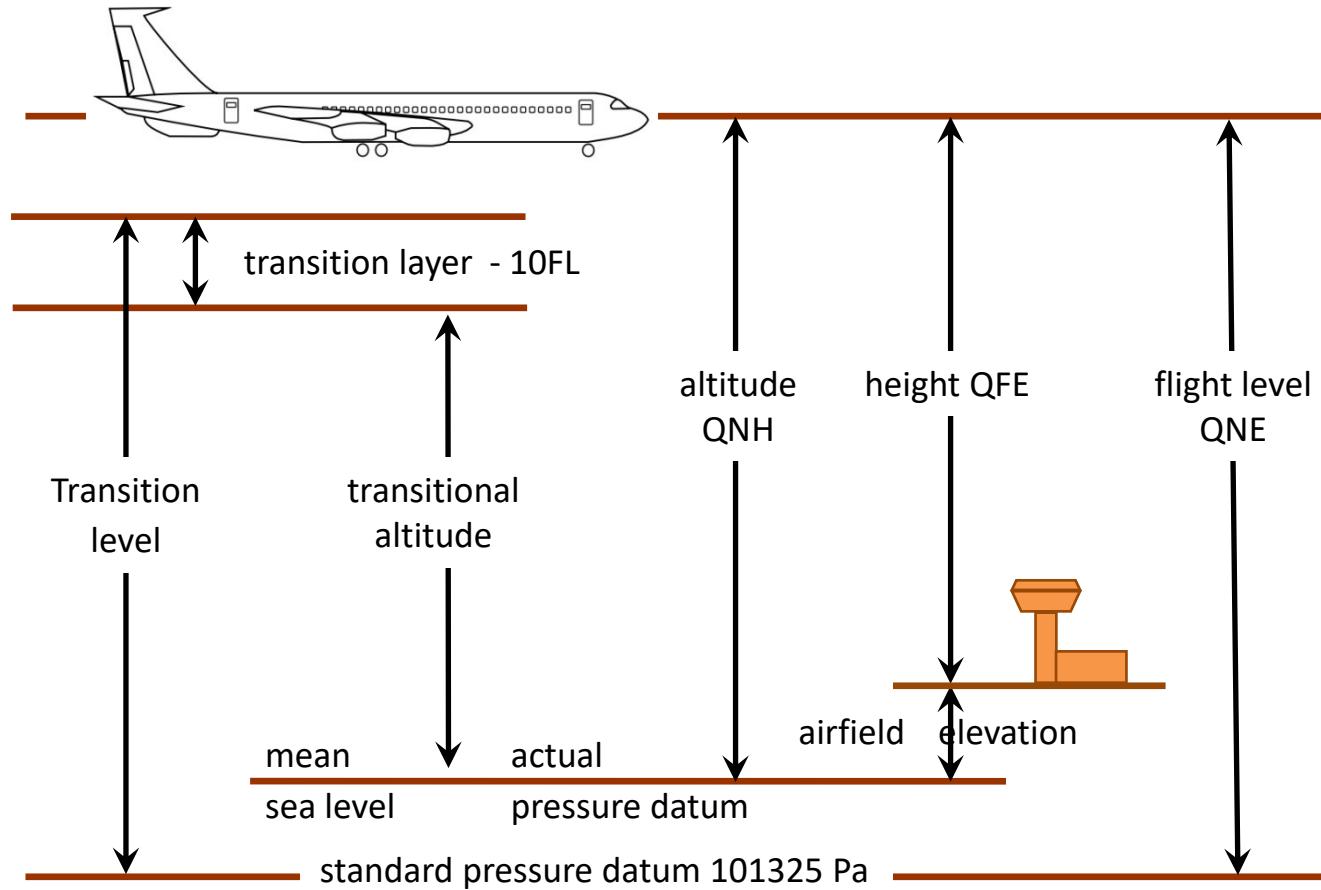
The choice of the reference pressure p_{ref} provides different estimates of the pressure altitude, each having its field of application.

QNE – The reference pressure is the ISA pressure at sea level. This value is also known as *standard pressure datum*.

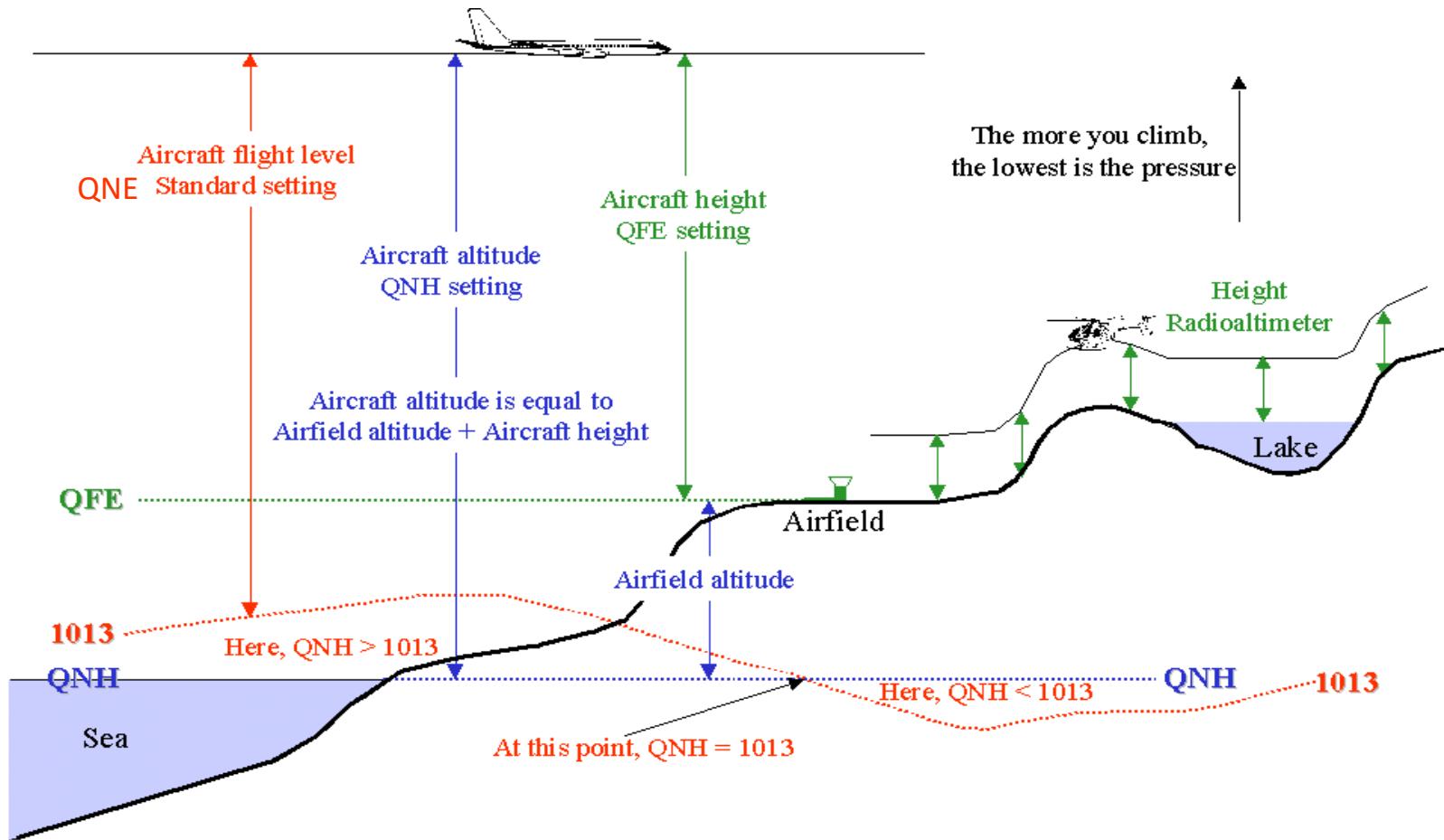
QFE – The pressure measured on the aerodrome is taken as a reference.

QNH – The pressure measured on the aerodrome is propagated to the sea level by means of the equations of the ISA. This pressure is then taken as reference.

Regulations of the Altimeter (I)



Regulations of the Altimeter (II)



The QNE Regulation

The **QNE** regulation, also known as *standard*, is used in the **nonterminal flight phases**, i.e. during: cruise, ascend, descend,

Purpose of this regulation is *not* to get an accurate estimate of the altitude, but to **ensure** a certain **vertical separation** between the airplanes.

The altitudes indicated in the QNE regulation are called **Flight Levels**, and measured in hundreds of feet.

The aeronautical regulations require that the minimum separation between two airplanes, operating in the same area, is 10 FL i.e. about 300 m.

The QNH Regulation

The **QNH** regulation is used in the **terminal flight phases** (approach, landing, take-off), i.e. when the airplane operates close to the aerodrome.

Purpose of this regulation is to get an **accurate estimate** of the altitude, so that the airplane will have a certain **separation from the ground** and **obstacles**.

The pressure altitude in the QNH regulation is simply called *altitude*, and measured in **feet**. When the airplane is at the aerodrome the pressure altitude coincides with the geometric altitude.

The QFE Regulation

As the QNH, the **QFE** regulation is used in the **terminal flight phases**, although is falling in disuse (the European and North America regulation requires the QNH).

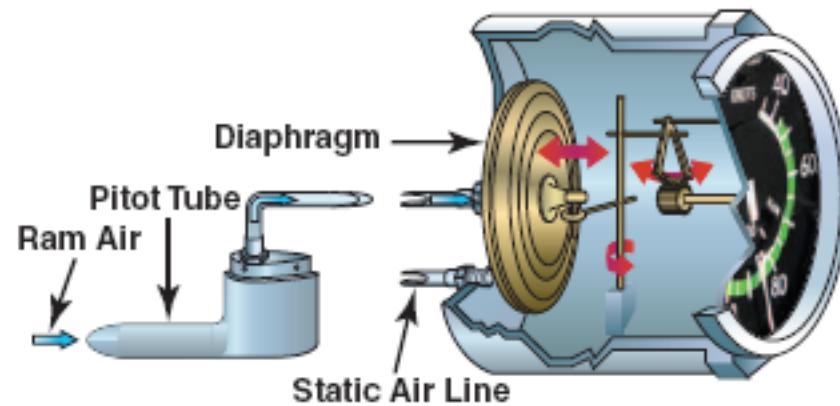
Purpose of this regulation is to get an accurate estimate of the altitude, so that the airplane will have a certain **separation from the ground and the obstacles**.

The pressure altitude in the QFE regulation is simply called *height*, and measured in feet. When the airplane is in the aerodrome the pressure altitude is equal to 0.

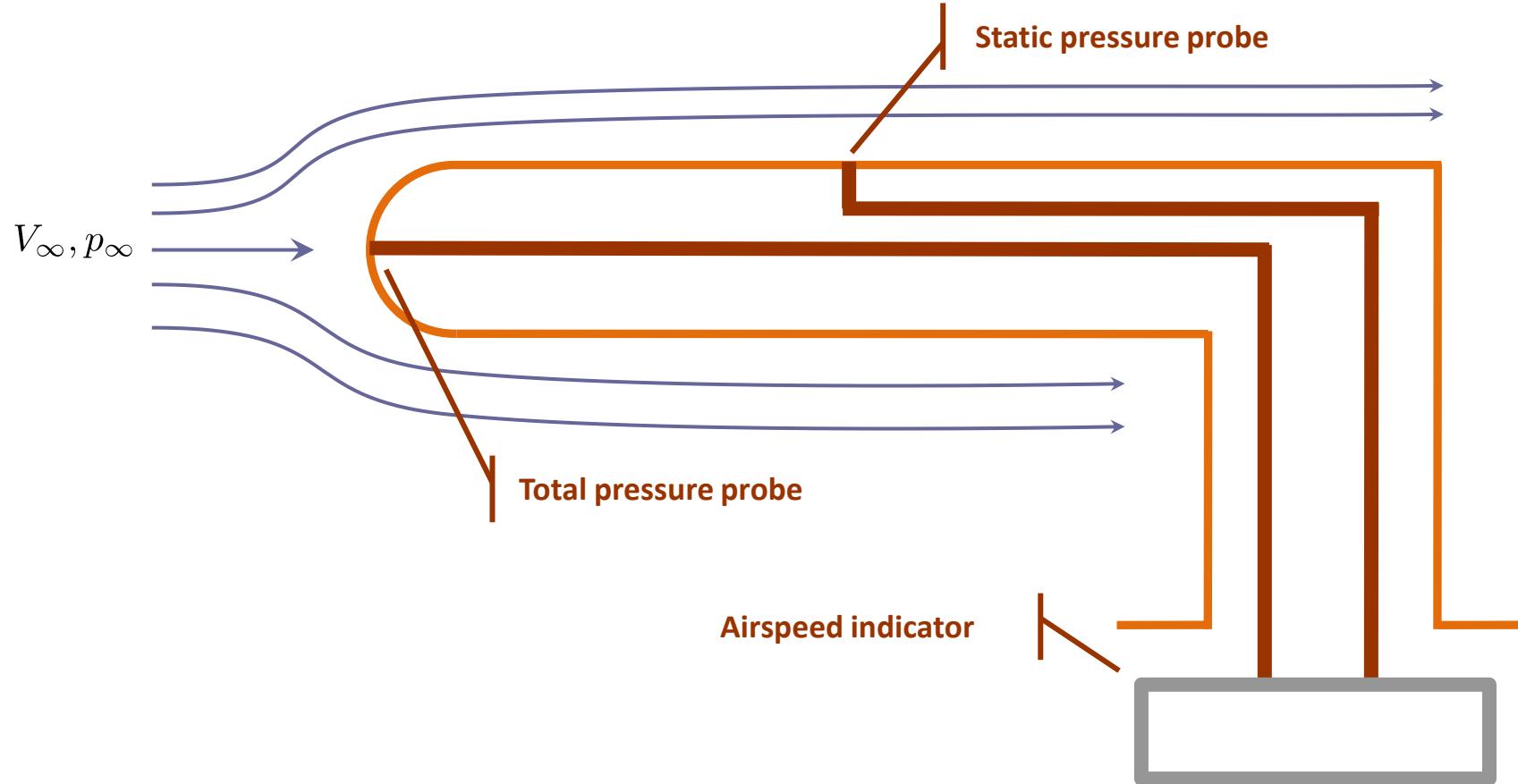
Airspeed Indicator

This tool is based on the principle of the differential barometer, i.e. it measures the difference between two pressures.

The two pressures are provided by the **Pitot tube**. A mathematical expression relates the flight speed to the differential (i.e. impact) pressure.



The Pitot Tube

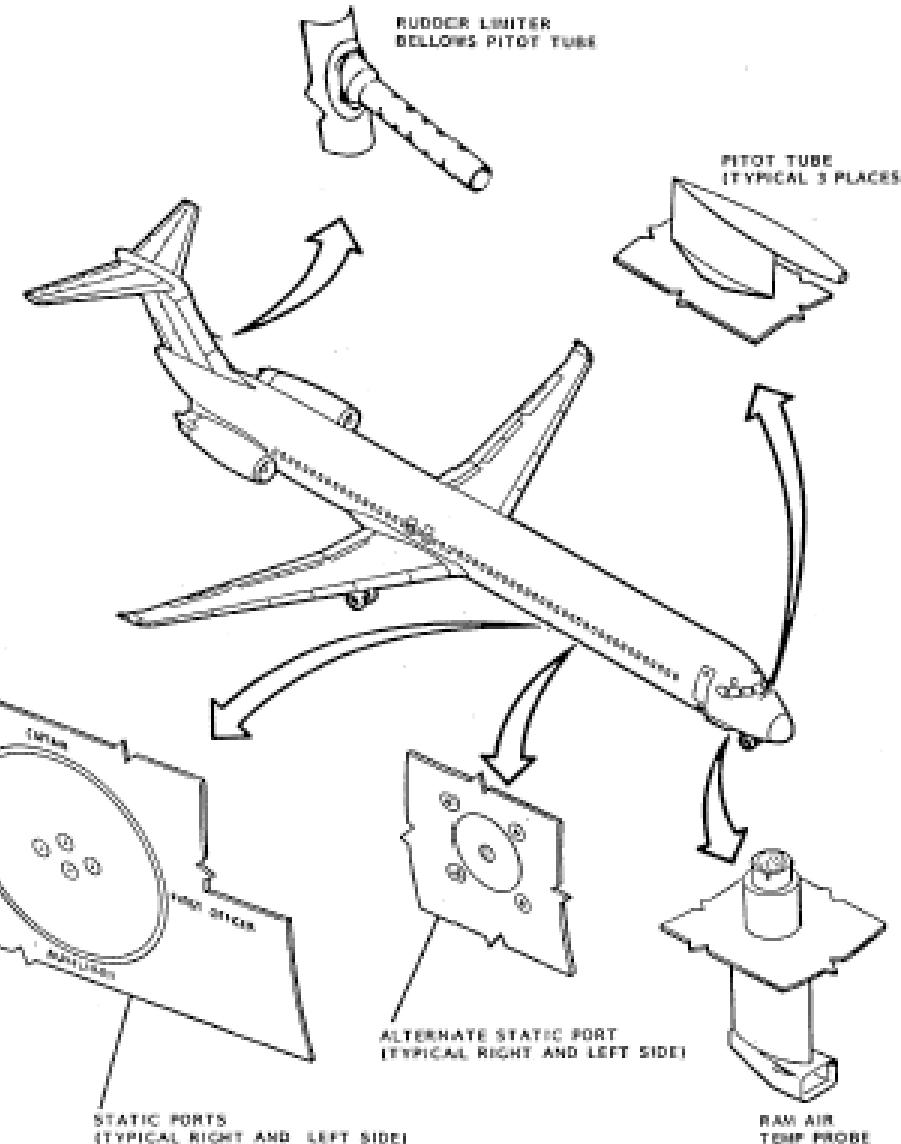




Static pressure probes on A330



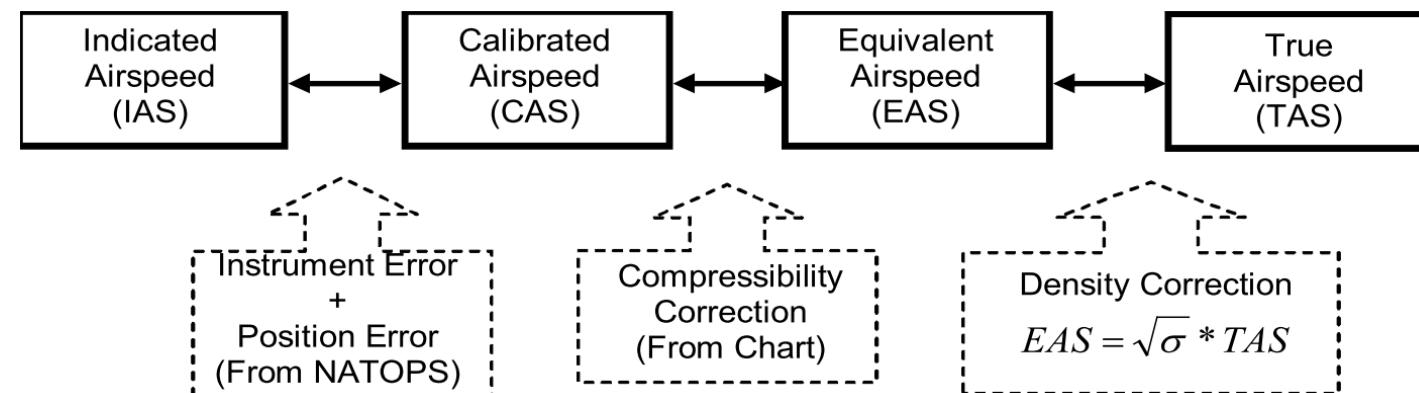
Pitot Probe on Bombardier Global 600



Pitot Probes on the DC9/80

Definitions of Flight Speed

Name	Description
Indicated airspeed IAS	Speed measured by the airspeed indicator, affected by the measurement errors of the Pitot tubes.
Calibrated airspeed CAS	Speed at which the aircraft should fly at sea level (ISA) to experience the same impact pressure (assuming a compressible flow). It is obtained from the IAS by correcting the measurement (position) error.
Equivalent airspeed EAS	Speed at which the aircraft should fly at sea level altitude (ISA) to experience the same dynamic pressure (assuming an incompressible flow). It is obtained by correcting the CAS with the air density.
True airspeed TAS	Speed of the aircraft with respect to the air. It is obtained from the EAS by adding the information on the true density.
Ground Speed GS	Speed with respect to the ground. It is obtained by vectorially summing the airplane and the wind velocities.



Equation of the Pitot tube in the incompressible regime

Let us write the **Bernoulli** theorem, for **incompressible** flows, between the undisturbed flow, denoted with $(\bullet)_\infty$, and the total pressure probe, denoted with $(\bullet)_{\text{ant}}$

$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p_{\text{ant}} + \frac{1}{2}\rho V_{\text{ant}}^2$$

Diagram illustrating Bernoulli's principle:

- Static pressure: p_∞ (undisturbed)
- Dynamic pressure: $\frac{1}{2}\rho V^2$ (due to velocity)
- Total pressure: $p_\infty + \frac{1}{2}\rho V_\infty^2$ (sum of static and dynamic pressures)
- At the stagnation point (probe tip):
 - Static pressure: p_{ant}
 - Dynamic pressure: $\frac{1}{2}\rho V_{\text{ant}}^2$ (0 because it's a stagnation point)
 - Total pressure: $p_{\text{ant}} + \frac{1}{2}\rho V_{\text{ant}}^2$ (sum of static and dynamic pressures at stagnation)

The static (i.e. lateral) pressure probe is positioned so that we have $p_{\text{lat}} \simeq p_\infty$, and by introducing the impact pressure $q_I = p_{\text{ant}} - p_{\text{lat}}$ we can estimate the airspeed as

$$V_\infty(q_I; \rho) = \sqrt{\frac{2q_I}{\rho}}$$

Equation of the Pitot tube in the compressible regime

Complex derivation...

We report here only the final outcome

$$V = \sqrt{\frac{2\gamma}{\gamma-1} \frac{p}{\rho} \left(\left(\frac{q_c}{p} + 1 \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)}$$

Relationship between the EAS and TAS

The **equivalent airspeed** is the speed at which the airplane should fly at the sea level, with the ISA, to experience the dynamic pressure measured at the true flight altitude.

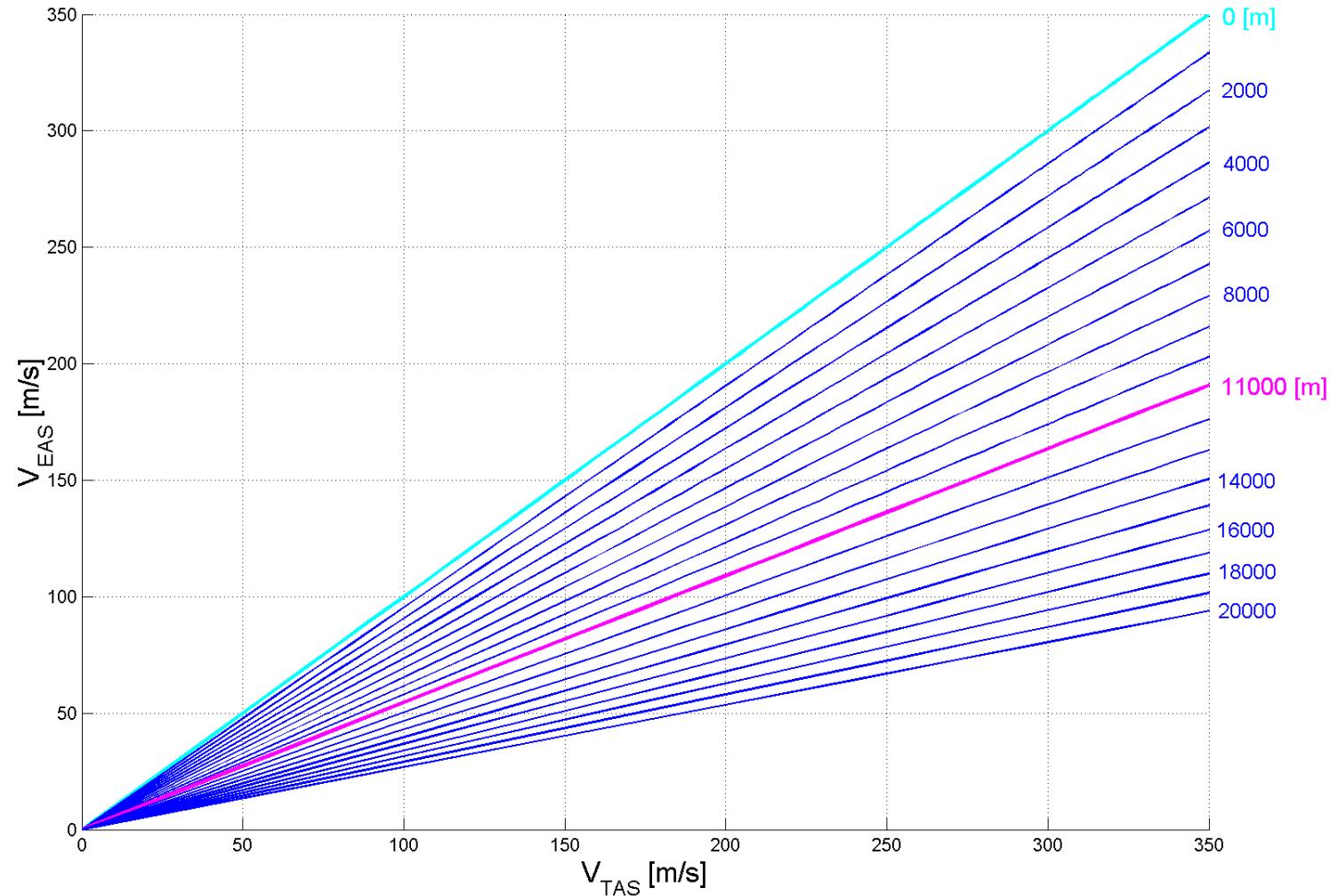
$$\frac{1}{2} \rho V^2 = \frac{1}{2} \rho_0 V_{EAS}^2$$

By solving for V_{EAS} we get

$$V_{EAS} = \sqrt{\frac{\rho}{\rho_0}} V$$

This process is known as altitude correction.

EAS – TAS Conversion Graph



Mach Number

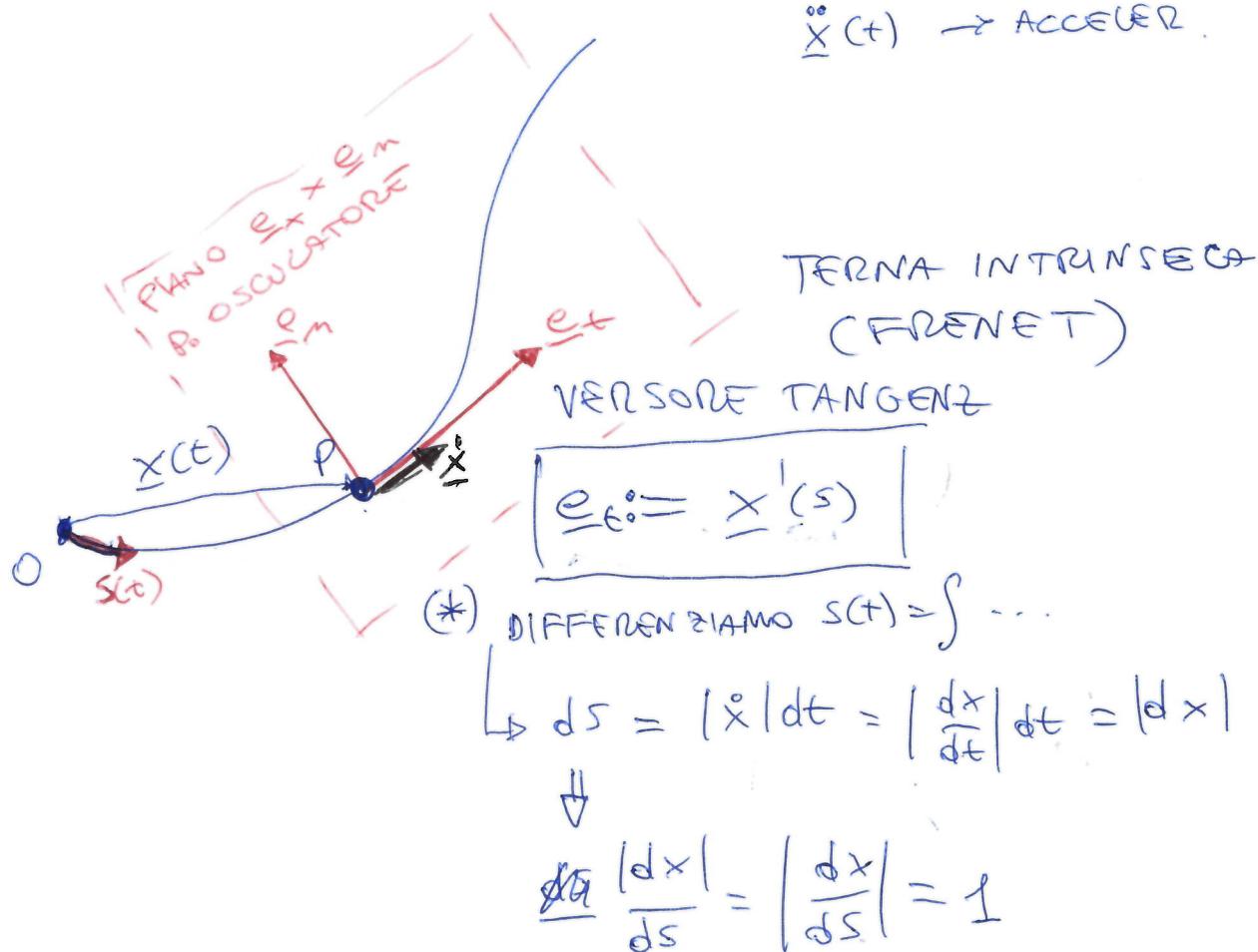
The Mach number is the ratio between the true air speed V and the speed of sound a

$$M = \frac{V}{a}$$

For an ideal gas $a = \sqrt{\gamma R \theta}$, and for the air $\gamma \simeq 1.4$.

RICHIAMI DI CINEMATICA

PUNTO MATERIALE \mathbf{P}



- $\underline{x}(t) \rightarrow \text{POSIZIONE}$
- $\dot{\underline{x}}(t) \rightarrow \text{VELOCITA'}$
- $\ddot{\underline{x}}(t) \rightarrow \text{ACCELER.}$

t : tempo (VAR. INDIP.)

s : coordinate curvilinee

$$s(\epsilon) = \int_{t_1}^{t_2} |\dot{x}(t)| dt$$

TANGENZIALE	e_t
NORMALE	e_n
BI-NORMALE	e_b

[REMARQUE: $\overset{\circ}{c} = \frac{d}{dt} c$

 $c' = \frac{d}{ds} c$

VERSORI NORMAUX

$$\underline{e}_m := \frac{\underline{x}''(s)}{\|\underline{x}''(s)\|}$$

DEF RAGGIO DI CURVATURA

$$R := \frac{1}{\|\underline{x}''(s)\|}$$

$$\begin{array}{l} \text{if } \\ \boxed{\underline{e}_m := R \underline{x}''(s)} \\ \text{(**)} \rightarrow \boxed{= R \underline{e}_t'} \end{array}$$

$$(**) \underline{x}''(s) = \frac{d}{ds} \underline{x}'(s) = \underline{e}_t'$$

VERSORI BINORMALI

$$\underline{e}_b = \underline{e}_t \times \underline{e}_m$$

VELOCITÀ E ACCELERAZIONE NELLA TERNA DI FRENÉT

$$\dot{\underline{x}} = \frac{d\underline{x}}{dt} = \frac{d\underline{x}}{ds} \frac{ds}{dt} \Rightarrow \boxed{\dot{\underline{x}} = \underline{x}' \dot{s}}$$

$$\Rightarrow \boxed{\dot{\underline{x}} = \dot{s} \underline{e}_t} \quad \begin{array}{l} \dot{s} \text{ modulo delle velocità} \\ \underline{e}_t \text{ direzione} \end{array}$$

$$\ddot{\underline{x}} = \frac{d}{dt}(\dot{\underline{x}}) = \ddot{\underline{x}}' + \dot{s} \frac{d}{dt} \underline{x}' = \ddot{\underline{x}}' + \dot{s} \frac{d}{ds} \underline{x}' \frac{ds}{dt} \Rightarrow \boxed{\ddot{\underline{x}} = \ddot{s} \underline{x}' + \dot{s}^2 \underline{x}''}$$

$$\ddot{\underline{x}} = \ddot{s} \underline{e}_t + \frac{\dot{s}^2}{R} \underline{e}_m$$

ACC. TANGENZIALE ACC. CENTRIPETA

VELOCITÀ ANG. $\omega = \frac{\dot{s}}{R}$

SISTEMI DI RIFERIMENTO

(S.d.R.)

SdR → ENTITA' GEOMETRICA composta da → ORIGINE
→ TERRA ORTOVORINATA

- EUNCIARE / DEFINIRE SdR PER LA MECCANICA DEL VOLO
- VARIABILI DI INTERESSE

① r^E

SOLIDALE ALLA TERRA
[EARTH - FIXED FR.
NAVIGATIONAL FR.]

- NOTE:
- A RIGORE NON INERZIALE
 - CON BUONA APPROSSIMAZIONE INERZIALE
PER IL VOLO ATMOSFERICO

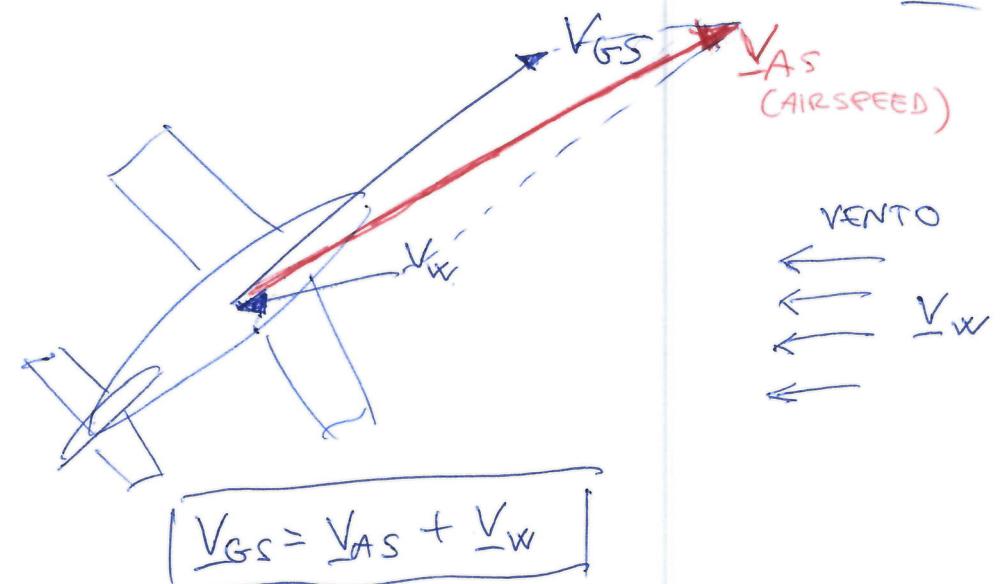
VARIABILI DI INTERESSE:

VELOCITA'

$$\underline{V}_{GS} = \dot{\underline{x}}(t)$$

GROUND SPEED

{ ORIGINE: PUNTO SULLA SUPERFICIE TERRESTRE
 $\underline{x}^E \underline{y}^E$: PIANO TANGENTE ALLA SUPERFICIE
 \underline{z}^E : ORTOGONALE A $\underline{x}^E \underline{y}^E$ (DI SOLITO DISCORDE) RISPETTO A \underline{g}



IPOTESI

$$\boxed{\underline{V}_W = 0}$$

ARIA CALMA



$$\underline{V}_{OS} = \underline{V}_{AS} = \dot{\underline{x}} = \underline{V} \Rightarrow \text{secondo i fermenti e to FRENTE}$$



$$\underline{V} = |\underline{V}| \underline{e}_t$$

$$\dot{\underline{V}} = |\dot{\underline{V}}| \underline{e}_t + \frac{|\underline{V}|^2}{R} \underline{e}_n$$

UNITÀ DI MISURA

SI → m/s

Km (NODO) = 1 MIGLIO NAUTICO (NM)

ALL'ORA $\Rightarrow 1 \text{ NM} = 1852 \text{ m}$

$$\boxed{1 \text{ Km} = 0.514 \text{ m/s}}$$

Km/h

mph → miles per hour

\hookrightarrow MIGLIO STATUARO $\approx 1609 \text{ m}$

②

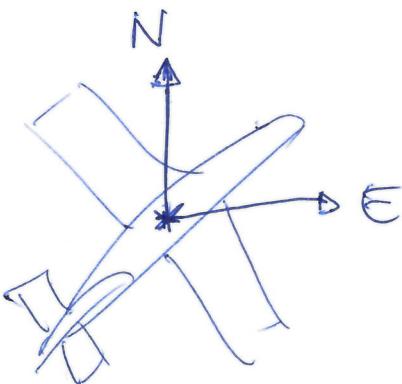
\vec{r}^N

ORIZZONTE LOCAL

LOCAL HORIZON FR.

NED (North - East - Down)

TERRA TERRESTRE MOBILE



ORIGINE: SU UN PUNTO DEL VEU VOLO

\underline{x}^N PUNTA VERSO NORD

\underline{y}^N PUNTA VERSO EST

\underline{z}^N PUNTA VERSO IL BASSO

(CONCERDE RISPESSO)

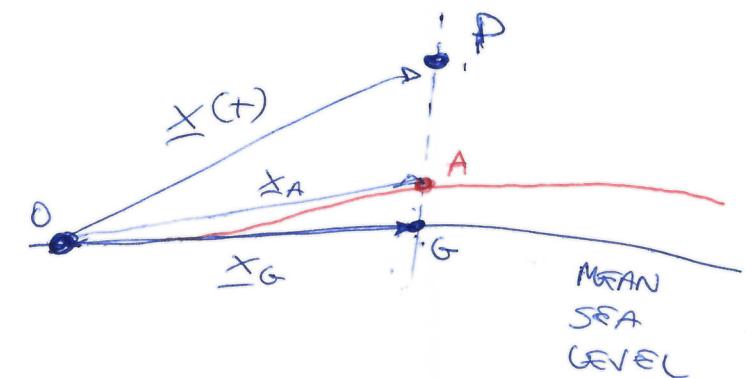
VOCABOLARIO INTERESSE

- QUOTA

$$h = \underline{z}^N \cdot (\underline{x}_G - \underline{x})$$

QUOTA VERA.

$$h_A = \underline{z}^N \cdot (\underline{x}_A - \underline{x})$$



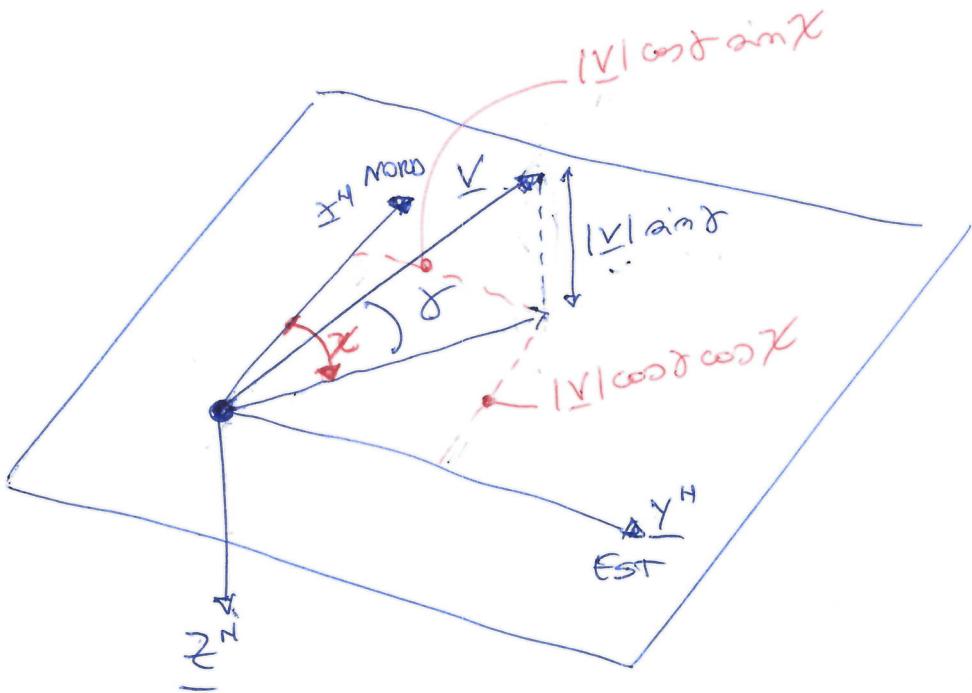
ANGOLI DI TRAIETTORIA

2 ANGOLI CHE DEFINISCONO COME SI MUOVE IL VELIVOLO

RISPESSO ALLA TERRA

- ANGOLO DI RAMPA γ
- ANGOLO DI ROTTA χ

ANGOLI DI RAMPA \rightarrow L'ANGOLI TRA IL VETTORE VELOCITA' E
 ANGLE OF CLIMB [FLIGHT PATH ANGLE] γ IL PIANO $\underline{x}^H \underline{y}^H$ (corz. velocità)
 POSITIVO SE DISCORDA CON GRAVITÀ
 $C > 0$ se il velivolo si muove verso l'alto)



$$|\underline{V}| \sin \gamma = -\underline{V} \cdot \underline{z}^H$$

$$\gamma = -\arcsin \left(\frac{\underline{V} \cdot \underline{z}^H}{\|\underline{V}\|} \right)$$

$$= -\arcsin (\underline{e}_t \cdot \underline{z}^H)$$

ANGOLI DI ROTTA \rightarrow ANGOLI TRA LA PROIEZIONE DI V SULL'ORIZ. (ORTO)
 TRACK ANGLE χ E IL NORD
 POSITIVO IN ACCORDO CON \underline{f}^H (VERSO ORARIO)

$$\chi = \arctan \left(\frac{\underline{V} \cdot \underline{y}^H}{\underline{V} \cdot \underline{x}^H} \right) = \arctan \left(\frac{\underline{e}_t \cdot \underline{y}^H}{\underline{e}_t \cdot \underline{x}^H} \right)$$

VELOCITÀ IN \mathbb{J}^H

$$V = |V| \cos \delta \cos \chi \underline{x}^H + |V| \cos \delta \sin \chi \underline{y}^H - |V| \sin \delta \underline{z}^H$$

VELOCITÀ ANGOCARE (IN \mathbb{J}^H)

CON LE NOTIZIONI $\delta \in X$

$$\omega = \frac{v}{R} = \dots = \sqrt{\dot{\delta}^2 + \dot{x}^2 \cos^2(\delta)}$$

(3)



ASSI CORPO

SISTEMA SOLIDALE COL CORPO

BODY FRAME

TIPICAMENTE \underline{z}^B È UN PIANO DI
SIMMETRIA MATERIALE E GEOMETRICA

{ ORIGINE: PUNTO MATERIALE
DEL VEICOLO (CG)

\underline{x}^B asse di rollio \rightarrow DIREZIONE VERSO PRUA

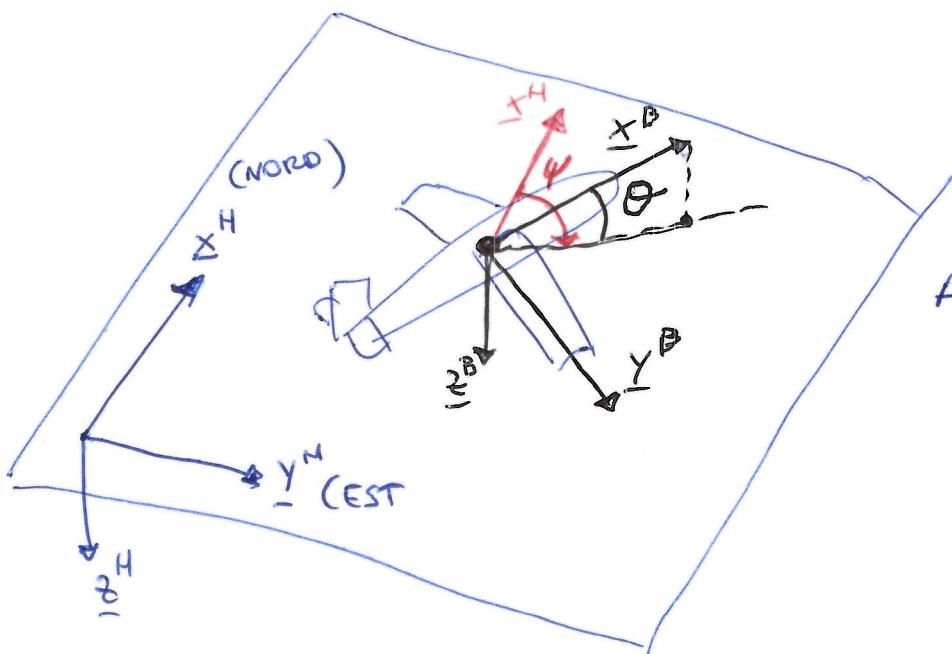
\underline{y}^B asse di beccheggio \rightarrow VERSO ALA DESTRA

\underline{z}^B asse di imbarolata \rightarrow VERSO IL VENTRE DEL VEICOLO

[ROLL / PITCH / YAW]

• ANGOLI DI ASSETTO

come è orientato il velivolo rispetto alla Terra.



A. DI BECCHEGGIO θ : Angolo tra l'asse \underline{x}^B (PITCH) e il piano $\underline{x}^H \underline{y}^H$

$$\boxed{\theta := -\arcsin(\underline{z}^B \cdot \underline{z}^H)}$$

A. DI PRUA: Angolo tra la proiezione di \underline{x}^B (HEADING ANGLE) ψ sul piano $\underline{x}^H \underline{y}^H$ e il NORD

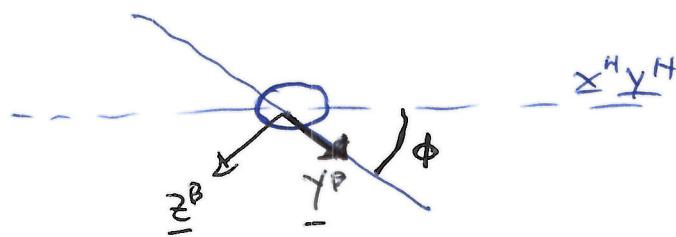
$$\boxed{\psi := \arctan\left(\frac{\underline{x}^B \cdot \underline{y}^H}{\underline{x}^B \cdot \underline{x}^H}\right)}$$

- ANGOLO DI ROLLO / INCUNAZIONE LATERALE
(ROLL ANGLE)
(BANK ANGLE)

$$\phi = \text{atan} \left(\frac{\underline{y}^B \cdot \underline{z}^H}{\underline{z}^B \cdot \underline{z}^H} \right) \quad (\text{da 3D ROTATION})$$

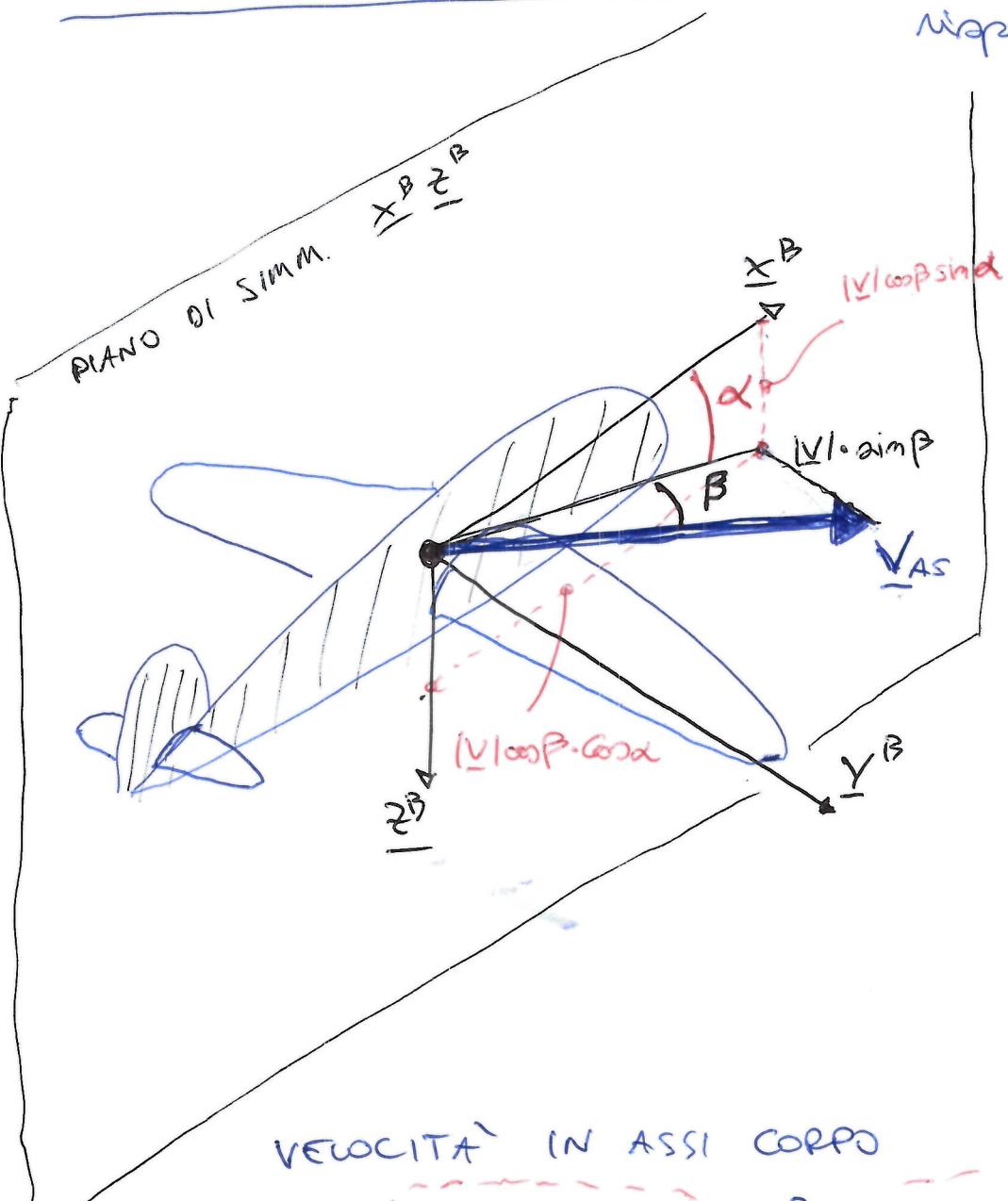
FORMULA SEMPLIFICATA con $\theta=0$ angolo tra \underline{y}^B e i PIANO $\underline{x}^H\underline{y}^H$

VISTA DALLA DIETRO



$$\phi = \text{atan}(\underline{y}^B \cdot \underline{z}^H) \quad (\text{solo se } \theta=0)$$

AN GOLI AERODINAMICI



VELOCITÀ IN ASSI CORPO

$$\underline{V} = (\underline{V} \cos \beta \cos \alpha) \underline{x}^B + (\underline{V} \sin \beta) \underline{y}^B + (\underline{V} \cos \beta \sin \alpha) \underline{z}^B$$

← $\underline{v} = m \underline{x}^B + n \underline{y}^B + w \underline{z}^B$

Come è orientato il velivolo
rispetto alla sua velocità (AIRSPEED)

- ANGOLI DI DERIVA: angolo tra \underline{V} e il piano di simmetria

$$\beta = \arcsin \left(\frac{\underline{V} \cdot \underline{y}^B}{|\underline{V}|} \right)$$

- ANGOLI DI INCIDENZA: angolo tra la proiezione di \underline{V} sul piano di simmetria e l'asse \underline{x}^B .

$$\alpha = \arctan \left(\frac{\underline{V} \cdot \underline{z}^B}{\underline{V} \cdot \underline{x}^B} \right)$$

• Epressione di \underline{V}
f($|\underline{V}|, \alpha, \beta$)

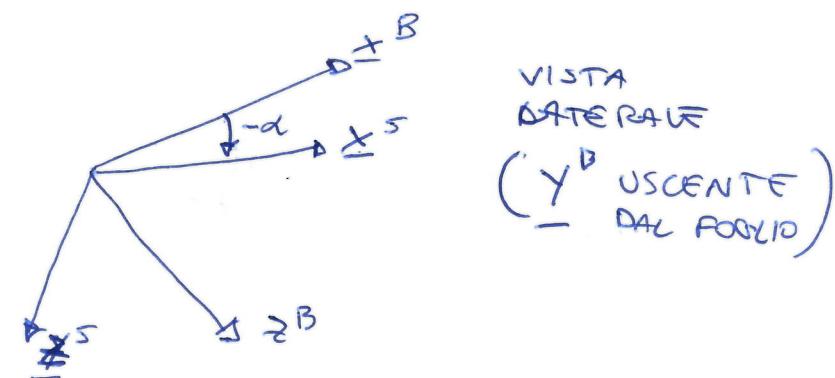
(4)

\mathcal{F}^A : RIFERIMENTO AERODINAMICO
(FLIGHT PATH FRAME)
(WIND FRAME)

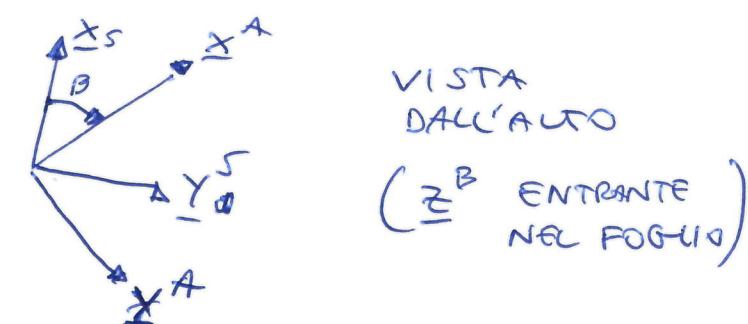
- ORIGINE IN UN PUNTO MATERIALE DEL VEICOLO
- \underline{x}^A ALLINEATO CON \underline{V}
- \underline{z}^A NEL PIANO DI SIMMETRIA

Definibile attraverso una sequenza di due rotazioni e partire dal \mathcal{F}^B

I \rightarrow RUOTO DI $-d$ RISPETTO A \underline{y}^B
(TRONCA TERNA DI STABILITÀ)



II \rightarrow RUOTO DI β RISPETTO A \underline{z}^S
(ARRIVO AL \mathcal{F}^A)



VELOCITÀ IN $\mathcal{F}^A \Rightarrow \underline{V} = |\underline{V}| \underline{x}^A$

CLASSIFICAZIONE REGIMI DI VOLO

- VOLO UNIFORME $\dot{v} = 0$
 $|v|$ COSTANTE
 - RETTILINEO $R = \infty \Rightarrow \omega = \frac{v}{R} = 0$
 SOLO SE $\delta = \dot{\chi} = 0$
 - ORIZZONTALE $\gamma = 0$
 - SALITA $\gamma > 0$
 - DISCESA $\gamma < 0$
 - NEL PIANO VERTICALE $\dot{\chi} = 0$
 $\omega = \dot{\gamma}$
 - MANOVRE CURVILINEE
 - $\dot{\gamma} > 0$: RICHIAMATA (PULL-UP)
 - $\dot{\gamma} < 0$: AFFONDATA (DIVE)
 - $\dot{\chi} > 0$: VIRATA POSITIVA (RIGHT TURN)
VERSO DESTRA
 - $\dot{\chi} < 0$: VIRATA NEGATIVA (LEFT TURN)
VERSO SINISTRA
-
- VOLO SIMMETRICO $\beta = 0$
 \Downarrow
 v_{AS} NEL PIANO
DI SIMMETRIA
 - VOLO LIVELLATO $\phi = 0$
 - VOLO SIMMETRICO
NEL PIANO ORIZZONTALE

$$\left\{ \begin{array}{l} \beta = 0 \\ \chi = 0 \\ \omega = \dot{\chi} \end{array} \right.$$
 - VOLO ORIZZ.
RETTO. UNIF.
(VORU)

$$\left\{ \begin{array}{l} \dot{M} = 0 \\ R = \infty \\ \delta = 0 \end{array} \right.$$

DESCRIZIONE FORZANTI

• EQUAZIONI CARDINALI $\rightarrow \begin{cases} \frac{d}{dt} \underline{Q} = \underline{F} + \underline{I} + \underline{W} \\ \frac{d}{dt} \underline{H}_G = \underline{M}_G + \underline{\Gamma}_G + \underline{\Sigma}_G \end{cases}$

$\underline{Q} \rightarrow$ Quantità di moto
 $\underline{H}_G \rightarrow$ momento delle
 \underline{Q} rispetto al polo G

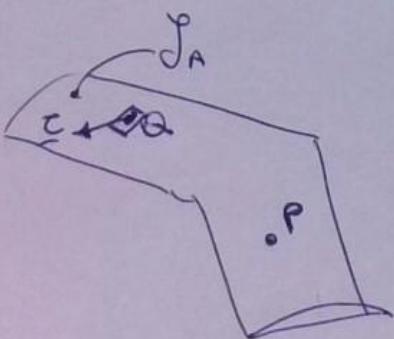
\underline{F} & \underline{M} .
 PESO

(Remark: $\underline{\Sigma}_G = 0$)
 Se $G \cong$ CENTRO DI GRAVITÀ

→ ELEMENTI DI AERODINAMICA

FORZA AERODINAMICA:

$$\underline{F} = \int_{S_A} \underline{\Sigma} dA \quad \underline{M}_P = \int_{S_A} \underline{\Sigma} \times (\rho - Q) dA$$



$$\underline{\Sigma} = \left(\underbrace{-\rho \underline{U}}_{\text{Pressione}} + \left(-\frac{2}{3} \mu \right) (\operatorname{div} \underline{\Sigma}) \underline{U} + 2 \mu \left(\operatorname{symm} \nabla \underline{\Sigma} \right) \underline{U} \right) =$$

PRESSURE
VERSORE USCENTE DALLA SUPERFICIE

VISCOSE

VISCOSE FLUIDO

$$= -\tilde{\rho} \underline{U} - \underline{f}$$

$$\underline{\tau} = \underline{\tau}(P, \underline{N}, M, \underline{m})$$

LEGAME FUNZIONALE DELLO SFORZO $\underline{\tau}$

$$\underline{F} = \underline{F}(P, \underline{N}, M, \underline{m}, S)$$

LEGAME VORTICO (P, V ... tensione
del punto sulla superficie)
LEGGE
GAS

$$\underline{F} = \underline{F}(g_\infty, \theta_\infty, M_\infty, V_\infty, \underline{m}, S)$$

VALORI DI MISERIMENTO
(FLUSSO INDISTURBATO)

$$P \rightarrow P_\infty \text{ e } P = PR\theta$$

$$V \rightarrow V_\infty$$

$$M \rightarrow M_\infty$$

$$= F(g_\infty, \alpha_\infty, M_\infty, V_\infty, \underline{m}, S) \leftarrow \begin{matrix} \text{VELOCITÀ} \\ \text{DEL SUONO} \\ \text{NEL FLUIDO} \end{matrix} \quad a = \sqrt{PR\theta}$$

$$\boxed{F = F(g_\infty, \alpha_\infty, M_\infty, |V|, \alpha, \beta, \underline{m}, S)} \leftarrow V_\infty = f(|V|, \alpha, \beta)$$

ESPRESSIONE FUNZIONALE DELLA FORZA AERODINAMICA

ANALISI DIMENSIONALE ATTRAVERSO IL TEOREMA DI BUCKINGHAM (Π)

Ogni equazione fisica, dipendente da n variabili fisiche $\{q_i\}$ esprimibili in termini di K quantità fisiche fondamentali, è rappresentabile come funzione di $n-K$ variabili adimensionali Π_j costante moltiplicando tra loro combinazioni delle variabili fisiche originali. $\Leftrightarrow f(q_1, q_2, \dots) = 0$
 $\Leftrightarrow g(\Pi_1, \Pi_2, \dots) = 0$

CASO DELLA FORZA AERODINAMICA:

5 VARIABILI FISICHE \rightarrow $|V|, \rho, \alpha, M, S$

3 QUANTITA' FISICHE FONDAMENTALI $3 \rightarrow L, T, M$ (lunghezza, tempo, massa)

$\hookrightarrow 5 - 3 = 2$ VARIABILI ADIMENSIONALI.

$$|F| = K \rho^{e_p} \alpha^{e_a} M^{e_M} V^{e_v} S^{e_s}$$

$$K = K(\alpha, \beta, \text{forma})$$

ANALISI DI MENSIONALE

$$[MLT^{-2}] = [ML^{-3}]^{e_p} [LT^{-1}]^{e_a} [ML^{-1}T^{-1}]^{e_M} [LT^{-1}]^{e_v} [L^2]^{e_s}$$

uguagliamo gli esponenti di M, L e T

$$M) \left\{ 1 = e_p + e_M \right.$$

$$L) \left\{ 1 = -3e_p + e_a - e_M + e_v + 2e_s \right.$$

$$T) \left\{ -2 = -e_a - e_M - e_v \right.$$

3 eqz \Rightarrow VARIABILI INDIPENDENTI
5 incognite e_p, e_a, e_M, e_v, e_s

SOLUZIONE

\Rightarrow

$$\begin{cases} e_p = 1 - e_M \\ e_v = -2 - e_M - e_a \\ e_s = 1 - \frac{1}{2}e_M \end{cases}$$

$$|\underline{F}| = \rho^{1-\epsilon_p} \alpha^{\epsilon_a} \mu^{\epsilon_r} V^{2-\epsilon_p - \epsilon_a} S^{1-\frac{1}{2}\epsilon_p} K(\alpha, \beta, \text{forma})$$

$$= \rho V^2 S \left(\frac{\alpha}{V} \right)^{\epsilon_a} \left(\frac{M}{\rho V \sqrt{S}} \right)^{\epsilon_p} K(\alpha, \beta, \text{forma})$$

↓
 VARIABLE π_1
 $\frac{1}{\text{Mach}}$

↓
 VARIABLE π_2
 $1/\text{Reynolds}$

$$= \rho V^2 S \tilde{C}_F(M_a, Re, \alpha, \beta, \text{forma})$$

$$\boxed{|\underline{F}| = \frac{1}{2} \rho V^2 S C_F(M_a, Re, \alpha, \beta, \text{forma}) = q_0 S C_F(\dots)}$$

Stesse procedure per i momenti

$$\boxed{|\underline{M}| = \frac{1}{2} \rho V^2 S e C_M(M_a, Re, \alpha, \beta, \text{forma}) = q_0 S e C_M(\dots)}$$

\uparrow
 lunghezza
 dei riferimenti

NOTA GENE

- 1) PROCEDURA
- 2) ADIMENSIONAL.
- 3) LEGGE COST.



GIUSTIF. TEORICA
GALLERIA VENTO

DEFINIZIONI PER LE FORZANTI AERODINAMICHE

FORZA	$D := -x^A \cdot F$ (Resistenza) $\rightarrow q_0 S C_D = D$	$F = -(D x^A + Q y^A + L z^A)$
	$Q := -y^A \cdot F$ (Derianza) $\rightarrow q_0 S C_Q = Q$	$Q = -q_0 S (C_B x^A + C_A y^A + C_Z z^A)$
	$L := -z^A \cdot F$ (Portanza) $\rightarrow q_0 S C_L = L$	

MOMENTI	$L_p := x^B \cdot M_p$ (momento di rotto) $\rightarrow q_0 S b C_{L_p} = L_p$	$b = \text{APERTURA}$ APARE
	$M_p := y^B \cdot M_p$ (" di BECCHEGGIO) $\Rightarrow q_0 S \bar{c} C_{M_p} = M_p$	$\bar{c} = \text{CORDA MEDIA}$ AERODINAMICA
	$N_p := z^B \cdot M_p$ (" di IMMATERIA) $\rightarrow q_0 S b C_{N_p} = N_p$	$\bar{c} = \left(\int_{\frac{D}{2}}^{\frac{b}{2}} c^2 \right) / S$

$$M_p = L_p x^B + M_p y^B + N_p z^B = q_0 S \left(b C_{L_p} x^B + \bar{c} C_{M_p} y^B + b C_{N_p} z^B \right)$$

FORZA IN ASSI CORPO

$$X := x^B \cdot F \quad (\text{FORZA LONGITUDINALE})$$

$$Y := y^B \cdot F \quad (\text{FORZA LATERALE})$$

$$Z := z^B \cdot F \quad (\text{FORZA VERTICALE TRASVERSALE})$$

PER ANGOLI PICCOLI $\alpha \ll 1$
 $\beta \ll 1$

$$X \approx -D + L \alpha$$

$$Y \approx -Q$$

$$Z \approx -L$$

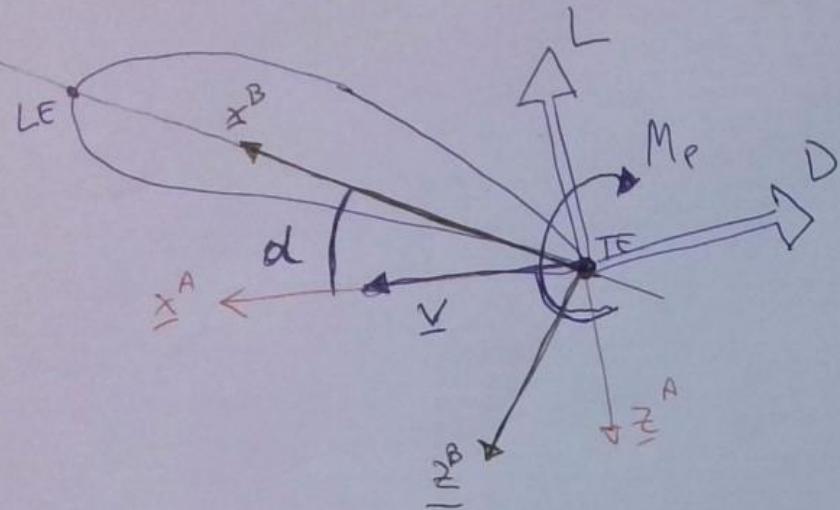
ANALISI DEL PROFILO

$$2D \Rightarrow \beta = 0$$

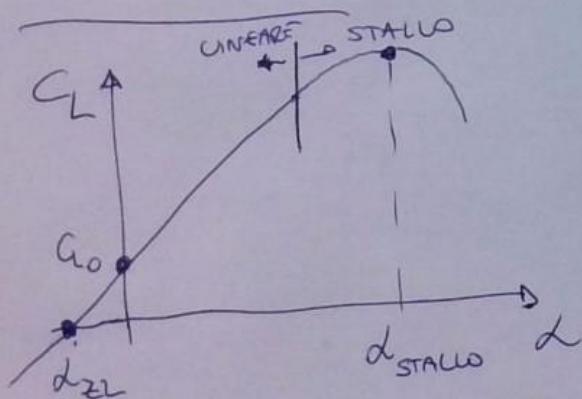
NOTA BENE \underline{Y}^A e \underline{Y}^B COINCIDENTI ED ENTRANTI

$$\underline{F} = - (D \underline{x}^A + L \underline{z}^A)$$

$$\underline{M} = M_p \underline{Y}^B$$



PORTANZA



PROFILO STANDARD CONNESSO $\rightarrow \alpha_0 > 0$
 $\rightarrow \alpha_{2L} < 0$

PROFILO SIMMETRICO

$$\rightarrow \alpha_0 = 0$$

$$\alpha_{2L} = 0$$

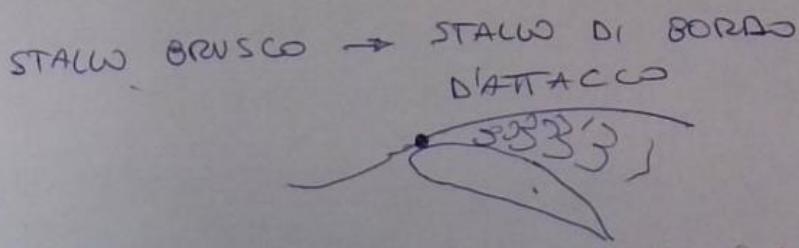
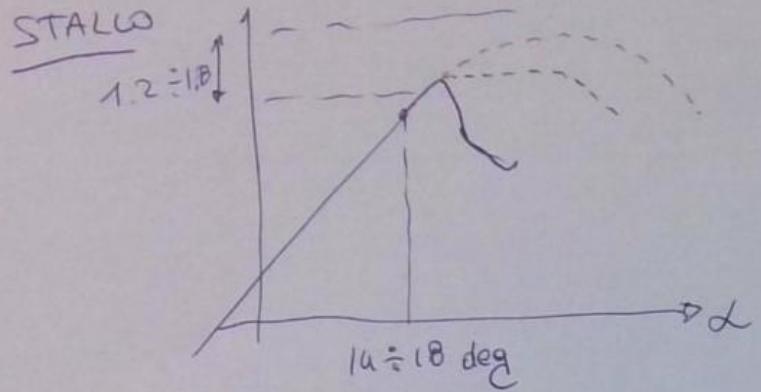
DERIVATA
DEL α_0 RISPETTO
AD d

PORTANZA IN REGIONE UNIFOR

$$C_L = C_{L0} + (C_{L12})d$$

$$= C_{L12}(\alpha - \alpha_{2L}) = C_{L12}(L_A)$$

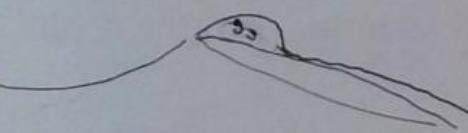
Angolo d_w
incidente
assoluto/
cerodinamico



STALLO DOLCI \rightarrow STALLO NEL BORDO DI USCITA



STALLO PIATTO \rightarrow BOLLA DI SEPARAZIONE

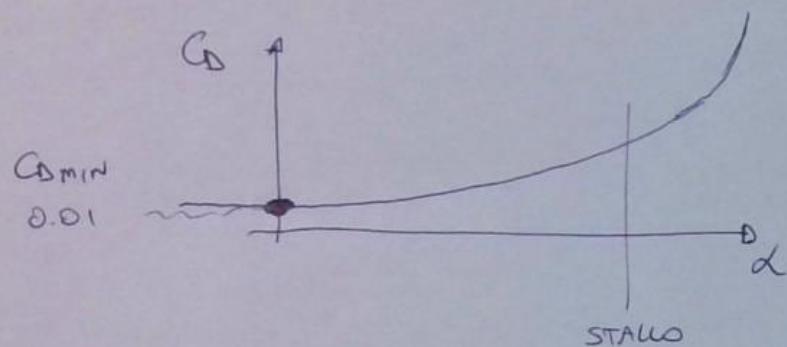


STALLO BRUSCO

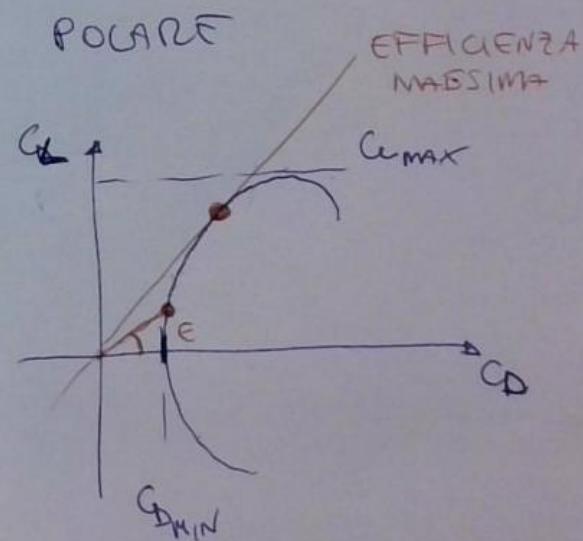
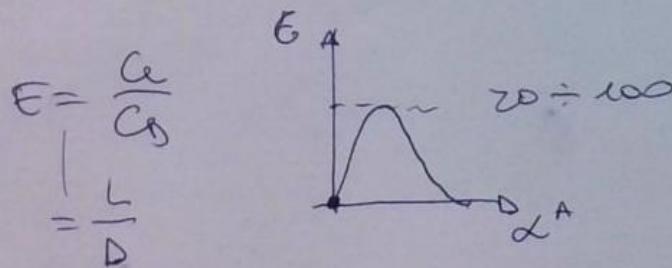


RESISTENZA DEL PROFILO

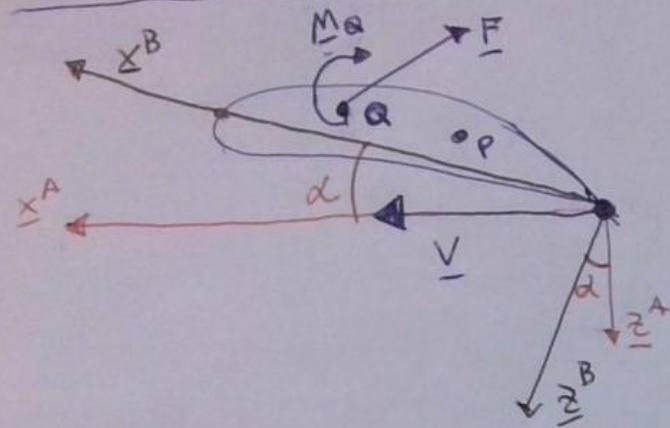
$D \rightarrow D_p$ resistenza di pressione (dovuta alla separazione) \rightarrow dipende da α
 $D \rightarrow D_f$ " attivo" \rightarrow dipende poco da α



EFFICIENZA (LIFT - TO - DRAG RATIO)



MOMENTO DI BECCNEGGLIO



$Q \Rightarrow$ POLO IN CUI PRODUCEMO I MOMENTI

$P \Rightarrow$ IN CUI DESIDERIAMO MISURARE FORZE E MOMENTI

$$\underline{F} = -(\underline{D} \underline{x}^A + \underline{L} \underline{z}^A)$$

$$\underline{M}_Q = M_Q \underline{y}^B = M_Q \underline{y}^A \quad \left(\begin{array}{l} \text{NOTA BENE} \\ \underline{2D} \Rightarrow \underline{\beta} = 0 \end{array} \right)$$

$$\underline{M}_P = \underline{(Q-P)} \times \underline{F} + \underline{M}_Q$$

$$* \underline{(Q-P)} = (\underline{x}_Q - \underline{x}_P) \underline{x}^B + (\underline{z}_Q - \underline{z}_P) \underline{z}^B = \underline{X} \underline{x}^B + \underline{Z} \underline{z}^B$$

$$** \underline{F} = -(\underline{D} \underline{x}^A + \underline{L} \underline{z}^A)$$

$$\begin{aligned} \text{VERSORI} \Rightarrow \underline{x}^A &= \cos \alpha \underline{x}^B + \sin \alpha \underline{z}^B \\ \underline{z}^A &= -\sin \alpha \underline{x}^B + \cos \alpha \underline{z}^B \end{aligned}$$

$$\underline{M}_P = -(\underline{X} \underline{x}^B + \underline{Z} \underline{z}^B) \times (\underline{D} \underline{x}^A + \underline{L} \underline{z}^A) + \underline{M}_Q$$

$$= -(\underline{X} \underline{x}^B + \underline{Z} \underline{z}^B) \times \left[(\underline{D} (\cos \alpha \underline{x}^B + \sin \alpha \underline{z}^B) + \underline{L} (-\sin \alpha \underline{x}^B + \cos \alpha \underline{z}^B)) \right] + \underline{M}_Q$$

$$\begin{array}{ll} \text{NOTA BENE} & \underline{x}^B \times \underline{x}^B = 0 \\ & \underline{x}^B \times \underline{z}^B = -\underline{y}^B \\ & \underline{z}^B \times \underline{x}^B = \underline{y}^B \\ & \underline{z}^B \times \underline{z}^B = 0 \end{array}$$

$$M_p = -Z D \cos \alpha Y^B + X D \sin \alpha Y^B + X L \cos \alpha Y^B + Z L \sin \alpha Y^B + M_Q Y^B$$

SOSTITUIAMO $E = \frac{L}{D} \Rightarrow D = \frac{L}{E}$

(relazioni di modulo di M_p)

$$\|M_p\| = -\frac{L}{E} Z \cos \alpha + \frac{L}{E} X \sin \alpha + L X \cos \alpha + L Z \sin \alpha + M_Q$$

$$\frac{\|M_p\|}{\|M_p\|} = \frac{L \cos \alpha \left(-\frac{Z}{E} + \frac{X}{E} \tan \alpha + X + Z \tan \alpha \right) + M_Q}{L \cos \alpha \left(X \left(1 + \frac{\tan \alpha}{E} \right) + Z \left(-\frac{1}{E} + \tan \alpha \right) \right) + M_Q}$$

SEMPLIFICAZIONI $\| \alpha \ll 1 \| \quad E \gg \theta \text{ (1)} \| \Rightarrow 1 + \frac{\tan \alpha}{E} \approx 1 \quad \| \Rightarrow M_p = L X + M_Q$
 $Z \left(-\frac{1}{E} + \tan \alpha \right) \ll X \| \|$

$$\|M_p\| = \boxed{M_p = M_Q - L(x_p - x_a)} \xrightarrow{\text{COEFF.}} \boxed{C_{M_p} = C_{M_Q} - (\xi_p - \xi_a) a_{1/d} d_A}$$

COEFF.

$$M = \frac{1}{2} \rho V^2 C^2 C_m$$

$$C_{M_Q} = \frac{M_Q}{\frac{1}{2} \rho V^2 C^2}$$

$$\frac{L(x_p - x_a)}{\frac{1}{2} \rho V^2 C} = C_2 (\xi_p - \xi_a)$$

$$\xi_{(k)} = \frac{x_{(k)}}{C}$$

CORDA PROFILE

$a_{1/d}$

pondenze curve portante

d_A

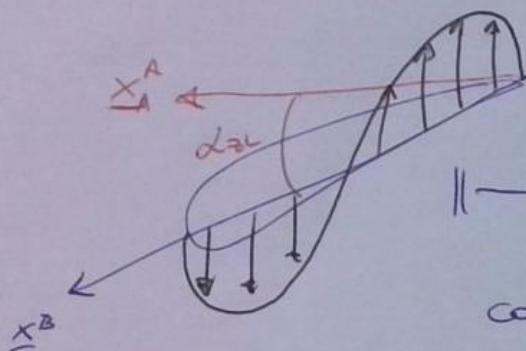
$A_0 A$ aerodinamico

$$C_{m_p} = C_{m_q} - (\xi_p - \xi_q) \alpha_{lx} \Delta_A$$

OSSERVAZIONE 1

se $\alpha_A = 0, \alpha = 0 \Rightarrow C_{m_p} = C_{m_q}$

PURA COPPIA
INDIPENDENTE DAL POLO



DISTRIBUZIONE CON
CONSTATAZIONE FENOMENOLOGICA

$$\begin{aligned} L &= 0 \\ M &\neq 0 \end{aligned}$$

<u>A PICCHIARE</u>	<u>PROFILI CONVESSI</u>
$C_{m_{zL}} < 0$	PROFI LI CONVES SI

<u>PROFI LI SIMMETRICI</u>

$$C_{m_{zL}}$$

OSSERVAZIONE 2

→ CONSIDERAMO IL CENTRO DI PRESSIONE $CP := C_{m_{CP}} = 0$

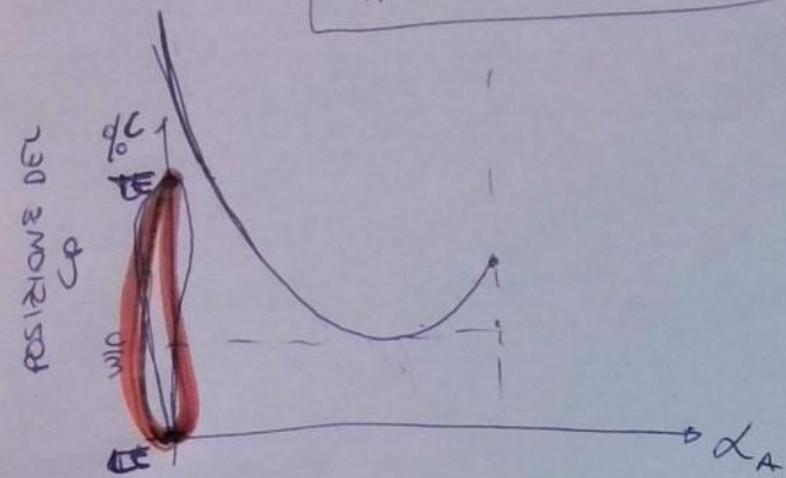
$$Q = CP \quad C_{m_p} = \cancel{C_{m_{CP}}} - (\xi_p - \xi_{CP}) \alpha_{lx} \Delta_A$$

→ CONSIDERAMO UN PROFILO CONVESSO E CALCOLIAMO IL NECESSARIO $C_{m_{zL}}$

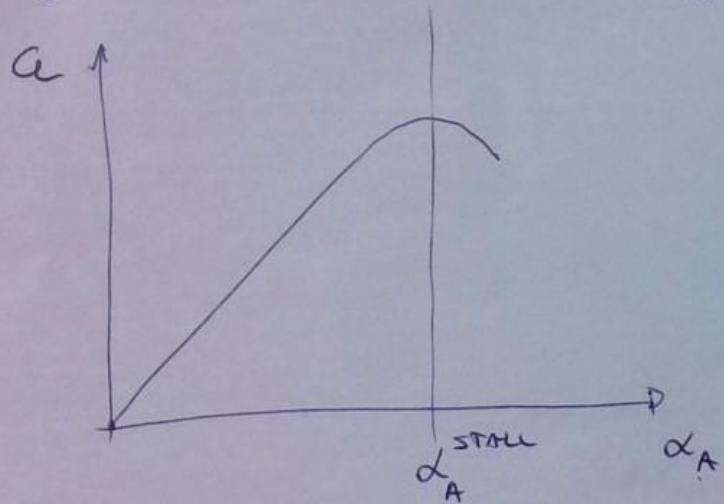
$$C_{m_{zL}} := C_{m_p} \Big|_{\alpha_A = 0} \Rightarrow C_{m_p} = \cancel{C_{m_{CP}}} - \cancel{(\xi_p - \xi_{CP})} \alpha_{lx} \cancel{\Delta_A} \xrightarrow[\substack{\cancel{x_{CP}} \rightarrow \infty]}{=} < 0$$

$$\lim_{\alpha_A \rightarrow 0} x_{CP} = -\infty$$

- CP non è costante con α
- CP NON È UN PUNTO MATERIALE!



CP non è un buon punto
per elaborare una funzione
del Cm in rapporto a α



CENTRO AERODINAMICO |

DEFINIZIONE

$$\exists \text{ A.C. t.c. } \left[\frac{Cm_{AC}}{\alpha} = 0 \right] \Rightarrow Cm_{AC} \text{ COSTANTE CON AOA}$$

AC COSTANTE CON AOA

Derrivate del coefficiente di momento, rispetto al centro del solo AC, rispetto all'angolo di incidenza uguale a α

$$Cm_p = Cm_{AC} - (\bar{\gamma}_p - \bar{\gamma}_{AC}) \overbrace{a_{1/2} \alpha_A}^{\alpha}$$

$$\frac{d}{d\alpha} Cm_p = \cancel{Cm_{AC}/\alpha} - (\bar{\gamma}_p - \bar{\gamma}_{AC}) a_{1/2} \Rightarrow \boxed{\bar{\gamma}_{AC} = \bar{\gamma}_p + \frac{Cm_p/\alpha}{a_{1/2}}}$$

SI VERIFICA $\checkmark P$



PUNTO
MATERIALE

PROPRIETA'

- TEORIA PROFILI SOTTIU $\Rightarrow x_{LE} - x_{AC} \approx \frac{c}{4}$
- VECCHI PROFILI NACA $\Rightarrow \approx \frac{c}{4}$
- MODERNI PROFILI $\Rightarrow \leq \frac{c}{4}$

• $a_A = 0$

$$Cm_{zL} = Cm_{AC} - (\bar{\gamma}_p - \bar{\gamma}_{AC}) \cancel{a_{1/2} \alpha} = 0$$

- PROFILI SIMMETRICI

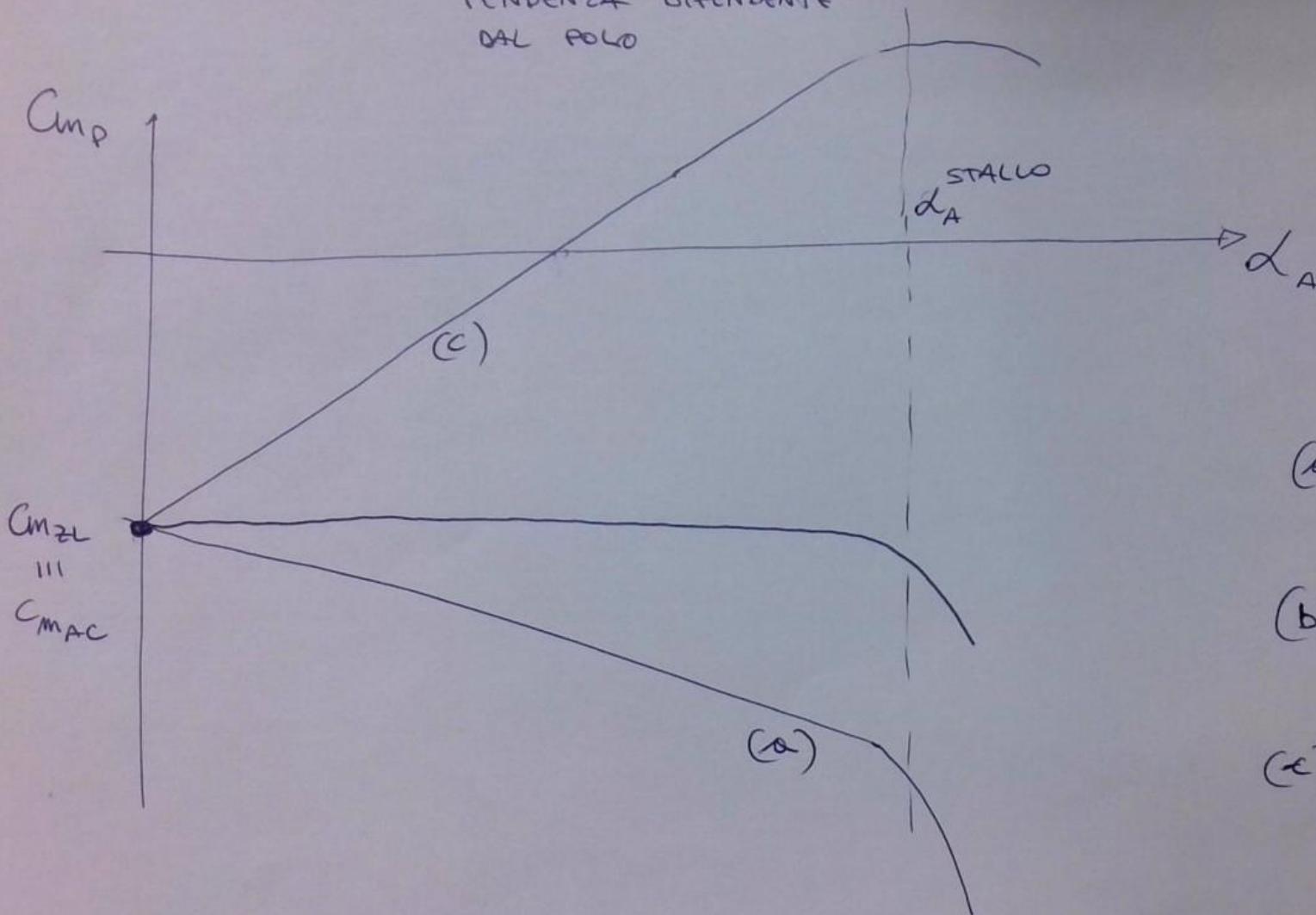
$$Cm_{zL} = 0 \Rightarrow Cm_{AC} = 0$$

$Cp = Ac$
SOLO PROF. SIMM.

CURVA DI MOMENTO

$$C_{m_p} = C_{m_{AC}} - (\xi_p - \xi_{AC}) \alpha_{12} d_A$$

PENDENZA DIPENDENTE
DAL POLO

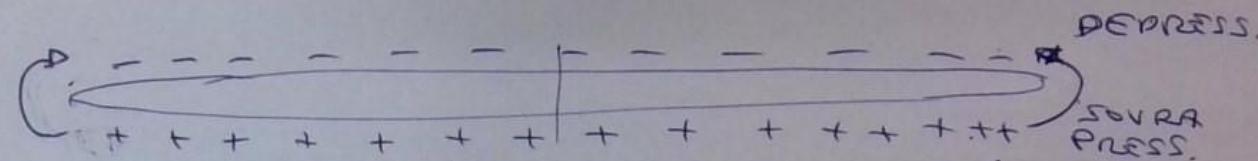


- (a) P DAVANTI AD AC
Pendenza negativa
- (b) P = AC
Pendenza nulla
- (c) P DIETRO AC

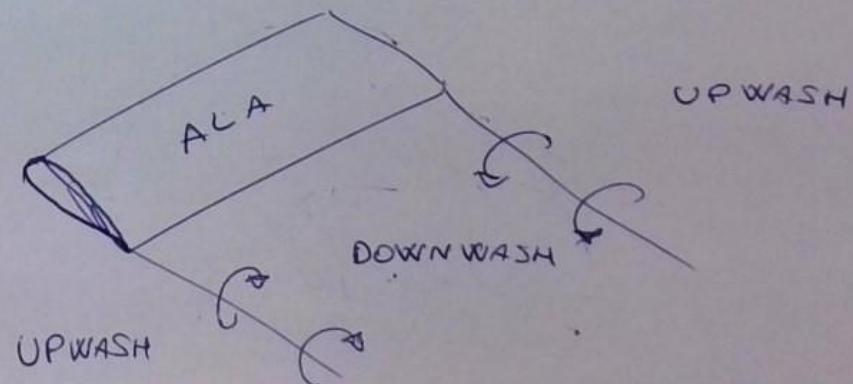
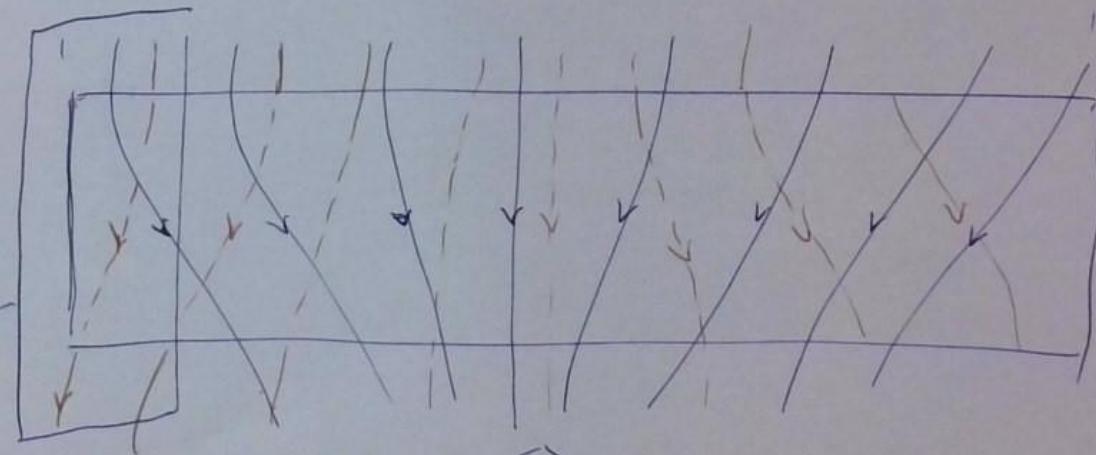
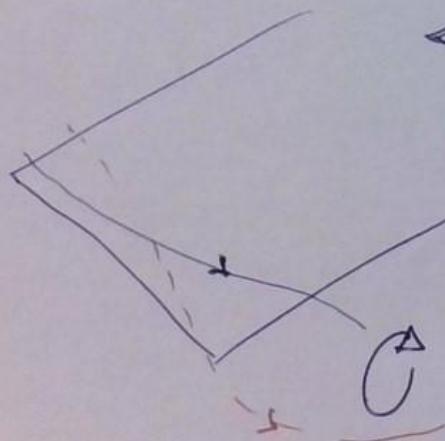
ALI DI APERTURA FINITA

- EFFETTI TRIDIMENSIONALI (particolarmente visibili nelle estreme altezze)

VISTA DAL DAVANTI



VISTA DA SOPRA



STUDIO DI ALI DI APERTURA FINATA

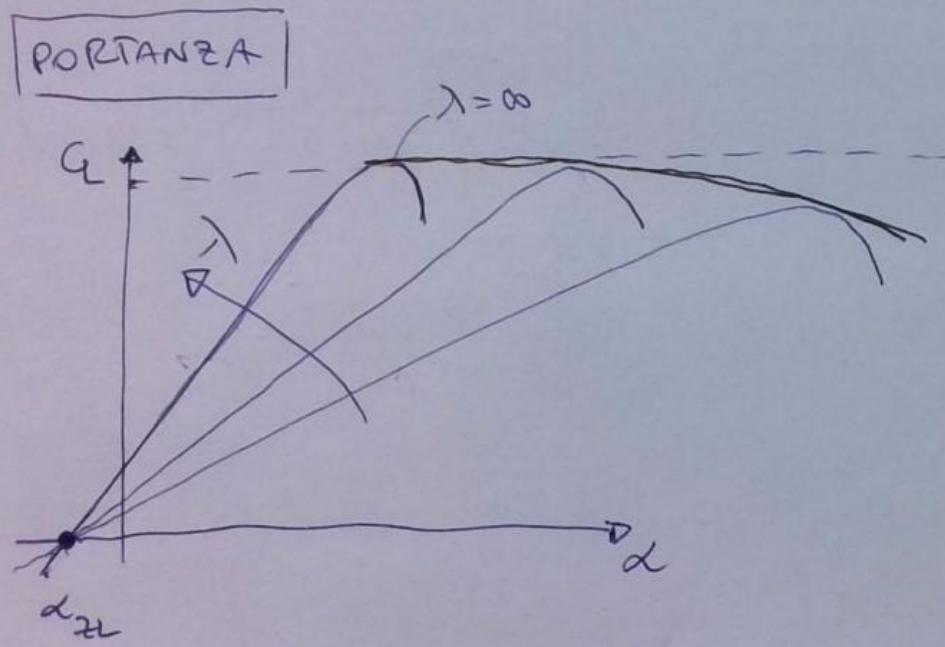
- TEORIA 2D + CORREZIONI ($\beta = 0$, ^{REGIME} ~~velo simmetrico~~)
- TEORIA 3D ($\beta \neq 0$)

VARIABILI DI INTERESSE

$$\lambda = \frac{b^2}{S} = \frac{(\text{apertura alare})^2}{\text{superficie}} \Rightarrow \text{AR}$$

Aspect Ratio
(Allungamento)

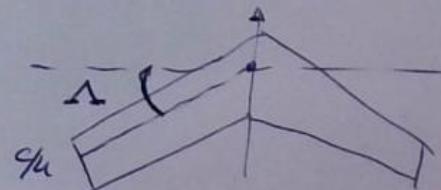
$\lambda = \infty \Rightarrow$ ALA DI ALUNG. INFINTO (2D/PROFILO)



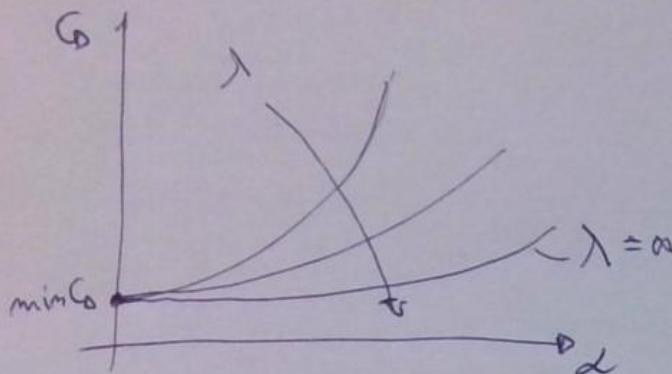
FORMULA SEMI-EMPIRICA

$$C_L/\alpha = \frac{2\pi}{\lambda^2 + \sqrt{\left(\frac{2}{\lambda}\right)^2 + \frac{1}{\cos\alpha} - M_\infty^2}}$$

\wedge angolo SW frecce

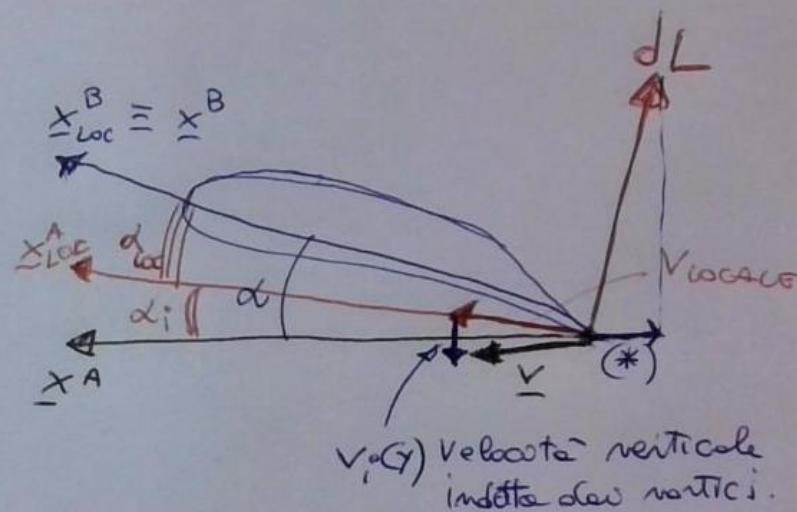
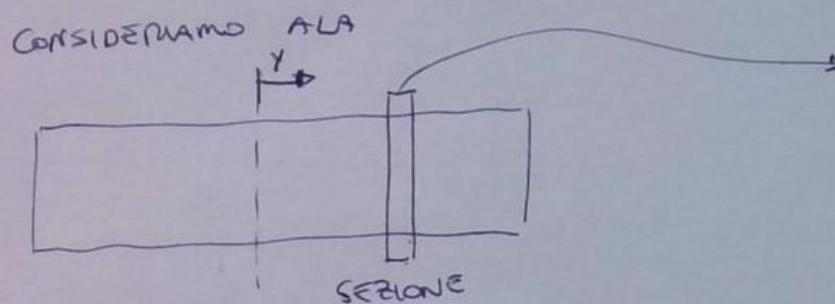


RESISTENZA



$$C_D = C_{D_p} + C_{D_i}$$

↑ PROFIL
↑ INDOTTO



$$(*) dL \sin \alpha_i = dD_i \rightarrow \text{componente aggiuntiva della resistenza indotta}$$

"BUONA" ASSUNZIONE $\alpha_i \propto C_L$ $\Rightarrow dD_p dC_{D_i} \propto (dC_L)^2$

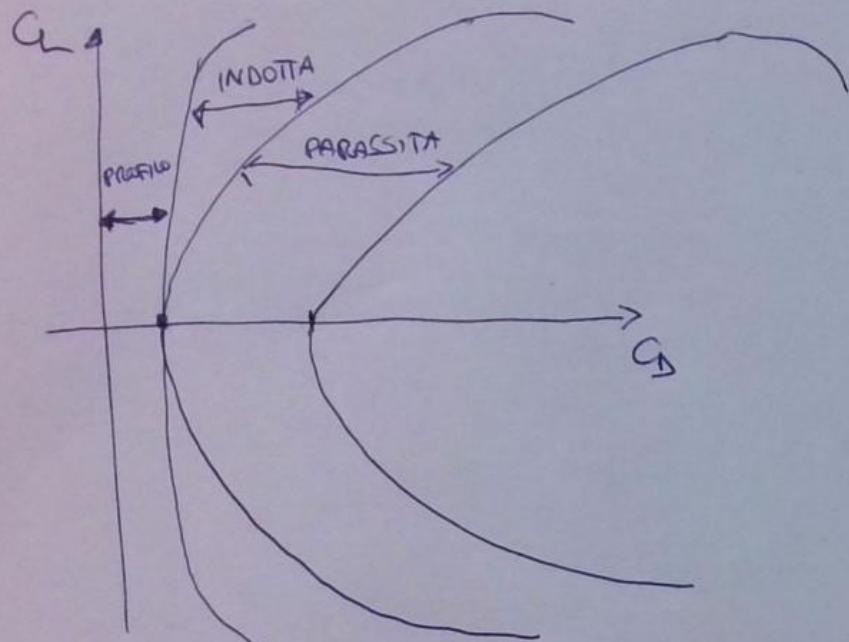
MODELLAZIONE POLARE ALA FINITA

$$C_D = C_{D_0} + K C_L^2 \quad | \quad K = \frac{1}{\pi \lambda} e$$

e = fattore OSVALD
 $0.6 + 0.9$

RESISTENZA DEL VELIVOLO COMPLETO (POLARE)

- PORTANZA DEL VELIVOLO COMPLETO = PORTANZA ALA
- RESISTENZA \Rightarrow RESISTENZA PROFILICO + RESISTENZA INDOTTA + RESISTENZA PARASSITA



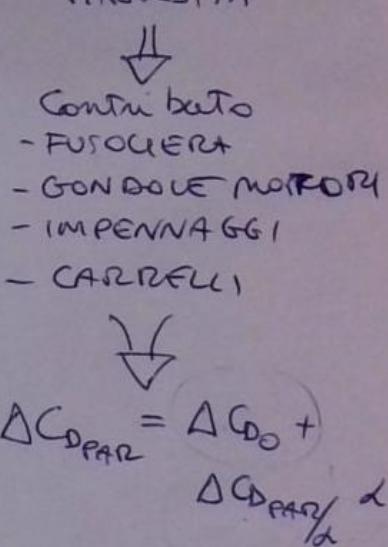
EFFICIENZA MAX

$$\epsilon_{MAX} = 17.8747$$

$$= 38 \quad \text{GLOBAL FLYER}$$

$$= 4.35 \quad \text{CONCORDE} \\ 7.14 \quad (\text{SUBSONIC / FASI TERMINALI}) \\ (\text{MACH } 2.0)$$

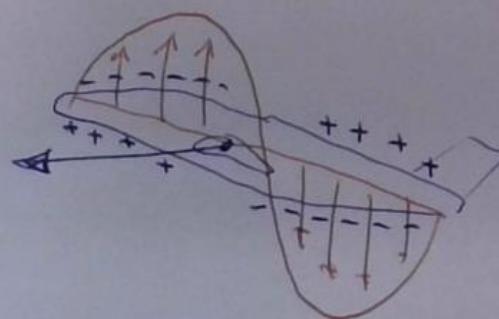
$$= 40 \div 60 \quad (\text{ACIANTE})$$



VELIVOLO
A GETTO
CONTRE FUSO/ERT
E ALA MOLTO ALLUNGATA
(CIRCA DEL MONDO)
NON STOP
2 GIORNI
19 ORE
2 MINUTI
46 SECONDI

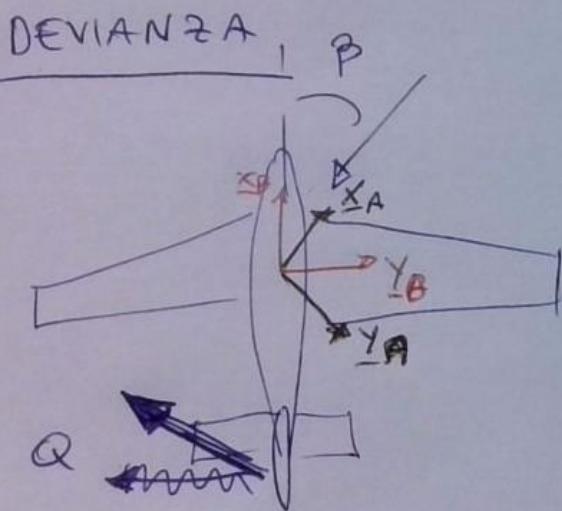
MOMENTO DI BECCHEGGIO VELIVO COMPLETO

$$C_{m_p} = C_{m_p \text{ pale}} + C_{m_p \text{ fusoliera}}$$



$$L = 0$$

$$M_p \neq 0$$



$$Q = Q/\beta \ \beta$$

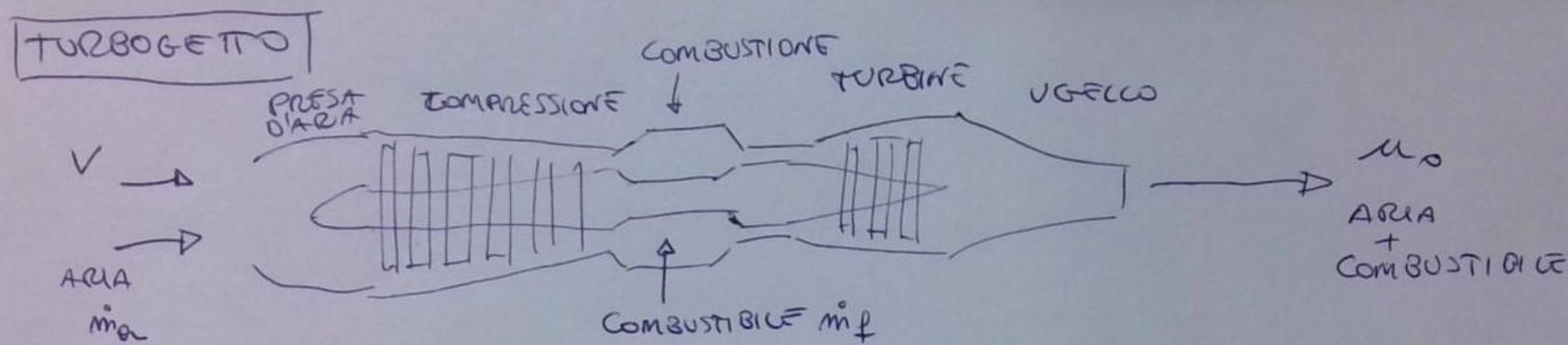
$$\Downarrow$$

$$C_Q = C_Q/\beta \ \beta$$

Q per $\beta=0$ è zero e Ø
DERIVE CON PROFILI SIMMETRICI

→ FORZE PROPULSIVE

- Formare una caratterizzazione generica per
 - TURBO GETTO
 - MOTOELICA CON PALE A PASSO VARIABILE



SPINTA: $T = \dot{m}_a (u_\infty - V) + \dot{m}_f u_\infty$

$$+ A = \frac{\dot{m}_f + \dot{m}_e}{\dot{m}_f} \Rightarrow \dot{m}_a = \dot{m}_f (A + 1) \quad A = 50 \div 100$$

$$\rightarrow T = \dot{m}_f [A \left(\frac{u_\infty}{V} - 1 \right) + 1] V$$

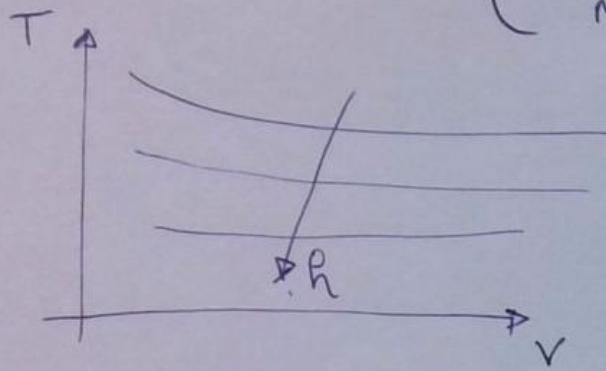
POTENZA $P_d = T V$

SPINTA \rightarrow DIPENDENTE

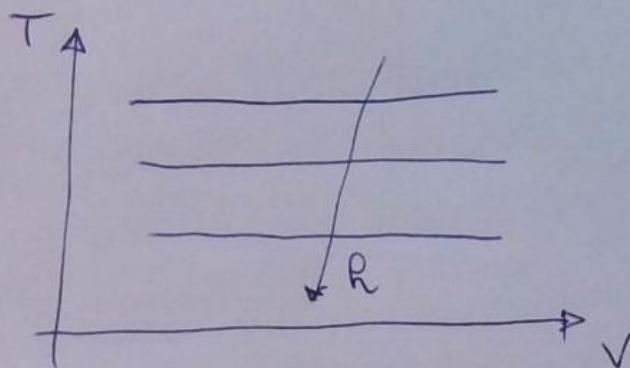
$$T = T(\rho, V, \underset{\uparrow}{\delta_T}, \alpha, \beta, \dots) \approx T(\underset{\uparrow}{h}, V, \delta_T)$$

PARZIAZZAZIONE
(MANETTA
MOTORE)

QUOTA

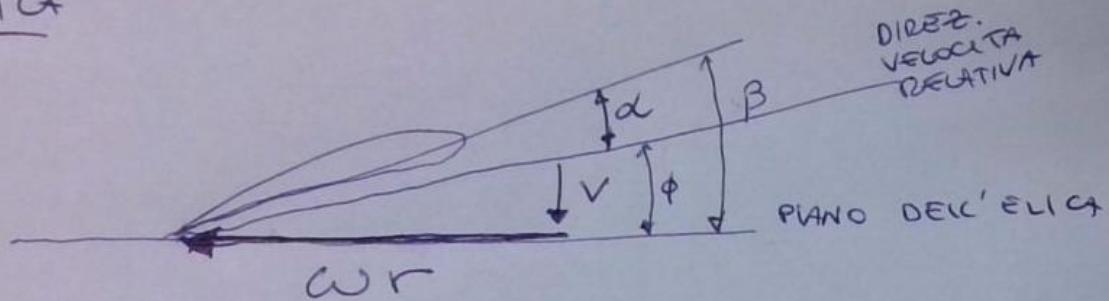


MODELLO SEMPLIFICATO DI TURBOGETTO T COSTANTE CON V



MOTORE EUCI → MOTORE A PISTONI ASSOCIAATO AD UN'ELICA

ELICA

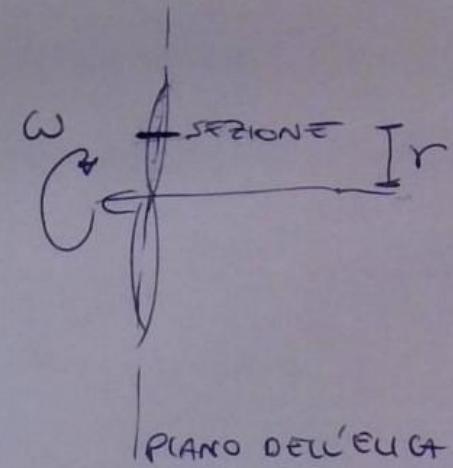


β PASSO LOCALE

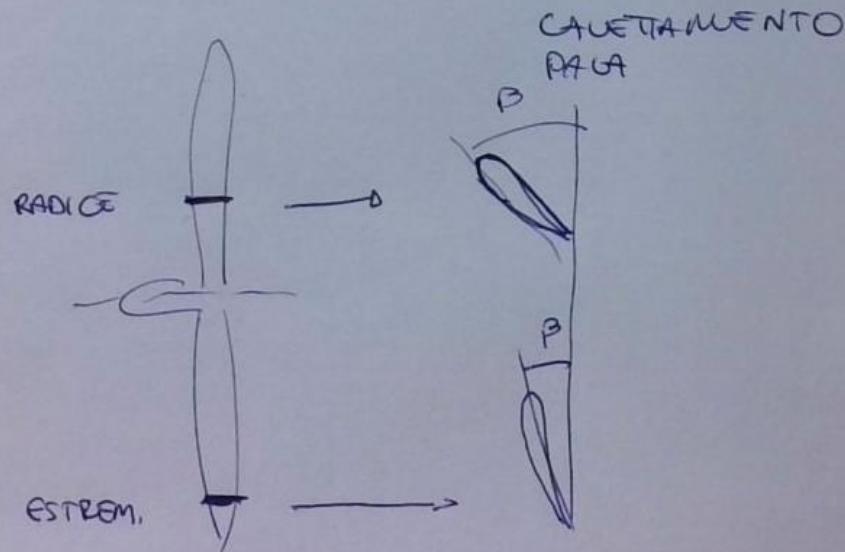
α ANGOLI DI INCIDENZA LOCALE $\boxed{\alpha = \beta - \phi}$

ϕ ANGOLI DI AVANTAMENTO LOCALE

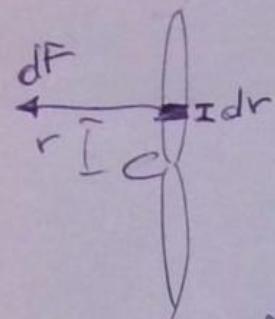
$$L_{patom} \left(\frac{v}{\omega r} \right)$$



PIANO DELL'ELICA



FORMULE DI RENARD



$$V_{REL}^2 = (\omega r)^2 + V^2$$

$$dF = \frac{1}{2} \rho V_{REL}^2 C dr c_f(\alpha)$$

DEF : ADVANCE RATIO

$$J = \frac{V}{ND}$$

$$N = \frac{\omega}{2\pi}$$

D = diametro
del rotore

R = raggio
del
rotore

$$V_{REL}^2 = (2\pi r N)^2 + (J N D)^2 = N^2 D^2 \left(J^2 + \pi^2 \left(\frac{r}{R} \right)^2 \right)$$

\bar{r} → coordinate
adimensionale

$$\alpha(r) = \beta(r) - \phi(r)$$

$$\phi = \arctan \left(\frac{V}{\omega r} \right) = \arctan \left(\frac{J}{\pi \bar{r}} \right) \Rightarrow \alpha(\beta, J, \bar{r})$$

$$\rightarrow \boxed{dF = \frac{1}{2} \rho V_{REL}^2 C dr c_f(\beta, J, \bar{r})}$$

$$= \rho N^2 D^2 \left[\frac{1}{8} \left(J^2 + \cancel{\pi^2} \bar{r}^2 \right) C dr c_f(J, \beta, \bar{r}) \right] \Rightarrow$$

INTEGRATUT
~~SEZIONE~~
WINGO CT PAGA

SPINTA

$$T = \rho N^2 D^4 C + (\beta, \beta^*)$$

β^* \Rightarrow coltellamento in un punto
su riferimento oscurato
(75% del regolo)

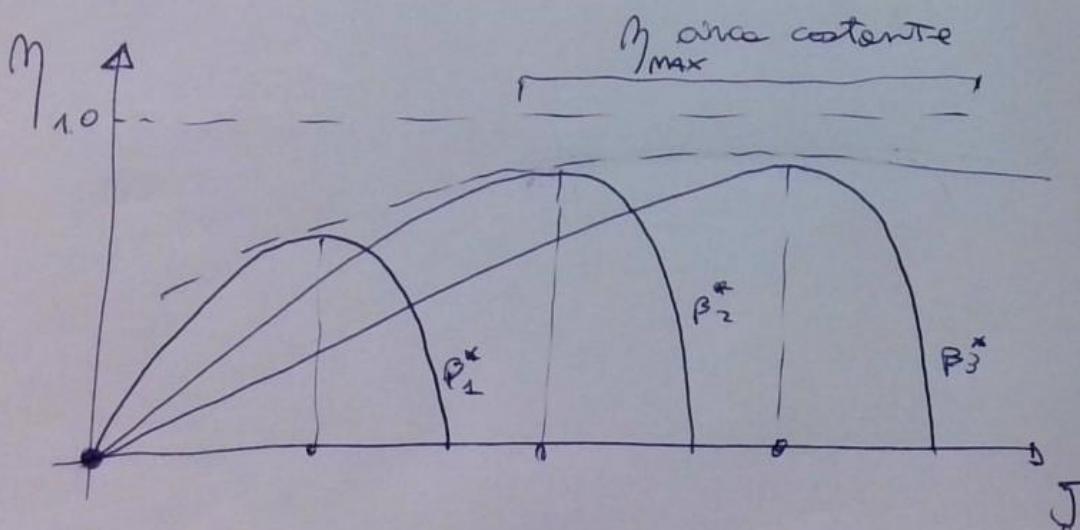
COPPIA

$$C = \rho N^2 D^5 C_c(\beta, \beta^*)$$

POTENZA

$$P = C \omega = \rho N^3 D^5 C_p(\beta, \beta^*)$$

Renoldi - Eliot: $\eta = \frac{r \cdot v}{\rho}$

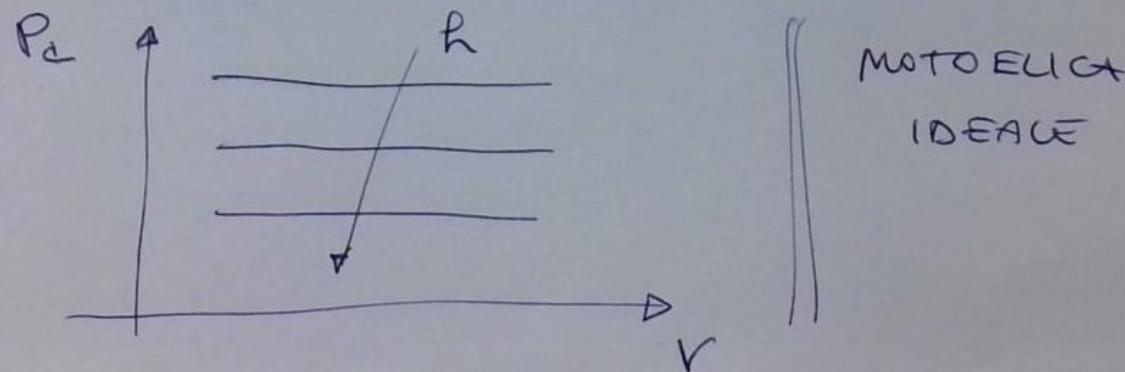


- POTENZA MOTORE A PISTONI COSTANTE (RATIONE VOLLENTE) CON VELOCITA'
- ~~SERIA~~ POTENZA SCENDE CON QUOTA ($\rho \downarrow$)
 - TURBOCOMPRESSORE
- EFFICIENZA FUORI A PASSO VARIAZIONE PUO' ESSERE CONSIDERATA COSTANTE CON VELOCITA'

$$P_2 = P_s \eta \Rightarrow P_2 \text{ COSTANTE con VELOCITA'}$$

↑
 Potenza disponibile
 ↓
 Potenza libera

EFFICIENZA



CONDIZIONI DI VOLO SIMMETRICO

NEL PIANO VERTICALE

- A partire dalle equazioni generiche, attraverso ipotesi e semplificazioni successive, si ottengono le equazioni per diversi regimi di volo (simmetria + piano vert.) fino al volo.

- EQUILIBRIO IN VOLO "CARICA SUCCESSIVI"
- STABILITÀ IN VOLO

\xrightarrow{x}

Quantità chi
metto

$$\frac{d}{dt} \underline{\Omega} = \frac{d}{dt} (\underline{m} \underline{V}) = \dot{\underline{m}} \underline{V} + \underline{m} \dot{\underline{V}}$$

$$\begin{cases} \underline{m} \dot{\underline{V}} = \underline{F} + \underline{T} + \underline{W} \\ \dot{\underline{M}_P} = \underline{M}_P + \underline{T}_P + \underline{Z}_P \end{cases}$$

$$\dot{\underline{m}} = -\frac{\dot{\underline{M}_P}}{\underline{T}} \quad \Rightarrow \text{essere assorbita}$$

portata
manica
combustibile
(carburante)

$$\text{ASSUNZIONI: } \rho = C_G \quad \Rightarrow \underline{Z}_G = 0$$

$$(\text{n.o. } \| \underline{V} \| = V)$$

$$\begin{cases} \underline{V} = \underline{V}_{\text{ext}} \\ \dot{\underline{V}} = \dot{\underline{V}}_{\text{ext}} + \frac{V^2}{R} \underline{e}_m \end{cases} \quad \omega = \frac{V}{R}$$

$$\underline{F} = -(\Delta \underline{x}_A + \alpha \underline{y}_A + L \underline{z}_A)$$

!

IPOTESI

$$\underline{T} = T \underline{x}_D$$

EQ2

VOCO SIMM + PIANO VERTICALE

$$\begin{cases} \beta = 0 \Rightarrow \alpha = 0 \\ \omega = \dot{\gamma} \\ f_{ca} = N_G = 0 \end{cases}$$

$\ddot{x} = 0 \Rightarrow x = \text{cost} \Rightarrow$ per semplicità $x = 0$

ovviamente la rotta verso nord

$$\begin{cases} f_{ca} = N_G = 0 \end{cases}$$

$$\begin{cases} m\ddot{V} = F + T + \underline{\psi} \\ M_{ca} = M_{ca} \underline{y}_B \end{cases} \quad \parallel \quad F = -(\Delta \underline{x}_A + L \underline{z}_A)$$

(*) \rightarrow BIANCO DI POTENZE

mostriamo o colossamente
la (*) per \underline{V}

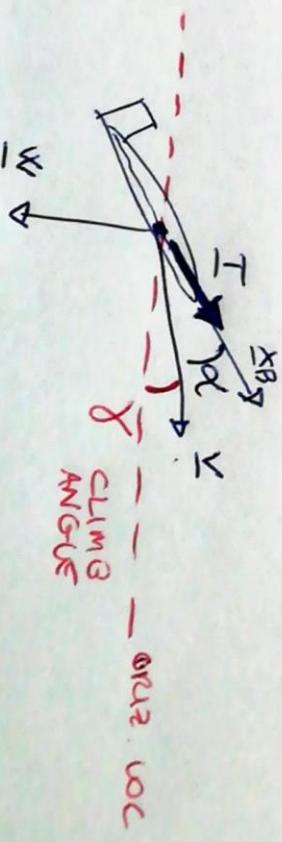
$$\underline{V} \cdot (m\ddot{V} = F + T + \underline{\psi})$$

$$m\underline{V} \cdot \dot{\underline{V}} = m\underline{V}\ddot{\underline{V}} \quad (\underline{V} = \underline{V}_{et}, \quad \dot{\underline{V}} = \dot{\underline{V}}_{et} + \frac{\underline{V}^2}{R} \underline{e}_n)$$

$$\underline{V} \cdot F = -\nabla V \quad (e_t = \underline{x}_A \text{ perché entrambi } \parallel \underline{a} \underline{V})$$

$$\underline{V} \cdot T = T \underline{V}_{cond}$$

$$\underline{V} \cdot \underline{\psi} = -\underline{\psi} \underline{V} \sin \gamma$$



DEFINIZIONI

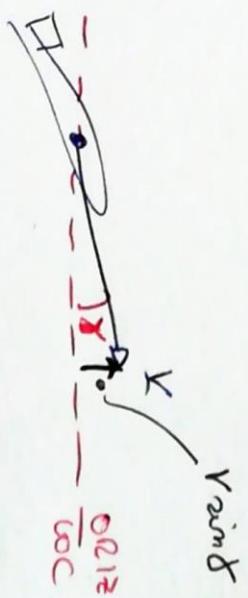
- $V_T = P_d$
- $V_D = P_r$

Potenza disponibile
(available power)
Potenza richiesta
(required power)

$$\underline{H_P} \Rightarrow d \ll 1 \Rightarrow \cos d \approx 1$$

$$V_{wind} = ?$$

↳ Velocità
verticale V_v



SEP Specific Excess Power

$$\begin{aligned} mV\dot{V} &= P_d - P_r - Vw \sin \gamma \\ \hookrightarrow mV\dot{V} + VwV_v &= P_d - P_r \\ \hookrightarrow \frac{V\dot{V}}{g} + V_v &= \frac{P_d - P_r}{w} \end{aligned}$$

$$\begin{aligned} SEP &\rightarrow \text{SALVE } V_v > 0 \\ &\left\{ \begin{array}{l} \text{ACCELERAZIONE } \dot{V} > 0 \\ \text{PARA ENTRANTI} \end{array} \right. \end{aligned}$$

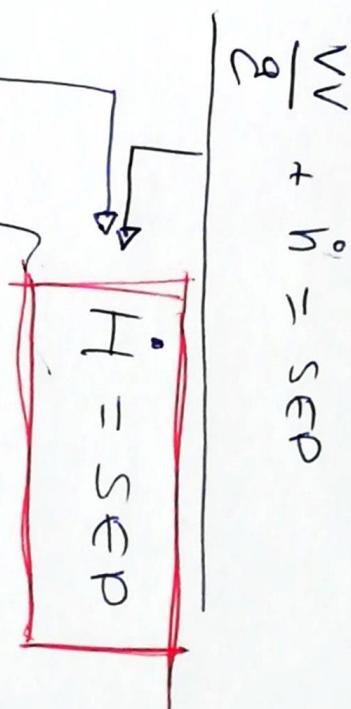
QUOTATOVALE

$$H := h + \frac{1}{2} \frac{V^2}{g}$$

$$\left[\begin{array}{l} E_{\text{TOT}} = \frac{1}{2} m V^2 + m g h \\ \frac{E_{\text{TOT}}}{m} = \frac{1}{2} \frac{V^2}{g} + h \end{array} \right]$$

$$\frac{dH}{dt} = H^{\circ} - h + \frac{V^{\circ} V}{g}$$

$$\left[\begin{array}{l} SEP = 0 \Rightarrow H^{\circ} = 0 \\ H = SEP \end{array} \right]$$



$$\frac{V^{\circ}}{g} + h = SEP$$

$$H^{\circ} \text{ dry calme } V^{\circ} = h$$

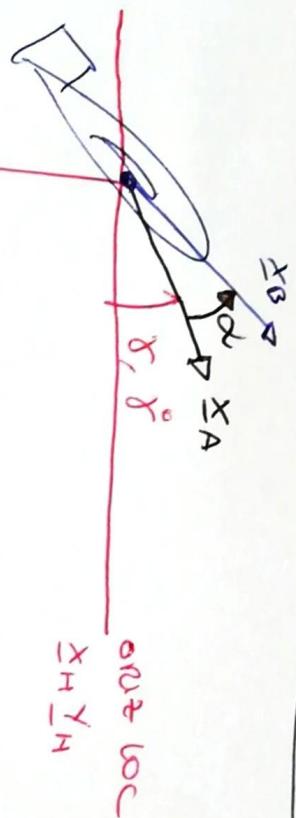
$$h^{\circ} = - \frac{V^{\circ} V}{g}$$

¶

$\alpha SEP = 0$ posso accelerare rendendo quote

α solle rendendo velocita'

EQUAZIONI DI EQUILIBRIO



(II) $\Sigma A \cdot (m \ddot{V} = F + T + W)$

$$\Sigma A \cdot (m \ddot{V} = F + T + W) \\ Y_A \cdot (H_{ca} = M_{cg})$$

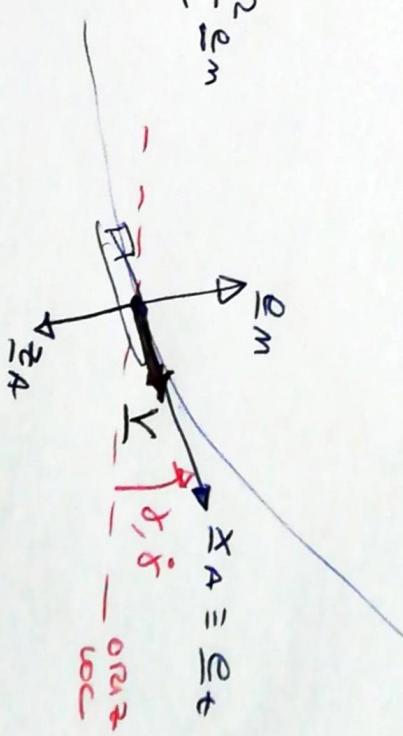
(I) \rightarrow come bilancio di potenze a meno di un fattore pari a V

$$m \ddot{V} = T - D - W \sin \gamma$$

(II)

$$\left\{ \begin{array}{l} \Sigma A \cdot \ddot{V} = -\frac{V^2}{R} = -\frac{V}{R} V = -\omega V \\ = -\ddot{\gamma} V \end{array} \right.$$

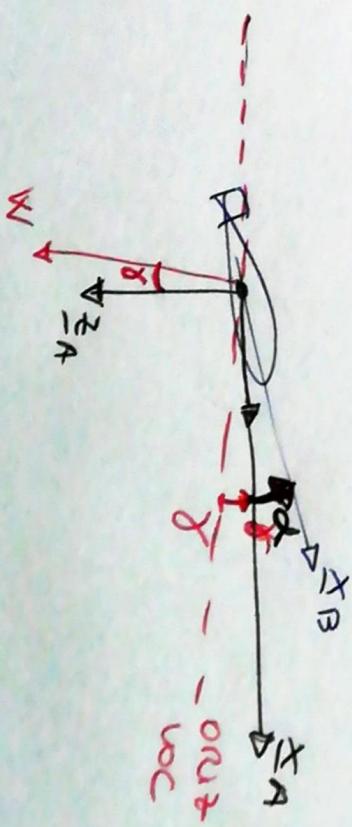
$$\ddot{V} = \ddot{V}_{eq} + \frac{V^2}{R} \omega_m$$



$\Sigma A \cdot F = -L$

$\Sigma A \cdot T = T \cos(\frac{\pi}{2} + \alpha) = -T \sin \alpha$

$\Sigma A \cdot W = W \cos(\alpha)$



$$\textcircled{II} - mV\dot{\gamma} = -L - T_{\text{airflow}} + W \cos\delta$$

$\boxed{H_p}$

$$\alpha \ll 1 \quad T_\alpha \ll L$$

$\frac{T_\alpha}{L}$ simile (o quasi) dello stesso volume
di grandezza (o $\frac{\alpha}{c}$)

$$mV\dot{\gamma} = L - W \cos\delta$$

\textcircled{III}

$$Y_A \cdot \left(\frac{H_{ca}}{M_{ca}} = \frac{M_{ca}}{M_{cg}} \right) \rightarrow J_Y \dot{\theta} = M_{cg}$$

momento
di inerzia
rispetto alle
asse di Beacheggio
 θ

VOLO
SIMM
+
PIANO
VERGAGE

$$\begin{cases} mV^0 = T - D - W \sin\delta \\ mV\dot{\gamma} = L - W \cos\delta \\ J_Y \ddot{\theta} = M_{cg} \end{cases}$$

+ MOTO STAZIONARIO

$$\frac{d(\cdot)}{dt} = 0$$

$\dot{V} = 0$ moto uniforme

$\delta = 0$ rettilineo

$\Theta = 0$ "centrato" o "trummatto"

(*)

$$T = D + W \sin \gamma$$

$$L = W \cos \gamma$$

$$M_{Ca} = 0$$

$$H = h \Rightarrow h = sep$$

— — — — —

volo uniforme e rettilineo

(*)

$$\frac{W}{T} = \frac{D}{W} + \gamma$$

$$L = \frac{4}{\pi}$$

$$\begin{cases} \frac{W}{T} = \frac{D}{W} + \gamma \\ \frac{W}{T} = \frac{1}{L} + \gamma \end{cases}$$

+ VOLO ORIZZONTALE $\delta = 0$

$$\theta_i = V_a \sin \delta = 0$$

$$\begin{cases} T = D \\ L = W \\ M_{Ca} = 0 \end{cases}$$

volo orizzontale
rettilineo
uniforme

$$\begin{cases} H = 0 \\ sep = 0 \end{cases}$$

+ VOLO ORIZZONTALE $\delta = 0$

$$\theta_i = V_a \sin \delta = 0$$

$$\begin{cases} T = D \\ L = W \\ M_{Ca} = 0 \end{cases}$$

volo orizzontale
rettilineo
uniforme

EQUILIBRIO IN V.O.R.U. (SIMMET.)

E STABILITÀ'

$$\left\{ \begin{array}{l} T = D \\ L = W \\ M_G = 0 \end{array} \right| \Rightarrow \begin{array}{l} D = D(p, V, \alpha) \\ L = L(p, V, \alpha) \\ T = T(p, V, \alpha, S_T) * \end{array}$$

G → BARICENTRO

$$M_G = M_G(p, V, \alpha)$$

l'unica che dipende da S_T
 \Downarrow
 $T = D$ può essere disaccoppiata

a valore della soluzione $L = W$ e $M_G = 0$
 possiamo sempre trovare un S_T
 che permette di realizzare $T = D$

(ASSUMIAMO DI POTER SEMPRE FARE
 OVVERO ABBIAMO UNA SPINTA
 "MOLTO ALTA")

$$\left\{ \begin{array}{l} L(\alpha, V, \alpha) = W \\ M_G(\alpha, V, \alpha) = 0 \end{array} \right.$$

INTRODUCIAMO
 LA V_{EAS}

$$\left\{ \begin{array}{l} L(V_{EAS}, \alpha) = W \\ M_G(V_{EAS}, \alpha) = 0 \end{array} \right. \quad \left. \begin{array}{l} \downarrow \\ \frac{1}{2} \rho V^2 = \frac{1}{2} \rho_0 V_{EAS}^2 \end{array} \right.$$

2 EQZ IN 2 INCognITE \Rightarrow 1 SOLUZIONE
 \hookrightarrow 1 SOLO
 POSSIBILITÀ
 DI VOLO

VEDI FIGL
 ALA ISOLATA

EQU. ALA ISOLATA

$$L = W \rightarrow \left\{ \begin{array}{l} L_{1/2} \alpha_A = W \\ M_G = 0 \end{array} \right.$$

$$\rightarrow \left\{ \begin{array}{l} M_G = m_{AC} - (x_G - x_{AC}) L_{1/2} \alpha_A = 0 \end{array} \right.$$

Centro aerodinamico

$$\Rightarrow \left\{ \begin{array}{l} x_G - x_{AC} = \frac{m_{AC}}{W} \\ \dots \end{array} \right.$$

$$x_G = x_{AC} + \frac{\frac{1}{2} \rho V_{EAS}^2 C_{mAC} c}{W/S}$$

→ COME PERMETTERE
IL SOLO SFACIMENTO
 $M_G = 0$ PER VARARE
VELOCITÀ DI VOLO (V_{EAS})

① CAMBIO x_G

- DELTA PIANO
(IL PILOTA SI SPOSTA)
AVANTI E INDIETRO

② CAMBIARE C_{mAC}

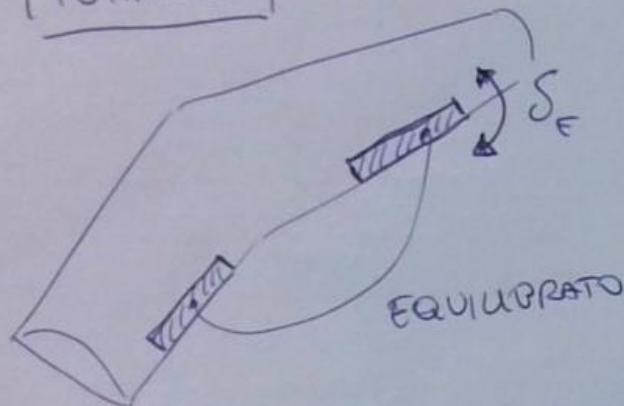
→ PROFILI DEFORMABILI

→ ULTERIORE SUPERFICIE
AERODINAMICA MOBILE

"EQUILIBRATO"

COSA È L'EQUILIBRATORE?

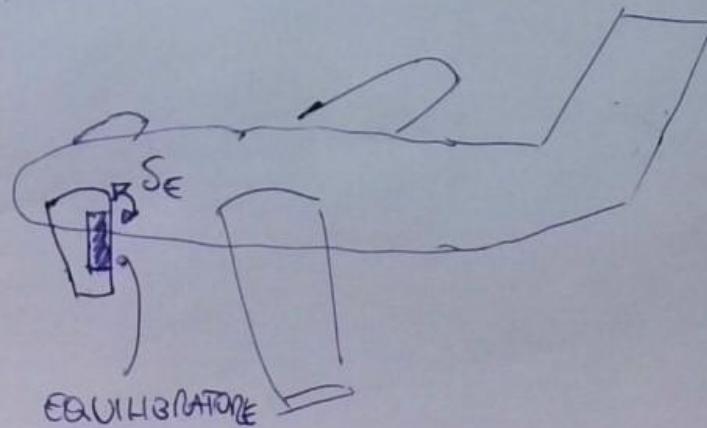
TUTTA-AUT



VELE VOLI CONVENZIONALI



VELIVOLI CANARD



S_E
STABIGATORI

LEGGI COSTITUTIVE CON L'EQUILIBRATORE

$$\begin{cases} L = L(\rho, V, \alpha, \delta_E) = L/\kappa \alpha + L_{/\delta_E} \delta_E + L_0 = L/\kappa (\alpha - \alpha_0) + L_{/\delta_E} (\delta_E - \delta_{E0}) \\ M_p = M_p(\rho, V, \alpha, \delta_E) = M_{p/\alpha} \alpha + M_{p/\delta_E} \delta_E + M_{p_0} = M_{p/\alpha} (\alpha - \alpha_0) + M_{p/\delta_E} (\delta_E - \delta_{E0}) \end{cases}$$

LEGGE NON OMOGENEA | LEGGE OMOGENEA

AERODIN.
LINEARE
(OK FINO ALL'
STALLO)

L_0 || FORZA E IL MOMENTO

M_{p_0} || PER $\alpha = 0$ $\delta_E = 0$

α_0 || INCIDENZA E DEFLESSIONE EQUILIB.
 δ_{E0} || TALI DA GENERARE PORTANZA NULLA
E MOMENTO NULLO

VANTO DI FORMA NON OMOGENEA

IN $\alpha > \alpha_0$ e $\delta_E = \delta_{E0}$

$$\begin{cases} L/\kappa \alpha_0 + L_{/\delta_E} \delta_{E0} = -L_0 \\ M_{p/\alpha} \alpha_0 + M_{p/\delta_E} \delta_{E0} = -M_{p_0} \end{cases}$$

|| SOLUZIONE
→

$$\alpha_0 = \frac{L_{/\delta_E} M_{p_0} - L_0 M_{p/\delta_E}}{\Delta_p}$$

$$\delta_{E0} = -\left(\frac{L_{/\alpha} M_{p_0} - L_0 M_{p/\alpha}}{\Delta_p} \right)$$

$$\Delta_p = L_{/\alpha} M_{p/\delta_E} - L_{/\delta_E} M_{p/\alpha}$$

TRAMMAGGIO LONGITUDINALE (CON EQUILIBRATORE)

$$\left\{ \begin{array}{l} L = w \\ M_G = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} L_{1/\lambda} (\alpha - \alpha_0) + L_{1/\delta_E} (\delta_E - \delta_{E0}) = w \\ M_{G/2} (\alpha - \alpha_0) + M_{G/2} (\delta_E - \delta_{E0}) = 0 \end{array} \right. \quad \left| \begin{array}{l} 2 \text{ EQUAZ} \\ 2 \text{ INCOGNITE} \rightarrow (\alpha - \alpha_0) \\ \rightarrow (\delta_E - \delta_{E0}) \end{array} \right.$$

$$(\alpha - \alpha_0) = \frac{M_{G/2}}{\Delta G} w$$

$$(\delta_E - \delta_{E0}) = - \left(\frac{M_{G/2}}{\Delta G} \right) w$$

$$\Delta G = L_{1/\lambda} M_{G/\delta_E} - L_{1/\delta_E} M_{G/\alpha}$$

$$\left| \begin{array}{l} \text{COEFF} \\ \rightarrow \end{array} \right| \quad \left| \begin{array}{l} (\alpha - \alpha_0) = \frac{w/s}{\frac{1}{2} \rho_0 V_{EAS}^2} \cdot \frac{C_m g / \delta_E}{\hat{\Delta} G} \\ (\delta_E - \delta_{E0}) = - \frac{w/s}{\frac{1}{2} \rho_0 V_{EAS}^2} \cdot \frac{C_m g / \alpha}{\hat{\Delta} G} \\ \hat{\Delta} G = C_{1/\lambda} C_m g / \delta_E - C_{1/\delta_E} \cancel{C_m g / \alpha} \end{array} \right.$$

CONSIDERAZIONI

- $\forall V_{EAS}$ ($\forall (q, v)$) TROVIAMO UN α e δ_E PER IL TRAMMAGGIO

- $V_{EAS} \uparrow \uparrow \Rightarrow \alpha - \alpha_0 \downarrow \downarrow$ (IN MODULO)
 $\delta_E - \delta_{E0} \downarrow \downarrow$
- $w/s \uparrow \uparrow \Rightarrow \alpha - \alpha_0 \uparrow \uparrow$ (IN MODULO)
 $\delta_E - \delta_{E0}$

• PUNTI CARATTERISTICI

(FORMULAZIONE ALLA BORRI)

$N \rightarrow$ PONTO NEUTRO (CENTRO AERODINAMICO DEL VECCHIO)

$$\exists N \text{ t.c. } M_{N/\alpha} = 0$$

$$M_N = m_p - (x_N - x_p)L \xrightarrow{\frac{d}{d\alpha}} \cancel{m_{N/\alpha}} = m_{p/\alpha} - (x_N - x_p)L_{1/\alpha}$$

$$\boxed{x_N - x_p := \frac{m_{p/\alpha}}{L_{1/\alpha}}}$$

$C \rightarrow$ PUNTO DI CONTROLLO

$$\exists C \text{ t.c. } M_{C/\delta_e} = 0$$

$$M_C = m_p - (x_C - x_p)L \xrightarrow{\frac{d}{d\delta_e}} \cancel{m_{C/\delta_e}} = m_{p/\delta_e} - (x_C - x_p)L_{1/\delta_e}$$

$$\boxed{(x_C - x_p) = \frac{m_{p/\delta_e}}{L_{1/\delta_e}}}$$

DA DEF

$$M_{P/L} = (x_N - x_P) L/2$$

$$M_{P/\delta_\epsilon} = (x_C - x_P) L/\delta_\epsilon$$

LEGAME COSTITUTIVO

$$\begin{cases} L = L_{12} (\alpha - \alpha_0) + L_{1S} (\delta_\epsilon - \delta_{\epsilon_0}) \\ M_P = \underbrace{(x_N - x_P) L_{12} (\alpha - \alpha_0)}_{M_{P/L}} + \underbrace{(x_C - x_P) L_{1S} (\delta_\epsilon - \delta_{\epsilon_0})}_{M_{P/\delta_\epsilon}} \end{cases}$$

LEGAME COSTITUTIVO EQUIVALENTE



SONO DIPENDENTI DA FORZE $(L_{12}(\alpha \dots), L_{1S}(\delta \dots))$
SENZA MOMENTI PURI

SCHEMA EQUIVALENTE A DUE FORZE

$$L^a := L_{1/2} (\alpha - \alpha_0)$$

Portanza di incidenza
(Attitude Lift)

[Quale parte delle portanze che dipende dalle incidenze]

$$L^c := L_{1/S_E} (S_E - S_{E_0})$$

Portanza di controllo
(control lift)

[Quale parte delle portanze che dipende dalle variabili di controllo S_E]

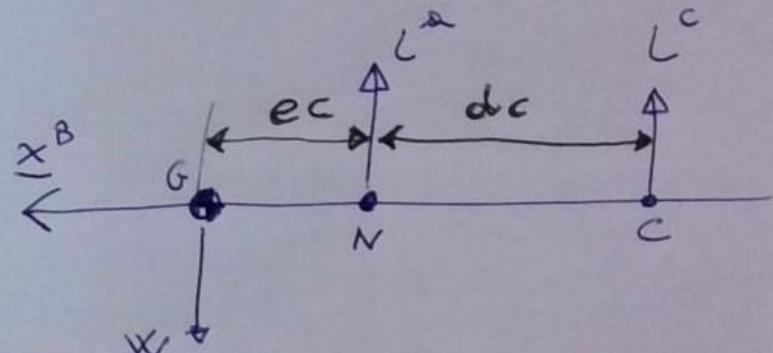
LEGAME COSTITUTIVO

$$\begin{cases} L = L^a + L^c \\ M_p = (x_N - x_e) L^a + (x_c - x_p) L^c \end{cases}$$

TRIM

$$\begin{cases} L^a + L^c = W \\ (x_N - x_G) L^a + (x_c - x_G) L^c = 0 \end{cases}$$

ATTENZIONE
(momento rispetto
BARICENTRO)



$$e = \frac{x_G - x_N}{c} \Rightarrow e = \bar{x}_G - \bar{x}_N$$

$$d = \frac{x_N - x_c}{c} \Rightarrow d = \bar{x}_N - \bar{x}_c$$

PARAMETRI DI
STABILITÀ ALTA-GORRI

$$\epsilon = \frac{e}{d}$$

MARGINE DI
STABILITÀ

LUNGHEZZA
AERODINAMICA
DEL VELIVOLI

$$\begin{cases} L^a + L^c = w \\ \epsilon L^a + (\epsilon + d) L^c = 0 \end{cases} \Rightarrow \begin{cases} \dots \\ \epsilon L^a + (\epsilon + 1) L^c = 0 \end{cases} \Rightarrow \begin{cases} \dots \\ L^c = -\frac{\epsilon}{\epsilon+1} L^a \end{cases}$$

$$\Rightarrow \begin{cases} L^a - \frac{\epsilon}{\epsilon+1} L^a = w \\ \dots \end{cases} \Rightarrow \begin{cases} \left(\frac{\epsilon+1}{\epsilon+1} - \frac{\epsilon}{\epsilon+1} \right) L^a = w \\ \dots \end{cases} \Rightarrow \boxed{\begin{cases} L^a = (1+\epsilon) w \\ L^c = -\epsilon w \end{cases}}$$

SOLUZIONE IN

TERMINI DI

PORTANZA DI INCIDENZA
E CONTRASLLO

$$\boxed{\begin{aligned} (d-d_0) &= \frac{x_g - x_c}{x_n - x_c} \frac{w}{L_{ds}} \\ (\delta_\epsilon - \delta_{\epsilon_0}) &= - \frac{x_g - x_n}{x_n - x_c} \frac{w}{L_{ds}} \end{aligned}}$$

SOLUZIONE IN TERMINI

DI d e δ_ϵ

STABILITÀ STATICA LONGITUDINALE (A COMANDI GLOCCATI)

→ Tendenze "in piccolo" a tornare verso la posizione di equilibrio dopo una perturbazione

- EQUILIBRIO $M_G(\alpha^*) = 0$

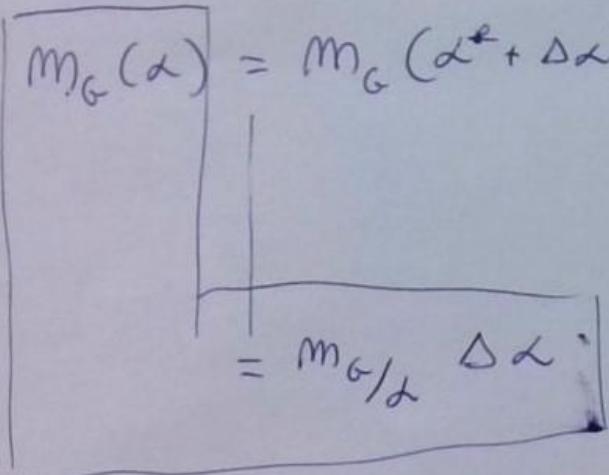
α^* angolo di insolvenza all'equilibrio.

- PERTURBAZIONE $\alpha = \alpha^* + \Delta\alpha$

δ_E COSTANTE

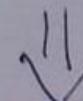
→ SVIWPPIRE IL MOMENTO AERODINAMICO CON TAYLOR

$$M_G(\alpha) = M_G(\alpha^* + \Delta\alpha) \cong M_G(\alpha^*) + \frac{M_G}{\alpha} \Big|_{\alpha=\alpha^*} \Delta\alpha + \dots$$



se $\Delta\alpha > 0$ STABILITÀ SE $\frac{M_G}{\Delta\alpha} \Delta\alpha < 0$

se $\Delta\alpha < 0$ STABILITÀ SE $\frac{M_G}{\Delta\alpha} \Delta\alpha > 0$



STABILITÀ SE $\frac{M_G}{\Delta\alpha} < 0$

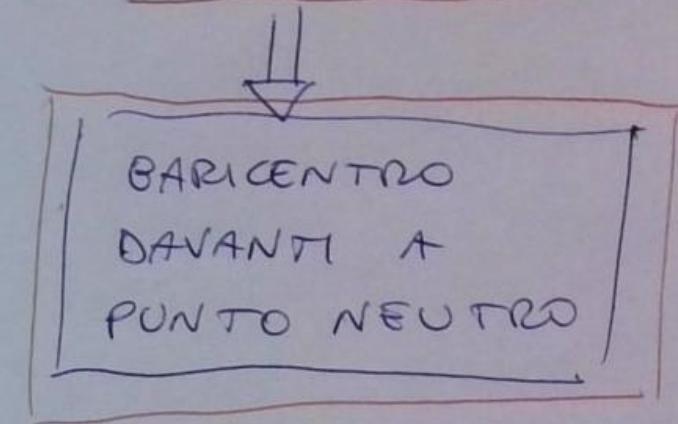
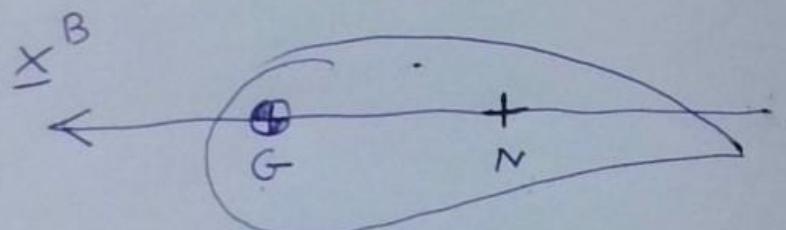
da legame costitutivo

$$M_G = L_{12} (\alpha - \alpha_0) (x_N - x_G) + L_{16} (\delta_\epsilon - \delta_{\epsilon_0}) (x_c - x_G)$$

$$\boxed{M_{G12} = L_{12} (x_N - x_G)} \rightarrow L_{12} > 0$$

Per la stabilità

$$x_N - x_G < 0$$



NOTE

- $\frac{x_G - x_N}{c} = e$ (da definizione) \Rightarrow se $e > 0$ velivolo stabile
 $e < 0$ velivolo instabile
(MARGINE DI STABILITÀ)

- N si chiama PUNTO NEUTRO proprio perché se G si trova in corrispondenza di N ha STABILITÀ NEUTRA.

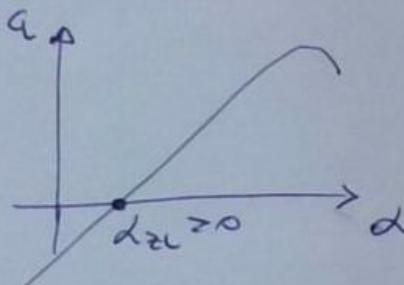
STABILITÀ E EQUILIBRIO DELL'ALA ISOCARTA

* EQL $\rightarrow M_G = 0 \Rightarrow x_G - x_{AC} = \frac{M_{AC}}{W}$ || $M_{AC} > 0$

* STAB $\rightarrow M_G/W < 0 \Rightarrow x_{AC} - x_G < 0$ || Per avere EQL + STAB

→ I PROFILI CON $M_{AC} \neq 0$ HANNO PROPRIETÀ AERODINAMICHE
NUOVAE

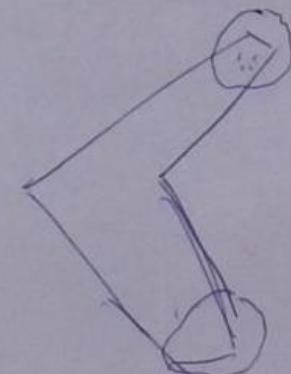
→ PROFILI CONCAVI
(PROFILI AUTO-STAB)



→ PROFILI REFLEX
(A DOPOIA CURVATURA)



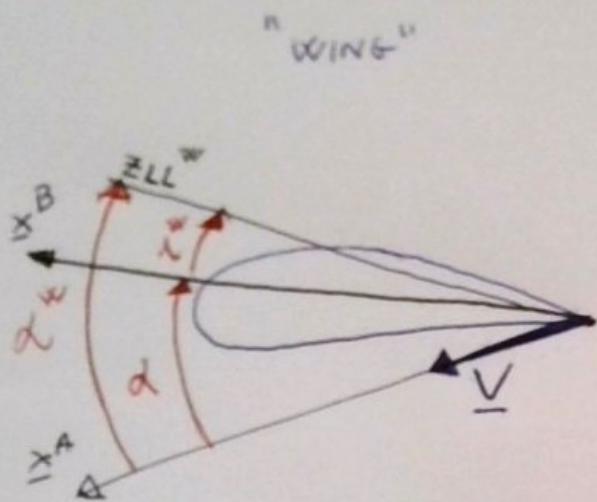
→ ALA A FRECCIA CON OPPORTUNO SVERGOLAMENTO



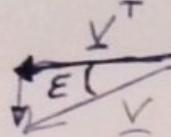
→ STABILIZZATORE

UTILIZZO UNA
SUPERFICIE AERODINAMICA
AGGIUNTIVA

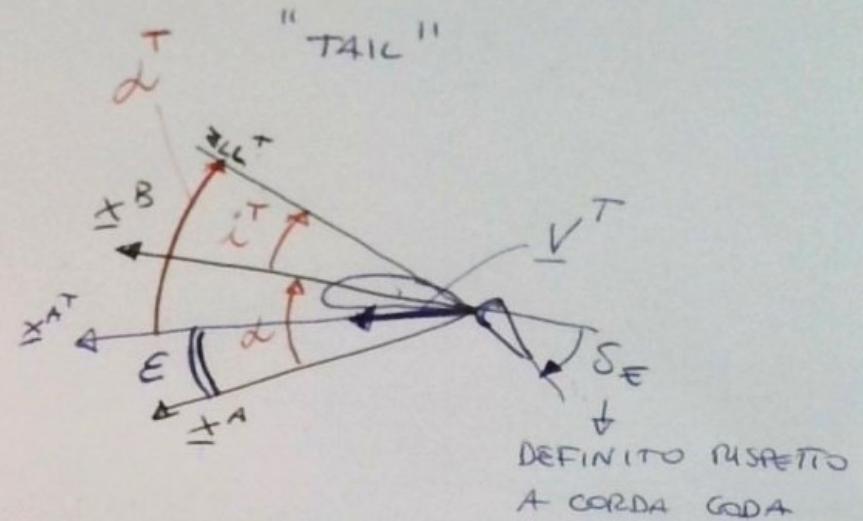
SCHEMA A DUE SUPERFICI



DOWNWASH



$$\epsilon = \epsilon_0 + \epsilon_{1/2} \alpha$$



α → angolo di incidenza del velivolo

$(\cdot)^w$ → VARIABILE RELATIVA ALLA ALA

$(\cdot)^T$ → VARIABILE RELATIVA ALLA CODA

$$\bullet q = q^w \quad \begin{cases} \text{PRESSIONE DINAMICA DI} \\ \text{RIFERIMENTO UOVALE} \\ \text{PRESSIONE DINAMICA ALA} \end{cases}$$

$$\bullet S = S^w \quad \begin{cases} \text{SUPERFICIE DI RIFERIMENTO} \\ \text{LA SUPERFICIE DI ALA} \end{cases}$$

$$\bullet L_{/\delta_e}^w = 0 \quad (\text{IPOTESI})$$

RAPPORTO TRA SUPERFICI

$$\bullet M = \frac{q^T}{q^w}; \quad G = \frac{S^T}{S^w} \quad ; \quad K = \frac{C^T}{C^w}$$

RAPPORTO
TRA CORDE
MEDIE
AERODINAMICHE

i^w → CALCOLO ALA
(INCIDENCE)

i^T → CALCOLO CODA

α^w → ANGOLO DI INCIDENZA ASSOLUTO
ALA ($\alpha^w = \alpha + i^w$)

ϵ = ANGOLO DI DOWN-WASH

$$\alpha^T \rightarrow \text{AoA CODA} \quad (\alpha^T = \alpha + i^T - \epsilon) \\ = \alpha + i^T - \epsilon_0 - \epsilon_{1/2} \alpha$$

PORTANZA

REMARK: $a := C_1/d$
 $b := C_1/s$

ESEMPPIO: $L^w = \frac{C_1}{d} \alpha^w$

rendenze
curve di
portanza dell'oleo

$$L = L^w + L^T$$

attenzione: $L^w \neq L^a!$
 $L^T \neq L^c!$

$$L^w = \underbrace{C_1}_{\alpha^w} d$$

$$L^c = \underbrace{C_0}_{\alpha^c}$$

$$L^w = q^w s^w \alpha^w \alpha^w = q^w s^w \alpha^w (\alpha^w + i^w) = q^w s^w \alpha^w \alpha^w + q^w s^w \alpha^w i^w$$

$$L^T = q^T s^T (a^T \alpha^T + b^T \beta_\epsilon) = q^T s^T (a^T (1 - \epsilon_{12}) \alpha^T + a^T (i^T - \epsilon_0) + b^T \beta_\epsilon)$$

$$= \eta \approx q^w s^w (a^T (1 - \epsilon_{12}) \alpha^T + a^T (i^T - \epsilon_0) + b^T \beta_\epsilon)$$

$$= L_{12}^T \alpha^T + L_{18}^T \beta_\epsilon + L_0^T$$

$$L_{12}^T = q^w s^w \eta \Gamma a^T (1 - \epsilon_{12})$$

$$L_{18}^T = q^w s^w \eta \Gamma b^T$$

$$L_0^T = q^w s^w \eta \Gamma a^T (i^T - \epsilon_0)$$

MOMENTI

$$M_p = M_p^w + M_p^T$$

$$\bullet M_p^w = M_{AC^w}^w - (x_p - x_{AC^w}) \underbrace{(L_{\alpha}^w \alpha + L_0^w)}_{L^w}$$

$$M_p^w = M_{p/\alpha}^w \alpha + M_{p_0}^w$$

$$M_{p/\alpha}^w = q^w s^w (x_{AC^w} - x_p) \alpha^w$$

$$M_{p_0}^w = M_{AC^w}^w + (x_{AC^w} - x_p) q^w s^w \alpha^w i^w$$

$$\bullet M_p^T = \underbrace{(M_{AC^T}^T)}_{-} - (x_p - x_{AC^T}) \underbrace{(L_{\alpha}^T \alpha + L_{/\delta_E}^T s_E + L_0^T)}_{L^T}$$

$$M_{AC^T/\delta_E}^T = q^T s^T c^T (m_{AC^T/\delta_E}^T)$$

$$\rightarrow M_{AC^T_0}^T + M_{AC^T/\delta_E}^T s_E$$

$\Rightarrow 0$ SE STABILIZATORE
 $\neq 0$ SE STABILIZZATORE + EQUILIBRATORE

$$M_p^T = M_{p/\alpha}^T \alpha + M_{p/\delta_E}^T s_E + M_{p_0}^T$$

$$M_{p/\alpha}^T = q^w s^w \gamma G (x_{AC^T} - x_p) (1 - \epsilon_{/\alpha}) \alpha^T$$

$$M_{p/\delta_E}^T = q^w s^w \gamma G [(x_{AC^T} - x_p) b^T + c^T \underbrace{(m_{AC^T/\delta_E}^T)}_{-}]$$

$$M_{p_0}^T = M_{AC^T_0}^T + (x_{AC^T} - x_p) \gamma G q^w s^w \alpha^T (i^T - \epsilon_0)$$

$$L = L_{1\alpha}^w + L_{/\delta_E}^{\delta_E} + L_0 \rightarrow \text{ADIMENSIONALIZ. } L = q^w s^w c_L \\ = q s c$$

$$\tilde{L}_{1\alpha} = \tilde{L}_{1\alpha}^w + \tilde{L}_{1\alpha}^T$$

$$\tilde{L}_{/\delta_E} = \tilde{L}_{/\delta_E}^T$$

$$\tilde{L}_0 = \tilde{L}_0^w + \tilde{L}_0^T$$

$$a = a_{1\alpha} \alpha + a_{/\delta_E} \delta_E + a_0$$

$$a_{1\alpha} = \alpha^w + \eta \sigma (1 - \varepsilon_{1\alpha}) \alpha^T$$

$$a_{/\delta_E} = \eta \sigma b^T$$

$$a_0 = \alpha^w i^w + \eta \sigma \alpha^T (i^T - \varepsilon_0)$$

$$M_p = M_{p/d} \alpha + M_{p/\delta_E} \delta_E + M_{p_0} \xrightarrow{\text{ADIM.}} M_p = q^w s^w c^w Cm_p$$

$$\underline{M_{p/d}} = \underline{M_{p/d}^w} + \underline{M_{p/d}^T}$$

$$\underline{M_{p/\delta_E}} = \underline{\cancel{M_{p/\delta_E}}} + \underline{M_{p/\delta_E}^T}$$

$$\underline{M_{p_0}} = \underline{M_{p_0}^w} + \underline{M_{p_0}^T}$$

$$Cm_p = Cm_{p/d} \alpha + Cm_{p/\delta_E} \delta_E + Cm_{p_0}$$

$$N.B. \quad \xi_{(c)} = \frac{x_{(c)}}{c^w}$$

$$Cm_{p/d} = (\xi_{AC^w} - \xi_p) \alpha^w + \eta \sigma (\xi_{AC^T} - \xi_p) b^T (1 - \varepsilon_{1/d}) \quad \rightarrow \kappa = \frac{c^T}{c^w}$$

$$Cm_{p/\delta_E} = \eta \sigma [(\xi_{AC^T} - \xi_p) b^T + (\kappa, Cm_{AC^T}/\delta_E)]$$

$$Cm_{p_0} = \eta \sigma (\kappa Cm_{AC^T_0} + (\xi_{AC^T} - \xi_p) \alpha^T (i^+ - \varepsilon_0)) + Cm_{AC^w} + (\xi_{AC^w} - \xi_p) \alpha^w i^w$$

CONSIDERAZIONI

NON-OMOGENEA

1) LEGAME COST.

$$\begin{cases} C_{1/\alpha} \lambda + C_{1/\delta_E} \delta_E + C_0 = C \\ C_{m_p/\alpha} \lambda + C_{m_p/\delta_E} \delta_E + C_{m_p,0} = C_{m_p} \end{cases} \quad \Rightarrow \quad \begin{cases} C = C_{1/\alpha} (\lambda - \lambda_0) + \\ + C_{1/\delta_E} (\delta_E - \delta_{E_0}) \\ C_{m_p} = C_{m_p/\alpha} (\lambda - \lambda_0) + \\ + C_{m_p/\delta_E} (\delta_E - \delta_{E_0}) \end{cases}$$

$C_{1/\alpha} (\lambda - \lambda_0) \rightarrow$ ADIMENSIONALIZZ. DELLA PORTANZA DI INCIDENZA

$$C_{1/\alpha} (\lambda - \lambda_0) = [a^w \cancel{\alpha} + \eta \sigma (1 - \varepsilon/\lambda) \alpha^+] (\lambda - \lambda_0)$$

PORTANZA DI INCIDENZA \neq PORTANZA A CA

$C_{1/\delta_E} (\delta_E - \delta_{E_0}) \rightarrow$ ADIMENSIONALIZZ. DELLA PORTANZA DI CONTROLLO

$$\hookrightarrow D = \eta \sigma b^T (\delta_E - \delta_{E_0})$$

PORTANZA CODA \neq PORTANZA DI CONTROLLO

2) STABILITÀ DEL VELIVOLO A DUE SUPERFICI

$$C_{mG/\alpha} < 0 \rightarrow C_{mP/\alpha} \Big|_{P \equiv G}$$

$$C_{mG/\alpha} = \underbrace{(\xi_{AC^w} - \xi_G) \alpha^w}_{\text{CONTRIBUTO ALA}} + \underbrace{(\xi_{AC^T} - \xi_G) \eta \sigma (1 - \epsilon_{1d}) \alpha^T}_{\text{CONTRIBUTO DELLA CODA}}$$

$$(\xi_{AC^T} - \xi_G)$$

$$\eta > 0$$

$$\sigma > 0$$

$$(1 - \epsilon_{1d}) > 0 \text{ perche' } \epsilon_{1d} \text{ numero piccolo}$$

$$\alpha^T > 0$$

SE CODA DIETRO < 0



CONTRIBUTO CODA NEGATIVO



CODA E' STABILIZZANTE

SE CODA E' DAVANTI (CANARD) > 0



CONTRIBUTO CANARD POSITIVO



CANARD E' INSTABILIZZANTE

NOTA BENE:

→ VELIVOLO STABILE SE
SOMMA CONTRIBUTI ALA E
CODA DA' $C_{mG/\alpha} < 0$

SCHEMA A DUE SUPERFICI

- FORMULAZIONE ALLA BORRI -

POSIZIONE PUNTO NEUTRO E PUNTO DI CONTROLLO

$$(x_N - x_p) := \frac{M_p / \alpha}{L_{1/2}} = \frac{(x_{AC^W} - x_p) \alpha^W + (x_{AC^T} - x_p) \eta G (1 - \epsilon_{\alpha}) \alpha^T}{\alpha^W + \eta G (1 - \epsilon_{\alpha}) \alpha^T}$$

PARAMETRO $\tau = \eta G (1 - \epsilon_{\alpha}) \frac{\alpha^T}{\alpha^W} \rightarrow 0 < \tau < 1$

> 0	> 0	prossimo
≤ 1	< 1	$\alpha \perp$

$$(x_N - x_p) := \frac{(x_{AC^W} - x_p) + (x_{AC^T} - x_p) \underbrace{\eta G (1 - \epsilon_{\alpha}) \frac{\alpha^T}{\alpha^W}}_{\tau}}{1 + \underbrace{(\eta G (1 - \epsilon_{\alpha}) \frac{\alpha^T}{\alpha^W})}_{\tau}} = \frac{(x_{AC^W} - x_p) + (x_{AC^T} - x_p) \tau}{1 + \tau}$$

$$= \frac{x_{AC^W} + \tau x_{AC^T}}{1 + \tau} - x_p \cancel{\frac{(1 + \tau)}{(1 + \tau)}} \Rightarrow \boxed{x_N - x_p = \frac{x_{AC^W} + \tau x_{AC^T}}{1 + \tau} - \cancel{x_p}}$$

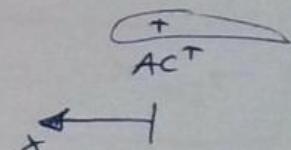
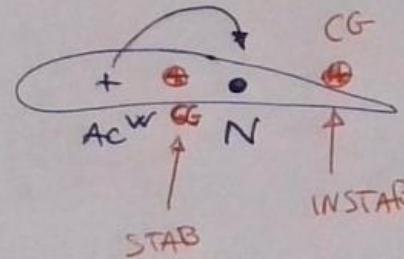
PUNTO NEUTRO NON DIPENDE DAL POGO
P USATO PER RIDURRE I MOMENTI

POSIZIONE	N	RISPETTO A	AC^W
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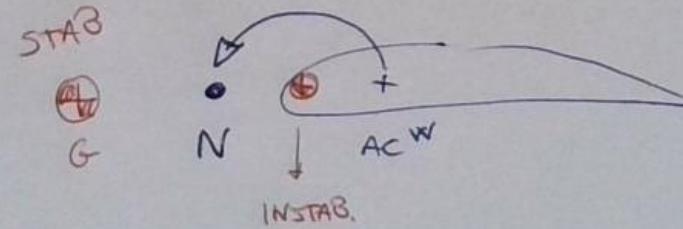
ASSUMIAMO $x_p = x_{ACT}$

$$x_N - x_{ACT} = \frac{x_{AC^W} - x_{ACT}}{1 + \tau} = \left(\frac{1}{1 + \tau} \right) (x_{AC^W} - x_{ACT}) < 1$$

VELIVOLO TRADIZIONALE



VELIVOLO CANARD



$$(x_N - x_{ACT}) = \left(\frac{1}{1 + \tau} \right) (x_{AC^W} - x_{ACT})$$

$$0 < \frac{1}{1 + \tau} < 1$$

$$\|x_N - x_{ACT}\| < \|x_{AC^W} - x_{ACT}\|$$

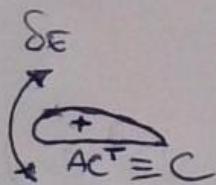
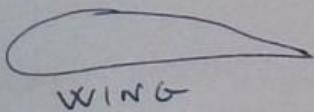
PUNTO CONTROLLO C^T

$$(x_c - x_p) := \frac{M_p / \delta_e}{L / \delta_e} = \frac{(x_{AC^T} - x_p) \gamma G b^T + \cancel{M_p / \delta_e} C^{AC^T} / \delta_e}{\gamma G b^T}$$

$$\boxed{x_c - x_p = x_{AC^T} - \cancel{x_p} + \cancel{\frac{C^{AC^T} / \delta_e}{b^T}}}$$

NON DIPENDE DA P.

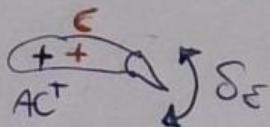
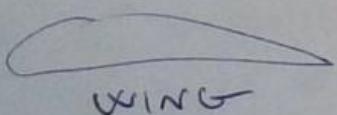
- STABILITÀ RE



$$C^{AC^T} / \delta_e = 0$$

$$\boxed{x_c = x_{AC^T}}$$

- STABILIZZATORE + EQUIVOLGATORE



$$C^{AC^T} / \delta_e \neq 0$$

$$\boxed{x_c \approx x_{AC^T}}$$

Perche' C^{AC^T} / δ_e
piccolo rispetto a b^T

SOLUZIONE ALL'EQUILIBRIO

$$(\alpha - \alpha_0) = \frac{x_G - x_C}{x_N - x_C} \frac{w/s}{\frac{1}{2} \rho_0 V_{EAS}^2}$$

$$(\delta_\epsilon - \delta_{\epsilon_0}) = - \frac{x_G - x_N}{x_N - x_C} \frac{w/s}{\frac{1}{2} \rho_0 V_{EAS}^2}$$

Relazione lineare
tra α e δ_ϵ

$$\frac{1}{a/\alpha}$$

$$\frac{1}{a/s}$$

$$\Rightarrow \frac{(\delta_\epsilon - \delta_{\epsilon_0})}{(\alpha - \alpha_0)} = - \frac{x_G - x_N}{x_G - x_C} \frac{a/\alpha}{a/s}$$



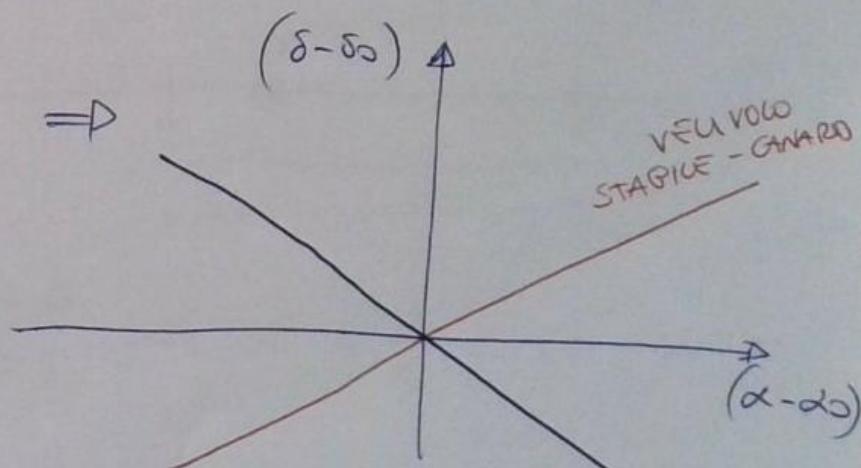
$$\delta_\epsilon - \delta_{\epsilon_0} = \left(\frac{x_G - x_N}{x_G - x_C} \frac{a/\alpha}{a/s} \right) (\alpha - \alpha_0)$$

$$\delta_\epsilon = A\alpha + \delta_{\epsilon_0} - A\alpha_0$$

$$(\delta - \delta_0)$$

CODA POSTERIORE	$\rightarrow x_G - x_C > 0$
	$\rightarrow x_N - x_C > 0$
STABILE	$\rightarrow x_G - x_N > 0$
CANARD	$\rightarrow x_G - x_C < 0$ $\Rightarrow A > 0$

$$\Rightarrow A < 0 \Rightarrow$$



(*) $L^\alpha = (1+\epsilon)W$

$$L^C = -\epsilon W = \frac{-\epsilon}{1+\epsilon} L^\alpha$$

$$L_s (\delta_\epsilon - \delta_{\epsilon_0}) = -\frac{\epsilon}{1+\epsilon} L/\alpha (\alpha - \alpha_0)$$

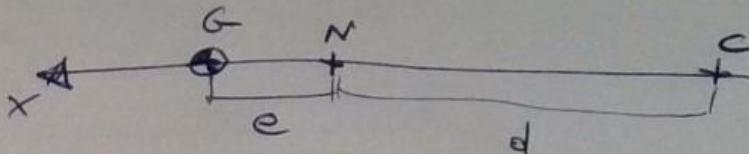
$$\delta_\epsilon - \delta_{\epsilon_0} = -\frac{\epsilon}{1+\epsilon} \frac{L/\alpha}{L/s} (\alpha - \alpha_0) = -\frac{\epsilon}{1+\epsilon} \frac{a/\alpha}{a/s} (\alpha - \alpha_0)$$

VEUVOLO
STABILE
 ϵ IN CONFIGUR.
TRAD.

ANALISI STABILITÀ E IL TRIM PER DIVERSI VELIVOLI (Per TIPOLOGIA)

→ VELIVOLO DUE SUPERFICI

- (A) → STABILE
- CONFIGURAZ. TRADIZIONALE



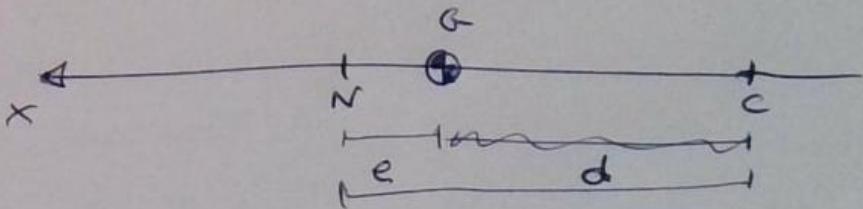
$$\frac{e}{d} > 0 \Rightarrow \epsilon = \frac{e}{d} > 0$$

$$L^a = (1 + \epsilon) W \Rightarrow L^a > W$$

$$L^c = -\epsilon W \Rightarrow L^c < 0$$

— x —

- (B) → INSTABILE
- CONF. TRADIZIONALE



$$\text{INSTAB} \Rightarrow \epsilon < 0$$

$$\frac{e}{d} > 0 \Rightarrow \epsilon = \frac{e}{d} < 0$$

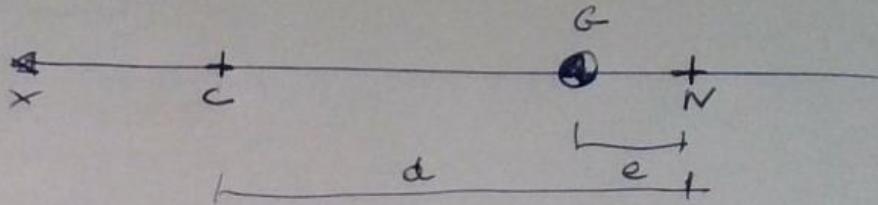
$$L^a < W !!$$

$$L^c > 0 !!$$

c)

→ STABILE

→ CANARD



~~$e > 0$~~ $e > 0$ || $\Rightarrow \epsilon = \frac{e}{d} < 0$
 $d < 0$

$$\begin{cases} L^a < W \\ L^c > 0 \end{cases}$$

MOLTO INTERESSANTE

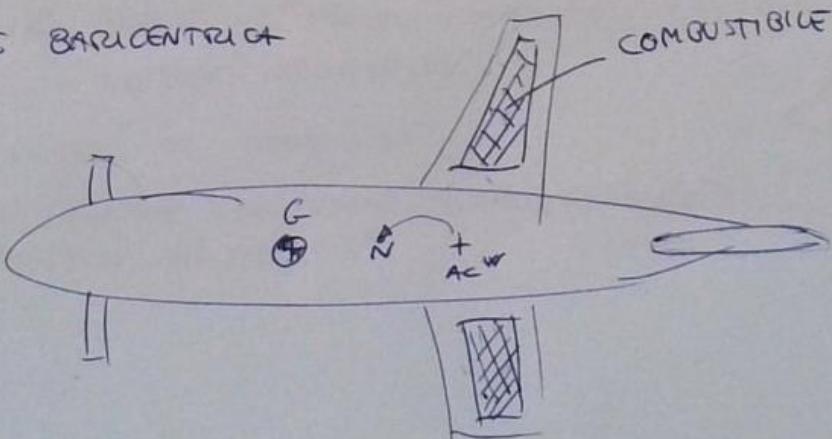
→ vantaggio del velivolo instabile ($L^a < W$)

→ STABILE

Perché ci sono pochi aerei canard?

1 → CANARD MI GENERA UN DOWNWASH SULL'ALA

2 → ESCURSIONE BAROCENTRICA



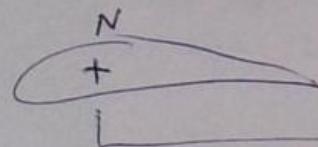
NOTA STABILITÀ ED EQUILIBRIO

- Velivolo STABILE E CON Coda POSTERURO

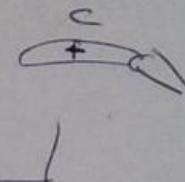
$$L^d = (1+\epsilon)W$$

$$L^c = (-\epsilon)W$$

$$\epsilon = \frac{e}{d} > 0$$

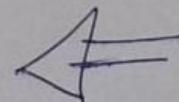


d



Per d che aumenta (fissato e) ottieni che L^d è diminuita

- A LIVELLO DI TRIM
CONVIENE AVERE L'EQUILIBRATORE IL PIÙ INDIETRO POSSIBILE
- A LIVELLO DI STABILITÀ
CONVIENE AVERE LA CODA (STABILIZZATORE) PIÙ INDIETRO POSSIBILE



L^d diventa più bassa ma sempre restando più maggiore del PESO

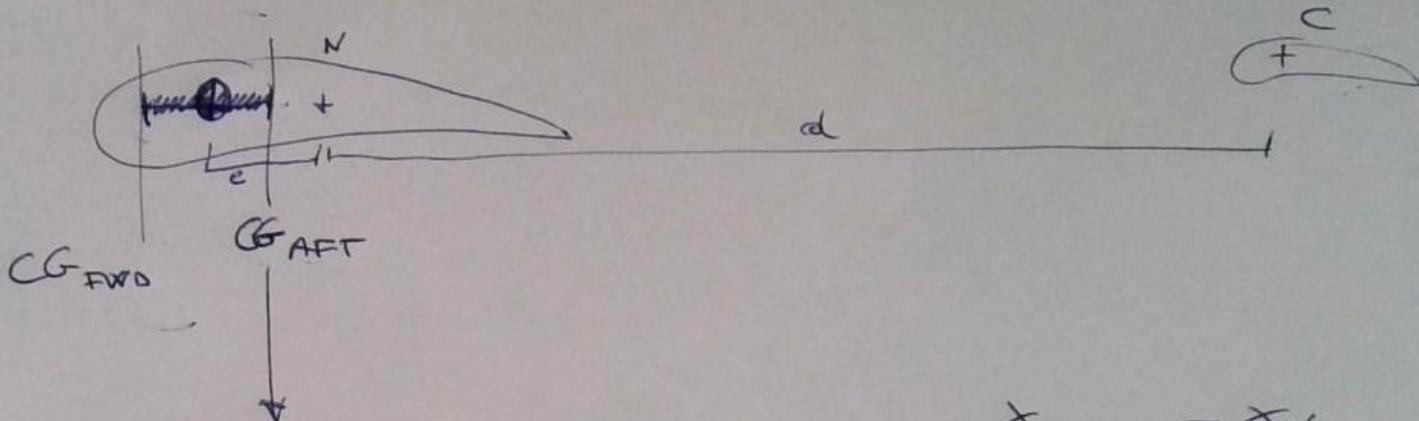
L^c sono sempre negative ma con modulo inferiore.

NOTA SU ESCURSIONE BARICENTRICA

Combinando

$x_{CG} \rightarrow$ VARIA $\epsilon \rightarrow$ VARIANO

L^a, L^c

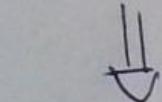


FWD $\rightarrow \underline{\epsilon}$ MASSIMO

LIMITE POSTERIORE \Rightarrow MARGINE DI STABILITÀ MINIMA 5% \rightarrow $\frac{x_{CG_AFT} - x_N}{C} > 0.05$

L^a PIÙ ALTA

$|L^c|$ PIÙ ALTA



$(d - d_0)$ È PIÙ ALTO



PIÙ RESISTENZA

\Rightarrow

CG_{FWD}

CONDIZIONE PIÙ GRAVOSA

PORTANZA E POLARE TRIMMATA

POSSO ESEMPLIFICARE LA PORTANZA E LA POLARE IN MODO DA INCLUDERE AL PROPRIO INTERNO L'EFFETTO DELLA VARIABILE DI CONTROLLO δ_e ?

$$\left\{ \begin{array}{l} L = L(V_{EAS}, \alpha, \delta_e) \\ D = D(V_{EAS}, \alpha, \delta_e) \\ M_G = M_G(V_{EAS}, \alpha, \delta) \end{array} \right. \quad \rightarrow \quad M_G = 0 \Rightarrow M_G(V_{EAS}, \alpha, \delta_e) = 0$$

(**) \downarrow

$$\boxed{\delta_e^* = \delta_e^*(V_{EAS}, \alpha)}$$

NOTA: "*" INDICA CHE L'EQUILIBRIO AI MOMENTI È VERIFICATO.

$$\left\{ \begin{array}{l} L = L(V_{EAS}, \alpha, \delta_e^*(V_{EAS}, \alpha)) = L^*(V_{EAS}, \alpha) \\ D = D(V_{EAS}, \alpha, \delta_e^*(V_{EAS}, \alpha)) = D^*(V_{EAS}, \alpha) \end{array} \right. \quad \rightarrow \quad \alpha = \alpha^*(V_{EAS}, L^*)$$

$$D^* = D^*(V_{EAS}, \alpha^*(V_{EAS}, L^*)) = D^*(V_{EAS}, L^*)$$

→ PASSO AI COEFFICIENTI

$$\left(\frac{1}{\frac{1}{2} \rho V_{EAS}^2 S} \right)$$

POLARE TRIMMATA

$$C_D = C_D^*(C_L, \text{Mach}, \text{Re})$$

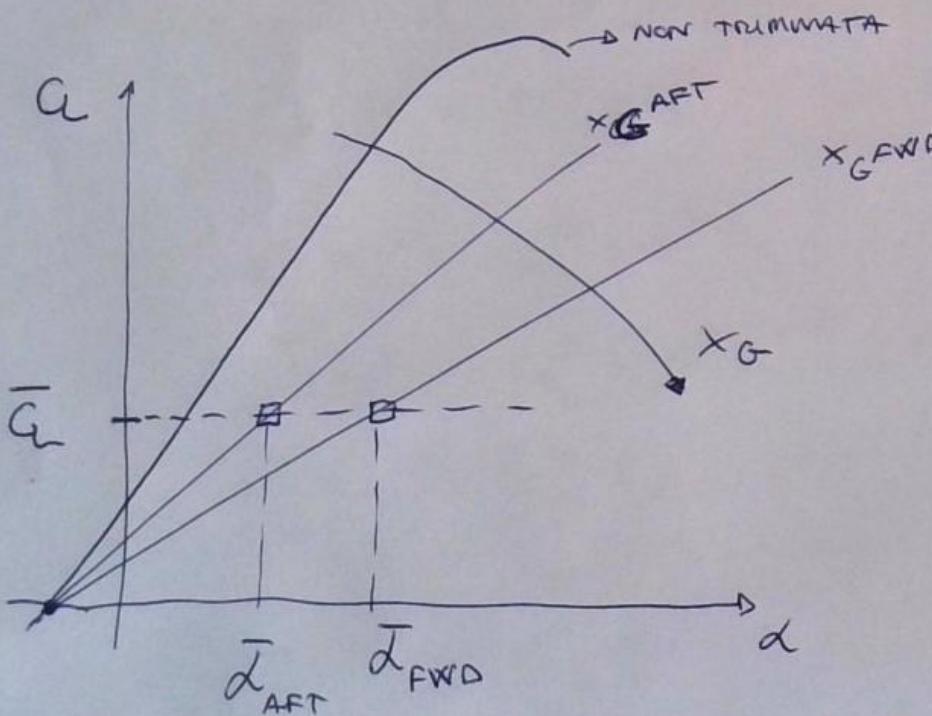
~~PER~~ [PORTANZA TRIMMATA]

$$L^* + L^c = L = \underbrace{\frac{1}{1+\epsilon} L^*}_{L^*} \Rightarrow L = \left(\frac{1}{1+\epsilon} L^* \right) (\alpha - \alpha_0)$$

L^*

$$\Rightarrow L = \underbrace{L^*}_{\frac{1}{1+\epsilon} L^*} (\alpha - \alpha_0)$$

$$L^* = \frac{1}{1+\epsilon} L_{1/2} \Rightarrow \boxed{a_{1/2}^* = \frac{1}{1+\epsilon} a_{1/2}} \Rightarrow \boxed{a_{1/2}^* = a_{1/2} \left(\frac{x_N - x_C}{x_G - x_C} \right)}$$



PIÙ AVANZA IL BARICENTRO
PIÙ SARÀ ALTO L'ANGOLONE DI
INCLINAZIONE CHE MI SERVE PER
GENERARE UN CERTO a
(È CONTEMPORANEAMENTE
~~VERIFICARE~~ L'EQUILIBRIO AI MOMENTI)
SODDISFARE

POLARE TRIMMATA

→ CASO VEROVOL 2 SUPERFICI

1) CONSIDERARE UN CERTO CG

→ ATTRAVERSO IL LEGAME COSTITUTIVO, CALCOLA α_0 , δ_{∞} | TACI DA ANNNULLARE C E C_{mg}

$$2) G_{1/\lambda}^* = G_{1/\lambda} \left(\frac{x_N - x_C}{x_0 - x_C} \right)$$

3) Per ogni λ

3.1) $D = D^W + D^T$ Resistenze sono la somma contributi ole e coda.

$$3.2) G_D^W = G_{D_0}^W + K^W a^W \lambda^2 \rightarrow (\text{POLARE ANALITICA ACA})$$

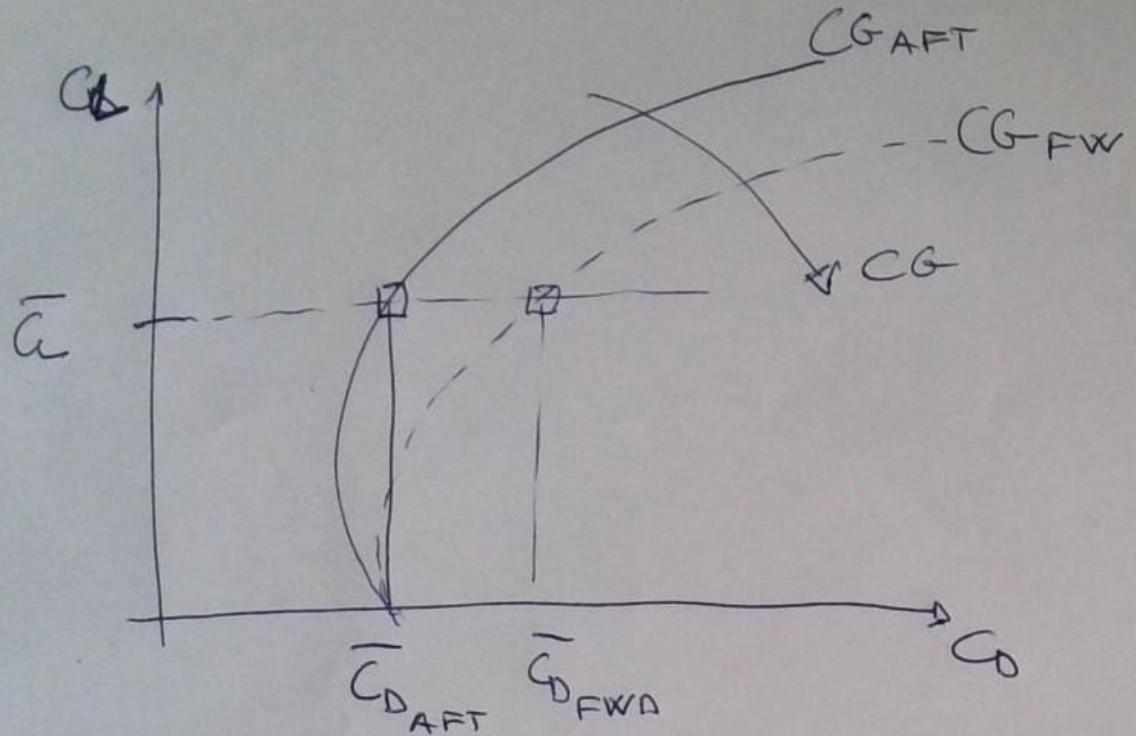
$$3.3) G_D^T = G_{D_0}^T + K^T a^T \lambda^2 \rightarrow (\text{POLARE ANALITICA CODA})$$

$$\bullet a^W = a_{1/\lambda}^W (\lambda + i^W)$$

$$\bullet a^T = a_{1/\lambda}^T (\lambda + i^T - c) + \underline{\delta_{1/\lambda}} \quad \text{REMARK: } \overbrace{\delta = A\lambda - A\alpha_0 + \delta_0}^{\text{DA EQUILIBRIO}}$$

$$= a_{1/\lambda}^T (\lambda + i^T - \varepsilon) + \underline{\delta_{1/\lambda}} (A\lambda - A\alpha_0 + \delta_0)$$

$$4) \nexists \dots = G_D = G_D(\lambda, \lambda^2, G_{D_0}) \text{ and } \begin{aligned} & \text{SICCOME } G_L \propto \lambda \Rightarrow \\ & \Rightarrow \text{POLARE ANALITICA PARABOLICA} \end{aligned}$$



$$\overline{C}_{D_{FW}} > \overline{C}_{D_{AFT}}$$

• PIÙ AVANZATO È IL
BARICENTRO

PIÙ ALTA SARÀ LA
RESISTENZA.

\Downarrow
AUMENTANO I CONSUMI !!

PER PERFORMANCE DA ORA IN POI USEREMO LA NOZIONE
DI POLARIS / PORTANZA TRAMMATA.

ESERCITAZIONE SU EQUILIBRIO E STABILITÀ

$$\begin{aligned} \textcircled{1} \quad S &= 87 \text{ m}^2 \\ C_{1d} &= 5.65 \text{ rad}^{-1} \quad C_{mg/d} = -0.82 \text{ rad}^{-1} \\ C_{1\delta} &= 0.38 \text{ rad}^{-1} \quad C_{mg/\delta_e} = -1.61 \text{ rad}^{-1} \\ C_0 &= -0.12 \quad C_{mg_0} = 0.128 \end{aligned}$$

TUTTO CIÒ CHE SERVE PER DEFINIRE LA LEGGE COSTITUTIVA (FORMA NON OMogenea)

$$\textcircled{*} \quad \text{DETERMINARE } \alpha \text{ E } W \text{ TALI DA AVERE EQUILIBRIO A } V_{EAS} = 125 \text{ m/s} \\ \epsilon \delta_e = 1.7^\circ = 0.02967 \text{ rad}$$

$$\text{EQL MOMENTI} \rightarrow C_{mg/d} \alpha + C_{mg/\delta_e} \delta_e + C_{mg_0} = 0 \Rightarrow \text{CONOSCO TUTTO TRAMME } \alpha$$

$$\alpha = \frac{-C_{mg/\delta_e} \delta_e + C_{mg_0}}{C_{mg/d}} = \frac{-(0.02967(-1.61) + 0.128)}{-0.82} = \\ = 0.09784 \text{ rad} = 5.6059 \text{ deg}$$

Conoscendo α e S ricavo C

$$C = C_{1d} \alpha + C_0 \delta + \xi_0 = 0.446$$

DAL Q RICAVO IL PESO (ATTENZIONE)
 V_{EAS}

$$W = L = \frac{1}{2} \rho_0 V_{EAS}^2 S C = 369,76 \text{ kN}$$

$$\begin{aligned}
 \textcircled{2} \quad S^w &= 87 \text{ m}^2 & q_{AC^w} &= -0.038 & M &= \frac{q^+}{q^w} = 0.95 \\
 S^T &= 19.25 \text{ m}^2 & \xi_{AC^+} &= -4.8 & C &= \frac{S^T}{S^w} \\
 a_{12}^w &= a^w = 4.70 \text{ rad}^{-1} & \epsilon_{12} &= 0.35 & \text{calcolare} \\
 a^+ &= 3.32 \text{ rad}^{-1} & & & \text{downwash w.r.t. AoA} \\
 C_{12}^T & & & &
 \end{aligned}$$

• LE COORDINATE SONO ADIMENSIONALIZZATE RISPETTO MAC DELL'AUT
 • ASSE X PUNTA VERSO LA PRNA.

(*) DETERMINARE LA POSIZIONE ADIMENSIONALE DEL BARICENTRO
 TALE DA FAR OTTENERE UN MARGINE STATICO DEL 16% (0.16)

$$\text{MARGINE STATICO} \Rightarrow S.M. = \xi_G - \xi_N \stackrel{\substack{\uparrow \\ \text{DA OTTENERE}}}{=} 0.16 \quad \rightarrow \text{OCORRE} \quad \text{CALCOLARLO}$$

$$C_{mN/2} = a_{12}^w (\xi_{AC^w} - \xi_p) + \eta C a_{12}^+ (1 - \epsilon_{12}) (\xi_{AC^+} - \xi_p)$$

$$\text{SE } p = N \Rightarrow C_{mN/2} = 0$$

$$\begin{aligned}
 a_{12}^w (\xi_{AC^w} - \xi_N) + \eta C a_{12}^+ (1 - \epsilon_{12}) (\xi_{AC^+} - \xi_N) &= 0 \Rightarrow \text{UNICA INCognITA} \\
 \xi_N &= \frac{a_{12}^w \xi_{AC^w} + \eta C a_{12}^+ (1 - \epsilon_{12}) \xi_{AC^+}}{a_{12}^{w*} + \eta C a_{12}^+ (1 - \epsilon_{12})} = -0.457 \quad | \quad \xi_G = \xi_N + 0.16 = \\
 &\quad | \quad = -0.297
 \end{aligned}$$

(3)

$$S^* = 86.8 \text{ m}^2$$

$$\epsilon_{12} = 0.35$$

$$\xi_{ACT} = -6.308$$

$$S^T = 18.4 \text{ m}^2$$

$$\eta = 0.95$$

$$\xi_G = -0.254$$

$$\omega_w = \alpha_w = 4.88 \text{ rad}^{-1}$$

$$\xi_{ACW} = -0.039$$

TRIPOLOGIA CODA \rightarrow STABILITORE

$$\omega^T = \alpha^T = 3.39 \text{ rad}^{-1}$$

$$\xi_{ACT} = \xi_{ABC}$$

$$\omega_{12} = 0.42 \text{ rad}^{-1}$$

$$\delta_0 = -0.6 \text{ deg} \quad S_{E_0} = 4.1 \text{ deg}$$

$$\xi_{ACT} = \xi_{ABC}$$

* DETERMINARE MARGINE STATICO, w , δ + c. EQL $\alpha = 2.1 \text{ deg}$, $V_{EAS} = 160 \text{ m/s}$

→ TROVARE I COEFF. DEL LEGAME COSTITUTIVO

$$\omega_{12} = \omega_w + \eta G(1 - \epsilon_{12}) \omega_{12}^T = 5.32 \text{ rad}^{-1}$$

$$Gm_{G/12} = \omega_w (\xi_{ACW} - \xi_G) + \eta G(1 - \epsilon_{12}) \omega_{12}^T (\xi_{ACT} - \xi_G) = \frac{-0.766 \text{ rad}^{-1}}{\downarrow \text{STAT}}$$

$$\xi_N - \xi_G = \frac{Gm_{G/12}}{\omega_{12}} \Rightarrow \xi_G - \xi_N = -\frac{Gm_{G/12}}{\omega_{12}} = 0.1438 \Rightarrow \underline{14,38 \%}$$

$$\rightarrow \xi_C - \xi_G = \frac{Gm_{G/12}}{\cancel{\omega_{12}}} \Rightarrow Gm_{G/12} = \omega_{12} (\xi_C - \xi_G) = -1.704 \text{ rad}^{-1}$$

$$\rightarrow \text{IMPONGO EQL AI MOMENTI} \Rightarrow Gm_{G/12} (\alpha - \alpha_0) + Gm_{G/12} (\delta_E - \delta_0) = 0 \Rightarrow$$

$$\delta_E = \delta_{E_0} - \frac{Gm_{G/12}}{Gm_{G/12}} (\alpha - \alpha_0) = 2.9 \text{ deg}$$

→ AVENDO $\alpha \in \delta$ POSSO CALCOLARE IL a

$$a = a_{12}(\alpha - \alpha_0) + a_{1\delta_E}(\delta_E - \delta_{E0}) = 0.2420$$

$$\rightarrow W = \frac{1}{2} \rho_0 V_{EAS}^2 S/G = \underline{\underline{329.3678 \text{ KN}}} \quad \xrightarrow{\text{VARIANZA DI ADIMENSIONALIZ.}} \text{PARI A QUELLA DELL'ACT}$$

$$S = S^W$$

④ NOTI $w, s, a_{12}^*, a_{12}^+, a_{12}^T, \sigma^T, M, \epsilon_{12}, \xi_{ACT}, \xi_{ACT}^+, a_{12}^*$
 a_{12}^*, V_{EAS}

* $L^a, L^c, (\alpha - \alpha_0), (\delta - \delta_0) \Rightarrow$ CI SERVE $\epsilon = \frac{e}{d}$

NOTA $a_{12}^* = a_{12} \left(\frac{1}{1+\epsilon} \right)$

$$a_{12} = a_{12}^w + M^T \epsilon (1 - \epsilon_{12}) a_{12}^T \rightarrow \text{HO TUTTO, POSSO VANTARLA}$$

$$a_{12}^+ = a_{12} \left(\frac{1}{1+\epsilon} \right) \rightarrow \text{CONOSCO TUTTO TRAMMENTE } \epsilon \Rightarrow \text{CALCOLO } \epsilon$$

$$L^a = (1+\epsilon) W \rightarrow \text{HO TUTTO POSSO CALCOLARE } L^a, L^c$$

$$L^c = -\epsilon W$$

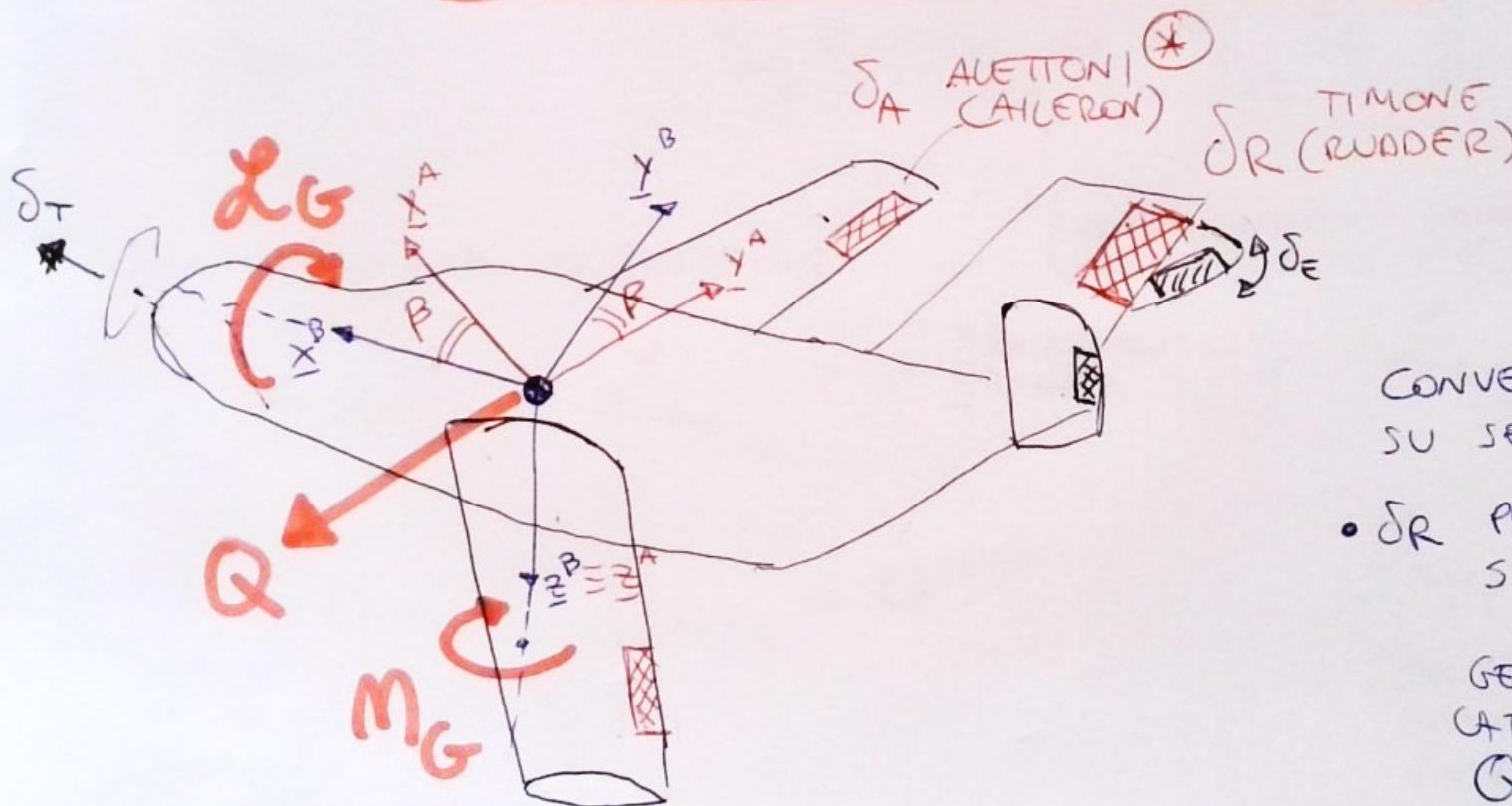
$$(\alpha - \alpha_0) = \frac{L^a}{L_{12}}, L_{12} = \frac{1}{2} \rho_0 V_{EAS}^2 S a_{12} \rightarrow \text{HO TUTTO, CALCOLO } (\alpha - \alpha_0)$$

$$(\delta_e - \delta_{e_0}) = \frac{L^c}{L_{18}} , \quad L_{18} = \frac{1}{2} \rho_0 V_{EAS}^2 \sigma_{18} , \quad \omega_{18} = \eta \sigma \omega_{18}^T$$

] HO TUTTO
POSso ACCOMPARRE
 $(\delta_e - \delta_{e_0})$

VOLO NON SIMMETRICO $(\beta \neq 0)$

EQUILIBRIO / CONTROLLO / STABILITÀ



MOVIMENTO ASIMMETRICO
 $\Delta x \uparrow \Rightarrow S_x \downarrow$

CONVENZIONE
SU SEGNI DEI CONTROLLI

- S_R POSITIVA SE VERSO SINISTRA
 \Downarrow
 GENERA UN FORZA LATERALE VERSO DESTRA
 $Q < 0$

- S_A POSITIVO SE AZIONE DESTRA VERSO IL BASSO
 $L_A < 0$

• LEGAME COSTITUTIVO PER L'AERODINAMICA CATERODIREZIONALE

$$\left\{ \begin{array}{l} Q = Q(\beta, \delta_A, \delta_R) = Q_{/\beta} \beta + Q_{/\delta_A} \delta_A + Q_{/\delta_R} \delta_R \\ L_p = L_p(\beta, \delta_A, \delta_R) = L_{p/\beta} \beta + L_{p/\delta_A} \delta_A + L_{p/\delta_R} \delta_R \\ M_p = M_p(\beta, \delta_A, \delta_R) = M_{p/\beta} \beta + M_{p/\delta_A} \delta_A + M_{p/\delta_R} \delta_R \end{array} \right. \quad \Rightarrow \quad \begin{array}{l} \text{MODELLO} \\ \text{LINEARE} \\ \text{OMOGENEO} \end{array}$$

DIAMO UN'OCCHIATA AI COEFFICIENTI ... (CONSIDERIAMO SOLO LA DERIVA)

$$L_{G/\delta_A} < 0$$

$$L_{G/\delta_R} > 0 \quad (\text{perché deriva in alto})$$

$$L_{G/\beta} < 0$$

$M_{G/\delta_A} > 0$
 (per lo sbilanciamento
 di resistenza indotta
 dalla portanza differente
 tra ala dx e sx)

$$M_{G/\delta_R} < 0$$

$$M_{G/\beta} > 0$$

(per effetto delle
 forze generate dalle
 derive)

$$Q_{/\delta_A} = 0$$

$$Q_{/\delta_R} < 0$$

$$Q_{/\beta} > 0$$

EQUILIBRIO IN VOLO A RETTILINEO CON $\beta \neq 0$ E $\phi \neq 0$

LATERODIREZIONALI

$$\begin{cases} Q = W \sin \phi \\ L_G = 0 \\ M_G = 0 \end{cases}$$

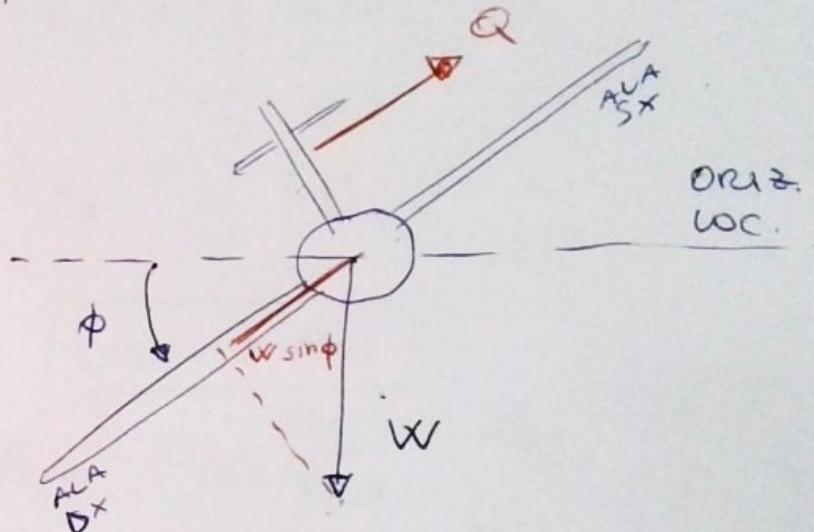


LONGITUDINALI

$$\begin{cases} T = \Delta \\ M_G = 0 \\ L = W \cos \phi \end{cases}$$

(SEMPRE
VERIFICATA)

VISTA DAL DAVANTI



$$① \frac{Q}{\rho} \beta + \frac{Q}{\rho S} \delta R = W \sin \phi$$

$$② \frac{L_G}{\rho} \beta + \frac{L_G}{\rho S} \delta A + \frac{L_G}{\rho S} \delta R = 0$$

$$③ \frac{M_{G/\rho}}{\rho} \beta + \frac{M_{G/S}}{\rho S} \delta A + \frac{M_G}{\rho S} \delta R = 0$$

$\Rightarrow \begin{cases} 3 \text{ EQZ} \end{cases}$

GIRNOGNITE ($\beta, \delta A, \delta R, \phi$)

INFINITE SOLUZIONI \Rightarrow INFINITE MODI

DI VOLARE

IN MANNERA

NON SIMMETRICA

SOLUZIONE

$$\textcircled{1} \quad \delta_R = \frac{w \sin \phi}{Q/\delta_R} - \frac{\partial \gamma P}{\partial \delta_R} \cancel{P} = (\delta R_0(\phi)) + (\delta R_1(\phi))$$

$$\textcircled{2} \quad \left\{ \begin{array}{l} L_{G/P} \beta + L_{G/\delta A} \delta A + L_{G/\delta R} [\delta R_0(\phi) + \delta R_1(\phi)] = 0 \\ M_{G/P} \beta + M_{G/\delta A} \delta A + M_{G/\delta R} [\delta R_0(\phi) + \delta R_1(\phi)] = 0 \end{array} \right.$$

CASO $\phi = 0$

SISTEMA OMogeneo

$$\Downarrow$$

$$\begin{array}{ll} \beta = 0 & \parallel \\ \delta A = 0 & \\ \delta R = 0 & \parallel \end{array}$$

SOLUZIONE BANALE

$$\Downarrow$$

VOLO SIMMETRICO

FISSO ϕ

$$\begin{cases} \beta = \beta(\phi) \\ \delta A = \delta A(\phi) \\ \delta R = \delta R(\phi) \end{cases}$$

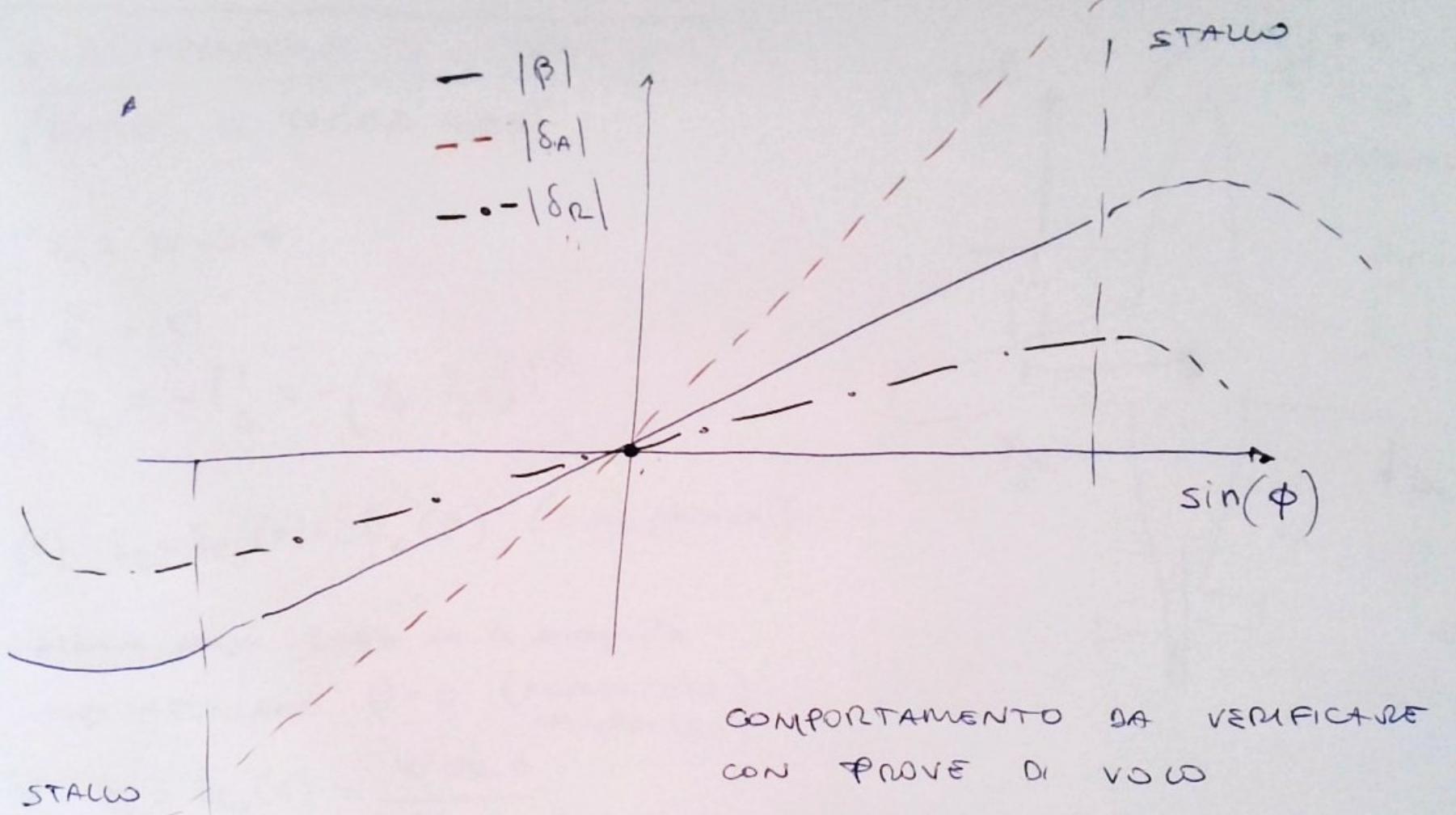
CASO $\phi \neq 0$

$$\begin{cases} L_{G/P} - L_{G/\delta R} \frac{\partial \gamma P}{\partial \delta R} \\ M_{G/P} - M_{G/\delta R} \frac{\partial \gamma P}{\partial \delta R} \end{cases}$$

$$\begin{cases} L_{G/\delta A} \\ M_{G/\delta A} \end{cases} \begin{cases} \beta \\ \delta A \end{cases}$$

$$= \begin{cases} L_{G/\delta R} \frac{w \sin \phi}{\partial \delta R} \\ M_{G/\delta R} \frac{w \sin \phi}{\partial \delta R} \end{cases}$$

2 EQUAZ / 2 INCONGRUI SISTEMA NON OMOG
 \Rightarrow 2 SOLUZIONI



COMPORTAMENTO DA VÉRIFICA
 CON PROVE DI VOLO

|| SSSS
 || (steady reading side-slip)

• ASIMMETRIA DI TRAZIONE I
(MOTORE DI DESTRA rotto)

$$\begin{cases} Q = W \sin \phi \\ L_G = 0 \\ M_G = -\Gamma_G = -(\gamma_G - \gamma_{je}) T^e \end{cases}$$

$$(1) \quad \delta_R = \delta_{R_0}(\phi) + S R_1(\beta) \quad (\text{come prima})$$

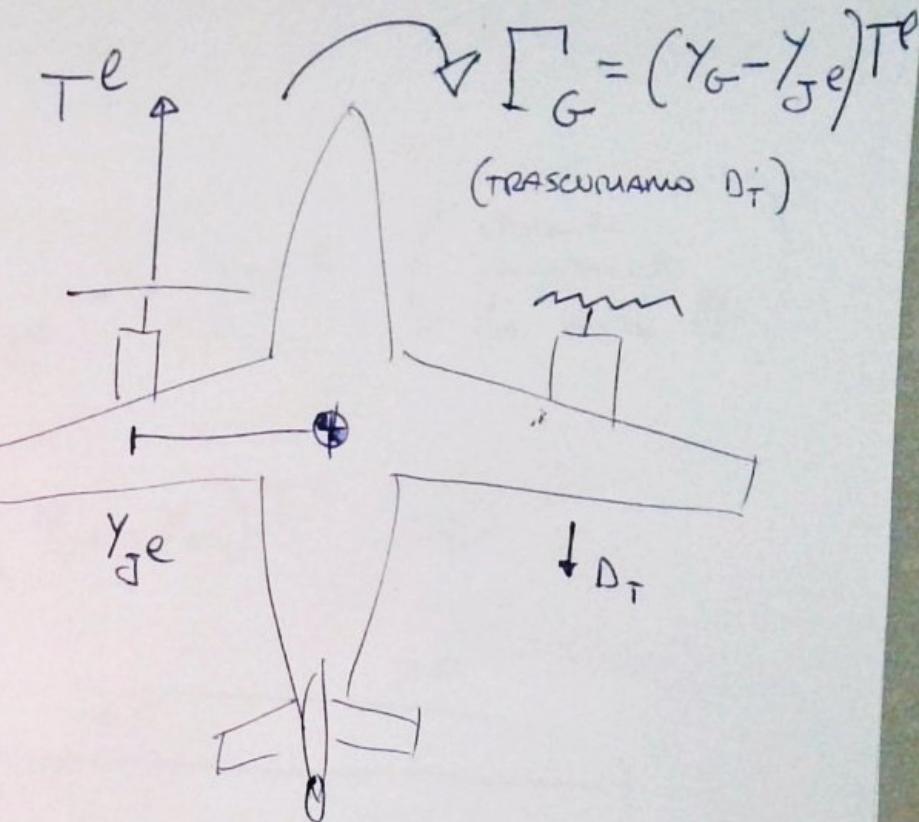
Abbiamo sempre 3 eqz in 4 incognite

\Rightarrow IMPONIAMO $\beta = 0$ (minimizza resistenza)

$$\textcircled{1} \quad \delta_R = \delta_{R_0}(\phi) = \frac{W \sin \phi}{Q/\delta_R}$$

$$\textcircled{2} \quad L_{G/S_A} \delta_A + L_{G/\delta_R} \frac{W \sin \phi}{Q/\delta_R} = 0 \quad \rightarrow \quad \delta_A = \delta_A(\phi)$$

$$\textcircled{3} \quad M_{G/\delta_A} \delta_A + M_0/S_R \frac{W \sin \phi}{Q/\delta_R} = 0 \quad (\gamma_{je} - \gamma_G) T^e$$



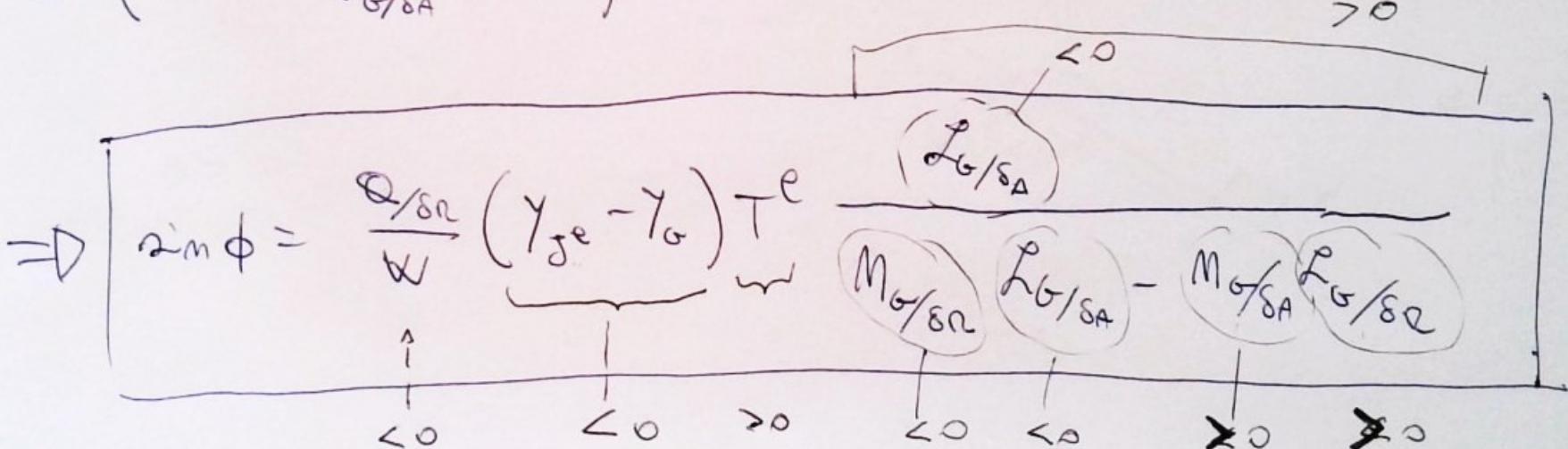
$$\Gamma_G = (\gamma_G - \gamma_{je}) T^e$$

(trascuriamo D_T)

$$\textcircled{2} \quad \delta_A = \frac{-L_G/s_A}{L_G/s_R} \frac{w \sin \phi}{Q/s_R}$$

$$\textcircled{3} \quad -M_{G/s_A} \frac{L_G/s_R}{L_G/s_A} \frac{w \sin \phi}{Q/s_R} + M_{G/s_R} \frac{w \sin \phi}{Q/s_R} = (\gamma_{je} - \gamma_o) T^e \quad \left(\begin{array}{l} \text{ottenuta} \\ \text{inserendo} \\ \delta_A \text{ dalla } \textcircled{2} \end{array} \right)$$

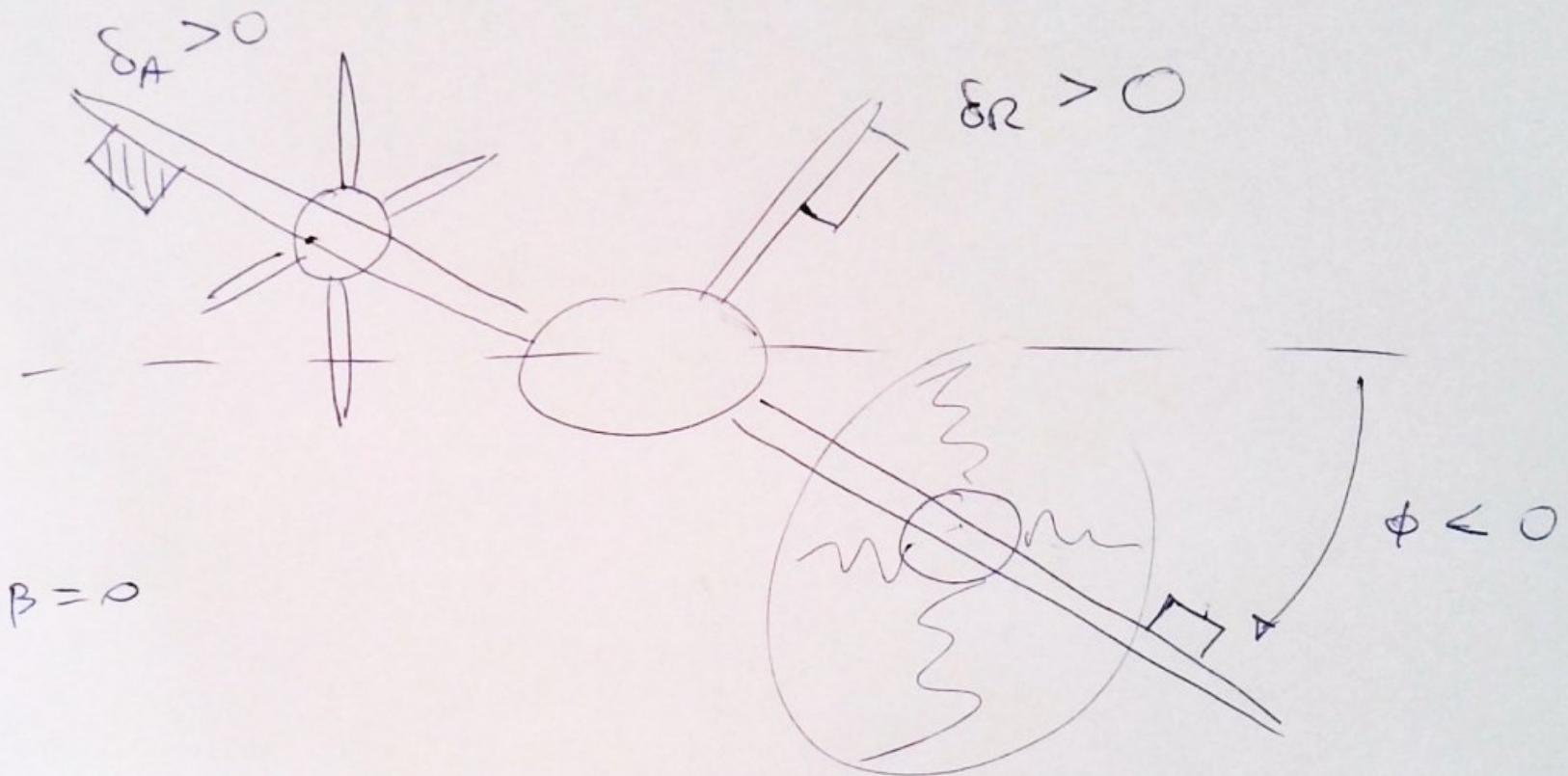
$$\Rightarrow \textcircled{3} \quad \left(M_{G/s_R} - \frac{L_G/s_R}{L_G/s_A} M_{G/s_A} \right) \frac{w}{Q/s_R} \sin \phi = (\gamma_{je} - \gamma_o) T^e \Rightarrow$$



$\sin \phi < 0$ se perdite d'aria
MOTORE DESTRO

$|M_{G/s_R}| > |L_G/s_R| \Rightarrow$ DENOMINATORE
 $|L_G/s_A| > |M_{G/s_A}| \Rightarrow$ > 0

VISTA FRONTALE



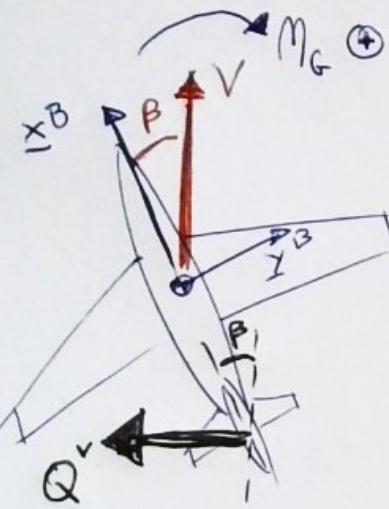
STABILITÀ STATICÀ LATERO DIREZIONALE

■ CONDIZIONE DI TRIM DI REFERIMENTO NON SIMMETRICO



DIREZIONALE

→ Reazione del velivolo in seguito a perturbazioni in β , attorno all'asse di imbarcata.



Per essere stabile direzionalmente, il velivolo deve generare un moto imbarcato che va a valle la perturbazione β



$$\Delta M_G > 0 \Rightarrow M_G/\beta > 0 \text{ se } \beta > 0$$

$$\Delta M_G < 0 \Rightarrow M_G/\beta < 0 \text{ se } \beta < 0$$

$M_G/\beta > 0$

EFFETTO STABILIZZANTE DELLA DERIVA

$$Q^v = \frac{1}{2} \rho V^2 S C_Q(\beta) = \frac{1}{2} \rho V^2 S C_{Q/\beta} \beta$$

$$M_G^v = (x_G - x_{AC^v}) Q^v \Rightarrow M_G^v / \beta = \underbrace{\frac{1}{2} \rho V^2 S}_{> 0} (x_G - x_{AC^v}) \underbrace{C_{Q/\beta}}_{> 0} \Rightarrow$$

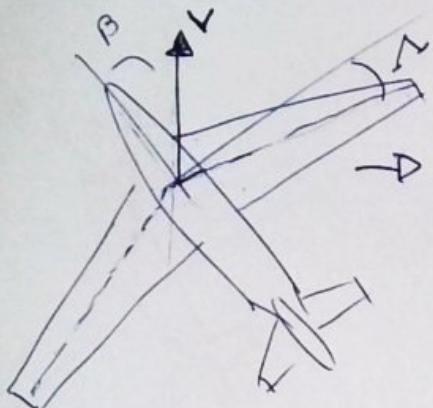
momento di imbarcata generato dalla deriva.

POSIZIONE LONGITUDINALE DEL CENTRO AERODINAMICO DELLA DERIVA

STABILIZZANTE SOLO SE x_{AC^v} DIETRO A x_G [$(x_G - x_{AC^v}) > 0$]
 "WEATHERCOCK STABILITY"

• ALTRI CONTRIBUTI ALLA STAB. DIREZIONALE

→ ALA + FRECCIA



→ ALA DX HA PIÙ PORTANZA

(Perche' più ortogonale al vettore V)

⇒ PIÙ RESISTENZA



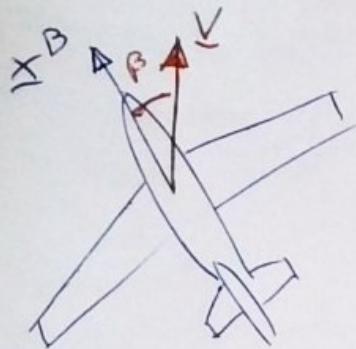
Stabilizzamento
di resistenza
genera un momento
imbarcatore
che tende a ridurre
 β (STABILIZZANTE)

FRECCIA POSITIVA → STAB/IZ.
FRECCIA NEGATIVA → INSTAB/IZ.

→ FUSOLIERA → di solito instabilizzante

STABILITÀ LATERALE

→ Derivate improprie L_G/β :

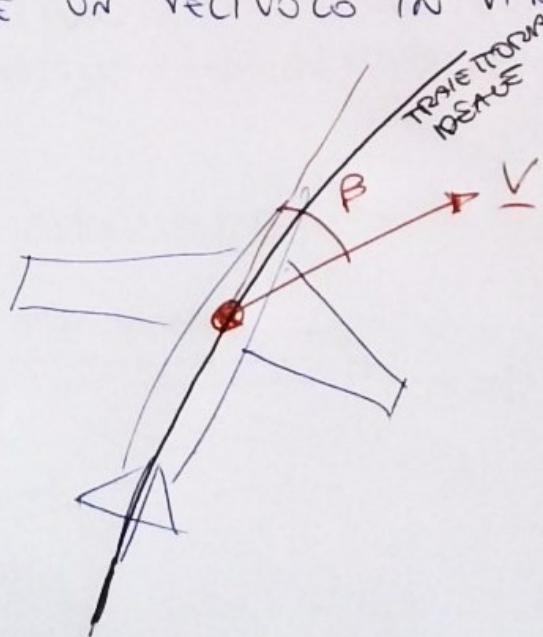


Se nel velivolo "Jerope", occorre che si generi un rollio che tende a far virare in direzione opposta

$$\text{DERAPATA} + \Delta x \Rightarrow \beta > 0 \Rightarrow \Delta L_G = L_{G/\beta} \beta < 0$$

$$L_{G/\beta} < 0$$

CONSIDERARE UN VELIVOLO IN VI RATA



\Rightarrow DERAPATA OTTOVUTA A UNA PERTURBAZIONE IN ϕ (ROLL ANGLE) (ϕ TROPPO ALTO)

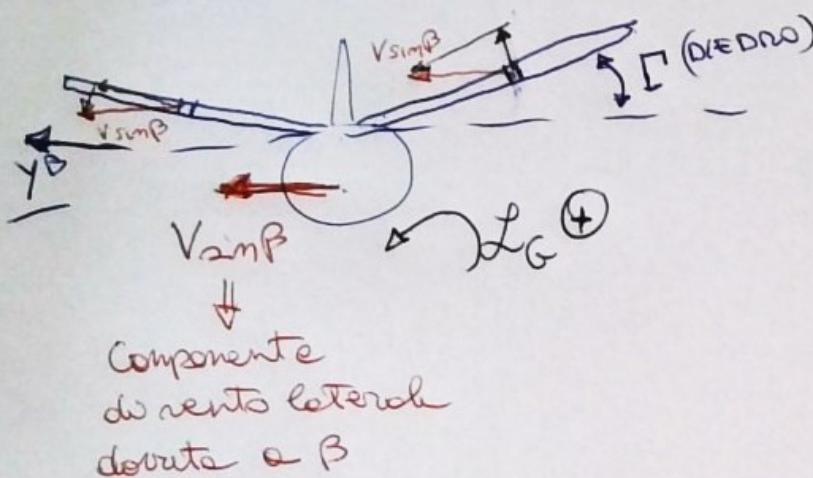
"SLIPPING IN TURN"

\uparrow
COMPORTAMENTO STABILE ~~ESTERNO~~ E'
QUELLO CHE MI PORTEREbbe A
RIDURRE $\phi \Rightarrow$

$$L_G < 0 \Rightarrow (L_{G/\beta} < 0)$$

EFFETTO DI EDRO

VISTA DAL DAVANTI



DIEDRO POSITIVO \rightarrow STABILIZZANTE

DIEDRO NEGATIVO \rightarrow INSTABILIZZANTE

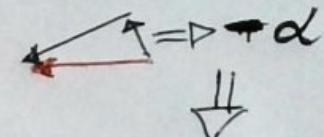
ALA DT



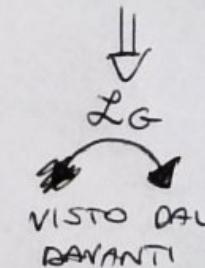
$$+ \alpha$$

INCREMENTO DI PORTANZA

ALA SX



DECREMENTO DI PORTANZA SULLA ALA SX



$$= D L_G < 0$$

EFFETTO ALA ALTA / ALA BASSA



\Rightarrow SI GENERA UN MOMENTO DI ROLLO

ALA ALTA STABILIZZANTE

ALA BASSA INSTABILIZZANTE

$$\begin{matrix} LG \\ \nearrow \\ + \end{matrix} \Rightarrow LG < 0$$

PERFORMANCE

OUTLINE

→ PUNTUALI

VORU
SALITA (ALIANTE)
VIRATA

→ INTEGRALI → CROCIERA (MASSA VARIABILE)
→ SALITA

→ TERMINALI → DECOLLO
→ ATTERRAGGIO

VELIVOLI PUNTO MATERIALE

AERODINAMICAMENTE È DESCRITTO
DALLA POGGIO TRIMMATA

PRESTAZIONI IN VORU

$$\left\{ \begin{array}{l} M_G = 0 \\ L = w \\ D = \Delta \end{array} \right| \text{EQZ EQL VORU}$$

$$T = T(\rho, v, \delta_T, \alpha) \approx \frac{T(\rho, v, \delta_T)}{\parallel}$$

Trascurando
il contributo
dell'Aero A

TIPICAMENTE
DA DATI Sperimentali
o DA MODELLI DI
PROPULSORE

1 $M_G = 0 \Rightarrow \delta_E = \delta_E^*(\alpha)$

$$M_G = M_G(\rho, v, \alpha, \delta_E)$$

2 $L = L(\rho, v, \alpha, \delta_E) = L(\rho, v, \alpha, \delta_E^*(\alpha)) =$
 $= L^*(\rho, v, \alpha)$

$$L = L(\rho, v, \alpha, \delta_E)$$

$$D = D(\rho, v, \alpha, \delta_E)$$

3 $L = L^*(\rho, v, \alpha) \Rightarrow \alpha = \alpha^*(\rho, v, L)$

4 $D = D(\rho, v, \alpha, \delta_E) = D(\rho, v, \alpha, \delta_E^*(\alpha)) =$
 $= D^*(\rho, v, \alpha)$

$$= D^*(\rho, v, \alpha^*(\rho, v, L))$$

$$= D^{**}(\rho, v, L) \Rightarrow \text{POLARE TRIMMATA}$$



$L = w \Rightarrow D = D^{**}(q, v, w)$

$$D = D^{**}(-) \Rightarrow \frac{\text{RESISTENZA CHE E' GENERATA IN VORU}}{(M_G = 0)} \\ L = w$$



SPINTA NECESSARIA (o RICHIESTA)

Perché è la spinta che è richiesta per mantenere il voru.

Da questo concetto deriva anche la misura della potenza richiesta (o necessaria) in voro (SPINTA NECESSARIA * VELOCITÀ DI VOLO)

$$T = D \Rightarrow \frac{T(q, v, \delta_T)}{\text{SPINTA DISPONIBILE}} = \frac{D(q, v, w)}{\text{SPINTA NECESSARIA}}$$

EQUILIBRIO TRA SPINTA NECESSARIA E DISPONIBILE

$$T(\rho, V, \delta_T) = D(\rho, V, W) \Rightarrow \begin{array}{l} 4 \text{ INCognite } (\rho, V, W, \delta_T) \\ 1 \text{ eqz} \end{array}$$

FISSO 3 INCognite e CALCOLO LA QUARTA

1) "VISIONE DEL PILOTA"

FISSO (ρ, V, W) e TROVO δ_T

$$\Rightarrow T(\rho, V, \delta_T) = \frac{1}{2} \rho V^2 S c_D(a), (*)$$

POLARE TRIMMATA

$L = W$ (equilibrio verticale per trovare a)

$$c_L = \frac{W/S}{\frac{1}{2} \rho V^2} \rightarrow \text{INSERIRE IL } a \text{ IN (*)}$$

$$T(\rho, V, \delta_T) = \frac{1}{2} \rho V^2 S c_D \left(\frac{W/S}{\frac{1}{2} \rho V^2} \right),$$

FUNZIONE

UNICA
INCognita
 δ_T

2) "VISIONE PROGETTISTA"

FISSO (ρ, V, δ_T) CALCOLO W

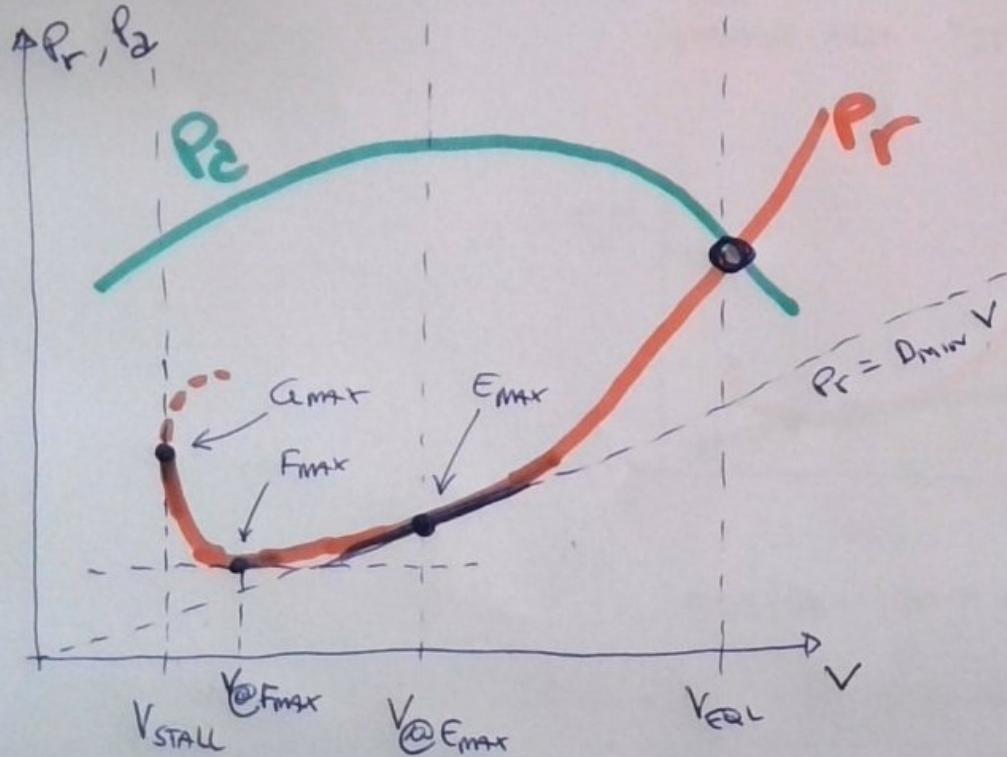
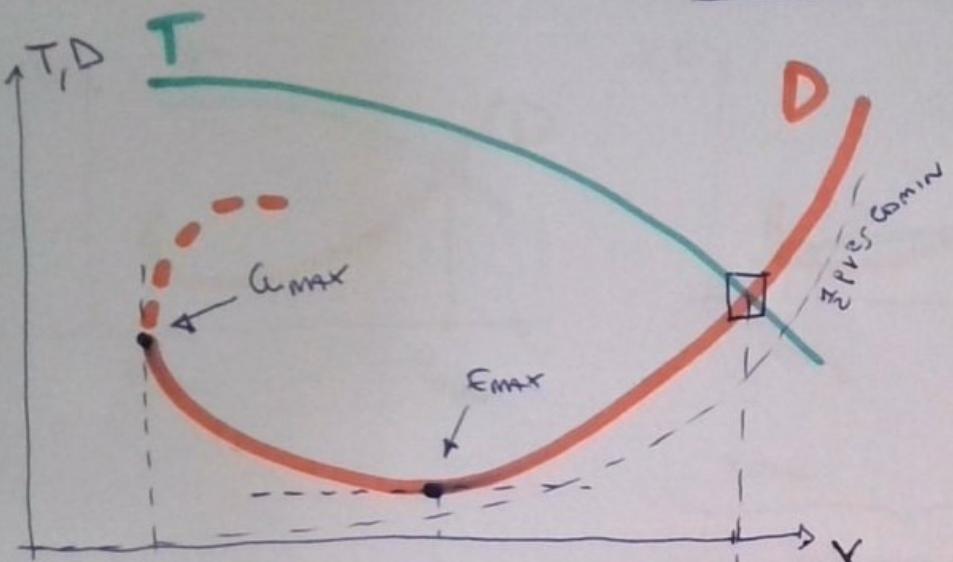
$$T(\rho, V, \delta_T) \Rightarrow \text{NOTA!} \Rightarrow T = \frac{1}{2} \rho V^2 S c_D(a) \Rightarrow c_D(a) = \frac{T}{\frac{1}{2} \rho V^2 S} \Rightarrow$$

$$\Rightarrow \text{INVERTIRE LA POLARE} \rightarrow c_L = f(c_D) \Rightarrow L = \frac{1}{2} \rho V^2 S c_L \Rightarrow \underline{W = L}$$

3) FISSO $(\rho, W, \delta_T) \rightarrow$ CALCOLO V ("PERFORMANCE") \Rightarrow DIAGRAMMI DI PEND

(SPINTA/POTENZA DISP/NEC VS V)

[DIAGRAMMI DI PENAUD]



$$i) D = \frac{1}{2} \rho V^2 S c_D(C_L) = \frac{1}{2} \rho V^2 S C_D \left(\frac{W}{2 \rho V^2 S} \right)$$

$$i.1) \exists C_{L\max} (\text{STALL}) \Rightarrow V_{\text{STALL}} = \sqrt{\frac{W}{\frac{1}{2} \rho C_{L\max} S}}$$

$$i.2) \exists \frac{C_L}{C_D} \max \Rightarrow E_{\max}$$

$$E = \frac{L}{D} \Leftrightarrow D = \frac{L}{E} \Rightarrow D = \frac{W}{E} \Rightarrow \begin{matrix} \text{con } E_{\max} \\ \text{e } \\ D_{\min} \end{matrix}$$

$$i.3) \exists c_{D\min} \Rightarrow D \geq \frac{1}{2} \rho V^2 S c_{D\min}$$

i.4) $T(\rho, V, \delta_T) \rightarrow$ disegnabile in funzione di V

$$ii) P_r = D V$$

ii.1) STALL

$$ii.2) E_{\max} \Rightarrow D_{\min} \Rightarrow P_r @ E_{\max} = D_{\min} V$$

(TANGENZA Γ_{P_r} $P_r = D_{\min} V \in P_r$)

$$ii.5) E = \frac{L}{D} \Rightarrow \frac{E}{V} = \frac{L}{D V} \Rightarrow \frac{E}{V} = \frac{W}{P_r} \Rightarrow$$

$$\Rightarrow \boxed{P_r = \frac{W V}{E}} \quad \leftarrow \boxed{V = \sqrt{\frac{W}{\frac{1}{2} \rho S C_L}}}$$

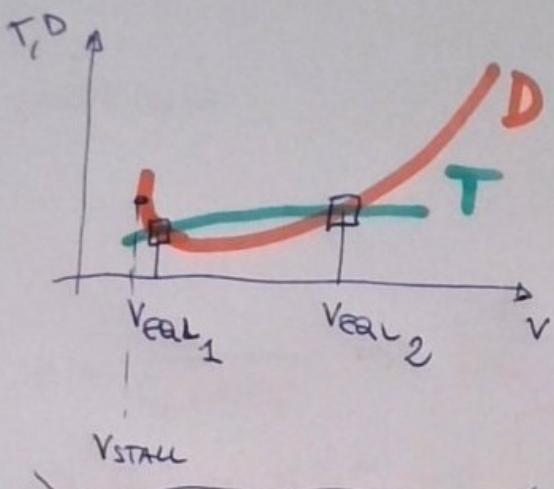
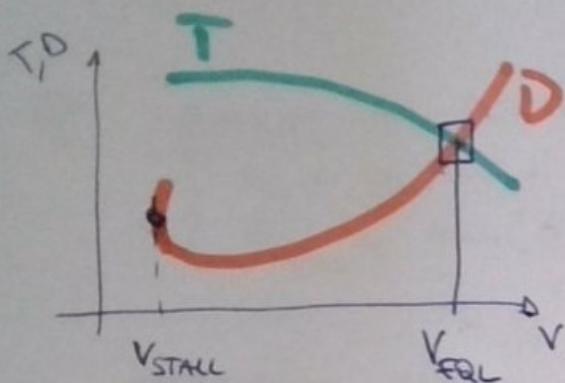
$$P_r = \frac{W^{\frac{3}{2}}}{\sqrt{\frac{1}{2} \rho S} \underbrace{E}_{F C_L}} = \frac{W^{\frac{3}{2}}}{\sqrt{\frac{1}{2} \rho S} F}$$

$F = E C_L$
(Power index)

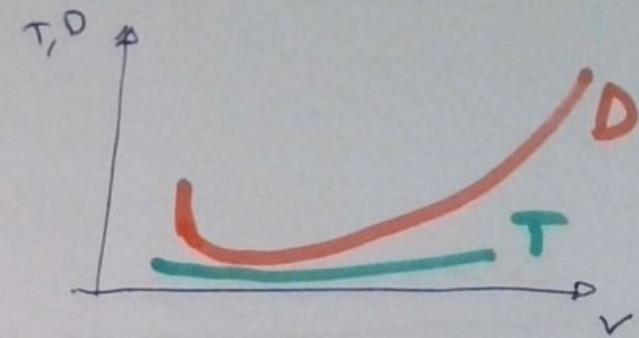
$$ii.6) P_r = T \cdot V$$

$\Downarrow P_{r\min} \leq F_{\max}$

CASI POSSIBILI



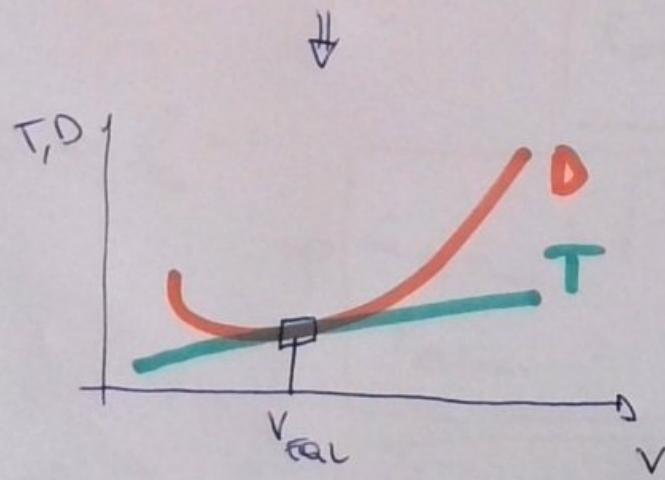
v_{MIN} DI VOLO NON NECESSARILMENTE
UGUALE ALLA v_{STALL}



$T < D \wedge v$



NESSUNA CONDIZIONE
DI EQUILIBRIO
POSSIBILE



TANGENZA $T = D$



1 SOLUZIONE

MODELLI ANALITICI PER POLARE E SPINTA (POTENZA)

POLARE → POLARE PARABOLICA

$$\begin{cases} C_0 = C_{D0} + K Q^2 \\ Q_{\max} = \max(Q) \end{cases}$$

• EFFICIENZA MASSIMA → $E = \frac{Q}{C_0 + K Q^2} \Rightarrow \frac{\partial E}{\partial Q} = 0 \Rightarrow \frac{C_{D0} + K Q^2 - 2 K Q^2}{(C_{D0} + K Q^2)^2} = 0$

$$\Rightarrow C_{D0} = K Q^2 \Rightarrow$$

$$Q_{\text{@ } E_{\max}} = \sqrt{\frac{C_{D0}}{K}}$$

$$Q_{\text{@ } E_{\max}} = \dots = 2 C_{D0}$$

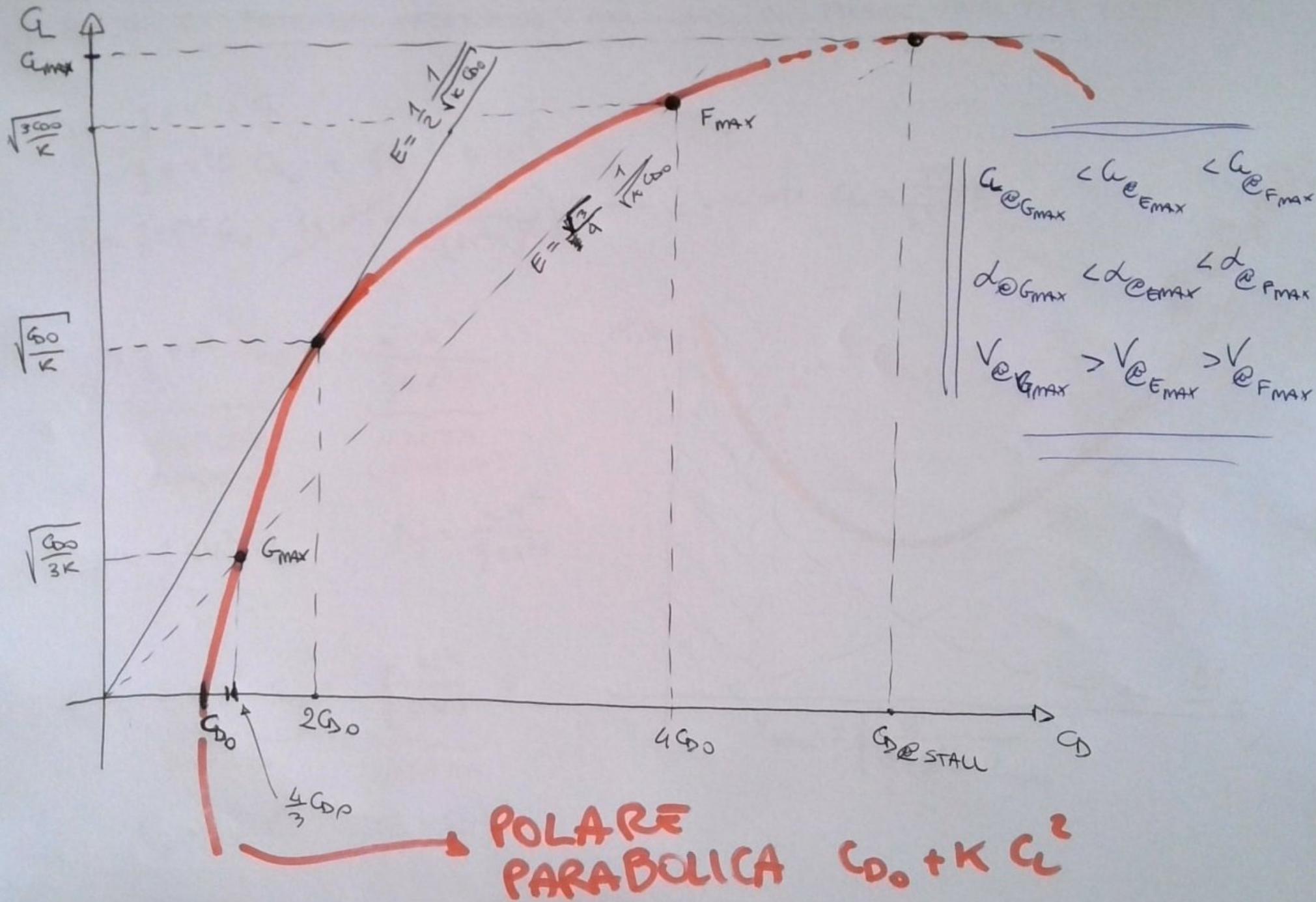
$$E_{\max} = \dots = \frac{1}{2} \frac{1}{\sqrt{K C_{D0}}}$$

• F_{\max} → $F = E \sqrt{Q} \Rightarrow \frac{\partial F}{\partial Q} = 0$

$$\Rightarrow \begin{cases} Q_{\text{@ } F_{\max}} = \sqrt{\frac{3 C_{D0}}{K}} & C_{D0} @ F_{\max} = 4 C_{D0} \\ E_{\text{@ } F_{\max}} = \frac{\sqrt{3}}{4} \frac{1}{\sqrt{K C_{D0}}} \end{cases}$$

• $G_{\max} \rightarrow G = \frac{E}{\sqrt{Q}} \Rightarrow \frac{\partial G}{\partial Q} = 0$

$$\Rightarrow \begin{cases} Q_{\text{@ } G_{\max}} = \sqrt{\frac{C_{D0}}{3 K}} & C_{D0} @ G_{\max} = \frac{4}{3} C_{D0} \\ E_{\text{@ } G_{\max}} = \frac{\sqrt{3}}{4} \frac{1}{\sqrt{K C_{D0}}} = E_{\text{@ } F_{\max}} \end{cases}$$



SPINTA E POTENZA NECESSARIA NEL CASO DI ROTARE ANALITICA

$$\begin{aligned} D &= \frac{1}{2} \rho V^2 S C_D \\ &= \frac{1}{2} \rho V^2 S C_{D_0} + \frac{1}{2} \rho V^2 S K C_L^2 \\ &= \frac{1}{2} \rho V^2 S C_{D_0} + \frac{1}{2} \rho V^2 S K \left(\frac{W^2}{(\frac{1}{2} \rho V^2 S)^2} \right) \end{aligned}$$

$$L = W \Rightarrow D \quad C_L = \frac{W}{\frac{1}{2} \rho V^2 S}$$

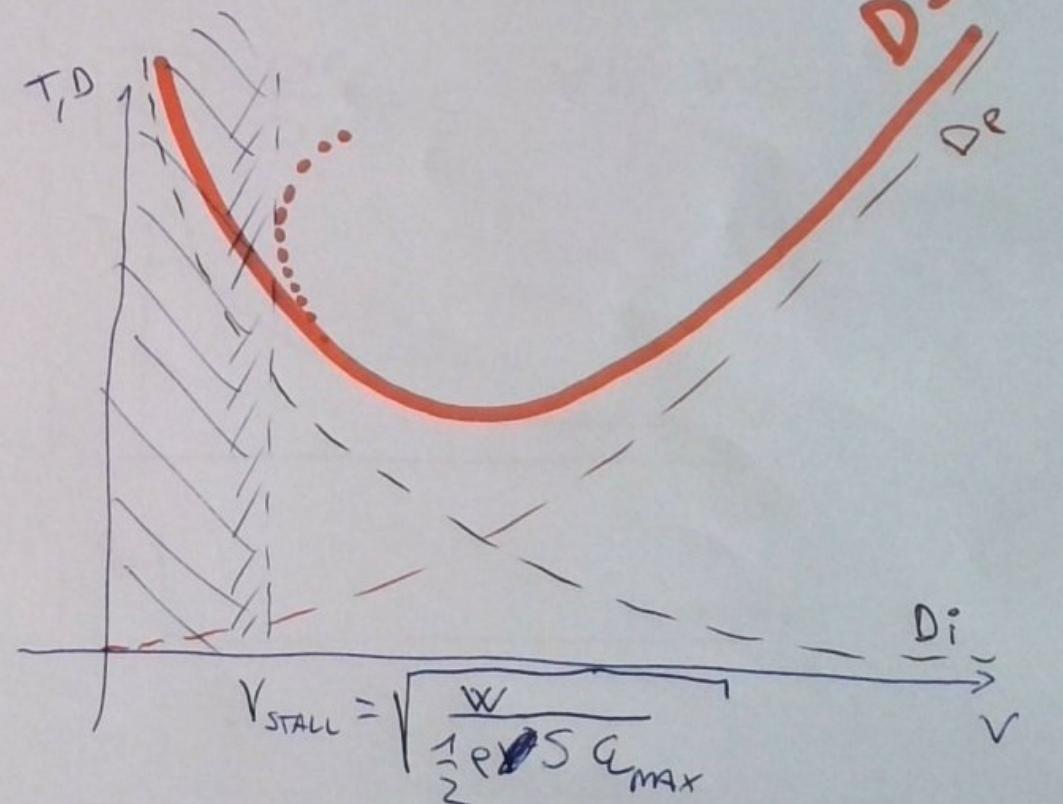
$$D = \underbrace{\frac{1}{2} \rho V^2 S C_{D_0}}_{\text{PARASSITA (PARABOLICA)}} + \underbrace{\frac{K W^2}{\frac{1}{2} \rho V^2 S}}_{\text{INDOTTA (IPERBOLICA)}}$$

$$D_p = \frac{1}{2} \rho V^2 S C_{D_0} \quad D_i = \frac{K W^2}{\frac{1}{2} \rho V^2 S}$$

$$P_r = \underbrace{\frac{1}{2} \rho V^3 S C_{D_0}}_{\text{PARASSITA}} + \underbrace{\frac{K W^2}{\frac{1}{2} \rho V^2 S}}_{\text{INDOTTA}}$$

$$P_{r_e} = D_p V$$

$$P_{r_i} = D_i V$$

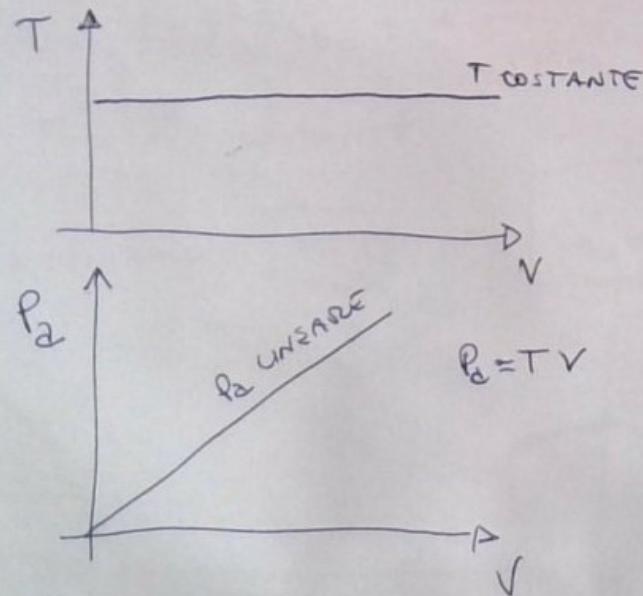


MODELLO ANALITICO PER SPINTA E POTENZA

① MOTORE A GETTO

"J" → TURBOGETTO IDEALE
CON SPINTA COSTANTE
AL VARIARE DELLA
VELOCITA'

$$\boxed{H_p} \quad \frac{\partial C}{\partial C} = 0 \quad \forall h, \forall \tau$$



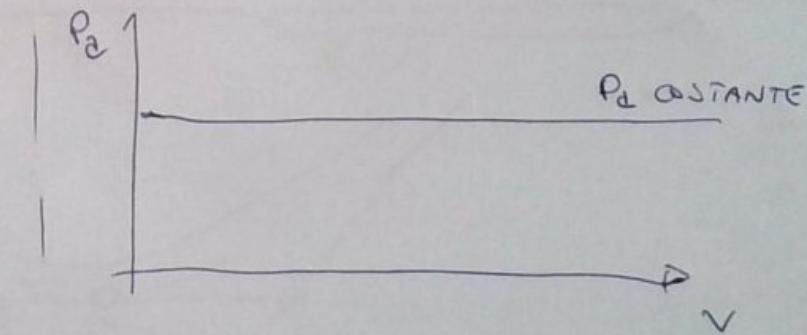
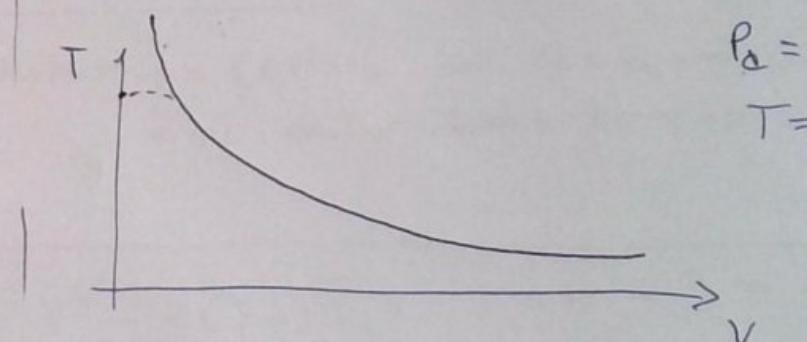
② MOTOELICA CON PALE A
PASSO VARIABILE

"P" → IDEALE CARATTERIZZATO
DA POTENZA COSTANTE
CON VELOCITA'

$$\boxed{H_p} \quad \frac{\partial P}{\partial V} = 0 \quad \forall h, \forall \tau$$

$$P_d = T \cdot V$$

$$T = \frac{P_d}{V}$$



EQUILIBRIO IN VORU

$M_G = 0 \Rightarrow$ GIÀ VERIFICATO CON POLARE TRIMMATA

$T(v, p, s_T) = D(v, p, w) \Rightarrow$ DA VERIFICARE

$\rightarrow H_p \rightarrow \begin{cases} \text{POLARE ANALITICA} \\ \text{PROSPULSIONE IDEALE} \end{cases}$

~~caso J~~ → CASO J (TURBOGETTO IDEALE)

$$\left\{ \begin{array}{l} D = \frac{1}{2} \rho v^2 S C_D + k \frac{w^2}{\frac{1}{2} \rho v^2 S} \\ T = \bar{T} \end{array} \right.$$

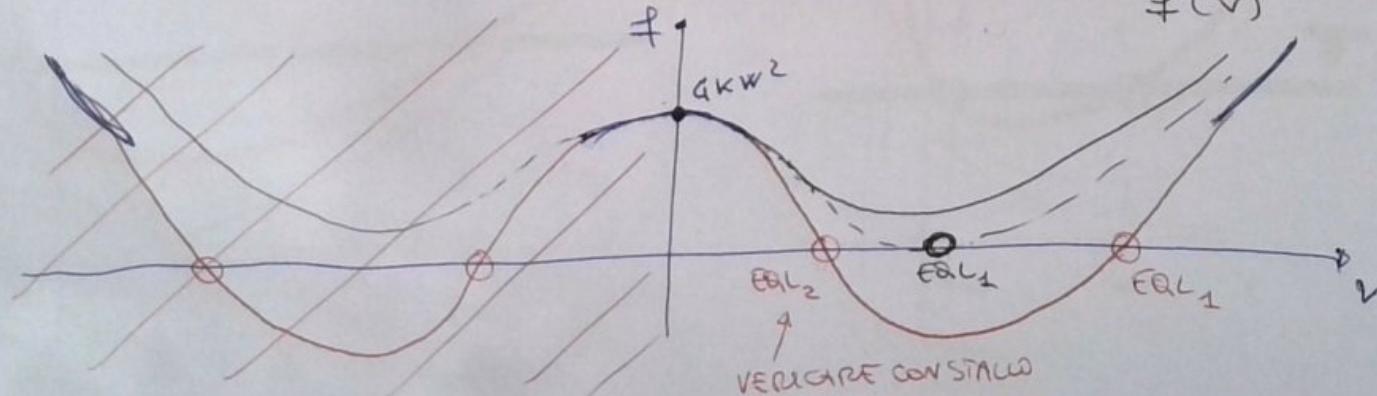
(MEMO: $D = \frac{1}{2} \rho v^2 S C_D$ con $C_D = C_{D_0} + k \alpha^2$
e α da equilibrio $L = w \Rightarrow \alpha = \frac{w}{\frac{1}{2} \rho v^2 S}$)

COSTANTE

IMPONIAMO

$$T = D$$

$$C_{D_0} + (\rho s)^2 v^4 - 2(\rho s) \bar{T} v^2 + k w^2 = 0$$



— A SOLUZIONI
SIMMETRICHE
 \uparrow \downarrow \uparrow \downarrow

→ NEGATIVE
NON FISICHE

→ EQL1 DA TENERE

→ EQL2 POSSIBILE
SOLUZIONE
DA VERIFICARE
CON STALLO

— NESSUNA SOLUZIONE
NON POSSO AVERE VORU

SOLUZIONE

$$G_0(\rho s)^2 V^4 - 2(\rho s) \bar{T} V^2 + 4Kw^2 = 0 \quad || \Rightarrow \text{BI-QUADRATICA}$$

$$t = V^2 \Rightarrow G_0(\rho s)^2 t^2 - 2(\rho s) \bar{T} t + 4Kw^2 = 0 \quad || \Rightarrow \text{QUADRATICA}$$

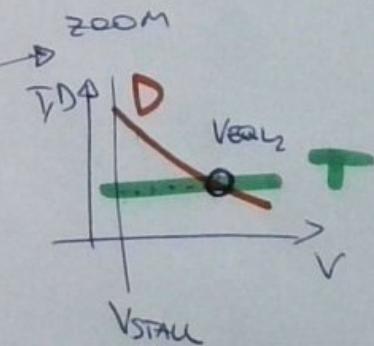
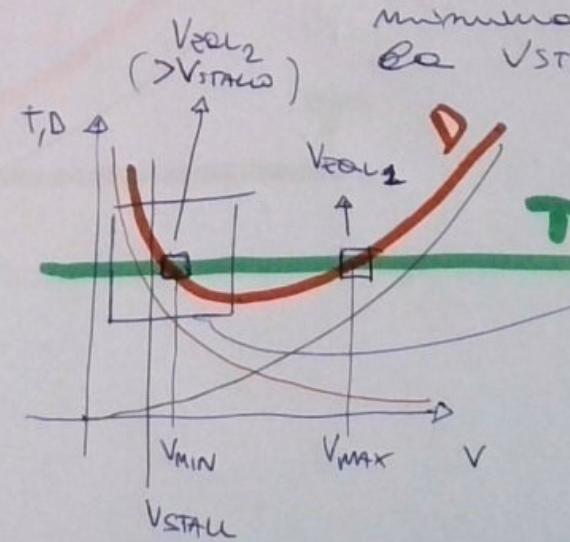
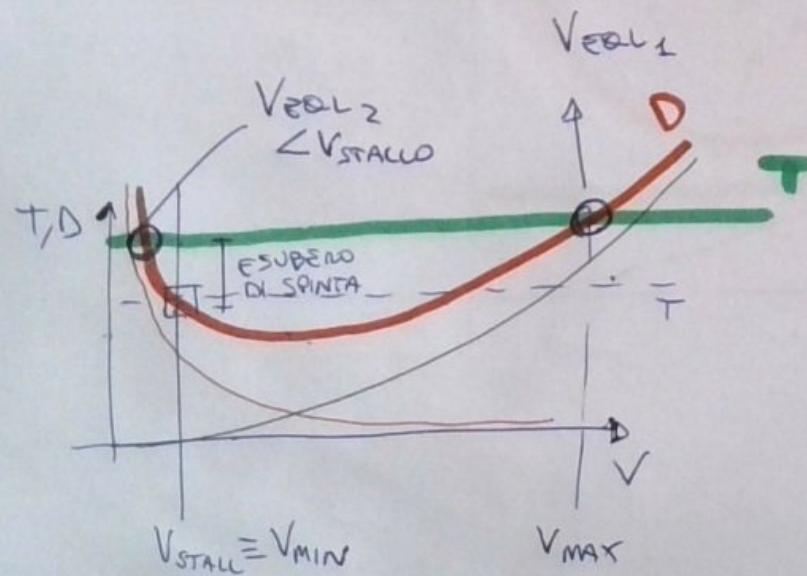
$$\boxed{t_{1,2} = V_{1,2}^2 = \frac{\bar{T}}{\rho s C_{D0}} \left(1 \pm \sqrt{1 - 4Kw^2 \left(\frac{\bar{T}}{\rho s C_{D0}} \right)^2} \right)}$$

$$V_{1,2} = \sqrt{t_{1,2}} \rightarrow \text{SCARTIAMO LE DUE SOLUZIONI NEGATIVE}$$

$$\rightarrow V_{\text{EQUAL}_1} (\cancel{>} > V_{\text{EQUAL}_2}) \text{ OK!}$$

$$\rightarrow V_{\text{EQUAL}_2} (< V_{\text{EQUAL}_1}) \rightarrow \text{dobbiamo verificare}$$

che sia > delle
Velocità di stall.
Altimenti la velocità
minima risulta essere
la VSTALL.



$$\max E = \frac{1}{2\sqrt{\kappa C_{D_0}}} ; \min D = \frac{W}{\max E} = W 2 \sqrt{\kappa C_{D_0}}$$

$$(\min D)^2 = 4 W^2 \kappa C_{D_0} \quad \textcircled{*}$$

$$\textcircled{*} \rightarrow t_{1,2} = V_{1,2}^2 = \frac{\bar{T}}{\rho S C_{D_0}} \left(1 \pm \sqrt{1 - \left(\frac{\min D}{\bar{T}} \right)^2} \right)$$

$$\text{se } \min D > \bar{T} \Rightarrow \sqrt{1 - \left(\frac{\min D}{\bar{T}} \right)^2} = \sqrt{(-)} = \sqrt{(<0)} \Rightarrow \begin{matrix} \text{NON} \\ \text{HO} \end{matrix} \\ \text{SOLUZIONI}$$



CASO P (MOTORELICIT IDEALE)

$$\left\{ \begin{array}{l} P_r = \frac{1}{2} \rho V^3 S C_{D_0} + \frac{K w^2}{2 \rho V S} \\ P_d = \overline{P}_d \text{ COSTANTE} \end{array} \right.$$

$$EQL \Rightarrow P_d = P_r$$

$$\frac{1}{2} \rho V^3 S C_{D_0} + \frac{K w^2}{2 \rho V S} = \overline{P}_d$$

moltiplichiamo tutto per $\frac{1}{2} \rho V S$

$$\underbrace{\frac{1}{4} \rho^2 S^2 V^4 C_{D_0}}_{\propto V^4} + \underbrace{\frac{K w^2}{2 \rho V S}}_{\propto V^0} = \overline{P}_d \underbrace{\frac{1}{2} \rho V S}_{\propto V} = 0$$

$P_d - N^{(I)}$ NON È UNA BIQUADRATICA



SOLUZIONE ITERATIVA!!!

$$\boxed{\text{ITERAZIONI}}$$

I) $V^{(I)} = \sqrt[3]{\frac{P_d}{\frac{1}{2} \rho S C_{D_0}}}$

TROVANDO L'INTERSEZIONE
TRA P_d e P_r

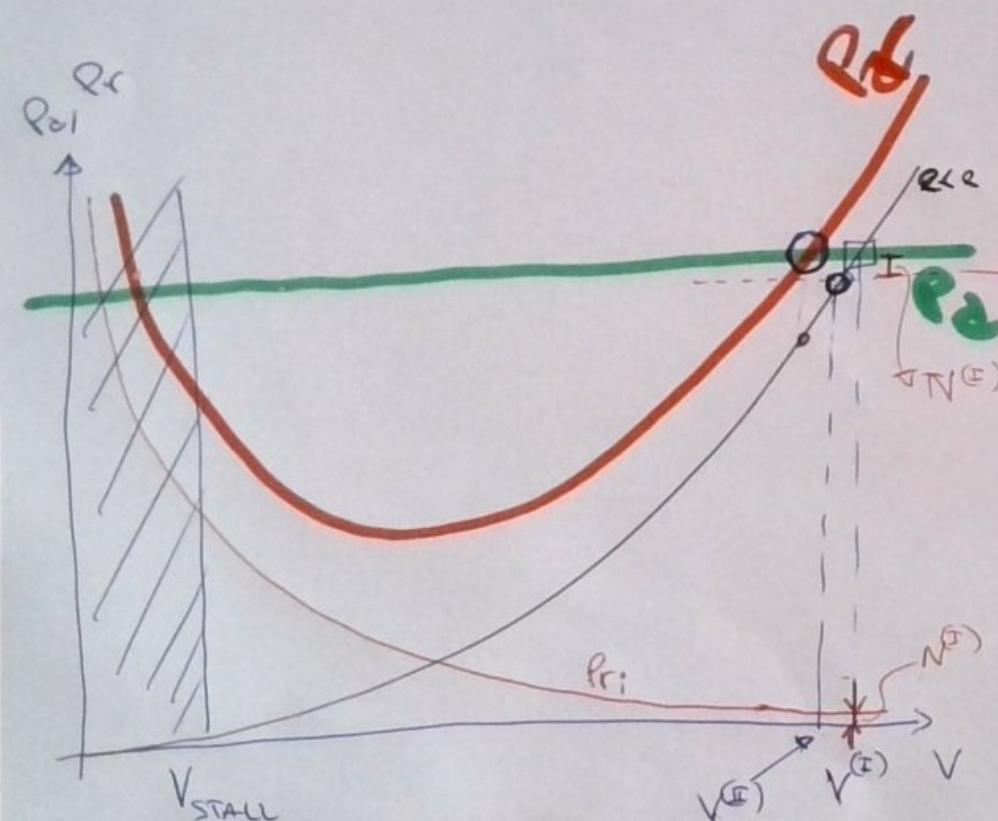
II) CALCOLO $N^{(I)} = \frac{K w^2}{\frac{1}{2} \rho V^{(I)} S}$

III) $V^{(II)} = \sqrt[3]{\frac{P_d - N^{(I)}}{\frac{1}{2} \rho S C_{D_0}}}$

IV) $N^{(II)} = \frac{K w^2}{\frac{1}{2} \rho V^{(II)} S}$

V) $V^{(III)} = \sqrt[3]{\frac{P_d - N^{(II)}}{\frac{1}{2} \rho S C_{D_0}}}$

COST FINO A CONVERGENZA

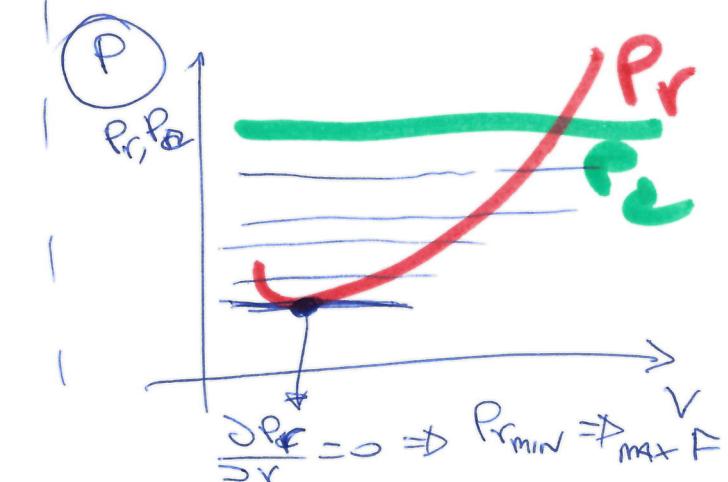
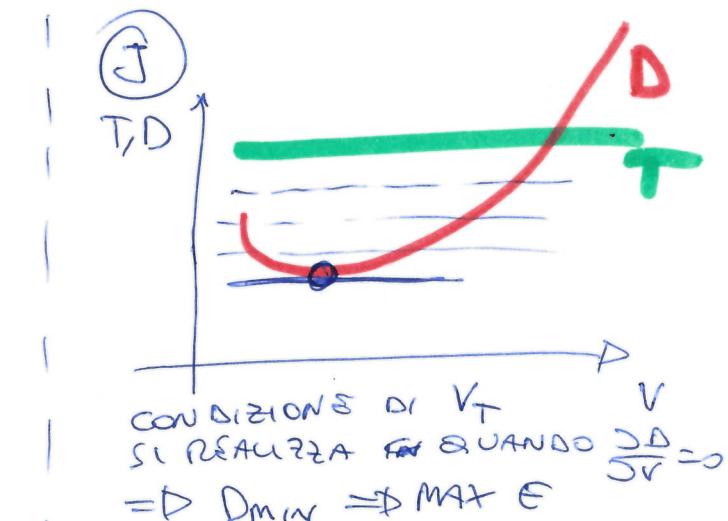
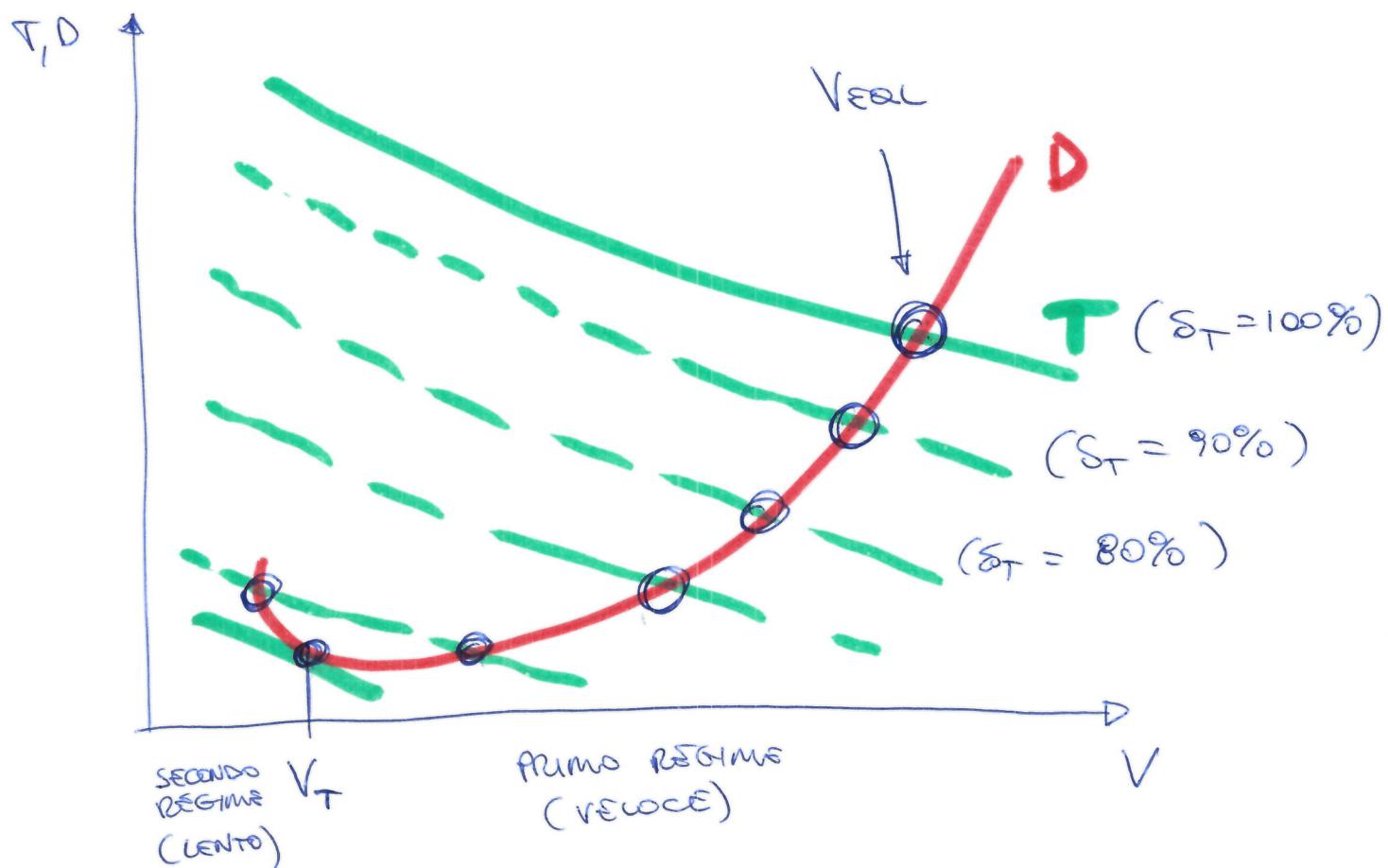


EFFETTO DI ALCUNI PARAMETRI SU DIAGRAMMI PENAUD (VORU)



MANETTA δ_T

(NON CONSIDERIAMO IPOTESI SU POGLIE E PROPELLISCE)



PESO

PROCEDURA:

- 1) CONSIDERARE UN PUNTO GENERICO SU D (Pr)
- 2) VAUTARE COME QUESTO PUNTO SI MUOVE ALL'INTERNO DEL DIAGRAMMA DI PÉNAUD, AD ASSETTO COSTANTE

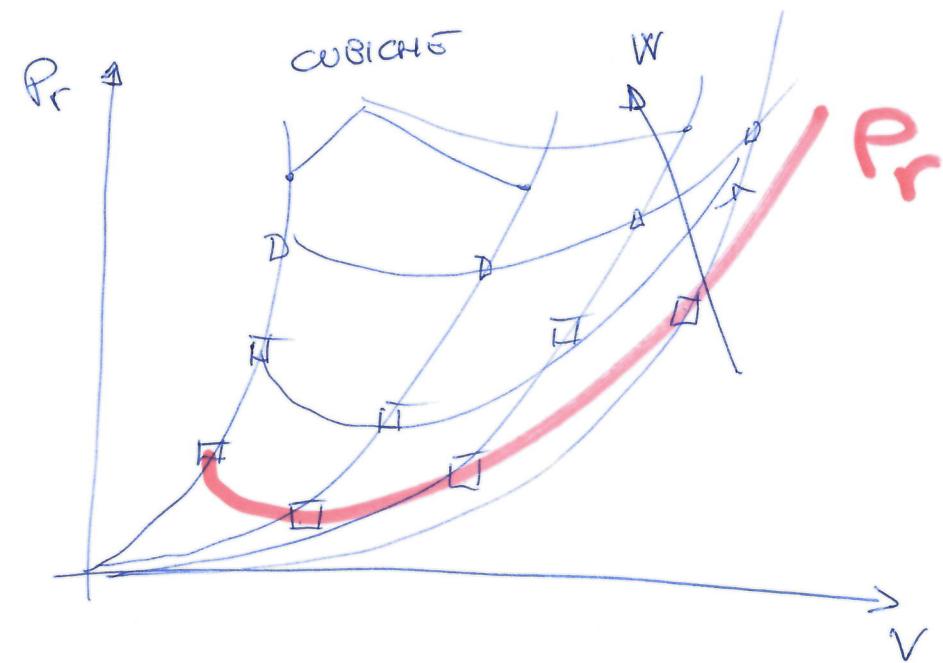
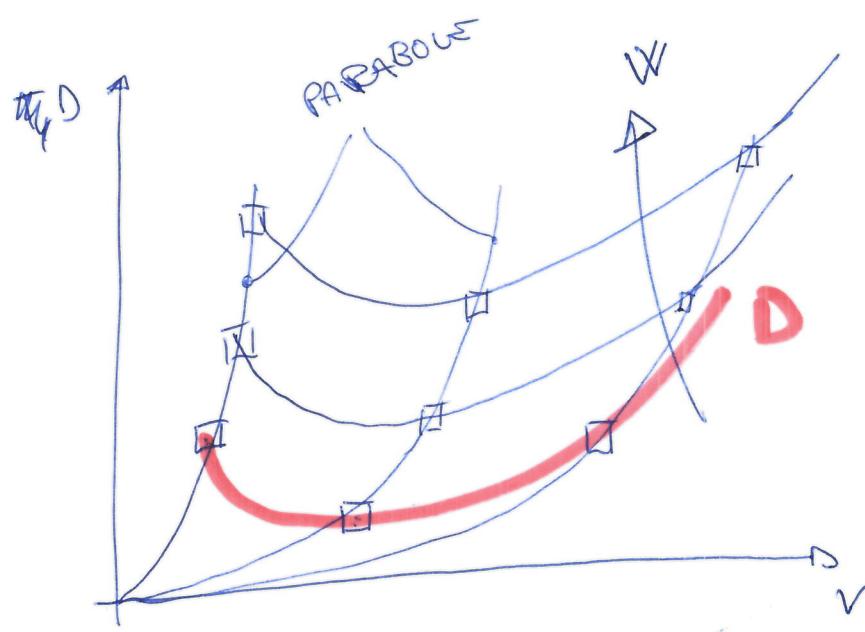
(α, E, a, g_0, F) COSTANTE

<p>COORDINATA V</p> $V = \sqrt{\frac{W/S}{\frac{1}{2} \rho a}}$	<p>W₁</p> $V_1 = \sqrt{\frac{W_1/S}{\frac{1}{2} \rho a}}$	<p>W₂</p> $V_2 = \sqrt{\frac{W_2/S}{\frac{1}{2} \rho a}}$ *
<p>COORDINATA D</p> $D_1 = \frac{W_1}{E}$	$D_2 = \frac{W_2}{E} \Rightarrow \frac{D_2}{D_1} = \frac{W_2}{W_1}$	
<p>COORDINATA Pr</p> $\frac{Pr_2}{Pr_1} = \frac{V_2^3}{V_1^3}$	* NOTIAMO CHE $\frac{W_2}{W_1} = \frac{V_2^2}{V_1^2}$	$\frac{D_2}{D_1} = \frac{V_2^2}{V_1^2}$

\Leftrightarrow

$$D_2 = \frac{D_1}{V_1^2} V_2^2$$

AD ASSETTO COSTANTE, AL VARIARE DEL PESO IL "PUNTO DI VOLO"
SI MUOVE SU PARABOLE NEL DIAGRAMMA DI PENAUD (SPINTE) ...



... SU DELLE CUBICHE NEL
DIAGRAMMA DI PENAUD PER
LE POTENZE

 QUOTA DI VOLO \Rightarrow VARIAZIONE DENSITÀ ρ

COORDINATA V

$$\frac{\rho_1}{\rho_2} = \sqrt{\frac{w/s}{\frac{1}{2} P_a}}$$

COORDINATA D

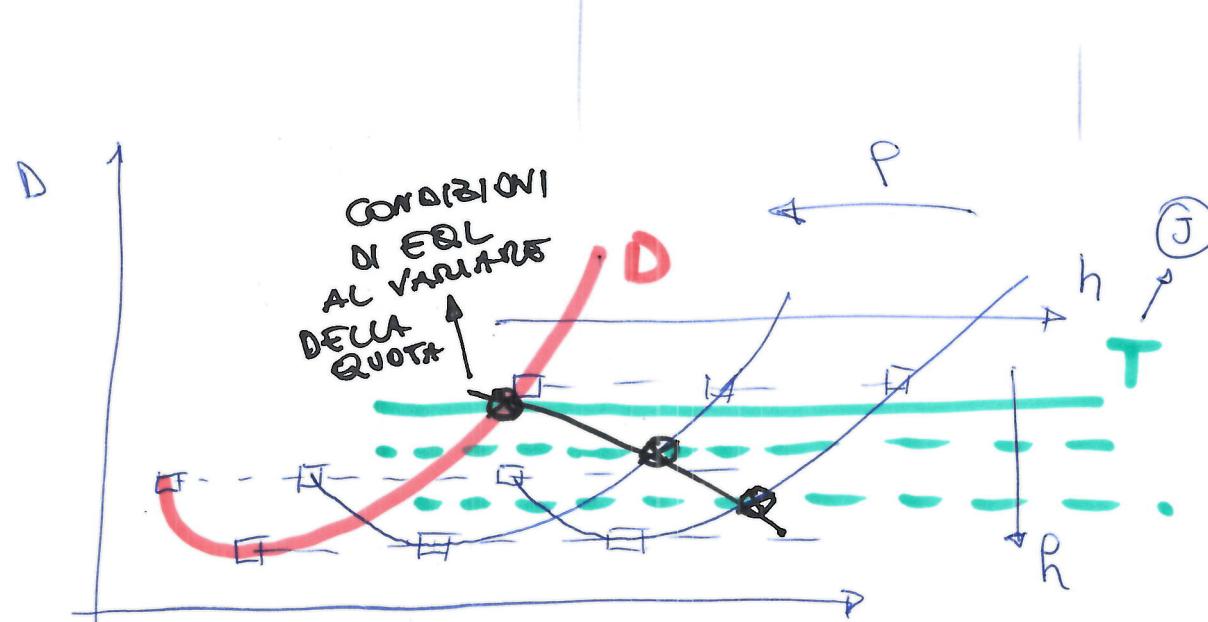
$$D_1 = \frac{w}{\epsilon}$$

COORDINATA P_r

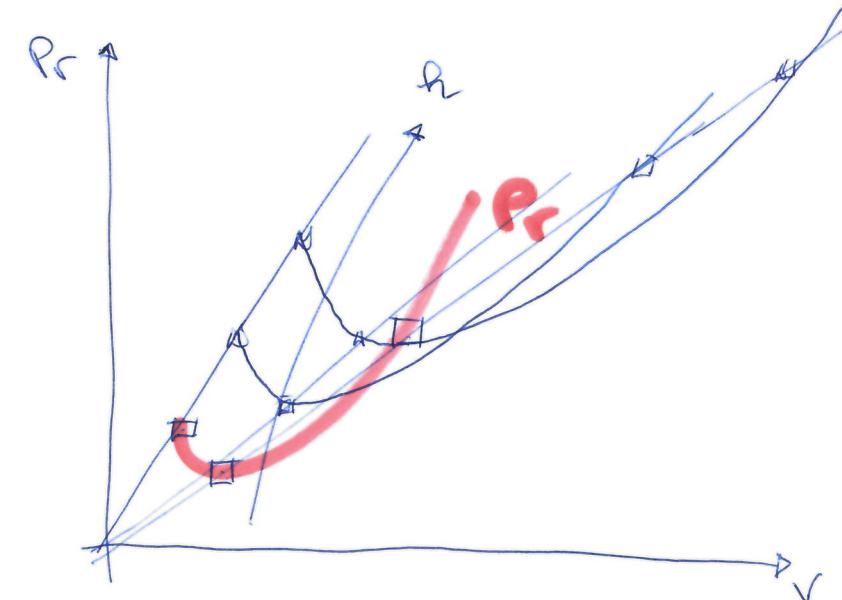
$$\frac{\rho_2}{\rho_1} = \sqrt{\frac{w/s}{\frac{1}{2} P_a}}$$

$$D_2 = \frac{w}{\epsilon} \Rightarrow D_1 = D_2$$

$$\frac{P_{r2}}{P_{r1}} = \frac{V_2}{V_1}$$



TUTTI I PUNTI A STESSO ASSETTO
SI MUOVONO SU RETTE ORIZ.

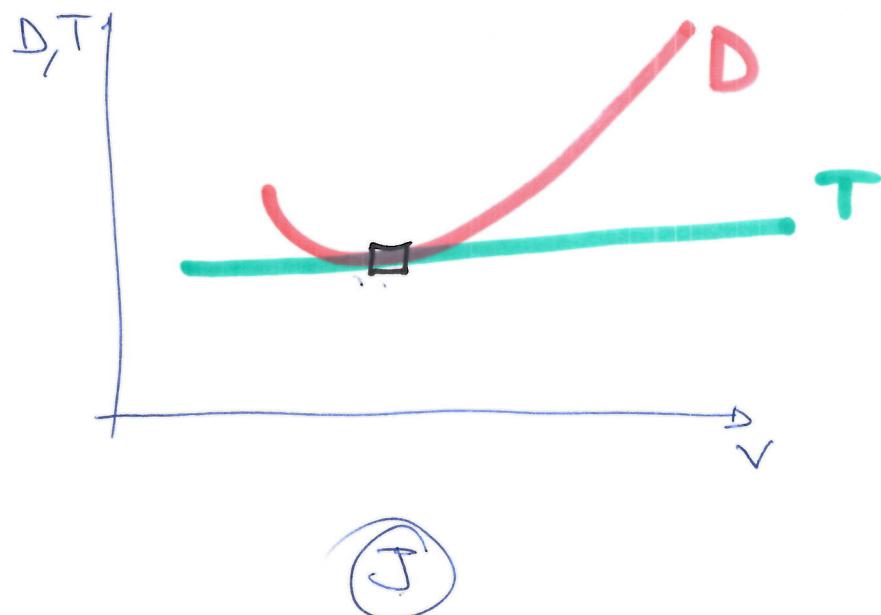


I PUNTI AD STESSO ASSETTO
SI MUOVONO SU DUE RETTE

OSSERVAZIONE

- ① D e P_r si modificheranno come abbiamo visto
 - ② T e P_d all'aumentare della quota diminuiranno
- $\left\{ \begin{array}{l} T = T_0 \left(\frac{P}{P_0} \right)^{\xi} \\ P_d = \dots \end{array} \right.$ $\xi = (0.75 \div 0.9)$

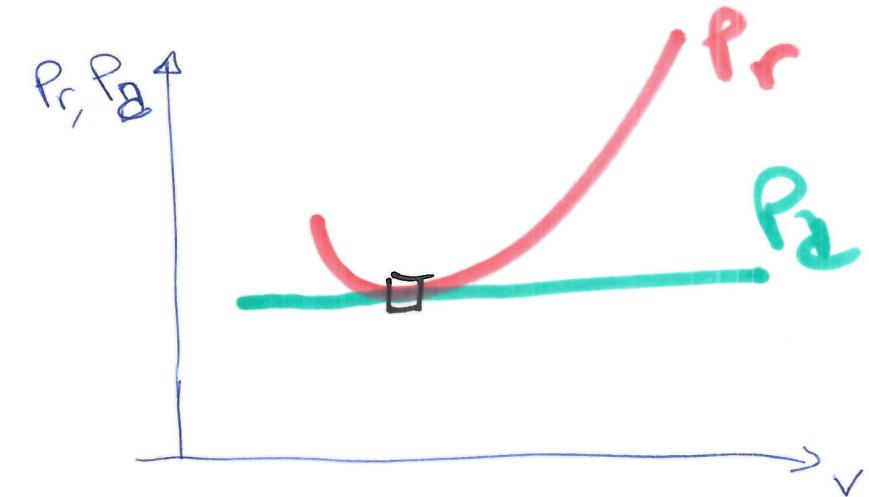
PER UNA CERTA QUOTA



□ TANGENZA TRA T E D

QUOTA PER CUI ~~è~~ SI REALIZZA
LA TANGENZA TRA LA CURVA DI
SPINTA NECESSARIA E QUELLA
DI SPINTA DISPONIBILE VIENE
CHIAMATA QUOTA DI TANGENZA

PER $J \Rightarrow$ INTERSEZIONE
LA TANGENZA AVVIENE IN
CONDIZIONI DI MASSIMA EFFICIENZA



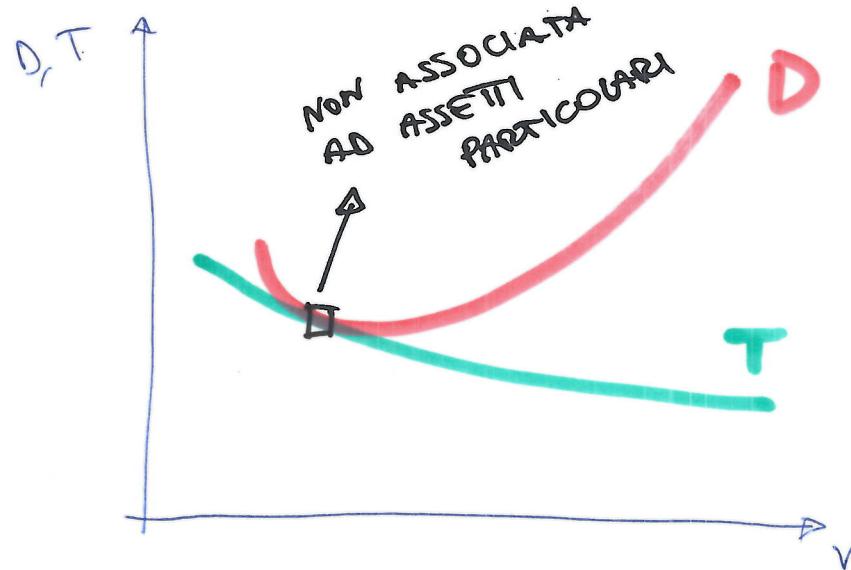
QUOTA DI TANGENZA



MASSIMA PRESTAZIONE

$$F = \frac{a}{c_0} \sqrt{a}$$

(P)



VENVOL
GENERALI

EFFETTO DELTA

V_{EAS}

SU

P_{RF} e D

$$D = \frac{1}{2} \rho v^2 S \underline{\underline{c}_D(a)}$$

POLARE
GENERALI

$$P_r = \frac{1}{2} \rho v^3 S \underline{\underline{c}_D(a)}$$

$$V_{EAS} = V \sqrt{\frac{\rho}{\rho_0}}$$

$$V = V_{EAS} \sqrt{\frac{\rho_0}{\rho}}$$

$$D = \frac{1}{2} \rho V_{EAS}^2 \frac{\rho_0}{\rho} S \underline{\underline{c}_D(a)}$$

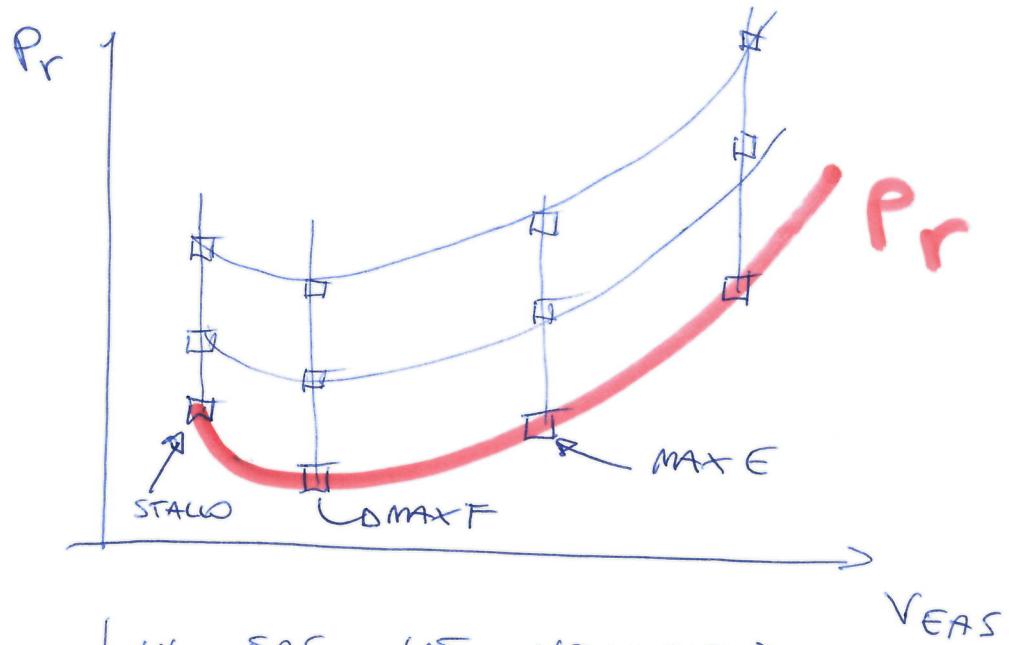
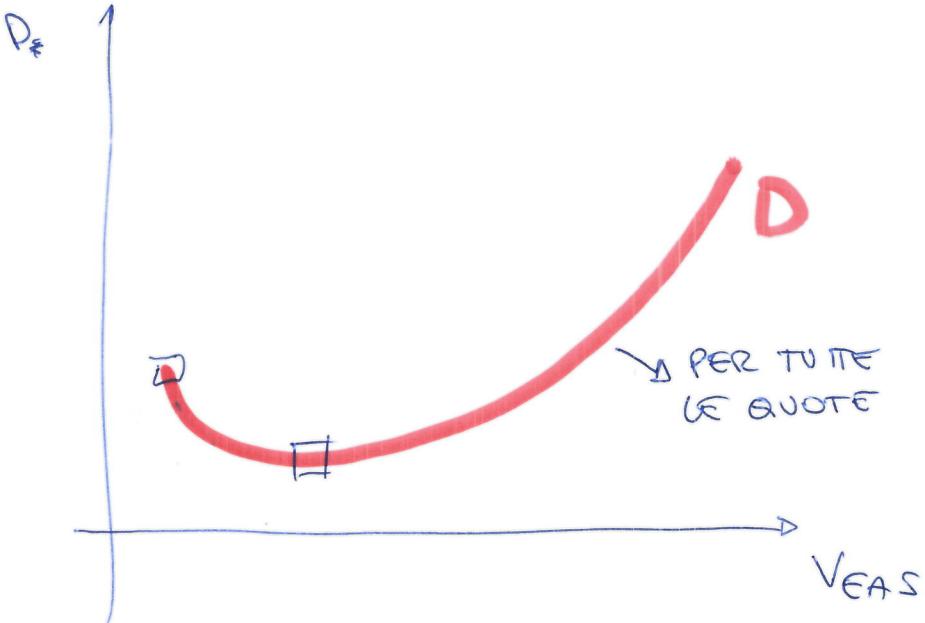
$$D = \frac{1}{2} \rho_0 V_{EAS}^2 S \underline{\underline{c}_D(a)}$$

UTILIZZANDO LA V_{EAS}
LA CURVA DI SPINTA
NECESSARIA NON VARIA
AL VARIARE DELLA QUOTA

$$P_r = \frac{1}{2} \rho V_{EAS}^3 \frac{\rho_0}{\rho} \sqrt{\frac{\rho_0}{\rho}} S \underline{\underline{c}_D(a)}$$

$$P_r = \frac{1}{2} \rho_0 V_{EAS}^3 \sqrt{\frac{\rho_0}{\rho}} S \underline{\underline{c}_D(a)}$$

L'EFFECTO DELTA QUOTA NON
SCOMPARSE



IN EAS LE VELOCITA'
CARATTERISTICHE RESTANO
COSTANTI AL VARIARE DELLA
QUOTA

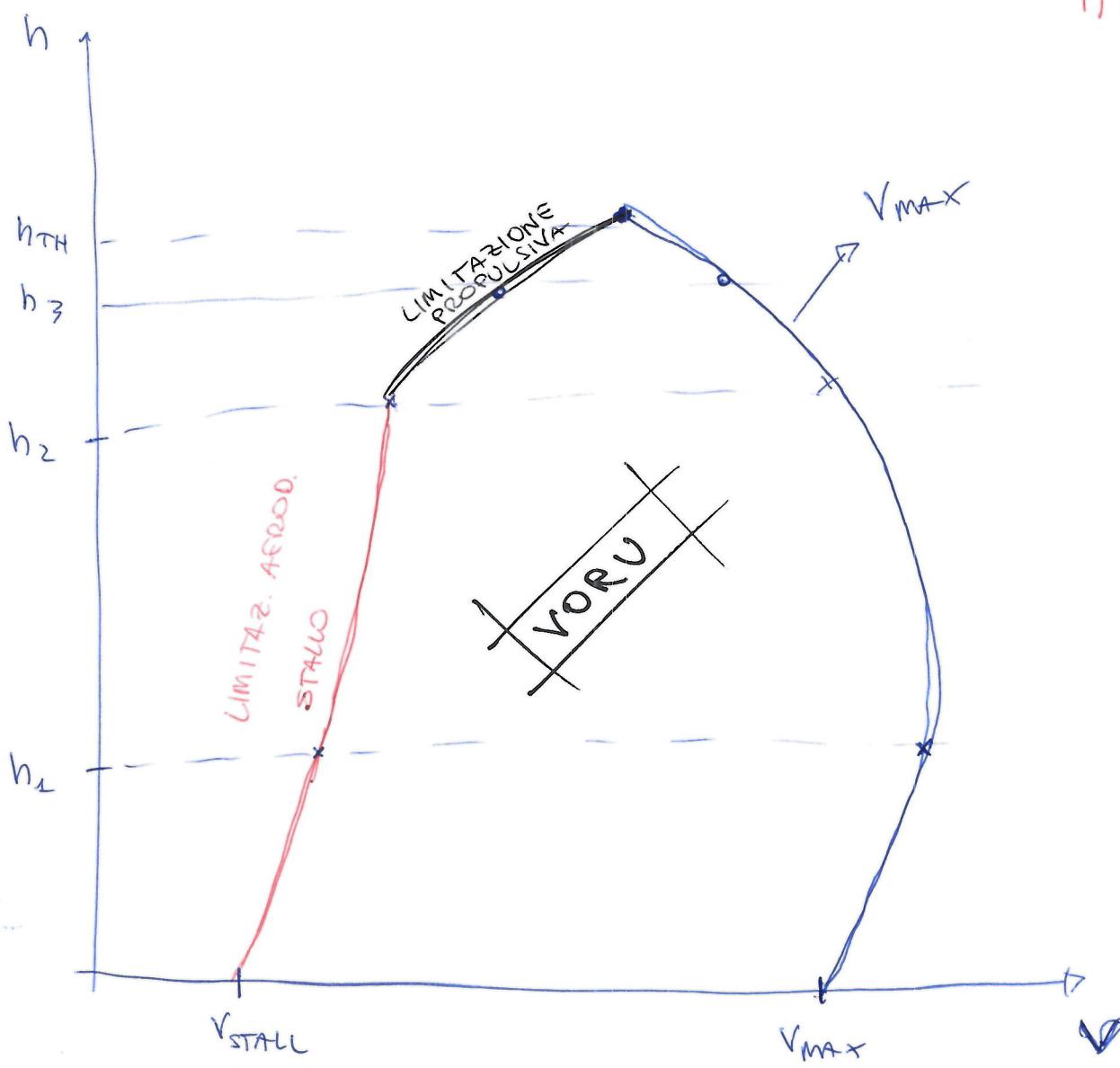
REMARK:
ANEMOMETRO DI
BORDO INDICA
LA EAS

IN VIVO PRO DI VOLO

IN VOLO

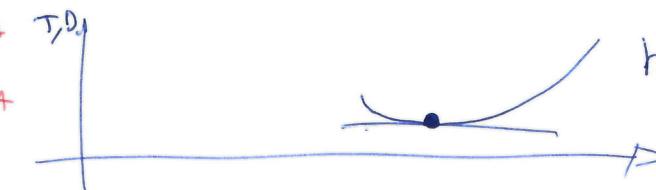
~~VELOCITÀ~~

GUARDIAMO LA VARIAZIONE DI VELOCITÀ MINIMA E MASSIMA IN VOLO CON LA QUOTA

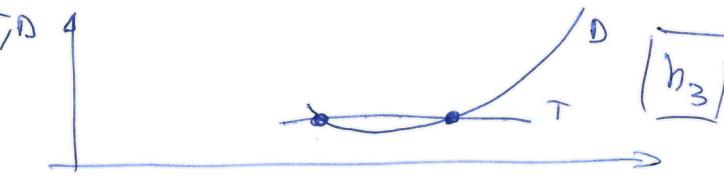


$h_{TH} \rightarrow$ QUOTA DI TANGENZA

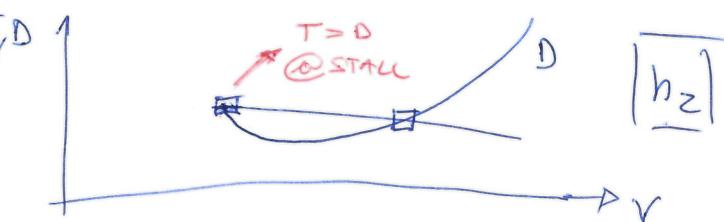
T,D



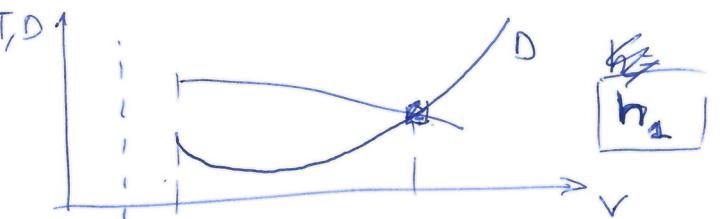
T,D



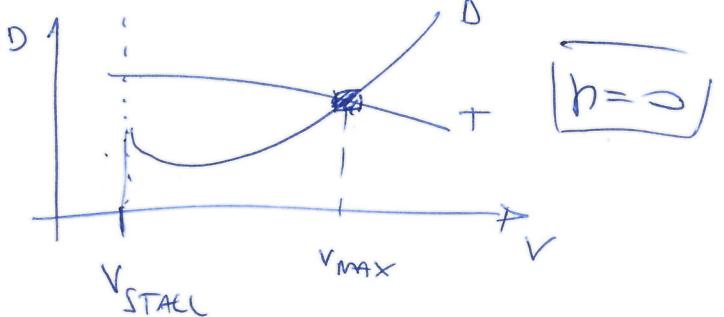
T,D



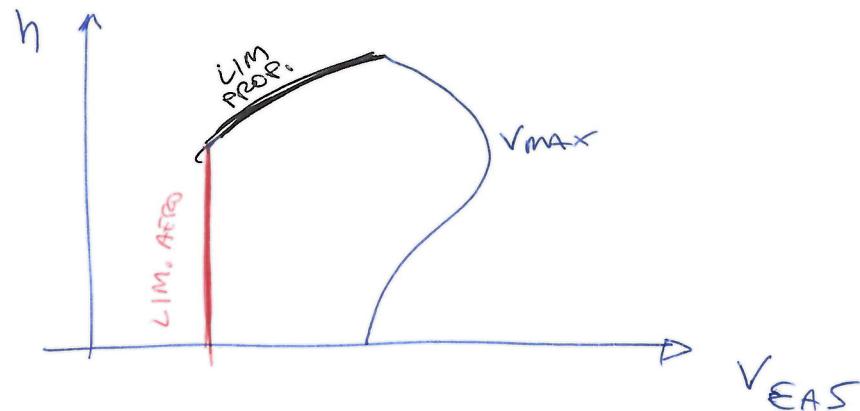
T,D



T,D



INVILUPPO DI VSO IN EAS

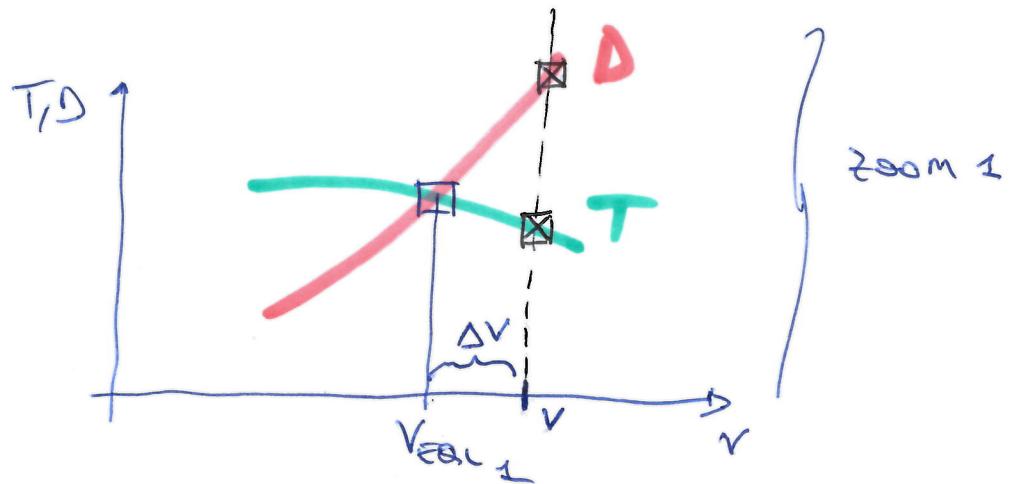
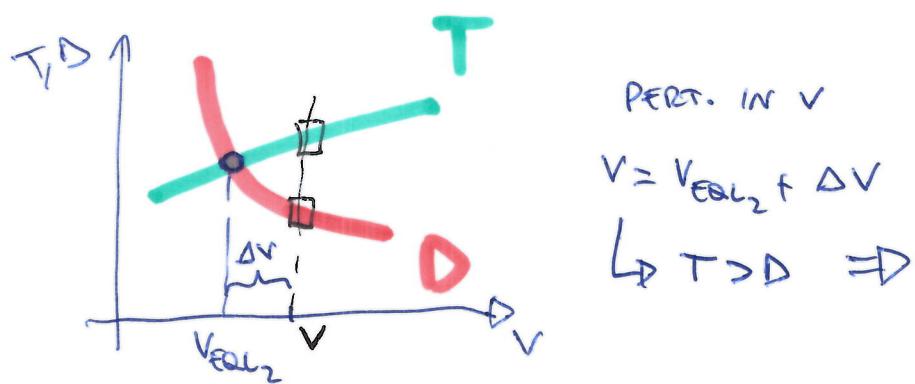
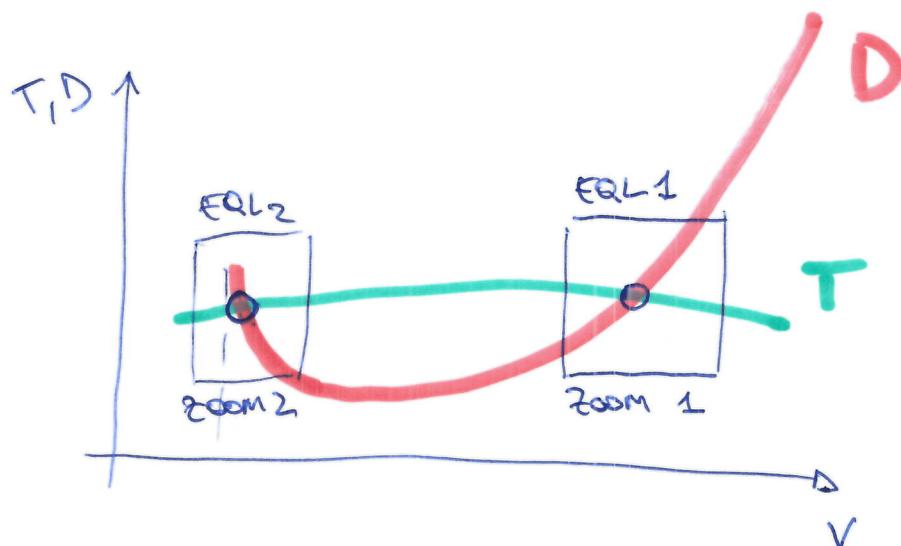


DOMANDA: È POSSIBILE VOLARE OLTRE LA QUOTA DI TANGENZA? SI MA NON IN VORU



STABILITÀ PROPELLENTE

CONSIDERIAMO UNA CONDIZIONE IN CUI ABBIAMO 2 INTERSEZIONI TRA T E D , ENTRAMBE A VELOCITÀ $> V_{STAB}$



PERTURBAZIONE IN V + c.

$$V = V_{EQL_1} + \Delta V$$

PER $V_{EQL_1} + \Delta V \rightarrow D > T \Rightarrow$

VELIVOLO
RALENTA

\Downarrow

STABILE

PERT. IN V

$$V = V_{EQL_2} + \Delta V$$

$\Downarrow T > D \Rightarrow$

VELIVOLO
TENDE AD
ACCELERARE

\Downarrow
INSTAB

PRESTAZIONI PUNTUALI IN VORU (ESERCITAZIONE)

ESERCIZIO 1

VELIVOLO A GETTO IDEALE

$$S = 93 \text{ m}^2$$

$$b = 28.6 \text{ m} \quad (\text{apertura alone})$$

$$\zeta_{D_0} = 0.019$$

$$C_{MAX} = 1.62$$

$$\text{ISA} \rightarrow \frac{P}{P_0} = \left(1 + h \frac{\lambda}{\theta_0}\right)^{-\left(1 + g/R\lambda\right)}$$

$$e = 0.86 \quad (\text{FATTORE DI ESVALDO})$$

$$w/S = 4170 \text{ N/m}^2$$

$$h = 5000 \text{ m} \quad (\text{quota del volo})$$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^\xi \quad \xi = 0.82$$

$$T|_{h=0} = 41000 \text{ N}$$

SPINTA
A QUOTAD

$$\text{con } \lambda = -0.0065 \text{ K/m}$$

$$g = 9.81 \text{ m/s}^2 \quad P_0 = 1.225 \text{ kg/m}^3$$

$$\theta_0 = 288.15 \text{ K}$$

$$R = 287.05 \text{ m}^2/\text{kg s}^2$$

COST. BOLTZ MANN

- *) DETERMINARE LA V_{MIN} E V_{MAX} DI VOLO IN VORU A 5000 m

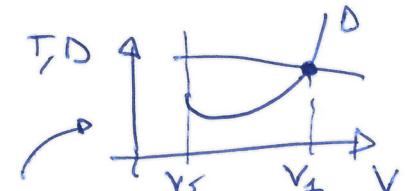
→ SOLUTIONE

$$P_{5000} \rightarrow \text{DA ISA} \Rightarrow P_{5000} = 0.736 \text{ kg/m}^3$$

$$T_{5000} \rightarrow \text{DA Modello SPINTA} \Rightarrow T_{5000} = T_0 \left(\frac{P_{5000}}{P_0} \right)^{\gamma} = 26998 \text{ N}$$

$$\text{CALCOLO } V_{\text{STALL}} \rightarrow W = \frac{W}{S} S = 1170 \cdot 93 = 387810 \text{ N}$$

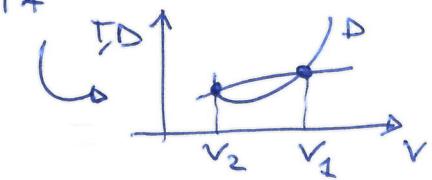
$$V_{\text{STALL}} @ 5000 \text{ m} = \sqrt{\frac{2 W/S}{P_{5000} C_{\max}}} = 83.64 \text{ m/s}$$



RISOLVO EQUILIBRIO ($T = D$) AVENDO SEMPRE IN MENTE LE DUE POSSIBILITA'

$$\rightarrow \text{CALCOLO DI POGLIE PARABOLICA } G_D = G_0 + K C^2$$

$$K = \frac{1}{\pi e \lambda_w} \quad \lambda_w = \text{ALUNGAMENTO ACARE} = \frac{b^2}{S} = 8.75 \Rightarrow K = \cancel{0.0421}$$



→ CONTROVERSI ESIBERÒ / DIFETTO DI SPINTA ALLO STALL

$$G_{\text{STALL}} = G_0 + K C_{\text{STALL}}^2 = 0.1294 \Rightarrow D_{\text{STALL}} = \frac{1}{2} P_{5000} V_{\text{STALL}}^2 S G_{\text{STALL}} = 30978 \text{ N}$$

$$D_{\text{STALL}} > T_{\text{real}} \Rightarrow 2 V_{\text{real}} > V_{\text{STALL}}$$

$$T = \Delta \Rightarrow T = \frac{1}{2} \rho V^2 S C_D$$

$$= \frac{1}{2} \rho V^2 S (C_{D0} + K C_L^2) \quad \leftarrow L = W \Rightarrow C_L = \left(\frac{W/S}{\frac{1}{2} \rho V^2} \right)$$

$$= \frac{1}{2} \rho V^2 S \left(C_{D0} + K \left(\frac{W/S}{\frac{1}{2} \rho V^2} \right)^2 \right)$$

$$= \frac{1}{2} \rho V^2 S C_{D0} + \frac{K W^2}{\frac{1}{2} \rho V^2 S}$$

$$T = \frac{1}{2} \rho V^2 S C_{D0} + \frac{K W^2}{\frac{1}{2} \rho V^2 S} \Rightarrow \text{MOLTIPLI CON PER} \frac{1}{2} \rho V^2 S \quad \underbrace{\frac{1}{2} \rho V^2 S T = \frac{1}{4} \rho^2 S^2 C_{D0} V^4 + K W^2}$$

$$\rightarrow \boxed{\frac{1}{4} \rho^2 S^2 C_{D0} V^4 - \frac{1}{2} \rho S T V^2 + K W^2 = 0}$$

$$x = V^2 \Rightarrow A x^2 + B x + C = 0 \quad A = \frac{1}{4} \rho^2 S^2 C_{D0} \quad B = -\frac{1}{2} \rho S T \quad C = K W^2$$

$$x_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\boxed{\sqrt{x_1} = V_1 = V_{\min} = 93.03 \text{ m/s}}$$

$$\boxed{\sqrt{x_2} = V_2 = V_{\max} = 181.29 \text{ m/s}}$$

ESERCIZIO 2

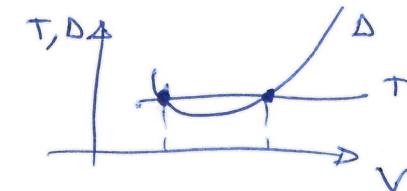
IDENTICO A ESERCIZIO 1 CON QUESTA DIFFERENZA \rightarrow NON VIENE FORNITA LA $T_{h=0}$
 MA VIENE FORNITA LA
 VELOCITÀ MINIMA DI VOLO IN VORM
 A 5000 m $V_{MIN} = 93 \text{ m/s}$
 ρ_{5000}

* RICHIESTA: SPINTA DISPONIBILE / ESUBERO/DIFETTO DI SPINTA ALLO STALLO

SOLUZIONE

$$C_{V_{MIN}} = \frac{2W/S}{\rho_{5000} V_{MIN}^2} = 1.31 \quad \text{con } \rho_{5000} = 0.736$$

$C_{V_{MIN}} < C_{STALL}$ \Rightarrow LIMITAZIONE PROSPULSIVO \Rightarrow



CALCOLO PONTE ANALITICO $C_D_0, K = \frac{1}{\pi e \lambda} = 0.0621$

$$\cancel{C_{STALL}} = \frac{C_D}{\rho V_{MIN}} = C_D_0 + K C_{V_{MIN}}^2 = \dots \Rightarrow T = D = \frac{1}{2} \rho_{5000} V_{MIN}^2 S C_{V_{MIN}} =$$

$$V_{STALL} = \sqrt{\frac{W/S}{\frac{1}{2} \rho_{5000} C_{MAX}}} = 83.64 \text{ m/s}$$

$$C_{D_{STALL}} = C_D_0 + K C_{STALL}^2 = 0.1294 \Rightarrow D_{STALL} = \frac{1}{2} \rho_{5000} V_{STALL}^2 S C_{D_{STALL}} = 3.09 \cdot 10^4 \text{ N}$$

$$T - D_{STALL} = -3.98 \cdot 10^3 \text{ N}$$

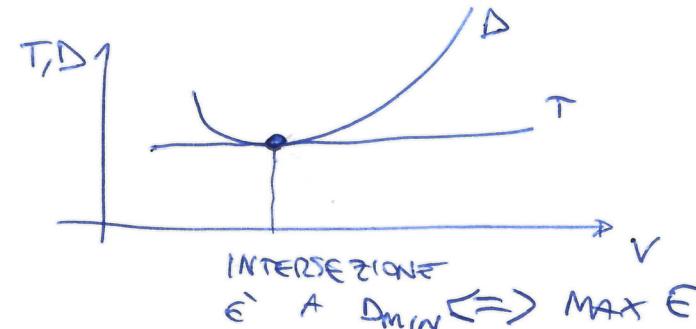
COSTANTE CON V
 (UGUALE ALLO STALL)

ESERCIZIO 3

STESSI DATI DELL'ESERCIZIO 2

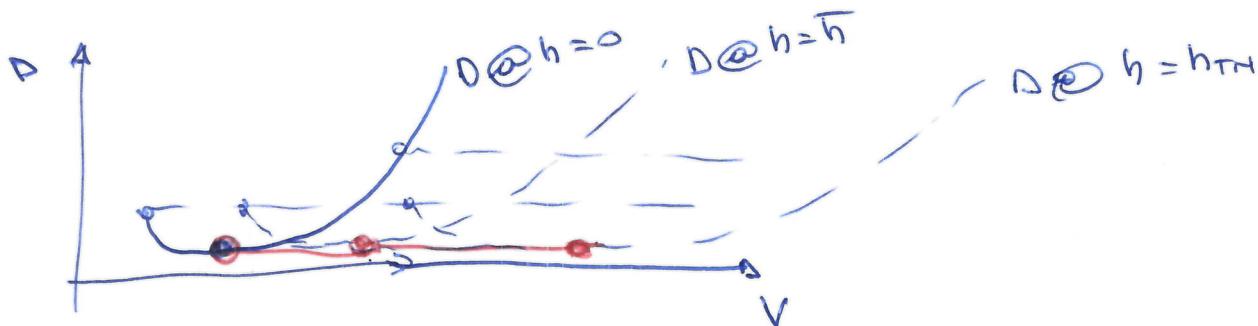
② RICHIESTA: CALCOLARE LA QUOTA DI TANGENZA

① ALLA QUOTA DI TANGENZA



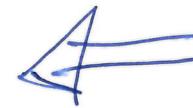
$$\begin{aligned} ① \text{ MAX } E \Rightarrow C_{D_{\max}} &= 2 C_{D_0} = 0.038 \\ C_{D_{\max}} &= \sqrt{k/C_{D_0}} = 0.6719 \quad || \Rightarrow E_{\max} = \frac{\sqrt{k/C_{D_0}}}{2 C_{D_0}} = \dots \end{aligned}$$

② D AL VARIARE DELLA QUOTA RESTA COSTANTE SE FISSO L'ASSETTO.



$$D_{\min} = D_{\min_{socca}} = \frac{W}{E_{\max}} = 2.193 \cdot 10^4 \text{ N}$$

A TUTTE LE QUOTE



ANCHE SE NON CONOSCIAMO LA QUOTA DI TANGENZA (E QUINDI LA DENSITÀ) POSSIAMO SAPERE QUANTO VALGONO LA D ALLA QUOTA DI TANG. NELLA CONDIZIONE DI MAX E

$$\textcircled{3} \quad T_{@h_{TH}} = D_{MIN} \Rightarrow T_{@h_{TH}} = 2.193 \cdot 10^4 \text{ N}$$

\textcircled{4} VAIUTO LA FUNZIONE DELLA SPANTA AL VARIARE DELLA QUOTA

$$\frac{T_{h_{TH}}}{T_{5000}} = \left(\frac{P_{h_{TH}}}{P_{5000}} \right)^{\frac{q}{k}} \Rightarrow P_{h_{TH}} = P_{5000} \left(\sqrt[k]{\frac{T_{h_{TH}}}{T_{5000}}} \right) = 0.5710 \frac{\text{kg}}{\text{m}^3}$$

\textcircled{5} INVERTIRE LE EQZ ISA PER TROVARE h_{TH} IN FUNZIONE DI P_{TH}

$$h_{TH} = \frac{\theta_0}{\lambda} \left(-1 + \left(\frac{P_{TH}}{P_0} \right)^{-\frac{R\lambda}{g+R\lambda}} \right) = \boxed{7276 \text{ m}}$$

ESERCIZIO 4

→ MOTOELICA

$$\epsilon = 0.88$$

$$G_0 = 0.019$$

$$\max G = 1.59$$

$$\eta_p = 0.94 \text{ rendimento elica}$$

$$\text{apertura elice } b = 32.1 \text{ m}$$

$$\text{superficie } S = 95.0 \text{ m}^2$$

$$\text{cavità elice } w/s = 3150 \text{ N/m}^2$$

$$P_b = \left(\frac{f}{\rho_0} \right) P_{b_0} \quad P_{b_0} = 6500 \text{ kW}$$

Potenze
all'albero

DATI: EQZ ISA

- *) 1) DETERMINARE LA ^{V_{EAS}} VELOCITÀ EQUIVALENTE MINIMA E MASSIMA (N VOLW)
 A UNA QUOTA DI 3000 m, UTILIZZANDO UNA TOLERANZA DI
 2 m/s (ANDROVE FOSSE NECESSARIO L'UTILIZZO DI UNA PROCEDURA ITERAT.)
- 2) DETERMINARE LA QUOTA DI TANGENZA TEORICA.

SOLUZIONE

CALCOLI PRELIMINARI

$$K = \frac{1}{\pi e \lambda} \quad \text{con } \lambda = \frac{b^2}{S} \quad \Rightarrow \quad K = 0.033$$

$$W = \frac{W}{S} S = 299250 \text{ N}$$

$$P_{3000} \rightarrow \text{DA VANTAZIONE ISAT} \quad P_{3000} = 0.9091$$

$$\frac{P_A}{P_0} = \gamma_p \quad \frac{P_b}{P_0} = \dots$$

$h = 0$

$$P_A = \gamma_p P_b = \gamma_p P_{b0} \left(\frac{P_{3000}}{P_0} \right) = 4534 \text{ kW}$$

$$V_{MAX} \rightarrow P_A = P_R = \frac{1}{2} \rho_{3000} V^3 S C_D$$

$$P_A = \frac{1}{2} \rho_{3000} V^3 S (C_{D0} + K A^2)$$

$$C_L = \frac{W}{\frac{1}{2} \rho_{3000} V^2 S}$$

$$P_A = \frac{1}{2} \rho_{3000} V^3 S C_{D0} + \frac{K W^2}{\frac{1}{2} \rho S V}$$

→ NO BICUAD.

if

PROCEDURA ITERATIVA

SOLUZIONE ITER.

① TRASCINO LA PARTE INDOITA

$$P_A = P_{ref} = \frac{1}{2} \rho V^3 S C_D \Rightarrow V_I = \sqrt[3]{\frac{P_A}{\frac{1}{2} \rho S C_D}} = 176.8 \text{ m/s}$$

② VALUTO TERMINE RESIDUO

$$P_L = \frac{kW^2}{\frac{1}{2} \rho S V_I^2} = 351 \text{ kW}$$

$$P_d - P_L = 4183 \text{ kW}$$

③

$$P_d - P_L = \frac{1}{2} \rho V^3 S C_D \Rightarrow V_{II} = \sqrt[3]{\frac{P_d - P_L}{\frac{1}{2} \rho S C_D}} = 171.56 \text{ m/s}$$

④ VALUTO TOLERANZA $|V_{II} - V_I| \approx 5 \text{ m/s} \rightarrow \text{TOLL}$

⑤

$$P_2 = \frac{kW^2}{\frac{1}{2} \rho V_{II}^2} = 403 \text{ kW}$$

⑥

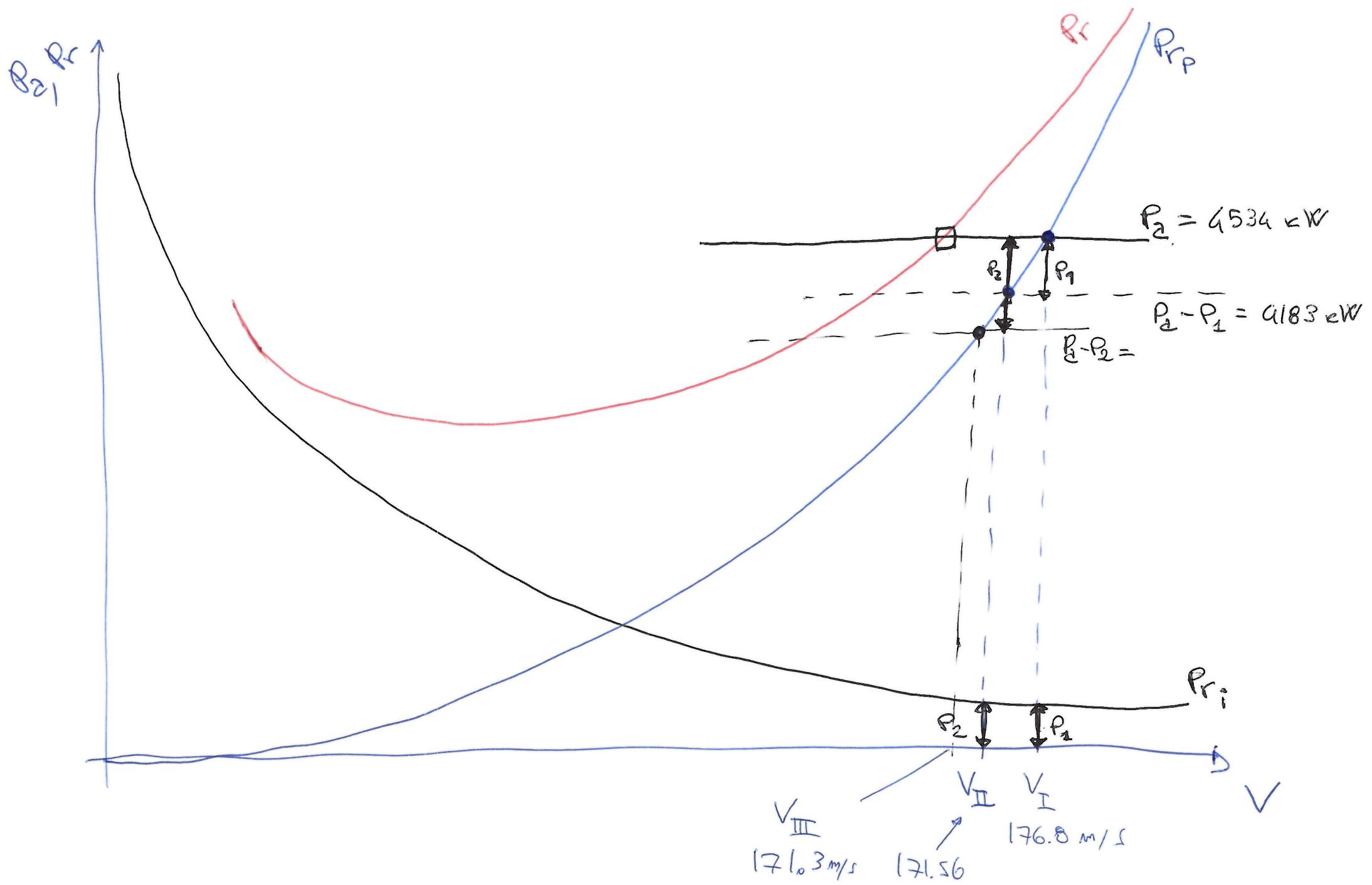
$$V_{III} = \sqrt[3]{\frac{P_A - P_2}{\frac{1}{2} \rho S C_D}} = 171.3 \text{ m/s}$$

⑦

$$|V_{III} - V_{II}| < \text{TOLL} \rightarrow$$

$V_{MAX,EAS} = V_{MAX} \sqrt{\frac{\rho}{\rho_0}} = 147.6 \text{ m/s}$

$V_{MAX} = V_{III} = 171.3 \text{ m/s}$



CALCOLO DELLA V_{MIN}

$$V_{STALL} = \sqrt{\frac{W/S}{\frac{1}{2} \rho_{3000} C_{M,STALL}}} = 66.02 \text{ m/s}$$

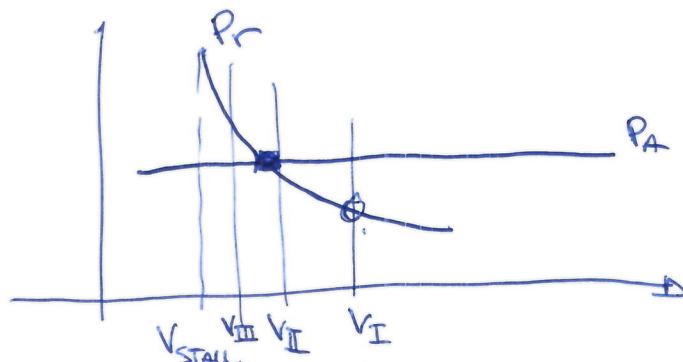
CALCOLO ESUBERO POTENZA AL VO STALL

$$C_D = C_{D_0} + k C_{STALL}^2 = 0.103$$

$$P_r|_{STALL} = \frac{1}{2} \rho_{3000} V_{STALL}^3 S C_D = 128.4 \text{ kW}$$

$$P_A = 453 \text{ kW}$$

SF AVESSIMO AVUTO UN DIFETTO DI POTENZIA

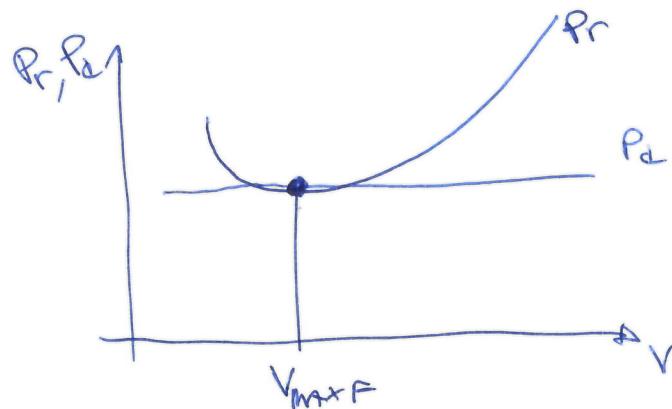


PROCEDERE PER BISEZIONE

$$\text{PRENDETE UNA } V_{II} = \frac{(V_{STALL} + V_I)}{2} \quad (\text{IPOTESI } P_r|_{V_{II}} - P_a < 0)$$

$$\text{PRENDETE UNA } V_{III} = \frac{V_{STALL} + V_{II}}{2}$$

QUOTA DI TANGENZA



INTERSEZ. TRA P_d E P_r ALLA QUOTA DI TANG.

$\min P_r$

$\max F = \left(\sqrt{a} \frac{a}{c_0} \right)$

$$c_0|_{\max F} = 4 c_0$$

$$a|_{\max F} = \sqrt{\frac{3 c_0}{K}}$$

$$P_A|_{H_{TH}} = P_r|_{\max F}$$

$$\eta_p P_{bo} \left(\frac{P_{H_{TH}}}{P_0} \right) = \frac{1}{2} P_{H_{TH}} \cdot \underbrace{V_{\max F}^3}_{S c_0|_{\max F}}$$

DA $L = W$

$$V_{\max F} = \sqrt[3]{\frac{W}{\frac{1}{2} P_{H_{TH}} S c_0|_{\max F}}}$$

$$\eta_p P_{bo} \left(\frac{P_{H_{TH}}}{P_0} \right) = \frac{1}{2} P_{H_{TH}} \left(\frac{W}{\frac{1}{2} P_{H_{TH}} S c_0|_{\max F}} \right)^{3/2} S c_0|_{\max F} \Rightarrow \text{UNICA INCOGNITA } P_{H_{TH}}$$

ELEVATO TUTTO ALLA $2/3$

$$\left(\eta_p \frac{P_{B_0}}{P_0}\right)^{2/3} = \left(\frac{1}{2} S G_{\max F}\right)^{2/3} \frac{2 W/5}{P_{H_{TH}} G_{\max F}}$$

$$P_{H_{TH}} = \frac{2 W/5}{G_{\max F}} \cdot \left(\frac{2 \eta_p P_{B_0}}{P_0 S G_{\max F}} \right)^{-2/3} = 0.3885 \text{ kg/m}^3$$

INVERSO LA ISA

$$h_{TH} = \frac{G_0}{\lambda} \left(-1 + \left(\frac{P_H}{P_0} \right)^{-\frac{R\lambda}{(g+R\lambda)}} \right) = 10.5 \text{ km}$$

ESERCIZIO SOLO ANALITICO (5)

SIANO NOTI $w, s,$ POLARE ANALITICA $G = G_0 + k_1 G + k_2 G^2$

G_{\max}

$$T = T_0 + T_2 V_{EAS}^2$$

$$\frac{1}{2} \rho_0 V_{EAS}^2 = \frac{1}{2} \rho V^2 \Rightarrow V_{EAS}^2 = V^2 \frac{\rho_0}{\rho}$$

P

* CALCOLO $V_{\min} \in V_{\max}$ IN VORO

$$\begin{cases} L = w \\ T = D \end{cases} \rightarrow a = \frac{w/s}{\frac{1}{2} \rho V^2} \quad \rho_0 = 1.225 \text{ kg/m}^3$$

$$\begin{aligned} T_0 + T_2 V^2 \frac{\rho_0}{\rho} &= \frac{1}{2} \rho V^2 s G_0 \\ &= \frac{1}{2} \rho V^2 s G_0 + \cancel{\frac{1}{2} \rho V^2 s k_1 \left(\frac{w/s}{\frac{1}{2} \rho V^2} \right)} + \frac{1}{2} \rho V^2 s k_2 \left(\frac{w/s}{\frac{1}{2} \rho V^2} \right)^2 \end{aligned}$$

$$\Delta \xrightarrow[\text{per } \frac{1}{2} \rho V^2 s]{\text{moltiplica}} T_0 \left(\frac{1}{2} \rho V^2 s \right) + T_2 V^2 \frac{1}{2} \rho s \frac{\rho_0}{\rho} = \frac{1}{4} (\rho s)^2 V^4 G_0 + k_1 w \left(\frac{1}{2} \rho V^2 s \right) +$$

$$\left| \left(\frac{1}{2} \rho^2 s^2 G_0 - \frac{1}{2} \rho^2 s T_2 \right) V^4 + \left(\frac{1}{2} \rho s k_1 w - \frac{1}{2} \rho s T_0 \right) V^2 + k_2 w^2 = 0 \right|$$

$$x = V^2 \Rightarrow Ax^2 + Bx + C = 0 \rightarrow \dots \sqrt{x_{1,2}} = V_1, V_2$$

BICUADRATICA
($V_1 < V_2$)

PARAGONARE $A > 0$
LA $V_{STALL} = \sqrt{\frac{w/s}{\frac{1}{2} \rho a_{\max}}}.$

PRESTAZIONI IN SALITA

PUNTAZIONI

- VOLO SIMMETRICO ($\beta = \omega$)
- PIANO VERTICALE ($\dot{x} = 0$)
- RETTILINEO ($\dot{\gamma} = 0$)
- $\alpha \ll 1$
- $T_d \ll L$

$$\Rightarrow \left\{ \begin{array}{l} \text{EQL} \\ m \ddot{v} = T - D - W \sin \gamma \\ 0 = L - W \cos \gamma \quad (+ T \cancel{\sin \alpha}) \\ \dot{h} = \frac{P_d - P_r}{W} \end{array} \right. \Rightarrow \text{derivate quote totale}$$

$$H = h + \frac{1}{2} \frac{v^2}{g}$$

$$\dot{H} = \dot{h} + \frac{vv'}{g} = \underset{\uparrow}{V_v} + \frac{vv'}{g}$$

Velocità
verticale

SALITA / DISCESA STAZIONARIA
($\dot{v} = 0$)

$$H_p \quad \dot{v} = 0 \Rightarrow \dot{h} = V_v$$

$$\Downarrow$$

$$SEP = V_v$$

$$\left\{ \begin{array}{l} \text{EQL} \\ T = D + W \sin \gamma \\ L = W \cos \gamma \end{array} \right.$$

PERFORMANCE DI RUVEVO IN SAUTA $\rightarrow \delta, V_v$

$$\sin \delta = \frac{T - D}{W}$$

$$V_v = \frac{P_d - P_r}{W}$$

$$\rightarrow \frac{T - D}{W} = SET$$

Specific excess of thrust

$$\rightarrow \frac{P_d - P_r}{W} = SEP$$

Specific excess of power

ANALITICAMENTE

POLARE
TRIMMATA

$$T = T(h, V, \delta_T)$$

$$L = W \cos \delta$$

$\overbrace{W}^{\text{NEW}}$

$$D = D(h, V, L) = D(h, V, W, \delta)$$

$$P_r = P_r(h, V, L) = P_r(h, V, W, \delta) = P_r(h, V, W, \delta_T)$$

$\overbrace{W}^{\text{NEW}}$

$$V_v = V \sin \delta$$

$$\sin \delta = \frac{T(h, V, \delta_T) - D(h, V, W, \delta)}{W}$$

$$V_v = \frac{P_d(h, V, \delta_T) - P_r(h, V, W, V_v)}{W}$$

LEGAME IMPLICITO

IN δ e V_v



RISOLUZIONE

NUMERICA --

SEMPLIFICAZIONE

$\gamma \ll 1$ angoli di rampe piccoli.



$$\cos \gamma \approx 1$$

$$\begin{array}{l} \text{ESL} \\ + \\ \gamma \ll 1 \end{array} \Rightarrow \left\{ \begin{array}{l} L = \cancel{W} \quad \circledast \\ D = D(h, V, W) \end{array} \right\} \parallel \Rightarrow$$

LEGAME COSTITUTIVO AERO

~~RESISTENZA IN VORU~~

$$\circledast C = \frac{W}{\frac{1}{2} \rho V^2 S} = C_{voru}$$

$$C_D = C_D(C) = C_D(C_{voru})$$

LA RESISTENZA D IN SALITA E' PARI ALLA SPINTA NECESSARIA
IN VORU. (CON L'ASSUNZIONE DI $\gamma \ll 1 \Rightarrow L = W$)

⇒ UTILIZZO DEI DIAGRAMMI DI PENAUD IN VORU
PER LA QUANTIFICAZIONE DELLE PERFORMANCE IN
SALITA

$$\gamma \approx 2m\delta = \frac{T - D}{W}$$

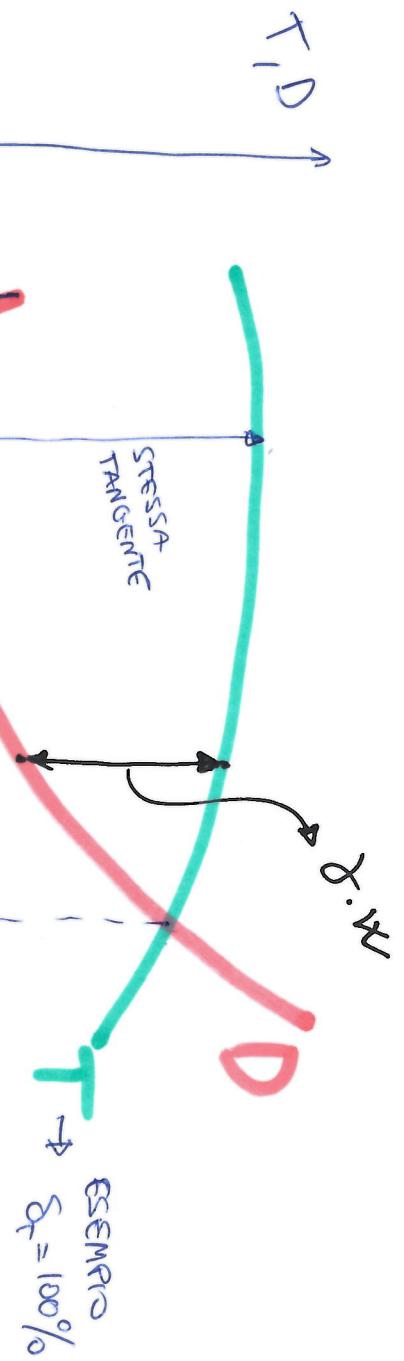
$$V_r = \frac{P_d - P_r}{W}$$

dove D e' la spinta necessaria
in voru

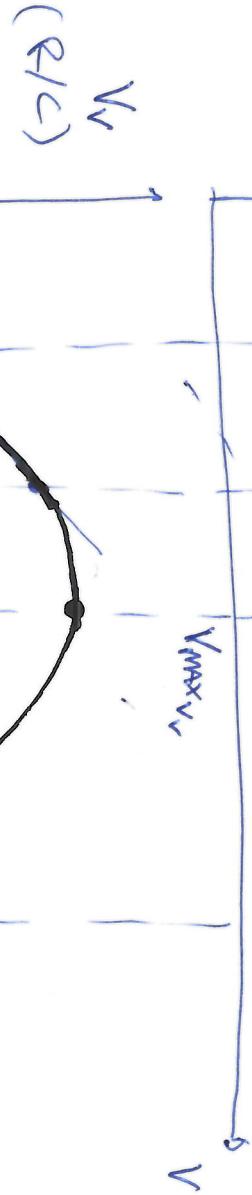
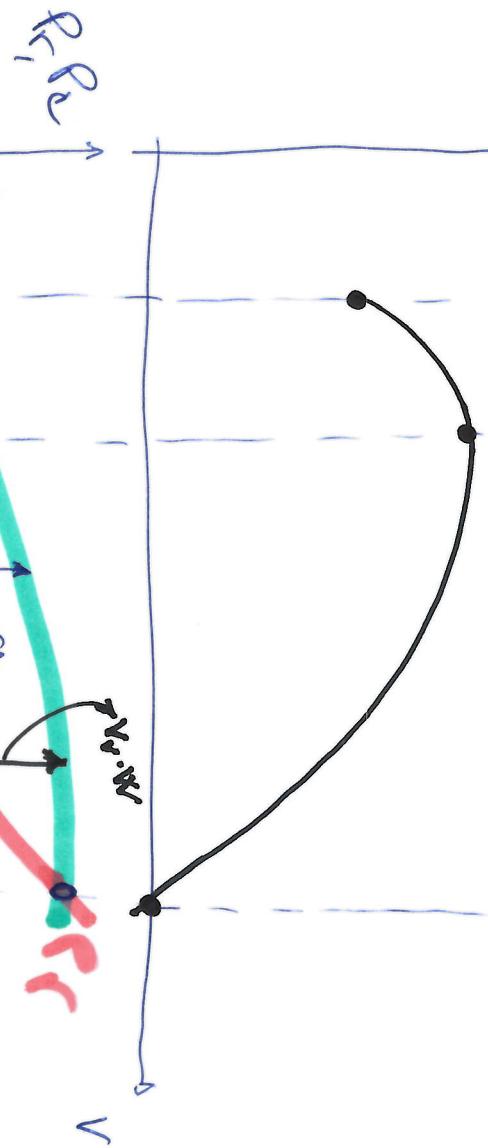
dove P_r e' la potenza richiesta
in voru

PRESTAZIONI IN SALITA PER VIA GRAFICA

NUOVO GENERICO



ESEMPIO
 $\sigma_y = 100\%$



$V_{min} = V_{stall}$

V_r
(R/C)

V_{max}

V_{max}

V_{max}

V

$$\gamma = \frac{T - D}{W} \Rightarrow \max \gamma \Rightarrow \frac{\partial \gamma}{\partial V} = 0 \Rightarrow \frac{\partial \left(\frac{T - D}{W} \right)}{\partial V} = 0 \Rightarrow \boxed{\frac{\partial T}{\partial V} = \frac{\partial D}{\partial V}}$$

MASSIMO ANGOLIO DI RAMPÀ QUANDO

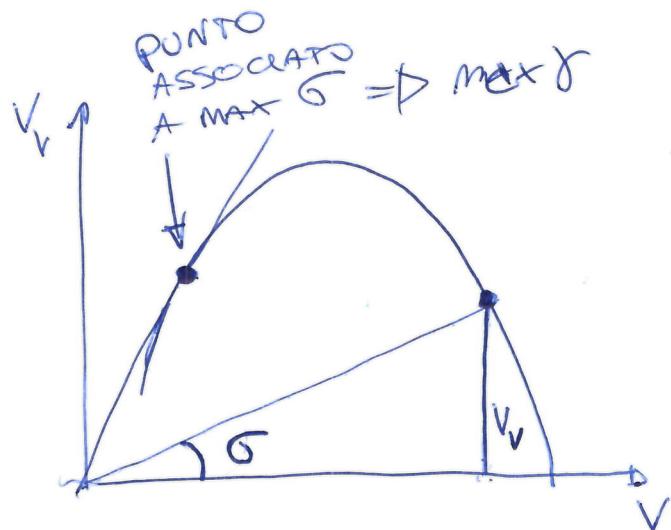
PER QUELLA V IN CUI TANGENTE DI T è UGUALE
ALLA TANGENTE DI D

$$V_r = \frac{P_c - P_r}{W} \Rightarrow \max V_r \Rightarrow \frac{\partial V_r}{\partial V} = 0 \Rightarrow \frac{\partial \left(\frac{P_c - P_r}{W} \right)}{\partial V} = 0 \Rightarrow \boxed{\frac{\partial P_c}{\partial V} = \frac{\partial P_r}{\partial V}}$$

massima V_r per quella V in cui le tangenti

di P_c e P_r sono uguali

VELOCITÀ
RAPIDA
 $\textcircled{*} V_{\max \gamma}$



$$\tan \gamma = \frac{V_r}{V} = \frac{V \sin \delta}{V} \Rightarrow \tan \gamma = \sin \delta$$

$$\Rightarrow \boxed{\tan \gamma = \sin \delta}$$

$$\gamma = \arctan(\tan \delta)$$

$$\gamma = \frac{\delta}{\sqrt{\delta^2 + 1}} \quad \begin{array}{l} \text{FUNZIONE} \\ \text{MONOTONA} \\ \text{CRESCENTE} \end{array}$$

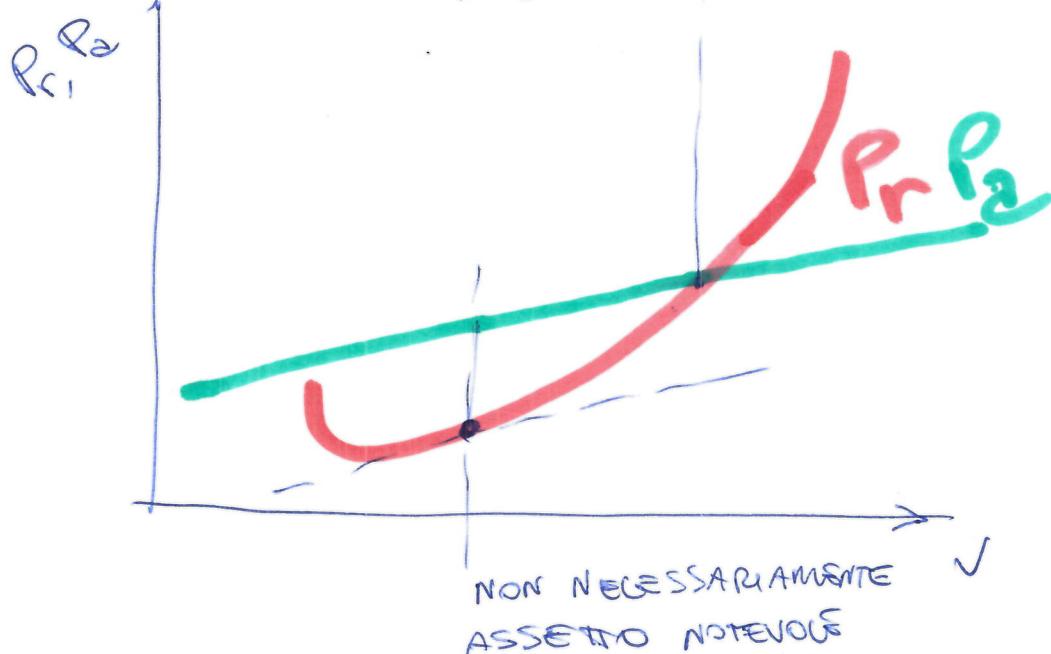
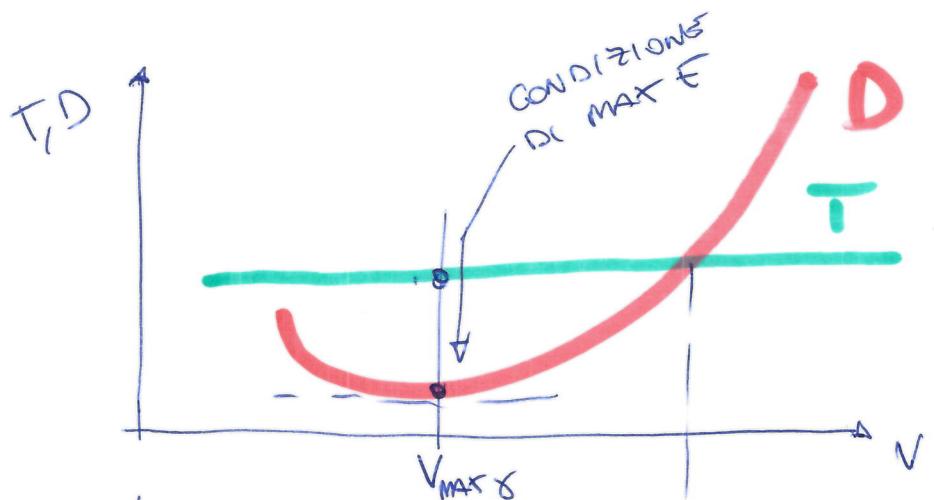
VELOCITÀ
RAPIDA
 $V_{\max V_r}$

$\max \gamma$ si ottiene
per il $\max \delta$

SOLUZIONI ANALITICHE PER LA SALITA

POLARE PARABOLICA
PROPELLSIONI IDEALI

$$J \quad \frac{\partial T}{\partial V} = 0 \Rightarrow \frac{\partial P_d}{\partial V} = T$$



VELOCITA' RAPIDA (V_{max_J})

$$\frac{\partial T}{\partial V} = \frac{\partial P_d}{\partial V} \Rightarrow \frac{\partial A}{\partial V} = 0$$

$$\Rightarrow V_{max_J} \Leftrightarrow V_{max_E} \Rightarrow C_{max} = \sqrt{C_0/k}$$

$$V_{max_J} = \sqrt{\frac{W/S}{\frac{1}{2} \rho \sqrt{C_0/k}}} : T$$

VELOCITA' RAPIDA (V_{max_VV})

$$\frac{\partial P_d}{\partial V} = \frac{\partial P_r}{\partial V} \Rightarrow T = \frac{\partial P_r}{\partial V}$$

$$T = \frac{\partial}{\partial V} \left(\frac{1}{2} \rho V^3 S C_D + \frac{k W^2}{\frac{1}{2} \rho V^2 S} \right)$$

$$T = \frac{3}{2} \rho V^2 S C_D - \frac{k W^2}{\frac{1}{2} \rho V^2 S}$$

$$\frac{1}{2} \rho S T V^2 = \frac{3}{4} (\rho s)^2 C_D V^4 - k W^2$$

$$\frac{3}{4} (\rho s)^2 C_D V^4 - \frac{1}{2} \rho S T V^2 - k W^2 = 0$$

BRAVAIO.

→ NO SOL
OPPURE
1 SOL
→ 0
E PIADE

SOMMARIO PER

(J)

V_{MAX_J} \Rightarrow MAX E (VALIDA INDIRENTAMENTE
DALLA FORMA POLARE)

\Rightarrow DIPENDE SOLO DALL'AERODINAMICA

$V_{MAX_{V_V}}$ \Rightarrow ASSETTO NON NOTEVOLI (NO MAX, MAX F O MAX G)
RISOLUZIONE BIQUADRATICA

\Rightarrow DIPENDE SIA DA AERODINAMICA CHE DA SPINTA

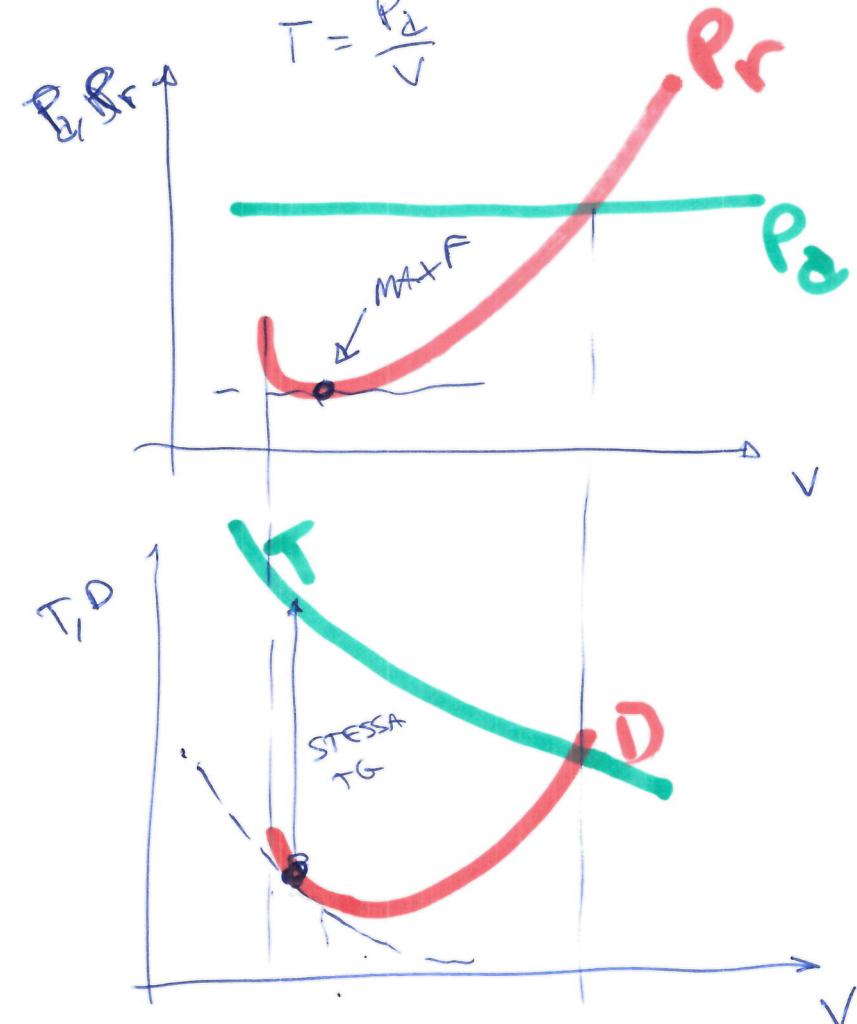
\Rightarrow DIPENDE DA FORMA POLARE

(P)

$$\frac{\partial P_c}{\partial V} = 0$$

(MOTORE G. IDEALE)

$$T = \frac{P_c}{V}$$



VELOCITÀ RAPIDA

$$\text{MAX } V_r \Rightarrow \frac{\partial P_c}{\partial V} = \frac{\partial P_r}{\partial V} \Rightarrow$$

$$\boxed{\frac{\partial P_r}{\partial V} = 0}$$

$$\Rightarrow \text{MAX } V_r = \text{MAX } F = \text{MAX} \left(\vec{F}_c \cdot \frac{\vec{a}}{c_0} \right)$$

(P)

$$V_{\text{MAX}_r} = \sqrt{\frac{w/s}{\frac{1}{2} C_L \sqrt{\frac{3 G_0}{K}}}}$$

↓
DIPENDE
SOLAMENTE
AERODINAMICA

$C_L \otimes \text{MAX } F$

VELOCITÀ RAPIDA

$$\text{MAX } \dot{Q} \Rightarrow$$

$$\boxed{\frac{\partial T}{\partial V} = \frac{\partial D}{\partial V}}$$

$$\text{SOLUZIONE} \quad \frac{\partial}{\partial C} = \frac{\partial}{\partial C}$$

$$\frac{P_d}{V} = T \Rightarrow \frac{\partial}{\partial V} = \frac{1}{V^2} - \frac{P_d}{V^2}$$

$$\frac{\partial}{\partial V} = \frac{\partial}{\partial V} \left(\frac{1}{2} \rho V^2 S G_{D_0} + \frac{K W^2}{2 \rho V^2 S} \right) = \rho G_{D_0} V - \frac{K W^2}{2 \rho S V^3}$$

$$\boxed{(\rho s)^2 G_{D_0} V^4 + \rho s P_d V - K W^2 = 0} \Rightarrow \begin{array}{l} \text{NO EQUAZIONE} \\ \Downarrow \\ \text{SOLUZIONE ITERATIVO} \\ (\text{BISEZIONE}) \\ \text{PER ESEMPIO} \end{array}$$

SOMMARIO PER \hat{P}

$V_{MAX_{VV}}^{(P)} \Rightarrow \text{MAX } F$, Dipende solo da AERODINAMICA

$V_{MAX_x}^{(P)} \Rightarrow \text{SOLUZIONE NUMERICA ITERATIVA}$
DIPIENDE DA AERODINAMICA E PROPULSIONE

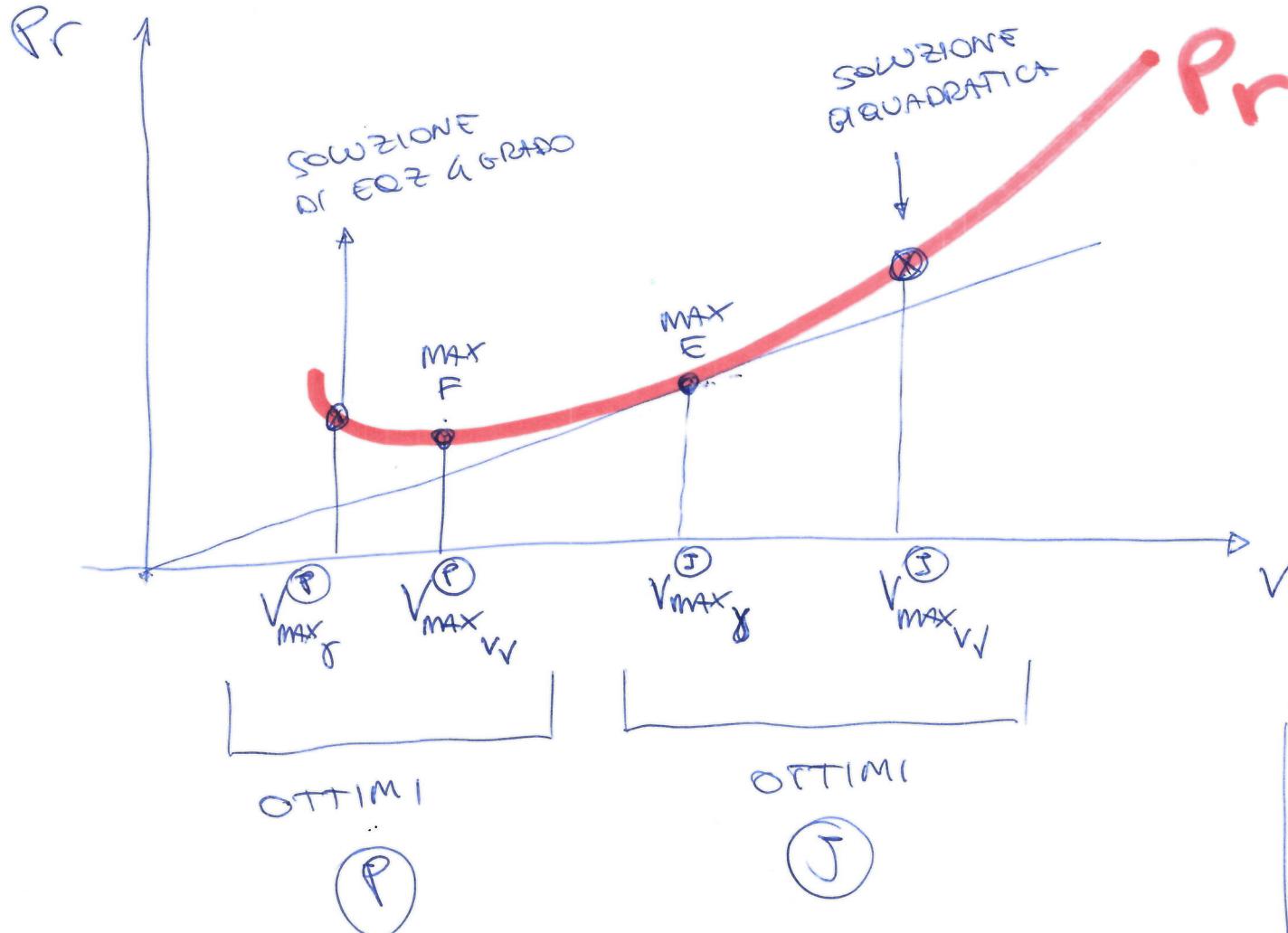
$V_{MAX_g}^{(P)}$
MOLTO PROSSIMA A STADIO

CONFRONTO

(J)

(P)

PER UNA SALITA



VEICOLO A GETTO
PROGETTATO PER
VELOCITA' PIU' ALTE
ASSETTI PIU' BASSI

VEICOLO MOTOCICLISTICO
PER VELOCITA' PIU' BASSE
ASSETTI PIU' ALTI

PRESTAZIONI INTEGRALI IN SALITA

[1] TEMPO DI SALITA

$$T_C := \int_{t_1}^{t_2} dt = \int_{h_1}^{h_2} \frac{1}{V_v} dh$$

(AREA CALCOLATA) $V_v = \frac{dh}{dt} \Rightarrow dt = \frac{dh}{V_v}$

[2] SPAZIO DI SALITA

$$S_C := \int_{t_1}^{t_2} V_H dt = \int_{h_1}^{h_2} \frac{V_H}{V_v} dh = \int_{h_1}^{h_2} \frac{1}{\tan \delta} dh$$

$t_1, t_2 \Rightarrow$ ISTANTI DI TEMPO
A INIZIO E FINE
SALITA

$h_1, h_2 \Rightarrow$ QUOTE A INIZIO E
FINE CROCIERA

$V_H \Rightarrow$ VELOCITÀ ORIZZONTALE
(componente delle $V \parallel x^H y^H$)



CONSUMO DI COMBUSTIBILE/ARGONANTE
IN SALITA

$$W_{FC} := \int_{t_1}^{t_2} \dot{W}_F dt = \int_{h_1}^{h_2} \frac{\dot{W}_F}{V_v} dh$$

REMARK: MASSA VARIABILE!!

MASSA PESO VARIA POCO...

CONSIDERO PESO VELIVOLO COSTANTE
PER TUTTA LA SALITA

SPAZIO DI SALITA

$$S_C(h_1, h_2) = \int_{h_1}^{h_2} \frac{dh}{\tan(\gamma(h))}$$

$$\min S_C = \int_{h_1}^{h_2} \frac{dh}{\tan(\max \delta h)} = \int_{h_1}^{h_2} \frac{dh}{\tan(\delta h)}$$

Volando AD OGNI QUOTA NELLA CONDIZIONE DI SALITA RAPIDA ($V_{MAX,g}$)
MINIMIZZIAMO IL SPAZIO DI SALITA



$V_{MAX,g} \Rightarrow$ PUO' VARIARE CON LA QUOTA \rightarrow

MINIMO SPAZIO DI SALITA

UTILISSIMA SE ABBIANO UN
OSTACO DA SUPERARE.

DEVO ACCELERARE // ATTENZIONE A
DECELERARE // NON STAZ.

Per il
momento
non consideriamo
(e non stazionare)

Se AUMENTO \downarrow A
VELOCITA' DI VOLO
PARTE DECESA SPINTA SERVE
PER INCREMENTARE LA
VELOCITA'.

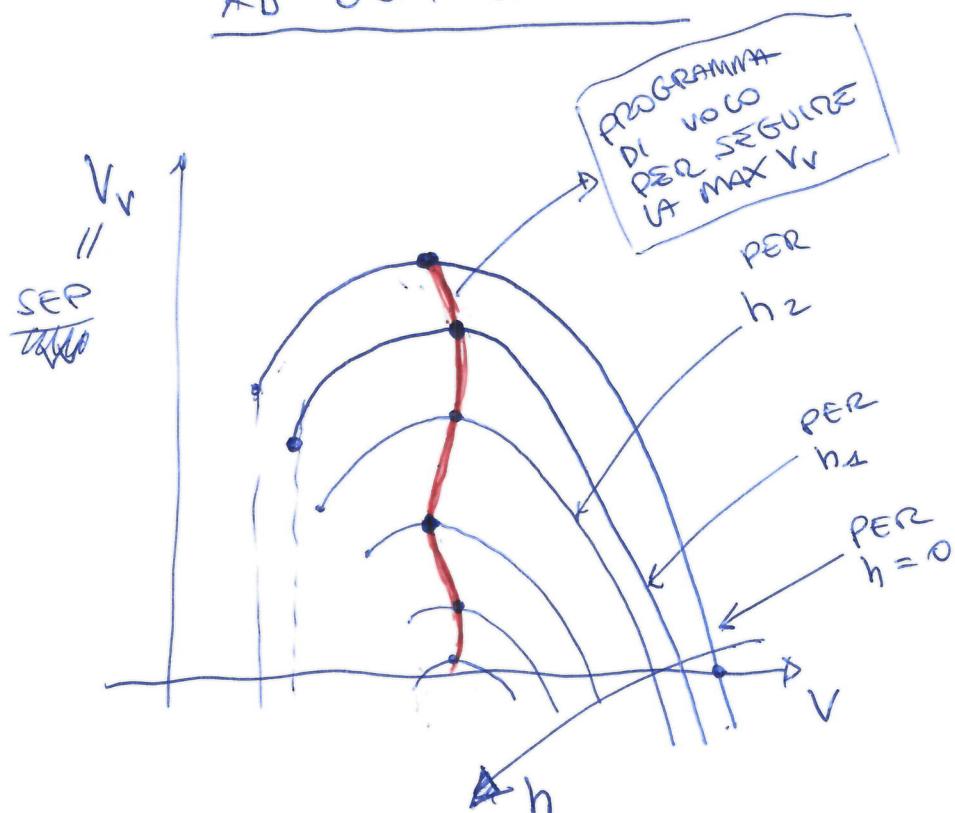
TEMPO DI SALITA

$$T_c := \int_{h_1}^{h_2} \frac{dh}{V_r(h)} \Rightarrow$$

$$\min T_c = \int_{h_1}^{h_2} \frac{dh}{\max V_r(h)}$$

VOLANDO NELLA CONDIZIONE DI

AD OGNI QUOTA OTTIENIAMO

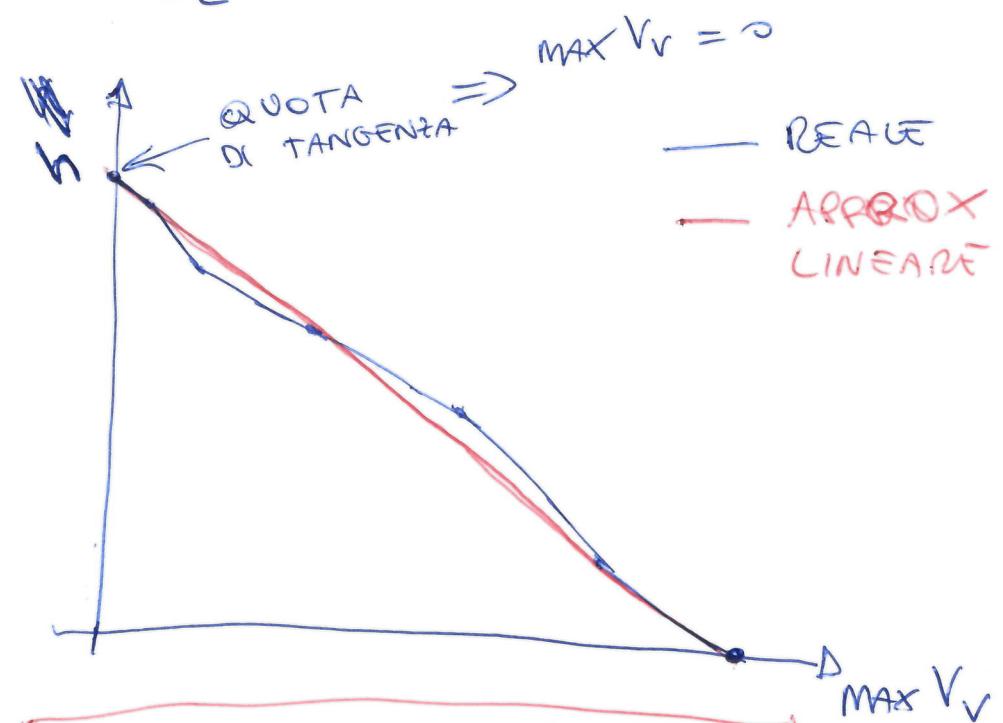


NOTA

- 1 → SIMILE ANDAMENTO PER MAX δ
- 2 → $V_{\max V_r}$ PUÒ VARIARE CON QUOTA!

SALITA RAPIDA ($V_{\max V_r}$)

IL T_c MINIMO.



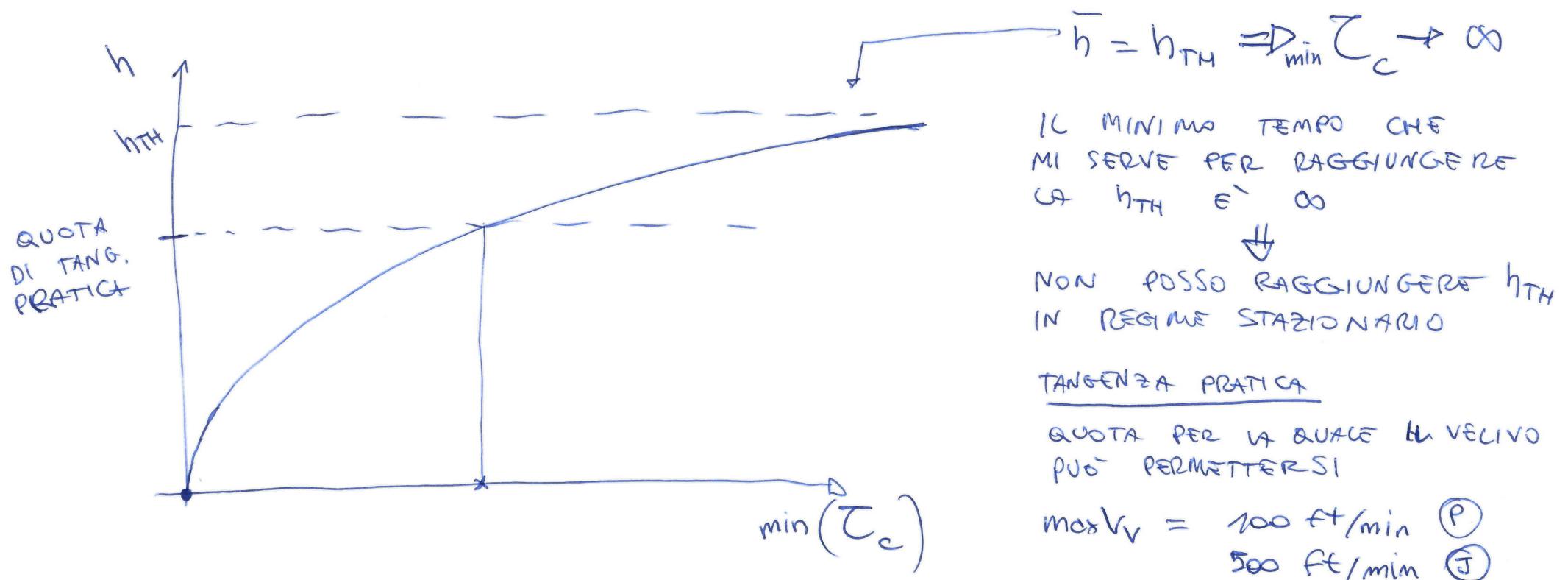
$$\left(* \right) \max V_r(h) = \left(1 - \frac{h}{h_{TN}} \right) \max V_{r0}$$

ossia V_r A QUOTA h .

$$\min \bar{C}_c := \int_{h_1}^{h_2} \frac{dh}{\max(V_v(h))} = \int_{h_1}^{h_2} \frac{dh}{(1-h/h_{TH}) \max V_{v0}}$$

consideriamo $h_1 = 0$ $h_2 = \bar{h}$

$$\boxed{\min \bar{C}_c := \int_{h_0=0}^{\bar{h}} \frac{dh}{(1-h/h_{TH}) \max V_{v0}}} = \dots = \boxed{\frac{\bar{h}/h_{TH}}{\max V_{v0}} \ln \left(\frac{1}{1-\bar{h}/h_{TH}} \right)}$$



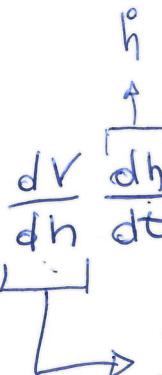
ANALISI NON STAZIONARIE TA'

NOTA: SAL VELOCITA' RAPIDA/RAPIDA VARIANO CON LA QUOTA

→ CASO SALITA RAPIDA |

$$SEP = \dot{h} \Rightarrow \dot{h} + \frac{V \ddot{V}}{g} = \dot{h} + \frac{V}{g} \frac{dV}{dh} \frac{dh}{dt}$$

↓ QUOTA TOTALE
 ↑ DERIVATA QUOTA GEOMETRICA
 ↓ DERIVATA VELOCITA' DI VOLO



VARIAZIONE DI VELOCITA' DI VOLO CON LA QUOTA

$$SEP = V_v \left(1 + \frac{V}{g} \frac{dV}{dh} \right) \rightarrow \begin{cases} \text{SE } \frac{dV}{dh} > 0 \rightarrow V_v < SEP \\ \text{SE } \frac{dV}{dh} < 0 \rightarrow V_v > SEP \end{cases}$$

$$\dot{h} = \frac{P_a - P_r}{w} \Rightarrow V_v = \frac{P_a - P_r}{w \left(1 + \frac{V}{g} \frac{dV}{dh} \right)}$$

$$\min C_c = \int_{h_1}^{h_2} \frac{dh}{\max V_v} = \int_{h_1}^{h_2} dh \min \left(\frac{w \left(1 + \frac{V}{g} \frac{dV}{dh} \right)}{(P_a - P_r)} \right)$$

OPTIMAL CONTROL THEORY

PROGRAMMA DI VOLO IN SALITA
A MINIMO TEMPO DI SALITA
"NON-STAZIONARIO" PER VENIRE
SUPER-SONICO TIPO F104

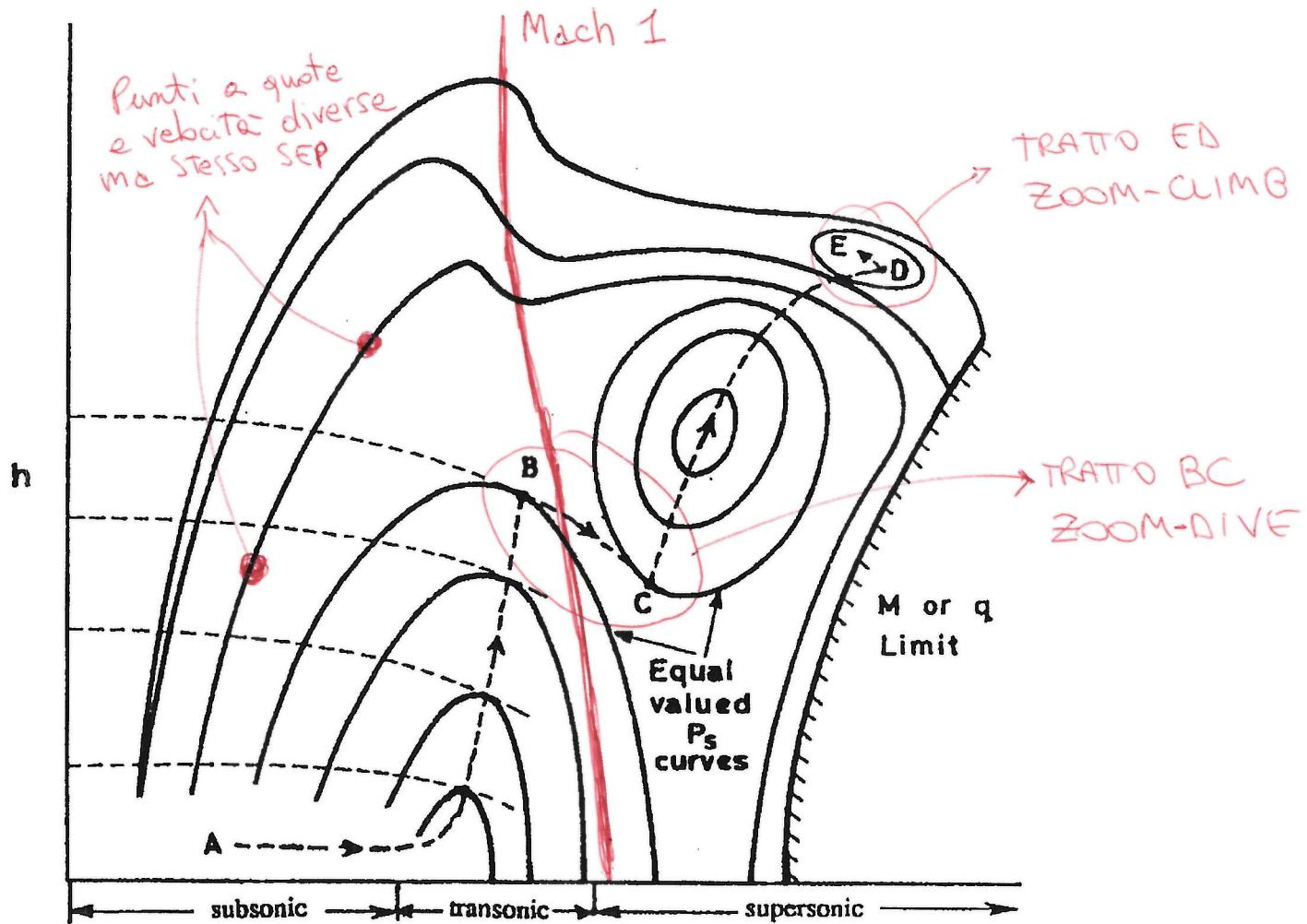


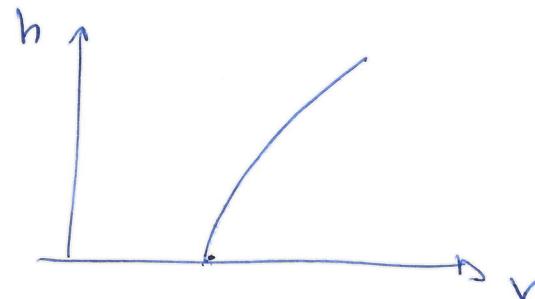
Fig. 2.20 Schematic illustration of minimum time to climb for a supersonic aircraft with limited excess-thrust capability.

PROGRAMMI DI SALITA REAISTICI

#1

EAS COSTANTE

$$\Downarrow \text{TAS} = \sqrt{\frac{P_0}{\rho}} \text{ EAS} \Rightarrow V \text{ AUMENTA} \boxed{V_v < SEP}$$



#2

MACH COSTANTE

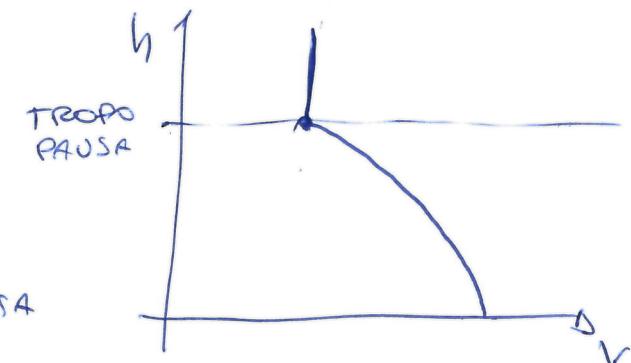
V_v SUONO \propto TEMPERATURA \Rightarrow si diminuisce con la quota

$$\Downarrow$$

TAS DIMINUISCE CON LA QUOTA (FINO A TROPOPAUSA)

TAS RESTA COSTANTE CON LA QUOTA DOPO LA TROPOPAUSA

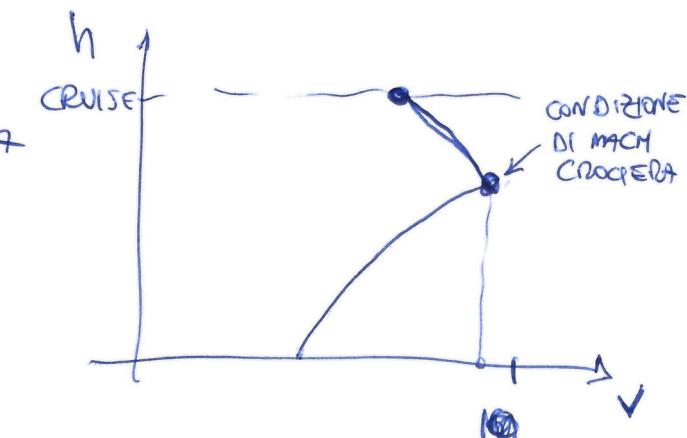
$$\boxed{V_v \geq SEP}$$



#3

TIPICO DEI VELIVOLI CIVILI

#1 + #2 \rightarrow EAS COSTANTE FINO A MACH CRUISE
MACH COSTANTE FINO A QUOTA CRUISE



SALITA ECONOMICA

MINIMIZZARE I CONSUMI



→ PESO DEL GARBORANTE / COMBUSTIBILE

MODELLO PER \dot{W}_F

$$\hookrightarrow \textcircled{j} \quad C_T := \frac{\dot{W}_F}{T}$$

$$\hookrightarrow \textcircled{p} \quad C_p := \frac{\dot{W}_F}{P_s}$$

POTENZA
ALL'ALBERO

TSFC (Thrust Specific Fuel Consumption)

BSFC (Brake Specific Fuel Consumption)

$$H_p: \frac{\partial G}{\partial h} = 0 \quad \frac{\partial G}{\partial T} = 0 \quad \left| \begin{array}{l} \text{VARIAZIONE DI} \\ G \in C_p \\ \text{CON QUOTA} \\ \text{NULLA!} \end{array} \right.$$

$$\textcircled{j} \quad W_{Fc} = \int_{h_1}^{h_2} \frac{C_T T}{V_r} dh = G \int_{h_1}^{h_2} \frac{T}{V_r} dh \Rightarrow \min W_{Fc} \Rightarrow \max V_r$$

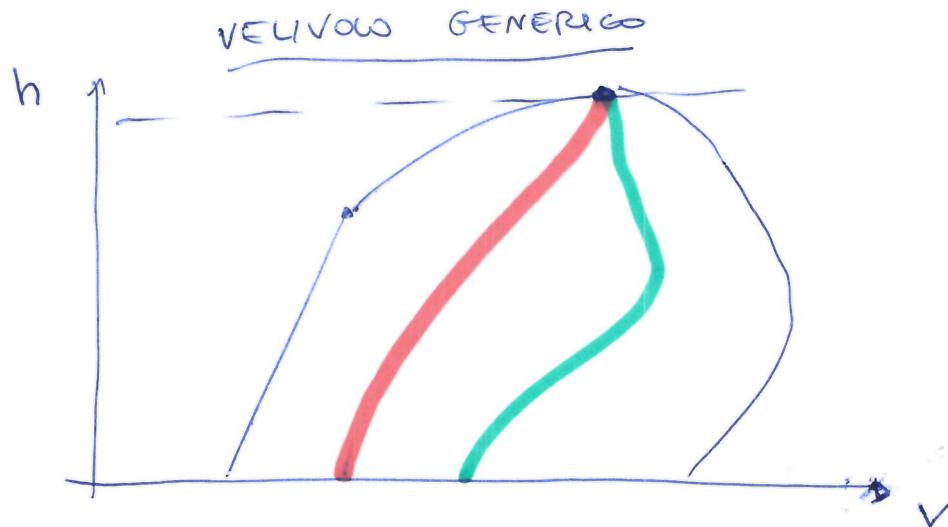
$$\textcircled{p} \quad W_{Fc} = \int_{h_1}^{h_2} \frac{C_p P_s}{V_r} dh = C_p \int_{h_1}^{h_2} \frac{P_s}{V_r} dh \Rightarrow \min W_{Fc} \Rightarrow \max V_r$$

↳ SALITA ECON.
V_r RAPIDA

$$W_{Fc} = \int_{h_1}^{h_2} \dot{W}_F \frac{dh}{V_r}$$

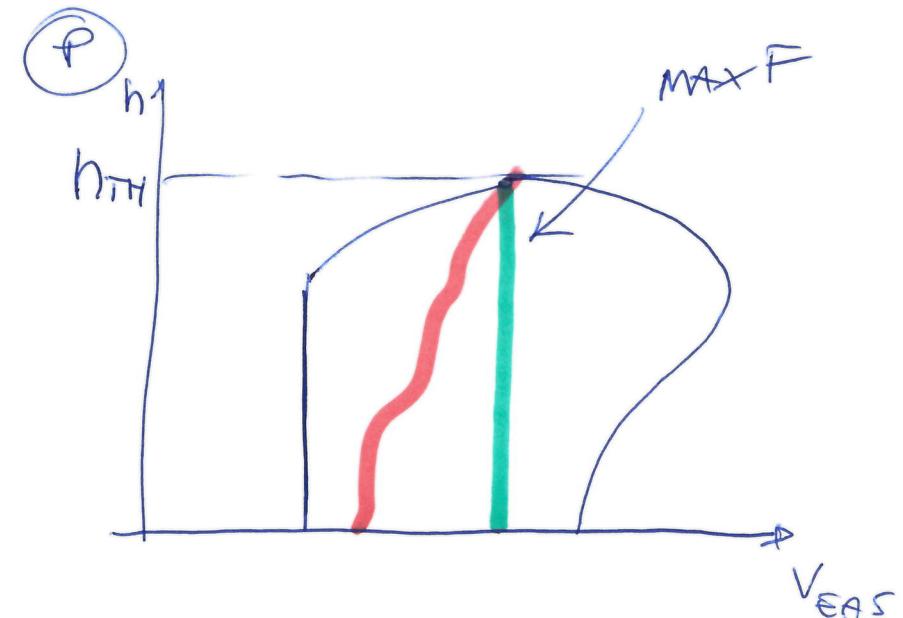
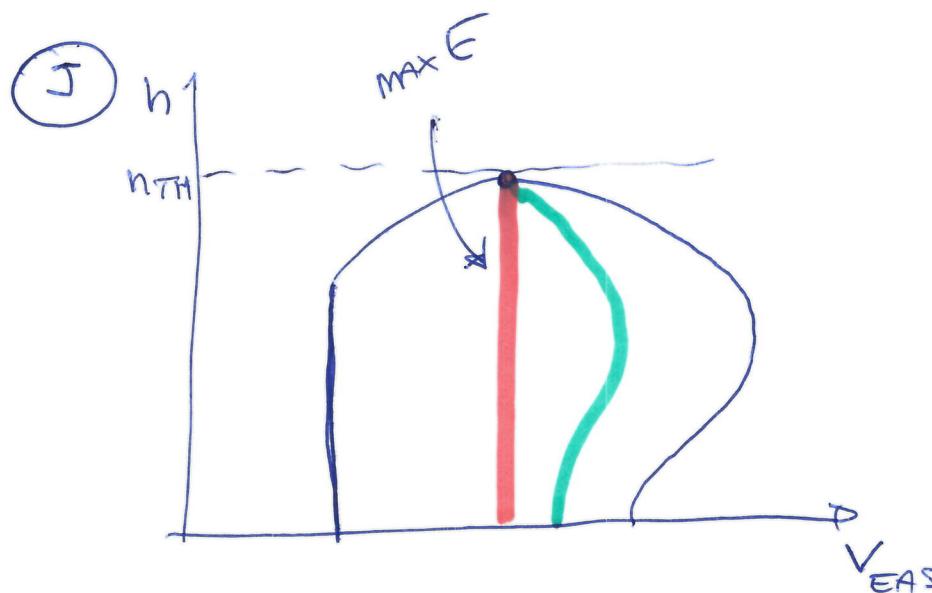
PRESTAZIONI IN SALITA NELL'INVIO DI VOLO

OTTIME



V. RIPIDA $V_{max\delta}$

V. RAPIDA $V_{max_{Y_V}}$



ESERCITAZIONE SU PRESTAZIONI PUNTAZI E INTEGRALI IN SALITA

ESERCIZIO 1

→ Prestazione puntuale

$$S = 93 \text{ m}^2$$

$$C_0 = 0.019$$

$$\text{MAXG} = 1.62$$

$$b = \text{spurio slope} = 28.6 \text{ m} \quad || \quad k = 0.0421$$

$$e = \text{fattore di OSWALD} = 0.86$$

$$w/S = 4170 \text{ N/m}^2$$

$$T_{5000} = 2.7 \cdot 10^4 \text{ N} (\text{Spinto a quota } 5000 \text{ m}) \Rightarrow \text{GETTO IDEALE}$$

$$\rho_{5000} = 0.736 \frac{\text{kg}}{\text{m}^3} (\text{Densità a quota } 5000 \text{ m})$$

* VELOCITÀ DI VOLO, COEFFICIENTE DI PORTANZA, VELOCITÀ VERTICALE E
ANGOLO DI RAMPA IN CONDIZIONE DI SAUTA RAPIDA A 5000 m

RISOLVIMENTO

SALITA RAPIDA $\Rightarrow \max(v_r) \Rightarrow \max(\text{SEP})$

$$\max\left(\frac{P_d - P_r}{w}\right) \Rightarrow \max(P_d - P_r)$$

$$\begin{aligned}
 P_d - P_r &= T_{5000} V - \frac{1}{2} \rho_{5000} V^3 S C_D \\
 &= T_{5000} V - \frac{1}{2} \rho_{5000} V^3 S C_D - \frac{1}{2} \rho_{5000} V^3 K \left(\frac{W/S}{\frac{1}{2} \rho V^2} \right)^2 \\
 &= T_{5000} V - \frac{1}{2} \rho_{5000} V^3 S C_D - K \frac{2 W^2}{\rho V S}
 \end{aligned}$$

$$\frac{\partial}{\partial V} (P_d - P_r) = T_{5000} - \frac{3}{2} \rho_{5000} V^2 S C_D + K \frac{2 W^2}{\rho_{5000} V^2 S}$$

STAZIONARITÀ \Rightarrow V RAPIDA $\Rightarrow \frac{\partial}{\partial V} (P_d - P_r) = 0$

$$\begin{aligned}
 T_{5000} - \frac{3}{2} \rho_{5000} V^2 S C_D + K \frac{2 W^2}{\frac{1}{2} \rho_{5000} V^2 S} &= 0 \Rightarrow \boxed{T V^2 - \frac{3}{2} \rho V^4 S C_D + K \frac{2 W^2}{\rho S} = 0} \\
 &\text{BIQUADRATICA}
 \end{aligned}$$

$$\Rightarrow aV^4 + bV^2 + c \rightarrow a = \frac{1}{2} - \frac{3}{2} \frac{P_{5000}}{S} \cancel{+ 5C_00} = -1.9508$$

$$b = T = 27000$$

$$c = K \frac{2W}{\frac{P}{S} S} = 1.8105 \cdot 10^8$$

$$(V_{max})^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SOL. POSITIVA $\rightarrow x^+ \rightarrow V_{max,vv} = \sqrt{x^+} = 137 \text{ m/s}$
 SOL. NEGATIVA \rightarrow OR SCARARE

$$G_{max,vv} = \frac{2 \text{ W/S}}{P V_{max,vv}^2} = 0.6037$$

$$G_{D,max,vv} = C_00 + K G_{max,vv}^2 = \cancel{0.003} \quad 0.034$$

$$D = \frac{1}{2} P_{5000} V_{max,vv}^2 S \quad G_{D,max,vv} = 2.207 \cdot 10^4 \text{ N}$$

$$P_{r,max,vv} = D \cdot V_{max,vv} = 303.2 \text{ kW}$$

$$P_2 = T_{5000} V_{\max vv} = 370.98 \text{ kW}$$

$$|\delta| = 2 \sin \left(\frac{T_{5000} - D}{w} \right) = 0.729 \text{ deg}$$

$$V_v = \left(\frac{P_2 - P_r}{w} \right) = 1.748 \text{ m/s} \rightarrow \text{E' ANCHE PARI ALLA MAX } V_v$$

ESERCIZIO 2 → Prestazione integrale.

$$S = 97.0 \text{ m}^2$$

$$b = 30. \text{ m}$$

$$c_{\infty} = 0.018$$

$$e = 0.89$$

$$\max A = 1.65$$

$$W/S = 3920 \text{ N/m}^2$$

$$T = \left(\frac{P}{P_0}\right)^{\xi} T_0 \quad \text{con } T_0 = 71000 \text{ N} \quad \text{e } \xi = 0.90 \quad T \text{ COSTANTE CON V}$$

ISA (eqz)

* A V_r INIZIALE E FINALE PER UNA SALUTA ALLA VELOCITA' DI
280 KNOTS (NOAI EAS) DAL LIVELLO DEL MARE A QUOTA
6000 m.

B SI DETERMINI IL TEMPO DI SALUTA DA 0 A 6000 m
CON UNA TOLLEGANZA DI 15 SECONDI

→ SI SUPponga W/S COSTANTE

→ SI RICORDA CHE UN MIGLIO NAUTICO E' PARI A 1852 m

$$A \quad V_r|_{h=0} = 16.78 \text{ m/s} \quad V_r|_{h=6000} = 7.21 \text{ m/s}$$

SOLUZIONE B

- $\lambda = \frac{b^2}{S} = 9.278 \Rightarrow k = \frac{1}{\pi e \lambda} = 0.0385$

- $W = \frac{W}{S} \cdot S = 380240 \text{ N}$

- $V_{EAS} \Big|_{h=0} = V_{KEAS} \underbrace{\frac{1852}{3600}}_{0.514} = 144.04 \text{ m/s}$

- $G = \frac{2W/5}{P_0 V_{EAS}^2} = 0.3085 \rightarrow \text{PER TUTTA LA CROCIERA}$

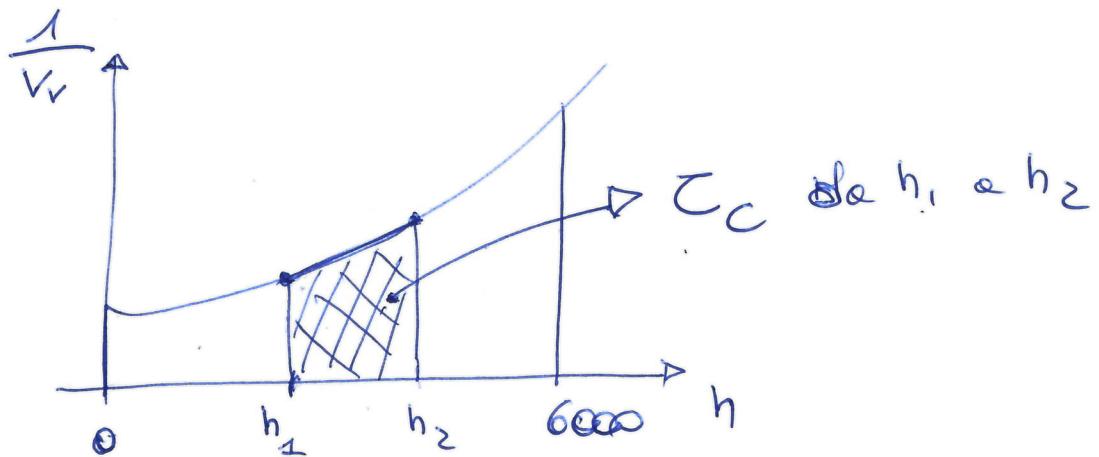
- $G_s = G_0 + k G^2 = 0.0217$

- PER UNA GENERICA QUOTA $h \rightarrow V_h = V_{EAS} \sqrt{\frac{P_0}{P_{r_h}}}$

$$\left. \begin{aligned} P_{r_h} &= \frac{1}{2} \rho_h V_h^2 S G_0 \\ P_{2h} &= T_h V_h \end{aligned} \right] \quad \left. \begin{aligned} V_{V_h} &= \frac{P_{2h} - P_{r_h}}{W} \end{aligned} \right]$$

TEMPO DI SALITA

$$T_c = \int_{h_1}^{h_2} \frac{1}{V_r} dh$$



CON LA REGOLA DEI TRAPEZI

$$T_c = \sum_{i=2}^n \frac{1}{2} (h_i - h_{i-1}) \left(\frac{1}{V_{r_i}} + \frac{1}{V_{r_{i-1}}} \right)$$

$h [km]$	$V [m/s]$	$P_r [kw]$	$P_d [kw]$	$V_r [m/s]$
0	144.04	3847.5	10227.1	16.78
1.5	159.99	4139.9	9645.1	14.48
3.0	167.21	4466.1	9077.1	12.13
4.5	180.89	4831.6	8523.5	9.71
6.0	196.29	5242.9	7986.3	7.21 [m/s]

$$T_c(0, 6 \text{ km}) = 9.915 \text{ min}$$

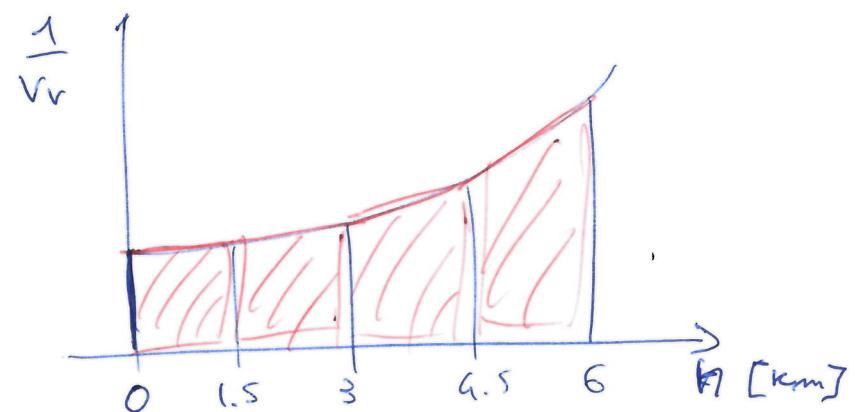
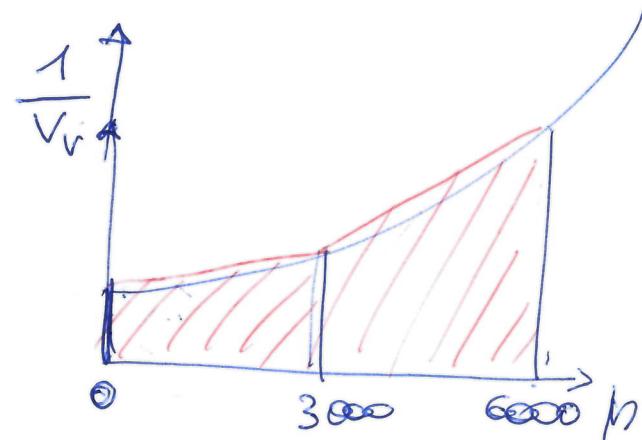
$$T_c(0, 3, 6 \text{ km}) = 9.081 \text{ min}$$

$$\Delta T \Rightarrow 50 \text{ seconds}$$

$$T_c(0, 1.5, 3, 4.5, 6 \text{ km}) = 8.842 \text{ min}$$

$$\Delta T = 14.3 \text{ seconds}$$

$$T_c(0 \rightarrow 6 \text{ km}) = 8.84 \text{ min}$$



ESERCIZIO 3) solo ANALITICO

NOT^E: $\bullet V_{V_{\max}}$ A QUOTA 0

$\bullet h_{TH}$ QUOTA DI TANGENZA

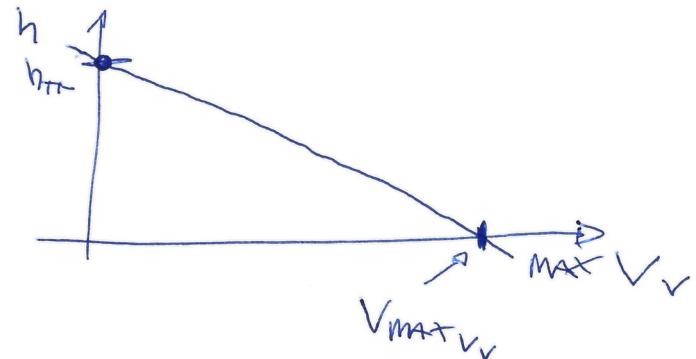
→ ASSUMENDO ANDAMENTO UNEARE DELLA VELOCITA' VERTICALE CON LA QUOTA

④ CALCOLO DEL MINIMO TEMPO DI SAUTA FINO AD UNA QUOTA $\overline{h} < h_{TH}$

SOLUZIONE

CON STAZIONE → $\bullet h_{TH} \quad V_V = 0$

$$\max V_V(h) = \left(1 - \frac{h}{h_{TH}}\right) \max V_V|_{h=0}$$

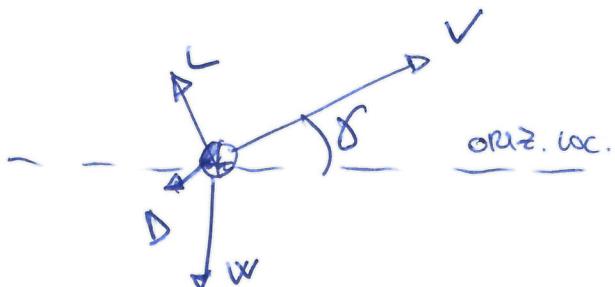


CON STAZIONE $\frac{\min Z_C}{\overline{h}}$ ASSOCIA TO A $\max V_V$

$$\boxed{\min Z_C = \int_0^{\overline{h}} \frac{1}{\max V_V(h)} dh = \dots = \boxed{\frac{h_{TH}}{\max V_V|_{h=0}} \cdot P_m \left(\frac{1}{1 - \overline{h}/h_{TH}}\right)}}$$

VOLO IN PLANATA (GLIDING FLIGHT)

CONSIDERIAMO LA SALITA STAZIONARIA



$$\begin{cases} T - D - W \sin \gamma = 0 \\ P_d - P_r - V_r W = 0 \\ L = W \cos \gamma \end{cases}$$

VOLO IN
PLANATA

$$\begin{cases} \sin \gamma = -\frac{D}{W} \\ V_r = -\frac{P_r}{W} \end{cases} \quad \left. \begin{array}{l} \gamma < 0 \\ V_r < 0 \end{array} \right\} \begin{array}{l} \text{ARIA CALMA} \\ \text{l'ALIANTE} \\ \text{PUÒ SOW} \\ \text{SCENDERE} \end{array}$$

PLANATA

- ALIANTI (GLIDERS)
- VELIVOLI
- "PERDONO" LA PROPULSIONE

$$T = P_d = \emptyset$$

INTRODUCIAMO DUE NUOVE
VARIAZIONI

- ANGOLO DI DISCESA $\gamma_D = -\gamma$
- VELOCITÀ DI DISCESA $V_D = -V_r$

PERFORMANCE OTTIMALI

• ANGOLI DI DISCESA

$$\sin \gamma_D = \frac{D}{W}$$

$$L = W \cos \gamma$$



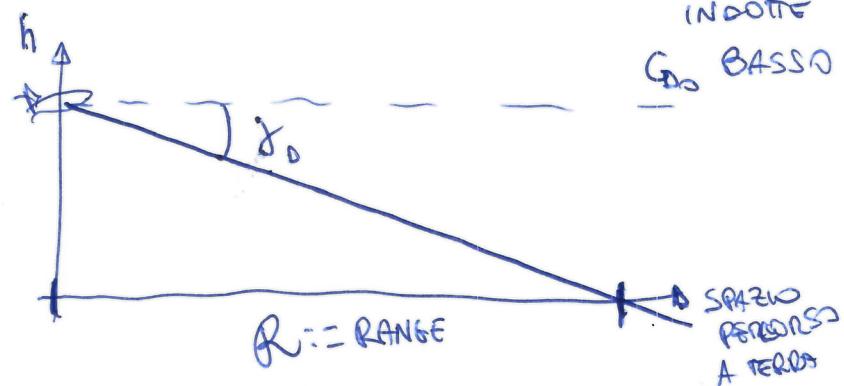
$$L = W \cos \gamma_D$$

(N.B. COSENZ
FUNZIONE PARI)

$$\tan \gamma_D = \frac{D}{L} \Rightarrow \boxed{\tan \gamma_D = \frac{1}{E}}$$

$$\min \gamma_D \Leftrightarrow \max E$$

$$\max R \Leftrightarrow \max E$$



DESIGN
ALTI
BASSI
RESISTENZE
INDOTTIVE

G_D BASSO

$$E_{\max} = 60$$

$$\gamma_D < 1 \text{ deg}$$

$$E_{\max} = 15$$

$$\gamma_D < 4 \text{ deg}$$

• VELOCITÀ DI DISCESA

$$V_D = \frac{Pr}{W} \quad \min V_D \Rightarrow \min Pr \Rightarrow$$

$\Rightarrow \max F$

$F = \sqrt{g} E$

NEGLI OLTRE
MAX F \Rightarrow MAX AUTONOMIA
OPARI
MAX E : (ENDURANCE)

CONSIDERAZIONE

MAX R e MAX E

\downarrow
MAX E \downarrow
MAX F

DIPENDONO SOLO

DALL'AERODINAMICA

(NON DAL PESO)

EQUILIBRIO

$$\left\{ \begin{array}{l} L = W \cos \delta_0 \\ D = W \sin \delta_0 \\ G = G_0 (a) \end{array} \right.$$

OSSERVAZIONI

①

$$L = W \cos \gamma \Rightarrow \frac{1}{2} \rho V^2 S C_L = W \cos \delta_0$$

$$= W \cos(\text{atan}(\frac{1}{E}))$$

$$V = \sqrt{\frac{W \cos(\text{atan}(\frac{1}{E}))}{\frac{1}{2} \rho S C_L}}$$

② BILANCIO EQZ. E INCONTRI

6 VARIABILI

$$(\rho, V, C_L, C_D, \delta_0, W)$$

3 EQZ



FISSO $\left\{ \begin{array}{l} \rho \text{ (QUOTA)} \\ W \end{array} \right.$



PER OGNI VELOCITÀ V
OTTENGO UN CERTO C_L, C_D
E UNA CERTA δ_0

SARANNO L'ASSETTO (FISSATO a e E)

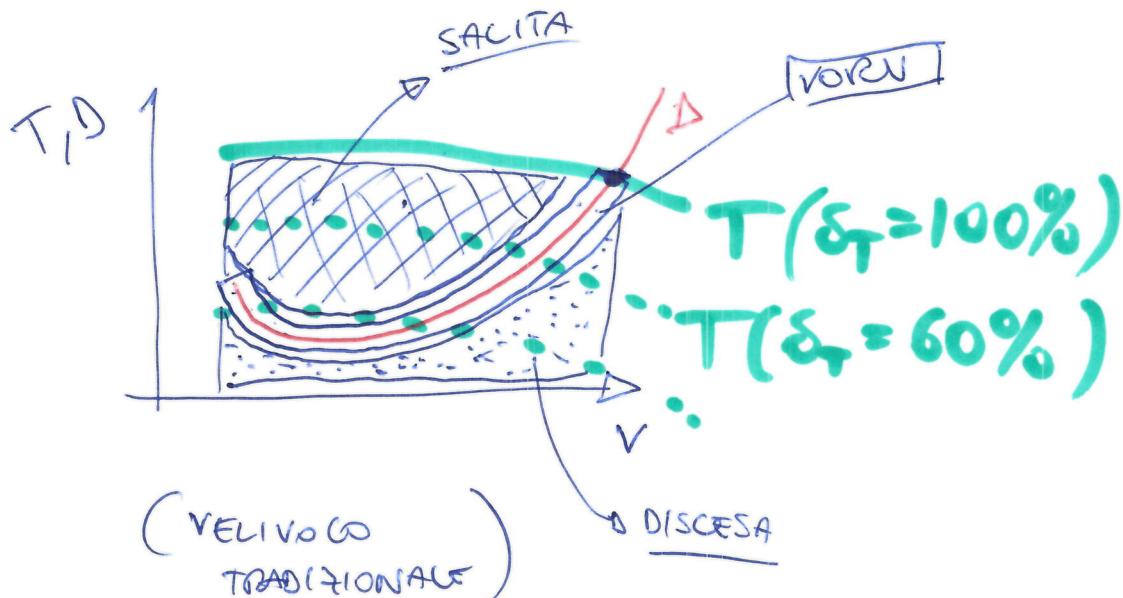
V DIPENDE DAL PESO

MAGGIORI E' IL PESO

MAGGIORI SARÀ LA VELOCITÀ ACC/EQL
(APPRESANTIRE L'AVANTE PER ANDARE PIÙ
VELOCE \Rightarrow ESEMPIO: ACQUA NELLE ALCI)

||| UNA TERNA $(a, G_0 \text{ E } \delta_0)$ È ASSOCIASTA
UNIVOCAMENTE AD UNA CERTA
VELOCITÀ DI VOLO

CONFRONTO VELIVOLO TRADIZIONALE, ALIANTE

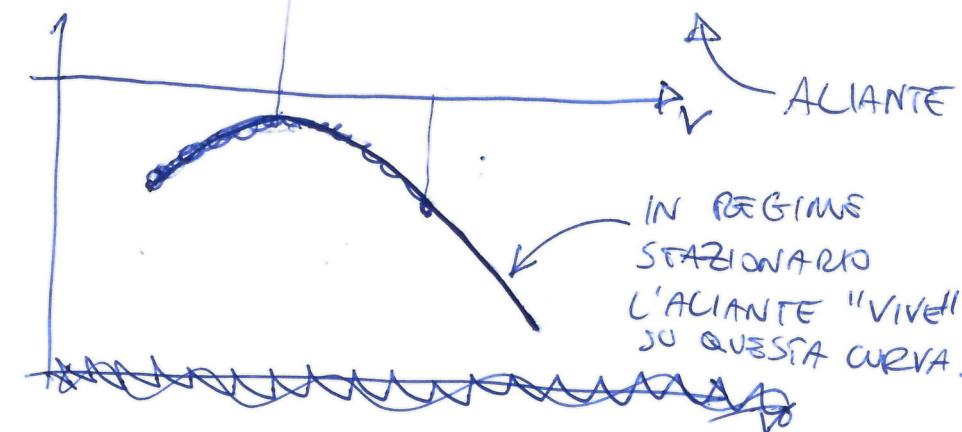
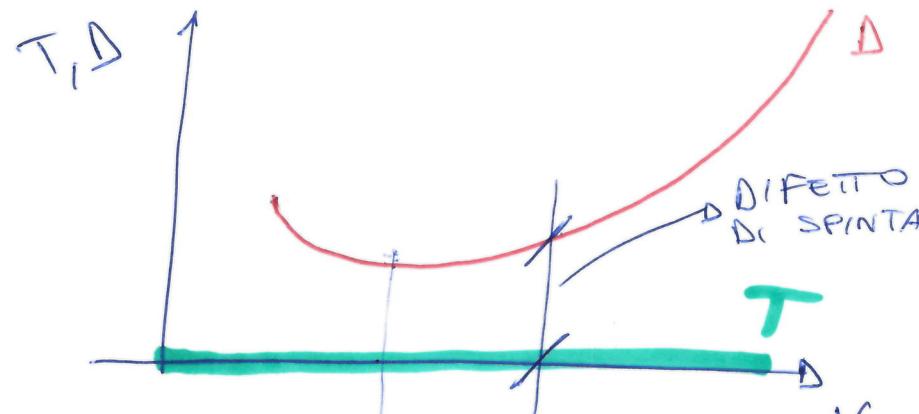


(VELIVOLO TRADIZIONALE)

$$\sin \alpha = \frac{T - D}{W}$$

$$SE T = 0 \Rightarrow$$

$$\Rightarrow \sin \alpha = \frac{-D}{W}$$

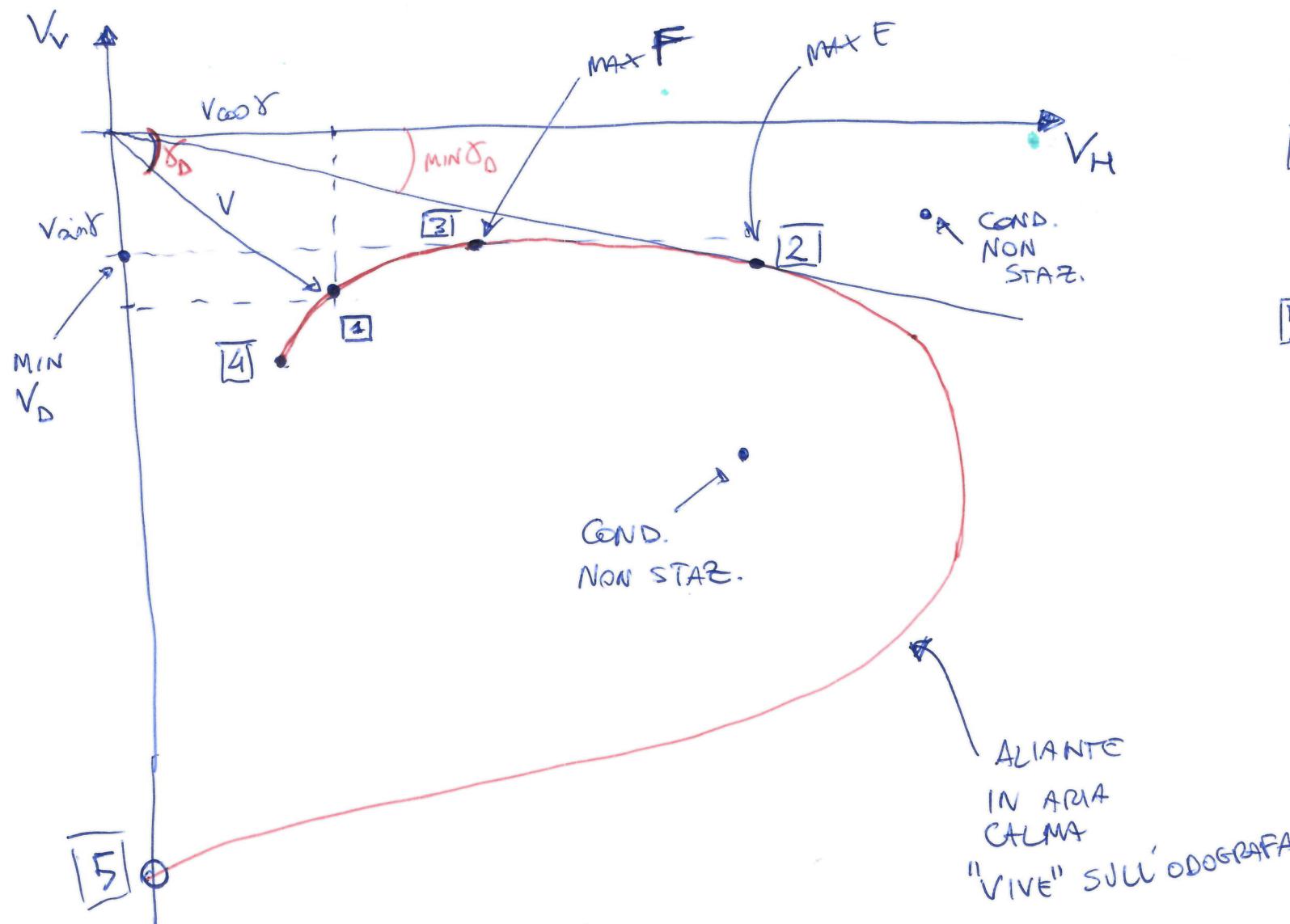


PER OGNI VELOCITA' NO UN ANGOLO α_0 DEFINITO
 (NON POSSO SCEGLIERE α_0 E V
 SEPARATAMENTE)

ODOGRAFA

GRAFICO V_v IN FUNZIONE V_h → VELOCITA' ORIZ.

($V \sin \delta$) ($V \cos \delta$)



[1] $V^2 = V_h^2 + V_v^2$

[2] MAX E \Rightarrow MIN δ_D

[3] MAX F \Rightarrow MIN V_h

[4] STALLO \Rightarrow MIN(V)

$E_{STALL} \Rightarrow \delta_D^{STALLO}$

[5] DIVE PURO
(PICCHIATA)
INUSUALE



$\delta_D = 90 \text{ deg}$

$L = 0$

$W = D$

$V = \sqrt{\frac{W}{\frac{1}{2} \rho S C_D}}$

PRESTAZIONI INTEGRALI IN ARIA (cavise)

$$T := \int_{t_1}^{t_2} dt \quad (\text{AUTONOMIA ORARIA})$$

(ENDURANCE)

t_1 e t_2 tempi di inizio
e fine crociera

$$S := \int_{t_1}^{t_2} v dt \quad (\text{AUTONOMIA CHILOMETRICA})$$

("RAGGIO"
RANGE)

Memo

$$C_T := \frac{\dot{W}_F}{T} \quad (\text{TSFC}) ; \quad C_P := \frac{\dot{W}_F}{P_S} \quad (\text{BSFC}) = \frac{\dot{W}_F}{\left(\frac{P_d}{M_p}\right)} \quad \begin{array}{l} \xrightarrow{\text{POTENZA}} \\ \text{ALL'ALBERO} \end{array}$$

$(0.7 \div 1.1 \frac{N/h}{N})$

\downarrow

$$(0.5 \frac{lb/h}{BHP})$$

P_d / M_p RENDIMENTO
ELICA
 $(P_d = \eta_p \cdot P_S)$

\downarrow POTENZA
DISPONIBILE

X
PROCEDURA GENERALE

$$\int dt \rightarrow \int dW$$

$$t_1 \rightarrow W_1$$

$$t_2 \rightarrow W_2$$

$\dot{W}_F = W_1 - W_2$

TIPIGANTE
NOTO

CONSERVAZIONE MASSA

$$\dot{W} + \dot{W}_F = 0 \Rightarrow \frac{dW}{dt} = -\dot{W}_F$$

$$\boxed{J = -C_T T}$$

$$\boxed{P = -\frac{C_P}{M_p} P_d}$$

COME EFFETTUARE LA CROCIERA: $L = W$ \rightarrow ATTENZIONE $W = W(t)$

CONDIZIONE QUASI-STATICA

IN CUI AD OGNI Istante il veicolo è in volo, con pesi "leggeramenti" differenti

PER OGNI Istante di tempo, il veivolo si trova in condizione volo (IPOTESI: PERDO PESO MA MOLTO LENTAMENTE)

$$W(t) = \frac{1}{2} \rho V^2 S a$$

DEVO VARIARE LA PORTANZA DURANTE LA CROCIERA

3 PROGRAMMI DI VOLO IN CROCIERA

#1 (α, h) COSTANTI $\Rightarrow V = V(t)$ CROCIERA DECCELERATA

#2 (α, V) COSTANTI $\Rightarrow h = h(t)$ CROCIERA IN SALITA

#3 (h, V) COSTANTI $\Rightarrow \alpha = \alpha(t)$ CROCIERA UNIFORME

$$a = a(t)$$

- $p = p(t)$
- OPPURE
- $V = V(t)$
- OPPURE
- $a = a(t)$

ANALISI PROFILI DI CICLO E RA GETTO (g)

$$\dot{C} = \int_{t_1}^{t_2} dt = - \int_{w_1}^{w_2} \frac{1}{C_T} dW = - \int_{w_1}^{w_2} \frac{1}{C_T} \frac{\epsilon}{w} dW = \int_{w_1}^{w_2} \frac{1}{C_T} \frac{\epsilon}{w} dW$$

$$\dot{S} = \int_{t_1}^{t_2} V dt = - \int_{w_1}^{w_2} \frac{1}{C_T} V dW = - \int_{w_1}^{w_2} \frac{1}{C_T} \frac{\epsilon V}{w} dW = \int_{w_2}^{w_1} \frac{1}{C_T} \frac{\epsilon V}{w} dw$$

DA
VAUTARE
IN
BASE
AL
PROGRAMMA
DI CICLO E RA

DEF: $\dot{W}_F = \frac{dW_F}{dt}$

$$dt = -\frac{dW}{\dot{W}_F} = -\frac{dW}{C_T \epsilon}$$

VORU
 $L = w$
 $T = D$ $\Rightarrow T = \frac{w}{\epsilon}$

#1 (α, h) cost

$$\dot{C}_{#1} = \frac{1}{C_T} \int_{w_2}^{w_1} \frac{\epsilon}{w} dW = \frac{\epsilon}{C_T} \int_{w_2}^{w_1} \frac{dw}{w} = \frac{\epsilon}{C_T} \left[\ln(w_1) - \ln(w_2) \right] = \frac{\epsilon}{C_T} \ln \frac{w_1}{w_2} = \frac{\epsilon}{C_T} \ln \left(\frac{1}{1-\xi} \right)$$

FRAZIONE
DI
COMBUSTIONE

$$\xi = \frac{W_F}{W_1}$$

$$\dot{S}_{#2} = \frac{1}{C_T} \int_{w_2}^{w_1} \frac{\epsilon V}{w} dW = \frac{\epsilon}{C_T} \int_{w_2}^{w_1} \frac{V}{w} dW = \frac{\epsilon}{C_T} \int_{w_2}^{w_1} \sqrt{\frac{w/15}{1/2 P}} \frac{1}{\sqrt{C_T}} \frac{dw}{w}$$

$$= 2 \frac{G}{C_T} \frac{1}{\sqrt{\frac{1}{2} P S}} (\sqrt{w_1} - \sqrt{w_2}) = 2 \frac{G}{C_T} \frac{1}{\sqrt{\frac{1}{2} P S}} (\sqrt{w_1/15}) (1 - \sqrt{1 - \xi})$$

#2 (λ, v) cost

$$C_{\#2} = C_{\#1} = \left[\frac{E}{G} \ln \left(\frac{1}{1-\xi} \right) \right]$$

$$S_{\#2} = C_{\#2} V_{\#2} = C_{\#2} \frac{\sqrt{w_1/s}}{\sqrt{\frac{1}{2} P_1} \sqrt{a}}$$

VELOCITA'

+ INIZIO CROCIERA
(ALTA QUOTA DI INIZIO
CROCIERA, CUI CORRISPONDE
DENSITA' PARL + P_L)

$$\left[\frac{G}{G} \frac{\sqrt{w_1/s}}{\sqrt{\frac{1}{2} P_L}} \ln \left(\frac{1}{1-\xi} \right) \right]$$

(N.B. $S_{\#2} > S_{\#1}$)

OSSERVAZIONI

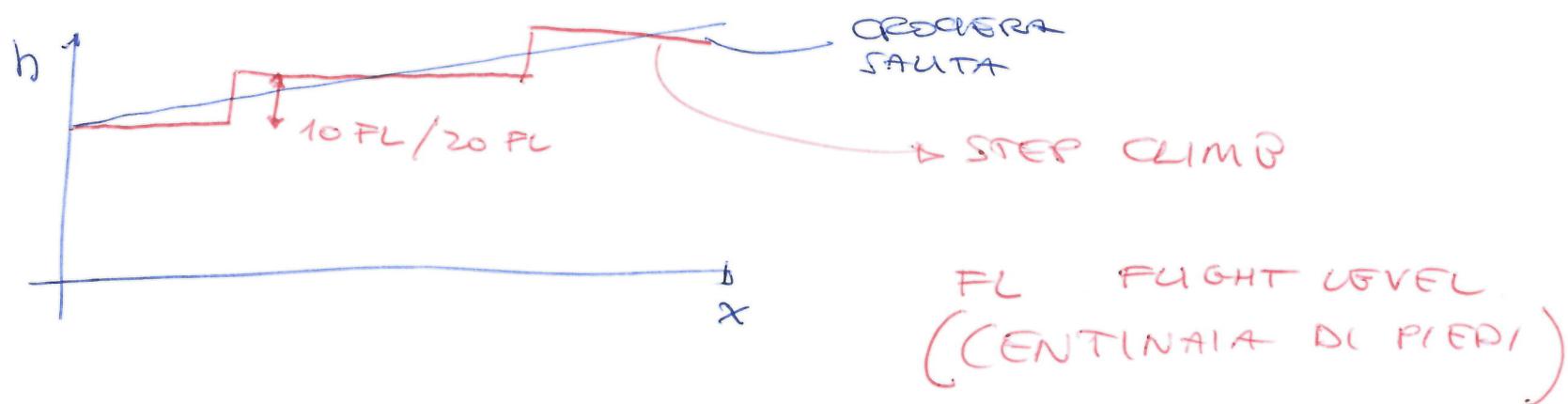
	$G \downarrow$	$\xi \uparrow$	$h \uparrow$	$w_1/s \uparrow$	
$C_{\#1}$	↑	↑	[-]	[-]	
$C_{\#2}$					
$S_{\#1}$	↑	↑	↑	↑	
$S_{\#2}$					

C OTTIMI
IN CONDIZIONE
DI MAX E

S OTTIMI
IN CONDIZIONE
DI MAX G

[STESSE CONSIDERAZIONI POSSONO
ESSERE FATTE PER #3]

CROCIERE REALISTICA



(Chi è interessato può consultare www.flighthware.com, per avere accesso alle informazioni sui piani di volo di relivoli reali)

PROFILO MISSIONE CROCIERA PER MODELLO

$$\bar{Z} = \int_{t_1}^{t_2} dt = - \int_{w_1}^{w_2} \frac{\eta_p}{C_p} \frac{dw}{P_a} = - \int_{w_1}^{w_2} \frac{\eta_p}{C_p} \frac{E}{V} \frac{dw}{w} = \int_{w_1}^{w_2} \frac{\eta_p}{C_p} \frac{E}{V} \frac{dw}{w} = \frac{\eta_p}{C_p} \int \frac{E}{V} \frac{dw}{w}$$

$$S = \int_{t_1}^{t_2} V dt = \dots = \dots = \dots = \frac{\eta_p}{C_p} \int \frac{E}{w} dw$$

↑
 VORW
 $L = w$
 $T = 0$
 $P_a = \frac{w}{E} V$

E' LO STESSO
 DEGLI ENDURANCE
 PER IL GETTO.
 DA VANTAGGIO
 SULLA BASE
 DEL PROGRAMMA
 DI CROCIERA

#1 (α, h) cost

$$\boxed{\bar{Z}_{\#1}} = \frac{\eta_p}{C_p} E \int_{w_2}^{w_1} \frac{1}{V} \frac{dw}{w} = \frac{\eta_p}{C_p} E \int_{w_2}^{w_1} \frac{\sqrt{\frac{1}{2} P S \alpha}}{\sqrt{w}} \frac{dw}{w} = \frac{\eta_p}{C_p} E \sqrt{\frac{1}{2} P S \alpha} \int_{w_2}^{w_1} \frac{dw}{w^{3/2}} =$$

$$= 2 \frac{\eta_p}{C_p} \frac{F \sqrt{\frac{1}{2} P \alpha}}{\sqrt{w_1 / S}} \left(\frac{1}{\sqrt{w_2}} - \frac{1}{\sqrt{w_1}} \right)$$

$$\boxed{S_{\#1}} = \frac{\eta_p}{C_p} E \ln \left(\frac{1}{1-\xi} \right)$$

$$= 2 \frac{\eta_p}{C_p} \frac{F \sqrt{\frac{1}{2} \alpha}}{\sqrt{w_1 / S}} \left(\frac{1}{\sqrt{1-\xi}} - 1 \right)$$

#2 (α , v) cost.

$$\begin{aligned} \bar{C}_{\#2} &= \left[\frac{\gamma_p}{C_p} \frac{v}{\sqrt{v}} \int_{w_2}^{w_1} \frac{1}{w} dw \right] = \frac{\gamma_p}{C_p} \frac{v}{\sqrt{v}} \left[\sqrt{\frac{1}{2} + \frac{1}{w}} \right] \ln \left(\frac{1}{1-\xi} \right) = \\ &= \frac{\gamma_p}{C_p} F \frac{\sqrt{\frac{1}{2} + \frac{1}{v}}}{\sqrt{w_1/5}} \ln \left(\frac{1}{1-\xi} \right) \end{aligned}$$

$$C_{\#2} > C_{\#1}$$

$$S_{P\#2}^1 = S_{\#2} = \left[\frac{\gamma_p}{C_p} v \ln \left(\frac{1}{1-\xi} \right) \right]$$

($\# \Rightarrow$ STESSA CONSIDERAZIONE)

OSSERVATIONI

	$C_p \downarrow$	$\gamma_p \uparrow$	$\xi \uparrow$	$b \uparrow$	$w_1/5$
C	\uparrow	\uparrow	\uparrow	\downarrow	\downarrow
S	\uparrow	\uparrow	\uparrow	\leftarrow	\leftarrow

$$+ \max C \Leftrightarrow \max F$$

$$\max S \Leftrightarrow \max E$$

SINTESI

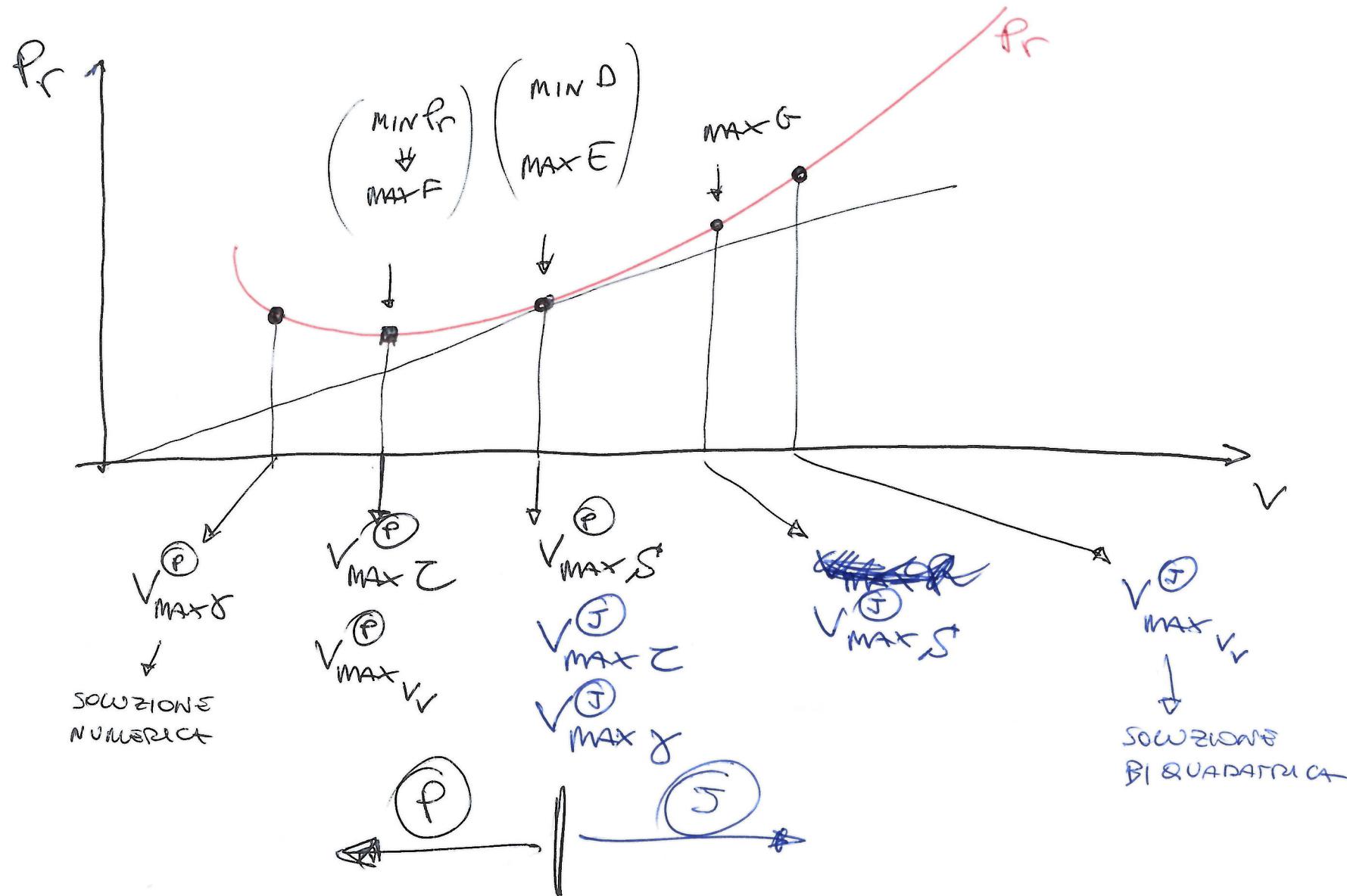
GETTO

VS

MOTORELICAT

(J)

(P)



ESERCIZIO SU PRESTAZIONI IN CROCIERA

$$S = 91 \text{ m}^2$$

$$W/S = 6840 \text{ N/m}^2$$

$$TSFC \Rightarrow G = 0.8 \text{ N/(Nh)}$$

$$T = (\rho / \rho_0)^{\frac{\gamma}{\gamma - 1}} T_0 \quad \text{con } T_0 = 68000 \text{ N} \quad \gamma = 0.75$$

POLARE
ANALITICA

$C_D = 0.019$
$E_{max} = 17.60$

STALLO

$$v_{FAS} = 68 \text{ m/s}$$

ISA eqz

$$h = 5600 \text{ m}$$

$$\rho_{5600} = 0.6893 \frac{\text{kg}}{\text{m}^3}$$

$$S = 3900 \text{ Km}$$

$$(\alpha, h) = \text{cost}$$

V_V e δ IN CONDIZIONI DI
FINE CROCIERA



Prima parte

Tutti i fogli consegnati devono essere scritti in modo chiaro.
Si riporti in stampatello su ogni foglio consegnato il proprio cognome, nome
e numero di matricola, assieme alla lettera identificativa del compito.
Nella soluzione degli esercizi, si riporti il procedimento completo.
Le risposte, a meno di diversa indicazione, vanno espresse in unità del sistema internazionale.

Problema 1 – Prestazioni

[valore 30%]

Si consideri un velivolo a getto caratterizzato dai seguenti dati:

superficie alare $S = 91 \text{ m}^2$ carico alare $W/S = 4840 \text{ N/m}^2$
coefficiente di resistenza passiva $C_{D0} = 0.019$ efficienza massima maxE = 17.60

(NB) \rightarrow velocità equivalente di stallo $V_{Eas\,stall} = 68 \text{ m/s}$

consumo specifico rispetto alla spinta (TSFC) $c_T = 0.8 \text{ N/(Nh)}$

La spinta disponibile è costante con la velocità di volo e dipende dalla quota secondo la formula

$$T = (C/\rho_0)^{\xi} T_0 \quad \text{con } T_0 = 68000 \text{ N} \quad \text{e} \quad \xi = 0.75$$

Legge di variazione della densità ρ con la temperatura θ nella troposfera (condizioni I.S.A.):

$$\rho/\rho_0 = (\theta/\theta_0)^{-(1+g/(R\lambda))} \quad \text{con} \quad \rho_0 = 1.225 \text{ kg/m}^3 \quad \theta_0 = 288.15 \text{ K}$$

$$g = 9.81 \text{ m/s}^2 \quad \lambda = -0.0065 \text{ K/m}$$

$$R = 287.05 \text{ m}^2/(\text{K s}^2)$$

Facendo uso delle usuali approssimazioni, si chiede quanto segue:

1.1) ~~Determinare raggio di virata, tempo di virata e fattore di carico per una virata corretta ad una velocità maggiore del 16% della velocità minima di volo alla quota $h = 9800 \text{ m}$. [valore 15%]~~

~~$[\min V = 133.1 \text{ m/s} > V_{stall} = 115.7 \text{ m/s} \Rightarrow V_{max} = 154.5 \text{ m/s}, R = 7510 \text{ m}, T_r = 153 \text{ s}, n = 1.10]$~~

1.2) Alla quota $h = 5600 \text{ m}$, il velivolo esegue una crociera di 3900 km ad incidenza e quota costanti. Supponendo condizioni di massima autonomia chilometrica con peso iniziale pari a

quello dato, determinare i valori massimi della velocità verticale e dell'angolo di rampa (in gradi) per la velocità ed il peso corrispondenti alla fine della crociera. [valore 15%]

$$\rho = 0.6093 \frac{\text{kg}}{\text{m}^3} \quad [G = 0.276, V_2 = 162.2 \text{ m/s} \Rightarrow \gamma = 4.18^\circ, V_v = 11.83 \text{ m/s}]$$

RISOLUZIONE

• CALCOLO K

$$E_{MAX} = \left. \frac{G}{G_0} \right|_{E_{MAX}} = \frac{1}{2} \frac{1}{\sqrt{K C_{D0}}} \Rightarrow K = \frac{1}{4 G_0 E_{MAX}^2} = 0.0425$$

$$\bullet \text{ MAX } S \Rightarrow \text{ MAX } G \rightarrow G = \frac{E}{\sqrt{a}} = \frac{\sqrt{a}}{G_0} = \frac{\sqrt{a}}{G_0 + k a^2}$$

$$\frac{\partial G}{\partial a} = \frac{\frac{1}{2} a^{-\frac{1}{2}} (G_0 + k a^2) - 2k a \sqrt{a}}{(\dots)} = 0 \Rightarrow \frac{\frac{1}{2} G_0 a^{-\frac{1}{2}} - \frac{3}{2} k a^{3/2}}{(\dots)} = 0$$

$$\frac{G_0 a^{-\frac{1}{2}} - 3 k a^{3/2}}{(\dots)} = 0$$

$$a^{1/2} = x$$

$$\frac{G_0}{x} - 3 k x^3 = 0$$

$$\Downarrow$$

$$G_0 - 3 k x^4 = 0 \Rightarrow x^4 = \frac{G_0}{3 k}$$

$$(*) \quad a = \sqrt{\frac{G_0}{3 k}} \Rightarrow G \Big|_{@ \text{MAX } G} = G_0 + k a^2 = \frac{4}{3} G_0$$

$$G \Big|_{@ \text{MAX } G} = 0.3861$$

$$G \Big|_{@ \text{MAX } G} = 0.0253$$

$$G_{MAX} > 24.53$$

- CONVERTIRE G IN UNITA' SI.

$$G = \frac{0.8}{3600} \frac{N}{N \cdot s} = 2.22 \cdot 10^{-4} \frac{N}{Ns}$$

- SOLUZIONE RANGE PER CROCIERA DECURATA

$$\boxed{\delta = \dots = 2 \frac{G}{\rho s} \sqrt{\frac{2 w_1}{\rho s}} (1 - \sqrt{1 - \xi}) \quad \xi = \frac{w_F}{w_1}}$$

$$\mu \doteq \frac{G \delta}{2 G} \sqrt{\frac{\rho s}{2 w_1}} \quad \Rightarrow \quad \sqrt{1 - \xi} = 1 - \mu \quad \Rightarrow \quad \boxed{\xi = 1 - (1 - \mu)^2}$$

$$\boxed{\xi = 0.2759} \quad w_F = \xi w_1 \quad \Rightarrow \quad \boxed{w_2 = w_1 - w_F = 3.18 \cdot 10^5 N}$$

- CALCOLIAMO LA VELOCITA' A FINE CROCIERA

$$v_2 = \sqrt{\frac{2 w_2 / s}{\rho \xi}} = 162.27 \text{ m/s}$$

ξ_{MAXG}

- CALCOLO LA SPINTA RESPONSABILE

$$T = \left(\frac{\rho_{3600}}{\rho_0} \right) \overline{T}_0 = 4.418 \cdot 10^4 \text{ N}$$

$$\cdot \boxed{\gamma} = \left(\frac{T - D}{w} \right) = \frac{T - \frac{1}{2} \rho_{5600} V_2^2 \cdot S \cdot C_D |_{\text{max } G}}{w_2} = \boxed{4.18 \text{ deg}}$$

A PINE
CALCOLA

$$\cdot \boxed{V_v} = \left(\frac{P_a - P_f}{w} \right) = \dots \quad \text{+ troppi CALCOLI} \dots$$

$$= V_2 \sin \gamma = \boxed{11.83 \text{ m/s}}$$

EQUILIBRIO IN VIRATA

→ FATTORE DI CARICO $m = \frac{\underline{F}_{massa}}{|\underline{w}|} = \frac{\underline{w} - m \dot{\underline{v}}}{|\underline{w}|}$

→ DINAMICA DEL VEICOLO $m \dot{\underline{v}} = \underline{F} + \underline{w} + \underline{I} \Rightarrow \underline{F} + \underline{I} = \underbrace{m \dot{\underline{v}} - \underline{w}}_{-F_{massa}}$

→ COMPONENTI FATTORE DI CARICO IN ASSI CORPO

$$m_x = -\frac{(T_x \beta + X)}{|\underline{w}|}; \quad m_y = -\frac{Y}{|\underline{w}|}; \quad m_z = -\frac{Z}{|\underline{w}|}$$

Fattore d. c. longitudinale

Fattore d. c. laterale

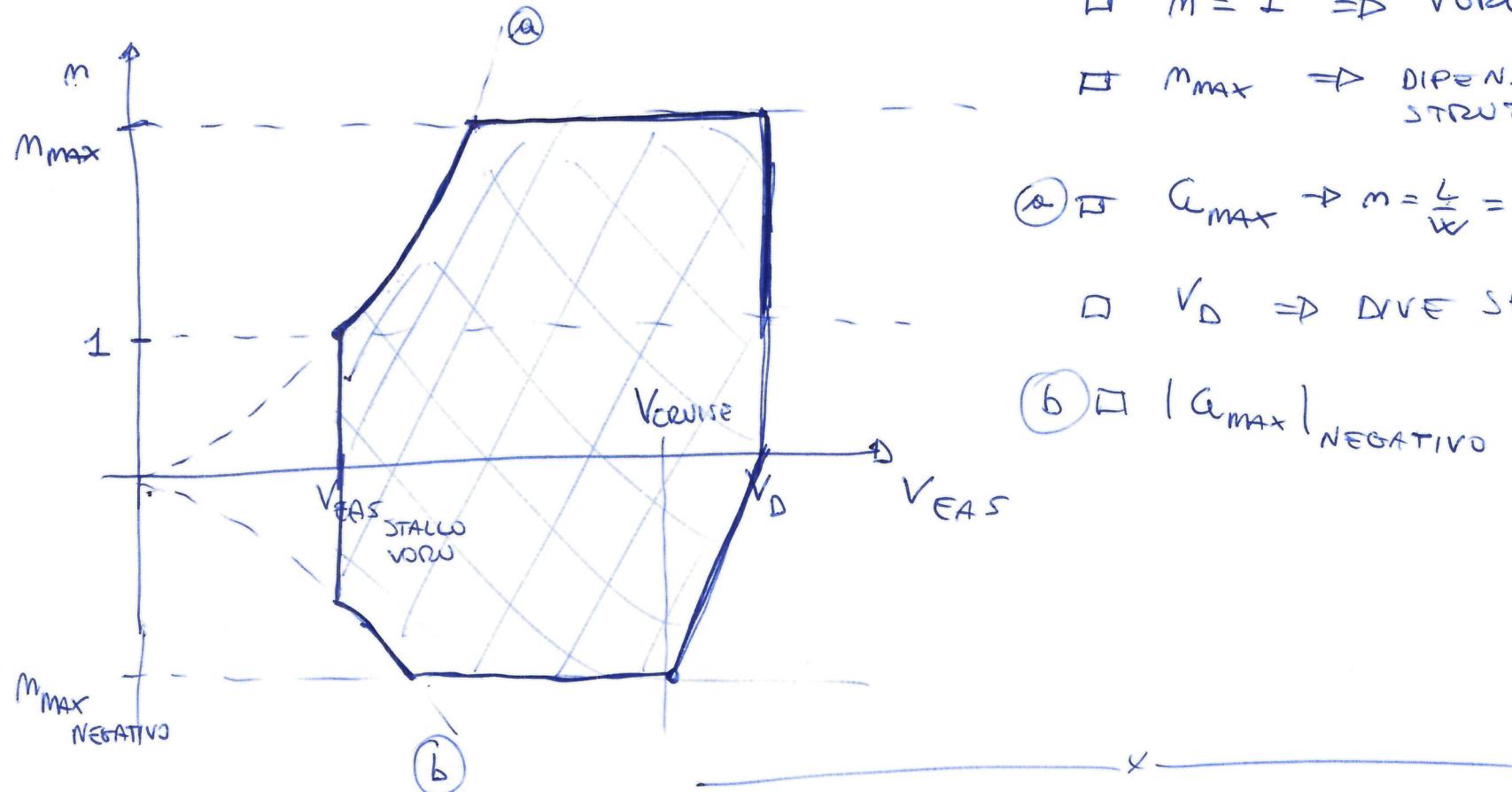
Fattore d. c. corico NORMALE

MEMO: $\alpha \ll 1$
 $\beta \approx 0$
 $L \gg D$
 $L \gg Q$

$$\Rightarrow L = -Z$$

$$m_z \triangleq m = -\frac{Z}{w} = \frac{L}{w}$$

DIAGRAMMA M-V



□ $M = 1 \Rightarrow V_{ORU}$

□ $M_{MAX} \Rightarrow$ DIPENDE DALLA STRUTTURA

① □ $G_{MAX} \rightarrow m = \frac{L}{w} = \frac{\frac{1}{2} \rho_0 V_{EAS}^2 S a_{max}}{w}$

□ $V_D \Rightarrow$ DIVE SPEED

② □ $|a_{max}|$ NEGATIVO

DEFINIZIONI

VIRATA : MANOVRA NEL PIANO VERTICALE ORIZZONTALE CON OBIETTIVO IL CAMBIO DI ROTTA ($\ddot{x} \neq 0$) $\ddot{x} > 0$ VIRATA DX / POSITIVA
 $\ddot{x} < 0$ " SX / NEGATIVA

- $\beta \neq 0 \rightarrow$ V. PIA TTA

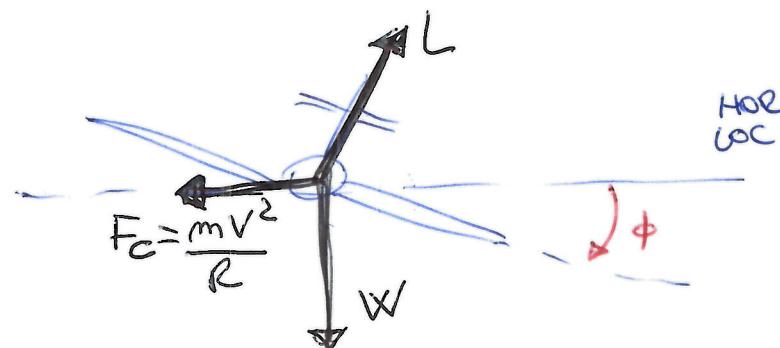
- $\beta = 0 \rightarrow$ V. COORDINATA // $\Rightarrow m_x \neq 0 \quad m_y = 0$

- $\dot{V} = 0 \rightarrow$ V. SOSTENNTA

$$m_z \neq 0$$

EQUILIBRIO VIRATA COORDINATA E SOSTENUTA
"CORRETTA"

VISTA DA DIETRO



$V \rightarrow$ Velocità a cui eseguiamo la virata

$R \rightarrow$ Raggio di curvatura (di VIRATA)

$$\text{EQZ} \quad \begin{cases} T = 0 \\ L \cos \phi = W \\ L \sin \phi = \frac{m V^2}{R} \end{cases}$$

$$\text{DEF} \quad \begin{cases} m = \frac{L}{W} \\ \text{POLARE} \quad c_0 = g_D(\alpha) \end{cases}$$

$D =$ RESISTENZA IN VIRATA

5 EQZ

\Rightarrow 9 VARIABILI

$(\rho, v, g, a, s_t, W, \phi, R, m)$

LEGAMI COSTITUTIVI

T, D, L

POSSIAMO SEMPLIFICARE IL PROBLEMA

\rightarrow DISACCOPPIAMO LE EQZ $T=D$ e $c_0=g_D(a)$ attraverso la variabile s_t .



IPOTIZZO DI AVERE SEMPRE UN s_t CHE PERMETTE DI SODDISFARE $T=D$ IN UNA QUALSIASI CONDIZIONE

3 EQZ

\Rightarrow VARIABILI $(\rho, v, a, W, \phi, R, m)$ || 4 gl. L.

$$\Rightarrow \begin{cases} L \cos \phi = W \\ L \sin \phi = \frac{m V^2}{R} \\ m = \frac{L}{W} \end{cases} \Rightarrow$$



FISSO

 $P, V, W, e \in \phi$

piccolo memo

$$m = \frac{1}{\cos\phi} \Rightarrow \cos\phi = \frac{1}{m}$$

$$\sin\phi = \sqrt{1 - \cos^2\phi} = \sqrt{1 - \frac{1}{m^2}} = \frac{\sqrt{m^2 - 1}}{m}$$

$$\tan\phi = \sqrt{m^2 - 1}$$

$$\left\{ \begin{array}{l} m = \frac{L}{W} = \frac{1}{\cos\phi} \\ a = \frac{mW}{\frac{1}{2}PV^2S} = \frac{W}{\cos\phi \cdot \frac{1}{2}PV^2S} \\ \tan\phi = \frac{mV^2}{R} \cdot \frac{1}{mg} = \frac{V^2}{RG} \Rightarrow R = \cancel{R} \cancel{\frac{V^2}{g}} \\ \Rightarrow R = \frac{V^2}{g} \cdot \frac{1}{\tan\phi} = \frac{V^2}{g\sqrt{m^2 - 1}} \end{array} \right.$$

$$\left\{ \begin{array}{l} \phi = \arctan\left(\frac{V^2}{gR}\right) \\ m = \frac{1}{\cos\phi} \\ a = \frac{mW}{\frac{1}{2}PV^2S} \end{array} \right.$$

$$\left\{ \begin{array}{l} m = \frac{L}{W} = \frac{\frac{1}{2}PV^2S}{W} \\ \phi = \arccos\left(\frac{1}{m}\right) \\ R = \frac{V^2}{g} \cdot \frac{1}{\tan\phi} \end{array} \right.$$

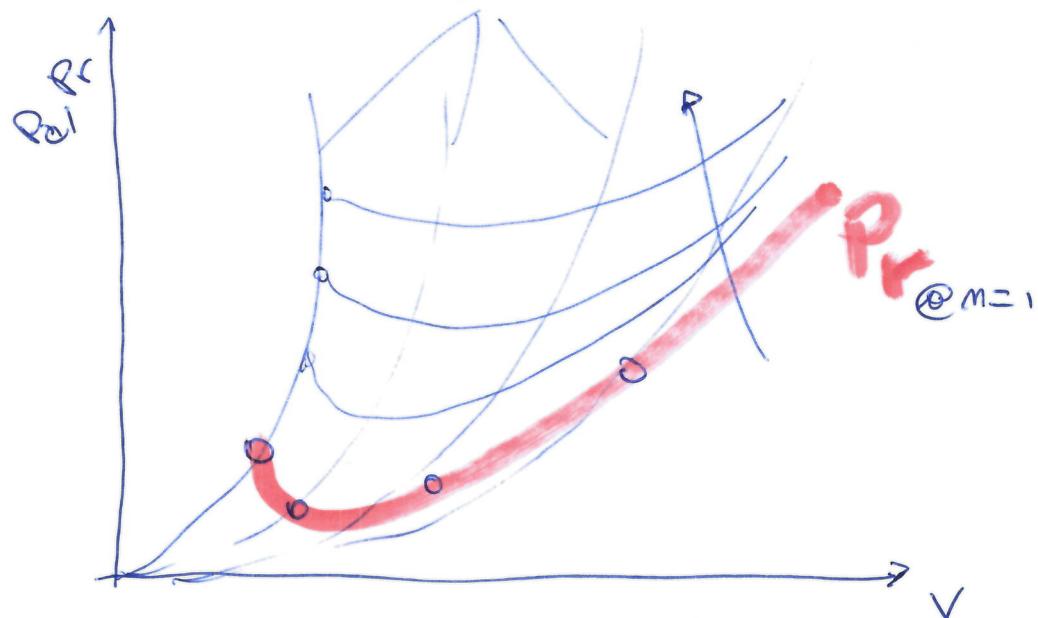
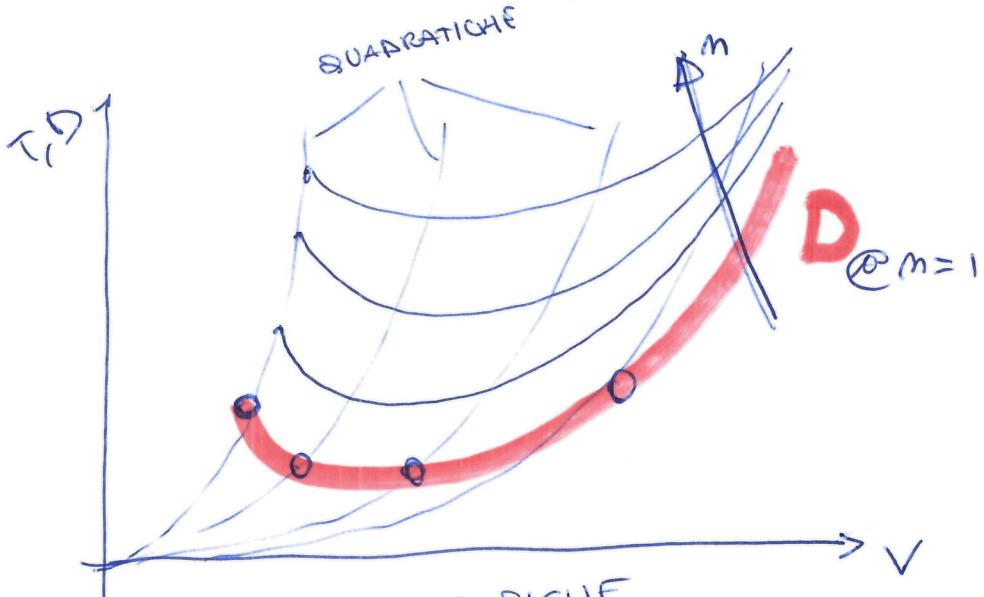
FISSO $P, V, W, e \in \mathcal{N}$

$$\left\{ \begin{array}{l} \phi = \arccos\left(\frac{1}{m}\right) \\ a = \frac{mW}{\frac{1}{2}PV^2S} \\ R = \frac{V^2}{g} / \left(g\sqrt{m^2 - 1}\right) \end{array} \right.$$

DIAGRAMMA DI PENAUD IN VIRATA

→ CONSIDERIAMO P, V, W FISSATE

→ Qual è l'effetto di m sui diagrammi?



PROCEDURA

→ FISSA L'ASSETTO, E CALCOLA D, P_r, V CON m DIVERSI

$$E = \frac{L}{D} \Rightarrow D = \frac{L}{E} = \frac{m W}{E}$$

$$L = m W \Rightarrow V = \sqrt{\frac{m W}{\frac{1}{2} P \sigma G}}$$

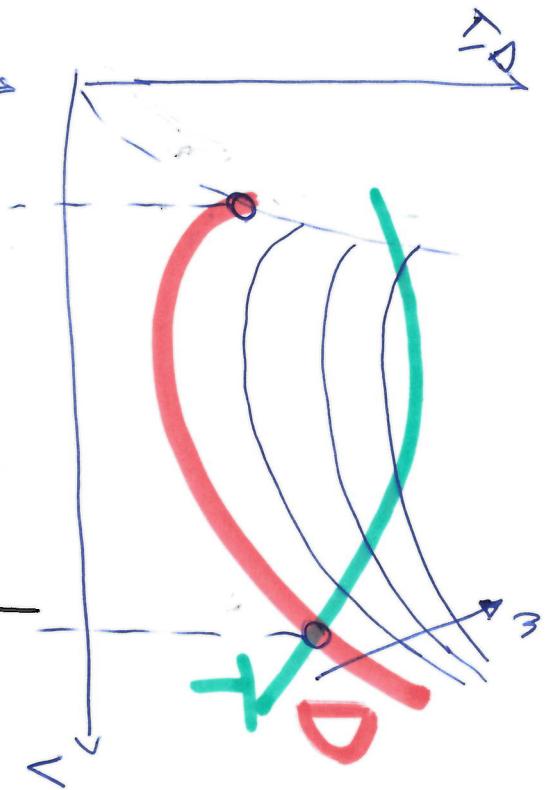
$$\frac{D_{m_1}}{D_{m_2}} = \frac{m_1}{m_2} = \left(\frac{V_{m_1}}{V_{m_2}} \right)^2$$

$$D \propto V^2$$

$$\frac{P_{r m_1}}{P_{r m_2}} = \left(\frac{V_{m_1}}{V_{m_2}} \right)^3$$

SOLUZIONI OTTIME VIA GRAFICA

HOTESI \Rightarrow non esiste m_{\max}



$$R = \frac{V^2}{g} \frac{1}{\sqrt{m^2 - 1}}$$

$$\begin{aligned} \Omega &= \text{Velocità} \\ &\text{angolare} \\ &= \frac{V}{R} \end{aligned}$$

$T_\pi =$ tempo di inversione

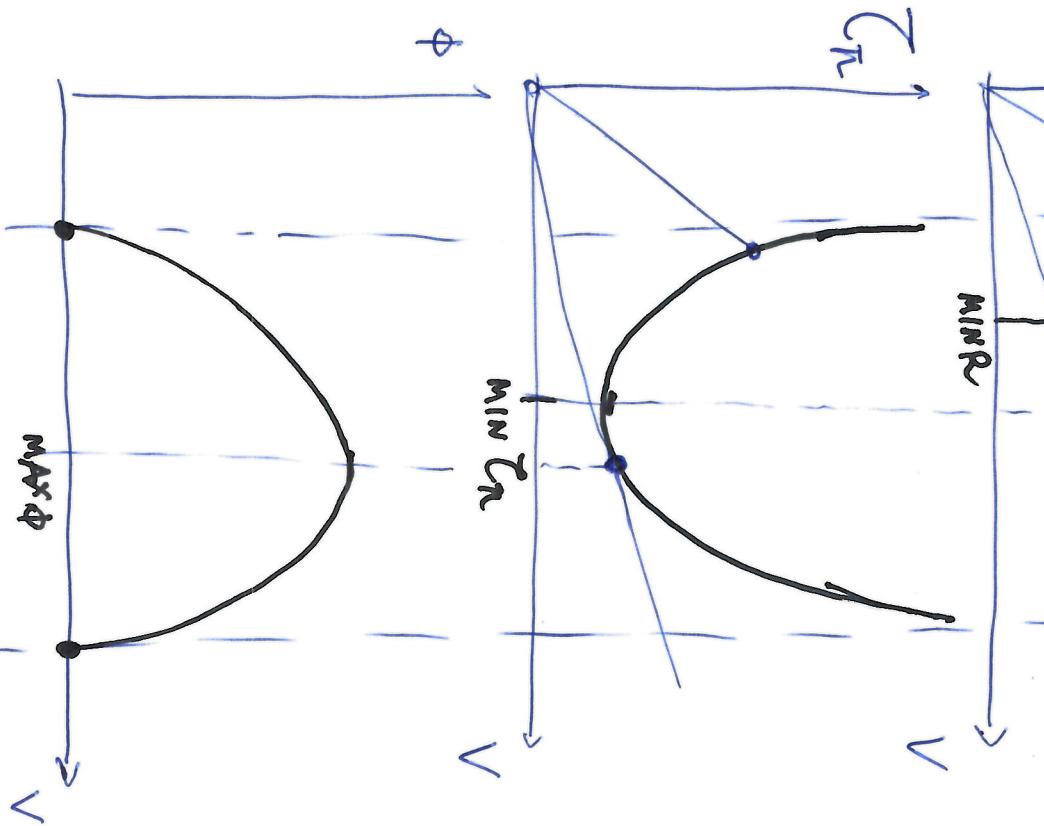
$$= \frac{\pi R}{V}$$

$$R = \frac{T_\pi}{\pi} V$$

COEFF. ANG.

$$T_\pi = \frac{\pi}{g} \frac{1}{\operatorname{tg}(\phi)} V$$

COEFF. ANG.



MIN COEFF. ANG.

MIN R

MIN T_π

MAX φ

V

φ

T_π

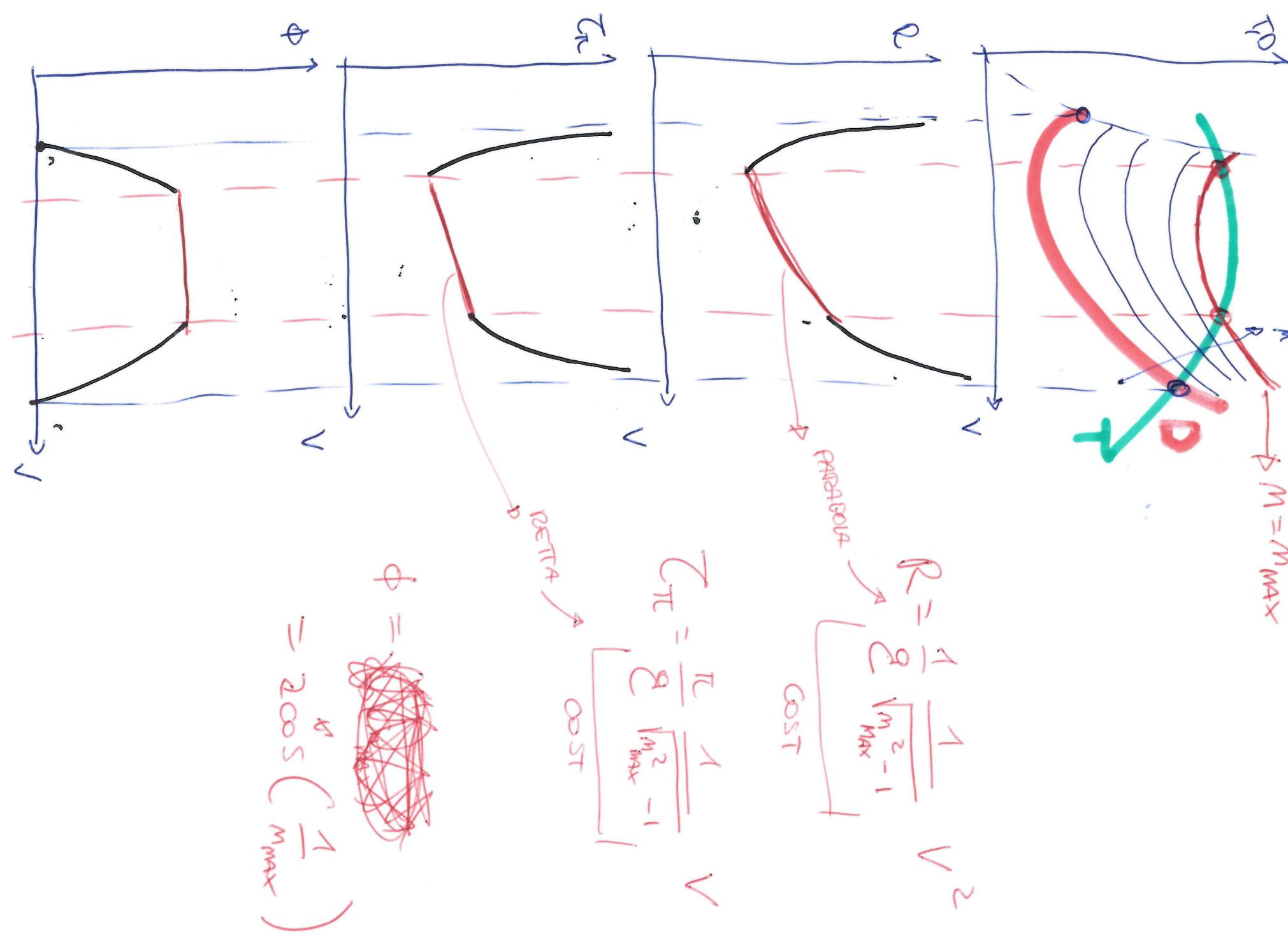
R

V

KP

PRESTAZIONI

SOLUZIONI GRAFICHE CON VINCOLO STRUTTURALE



PROCEDURA PI CALCOLI PER VIRATA

→ ASSEGNA UNA QUOTA (β), PESO (w), VELOCITÀ (v)

~~STEP 1)~~ FISSO $\delta_T \Rightarrow$ FISSO T

→ IMPONGO IL LIMITE ~~STAB~~ PROPULSIVO $T = D$

$$\Rightarrow C_{D_{TURN}} = \frac{T}{\frac{1}{2} \rho v^2 S} \quad (\text{VINCOLO ATTIVO: PROPULSIVO})$$

2) CALCOLI IL G DI POLARÉ

$$\text{ex: } G_{TURN} = \sqrt{(C_{D_{TURN}} - C_0)/k}$$

CHECK: $G_{TURN} \leq G_{MAX}$

$$\text{SE NO } G_{TURN} = G_{MAX}$$

(VINCOLO AERICO)

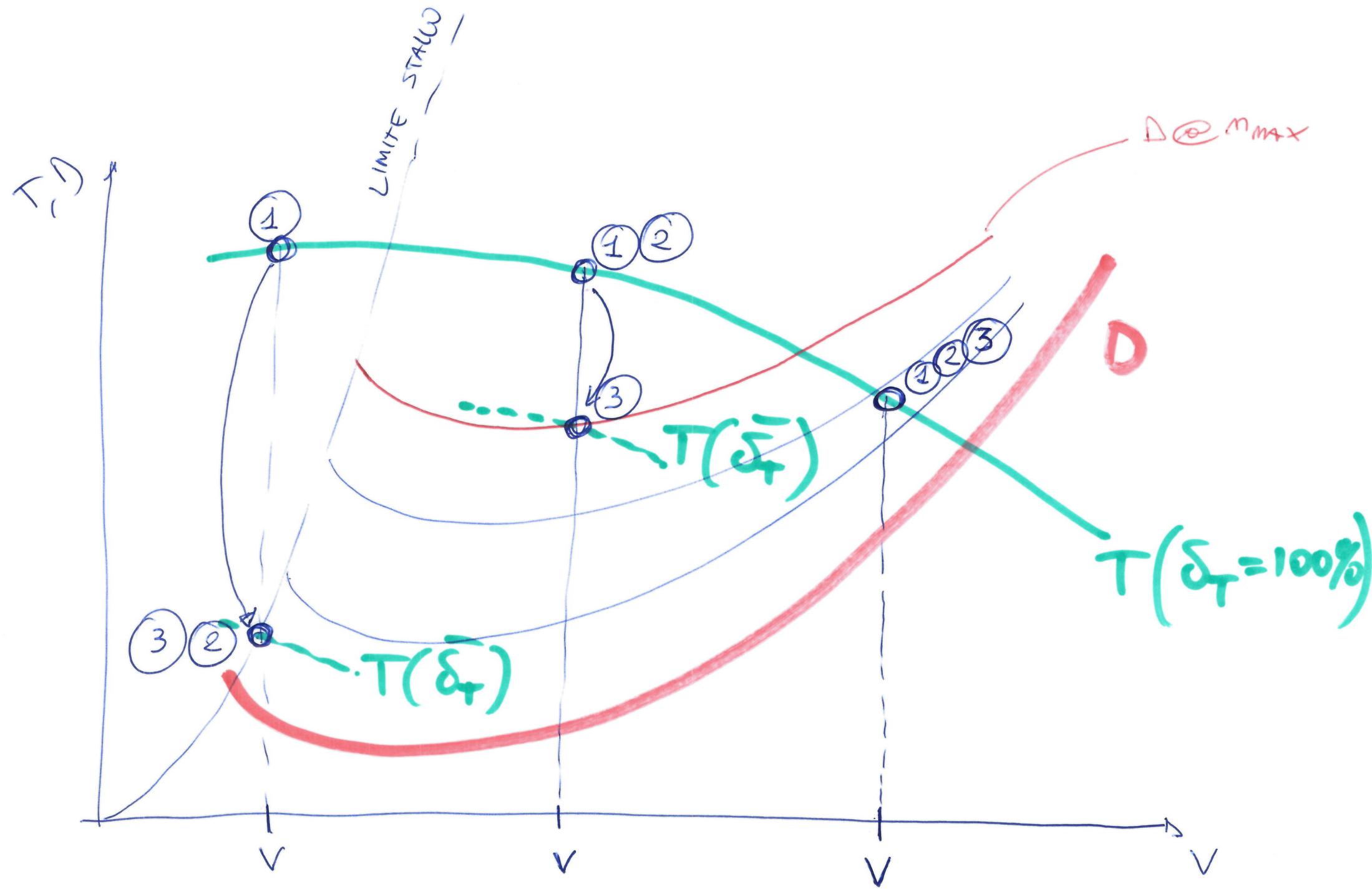
③ CALCOLO m

$$m_{TURN} = \frac{\frac{1}{2} \rho v^2 S G_{TURN}}{w}$$

CHECK: $m_{TURN} \leq m_{MAX}$

$$\text{SE NO } m_{TURN} = m_{MAX}$$

(VINCOLO STRUTTURALE)



ESERCIZIO SU VIRATA

$$w/s = 4410 \text{ N/m}^2$$

$$S = 94 \text{ m}^2$$

$$g_{d_0} = 0.021$$

$$K = 0.0492$$

$$q_{\max} = 1.1673$$

$$m_{\max} = 2.5$$

$$h = 500 \text{ m}$$

$$\bar{P}_{500} = 1.1673 \text{ kg/m}^3$$

$$T_{500} = 6.83 \cdot 10^4 \text{ N}$$

R_{TURN} , T_n , n migliori ottenibili

(tenuto conto di tutte le limitazioni) e in

virata corretta $h = 500 \text{ m}$ e $V = 162.27 \text{ m/s}$

SOLUZIONE

1) ASSUMO LIMITE PROPULSIVO

$$T = D \rightarrow G_{\text{TURN}} = \frac{T_{500}}{\frac{1}{2} \rho_{500} V^2 S} = 0.0473$$

2) CALCOLO A ED EFFETTUO VERIFICA AERO

$$a = \sqrt{\frac{G_{\text{TURN}} - g_0}{k}} = 0.7306 < a_{\max} (1.1673) \quad \text{OK!}$$

3) CALCOLO M ED EFFETTUO LA VERIFICA STRUTT.

$$m = \frac{L}{W} = \frac{\frac{1}{2} \rho_{500} V^2 S G_{\text{TURN}}}{W} = 2.54 > m_{\max} (2.5)$$

$$\rightarrow R_{\text{TURN}} = \frac{V^2}{g \sqrt{m_{\text{turn}}^2 - 1}} = 1.172 \text{ Km}$$

$$\rightarrow T_{\text{R}} = \frac{\pi V}{g \sqrt{m_{\text{turn}}^2 - 1}} = 22.68 \text{ s}$$

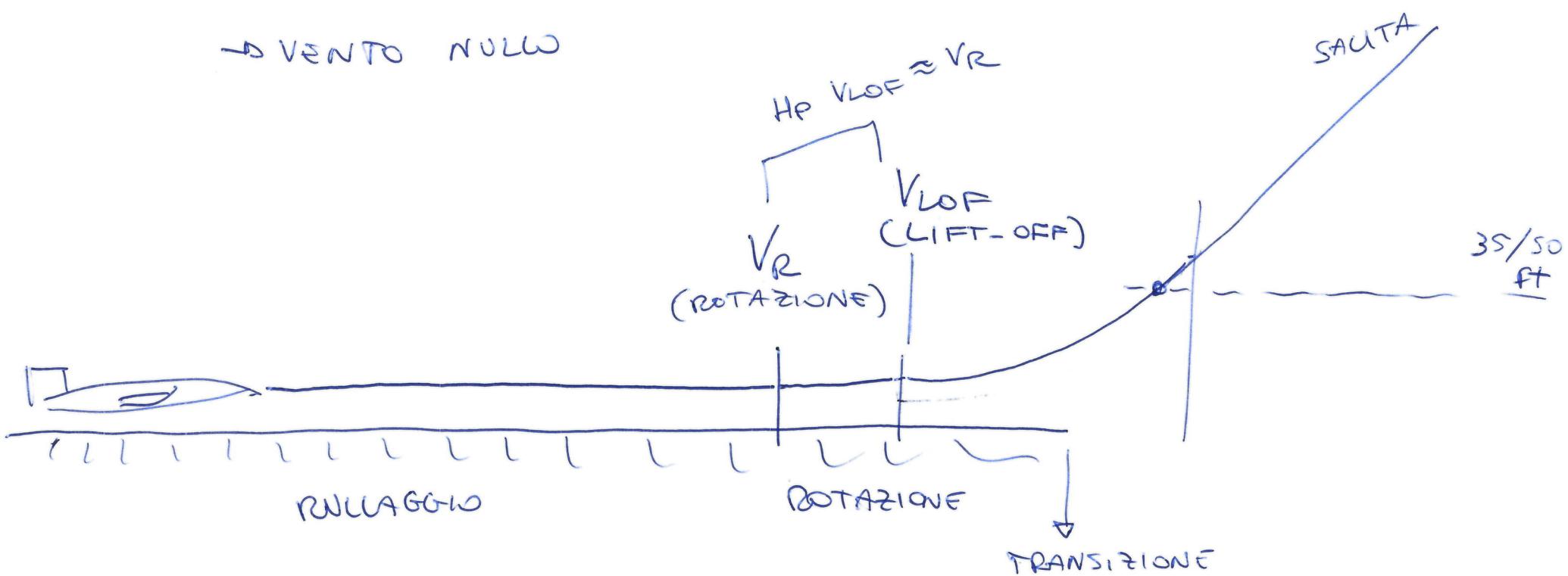
$\rightarrow +$ VINCOLO ATTIVO: ~~ASTRODINAMICO~~ STRETTURALE

$$\left\{ \begin{array}{l} m_{\text{turn}} = m_{\max} \\ G_{\text{turn}} = \frac{m_{\max} \cdot W}{\frac{1}{2} \rho_{500} V^2 S} = 0.7175 \\ G_{\text{turn}} = G_0 + k a_{\text{turn}}^2 = 0.0463 \\ D_{\text{turn}} = \frac{1}{2} \rho_{500} V^2 S G_{\text{turn}} \end{array} \right.$$

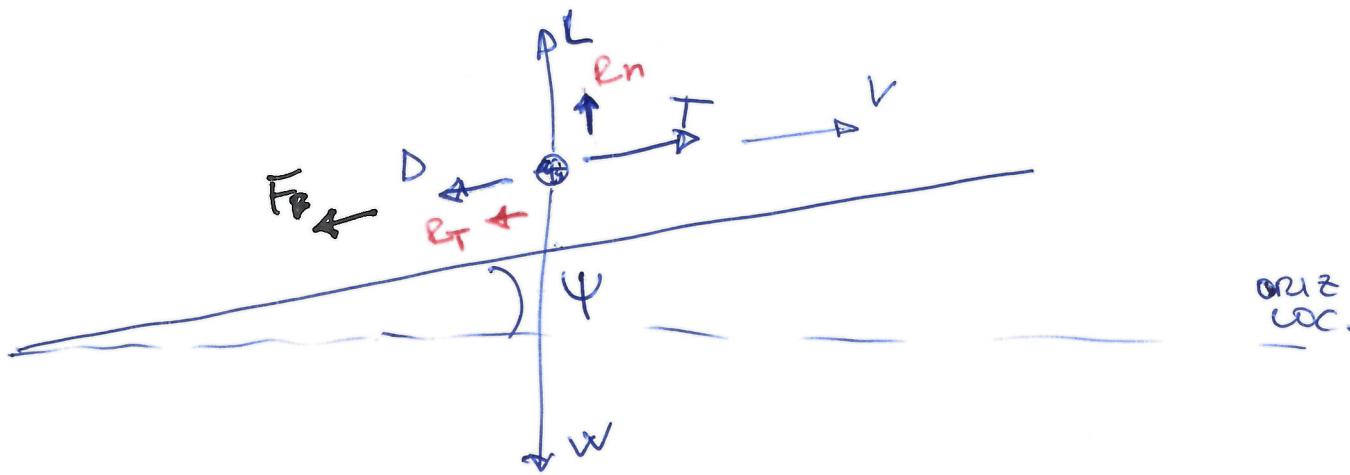
DECOLLO

→ FASE DELLA MISSIONE DI VOLO COMPRESA TRA LO STATO DI QUIETE A TERRA E VOLO AD UNA CERTA QUOTA → 35 ft FAR 25 (CS 23)
50 ft FAR 23 (CS 23)

- Hp → NO TAXI / NO CLIMB TO CRUISE
→ CONFIGURAZIONE VELIVOLO FISSATA
→ VENTO NULLO



MODELLO DI RULLAGGIO (GROUND RUN MODEL)



R_N = REAZIONE
VINCOLARE
NORMALE

R_T = $R_N \cdot V_0 \cdot \tan \psi$

$R_T = \mu R_N$
COEFF.
ATTUAUTO
VOLVENTE

EQU

// PISTA

$$m \ddot{v} = T - D - R_T - W \sin \psi - F_B$$

L PISTA

$$0 = L + R_N - W \cos \psi$$

$$R_T = \mu R_N$$

$|H_P| \psi$ piccolo

F_B = FORZA
FRENANTE

$F_B \neq 0$ solo se
PILOTA FRENA

$$\ddot{v}(t) = g \left(\frac{T}{W} - \frac{F_B}{W} - \frac{D - \mu L}{W} - (M + \psi) \right)$$

$$S_G = \int_0^{t_{LOF}} V \, dt = \int_0^{V_{LOF}} \frac{V}{\dot{V}} \, dV$$

CORSA
DECOLLO

$$\dot{V}(t) = \frac{dV}{dt} \Rightarrow dt = \frac{dV}{\dot{V}(t)}$$

$t_{LOF} \rightarrow$ Tempo
a cui
si raggiunge
 V_{LOF}

V_{LOF} DA NORMATIVA: VELOCITA A CUI MO

$$a = \frac{1}{1.1} a_{MAXTO}$$

$$S_G = \int_0^{V_{LOF}} \frac{V}{\frac{T}{W} - \frac{F_B}{W} - \frac{\Delta - M_L}{W} - (\mu + \psi)} \, dV$$

$$\xi = \frac{V}{V_{LOF}} \Rightarrow$$

$$dV = V_{LOF} \, d\xi$$

$$S_G = \frac{W/S}{g} \int_0^1 \frac{\xi \, d\xi}{\frac{1}{2} \rho \frac{1}{1.1} a_{MAXTO} \left[\frac{T}{W} - \frac{F_B}{W} - (\mu + \psi) \right] - \frac{1}{2} \rho \xi^2 (C_D - \mu a)}$$

PER

$S_G \downarrow \downarrow$ occurs $\frac{T}{W} \uparrow$ $M \downarrow$ $\psi \downarrow$ $W/S \downarrow$ $e \uparrow$ $a_{max, ro} \uparrow$

$$+ \boxed{G_D - M_a \text{ OTTIMO}} =$$

PISTA BLANDATA

→ considero l'avaria di un motore
nel caso di veicolo puramente

■ AEO (All Engines Operational)

$T \rightarrow FULL$

$F_B = 0$

$$|| S_{AEO} = - \int_0^{V_{EOF}} \dots dV$$

■ OEI (One Engine Inoperative)

$T \rightarrow REDOTTA$

$F_B = 0$

$$|| S_{OEI} = \int_V^{V_{EOF}} \dots dV$$

■ STOP (FRENO!)

$T = 0$ (oppure $T < 0$ se HO)
(INVERSIONE)

$F_B \neq 0$

$$|| S_{STOP} = \int_V^0 (\dots) dV$$

