# CUSUM technique

Using July through October daily-high-temperature data for Atlanta for 1996 through 2015, use a CUSUM approach to identify when unofficial summer ends (i.e., when the weather starts cooling off) each year. CUSUM (or cumulative sum control chart) is a sequential analysis technique used for monitoring change detection.

```
# Clear environment
rm(list = ls())
#install.packages(qcc)
library(qcc)
```

```
## Warning: package 'qcc' was built under R version 4.0.2
```

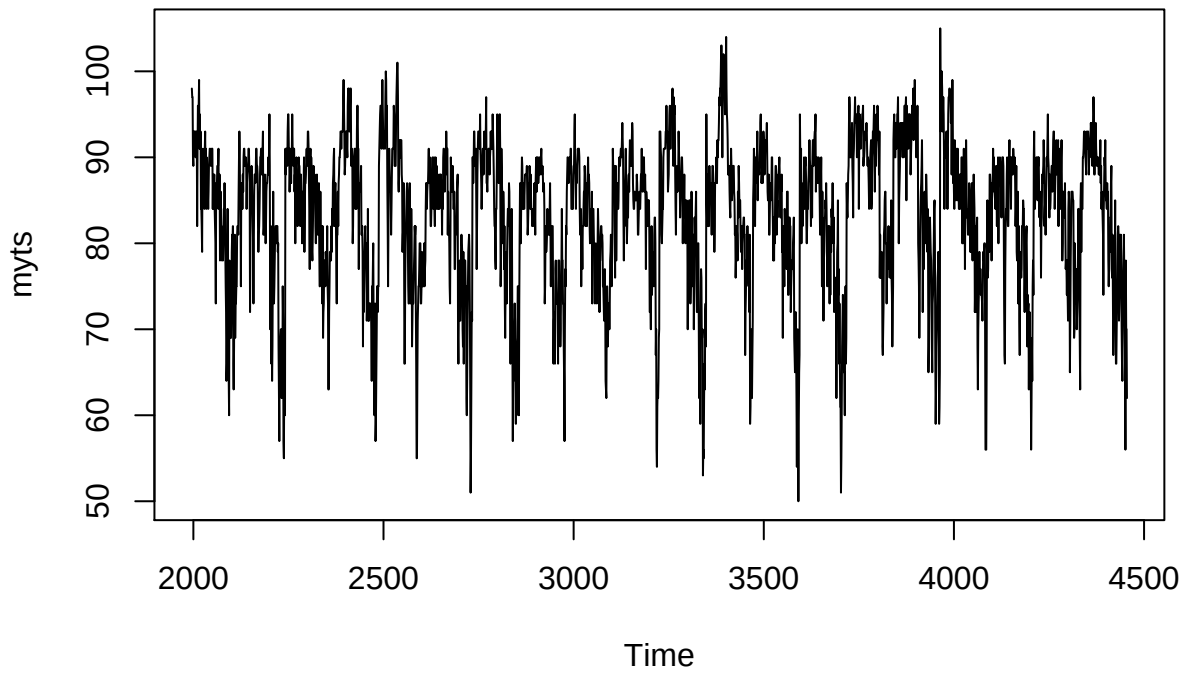
```
## Package 'qcc' version 2.7
```

```
## Type 'citation("qcc")' for citing this R package in publications.
```

```
# Reading the data
data <- read.table("temps.txt", stringsAsFactors = FALSE, header = TRUE)
head(data)
```

```
##     DAY X1996 X1997 X1998 X1999 X2000 X2001 X2002 X2003 X2004 X2005 X2006 X2007
## 1 1-Jul    98    86    91    84    89    84    90    73    82    91    93    95
## 2 2-Jul    97    90    88    82    91    87    90    81    81    89    93    85
## 3 3-Jul    97    93    91    87    93    87    87    87    86    86    93    82
## 4 4-Jul    90    91    91    88    95    84    89    86    88    86    91    86
## 5 5-Jul    89    84    91    90    96    86    93    80    90    89    90    88
## 6 6-Jul    93    84    89    91    96    87    93    84    90    82    81    87
##   X2008 X2009 X2010 X2011 X2012 X2013 X2014 X2015
## 1    85    95    87    92   105    82    90    85
## 2    87    90    84    94    93    85    93    87
## 3    91    89    83    95    99    76    87    79
## 4    90    91    85    92    98    77    84    85
## 5    88    80    88    90   100    83    86    84
## 6    82    87    89    90    98    83    87    84
```

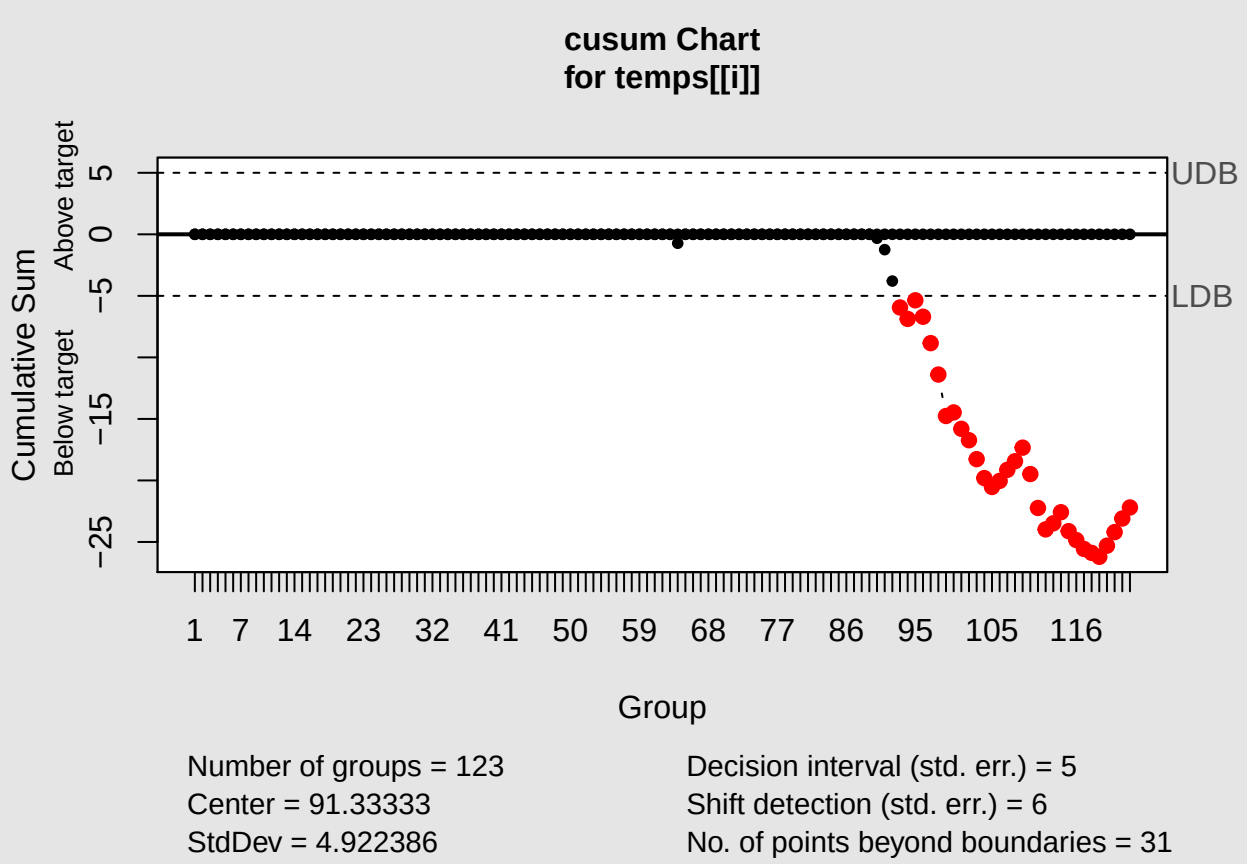
```
# Create matrix without day column
temps = matrix(data[,2:ncol(data)])
# Create a vector of this data
temps_vec <- as.vector(unlist(temps))
# Turn the vector into a time series object and view plot
myts <- ts(temps_vec,start=1996,frequency=1)
plot(myts)
```

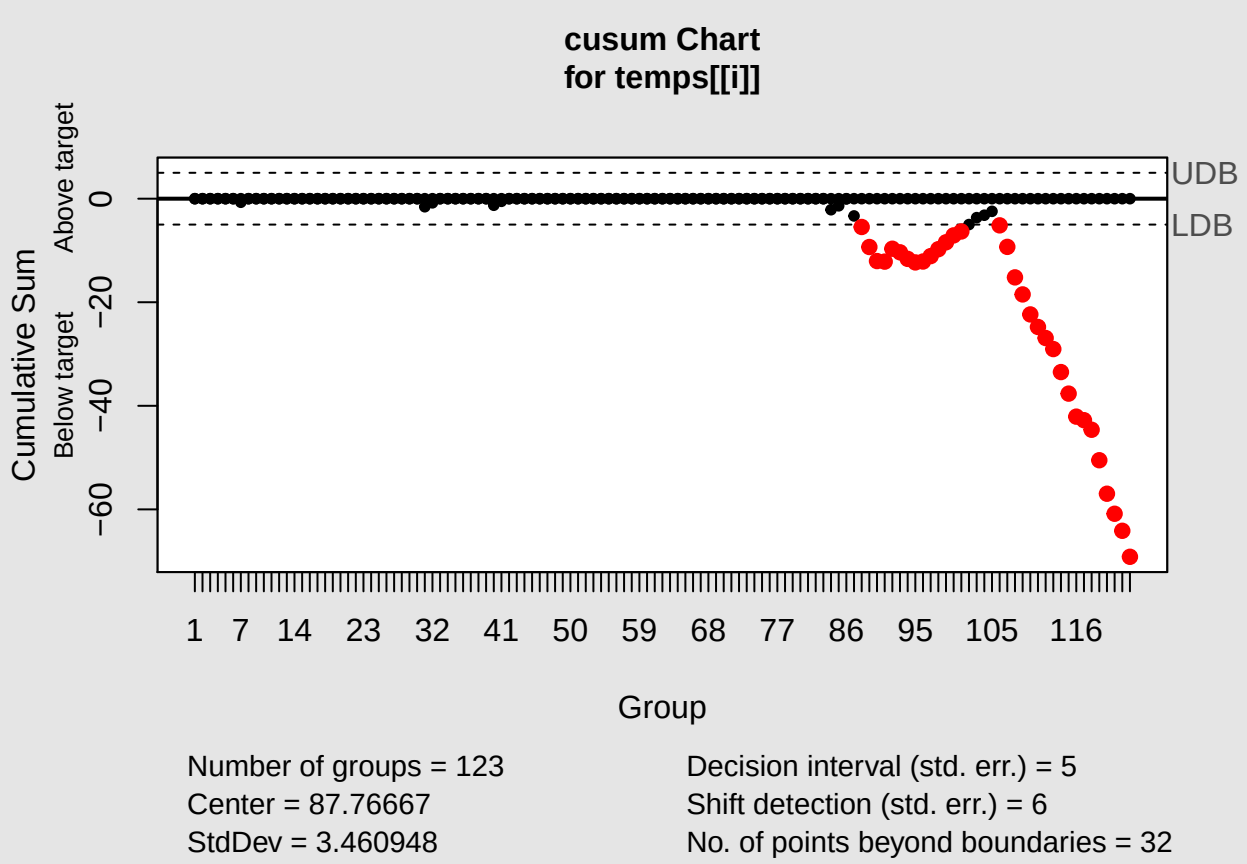


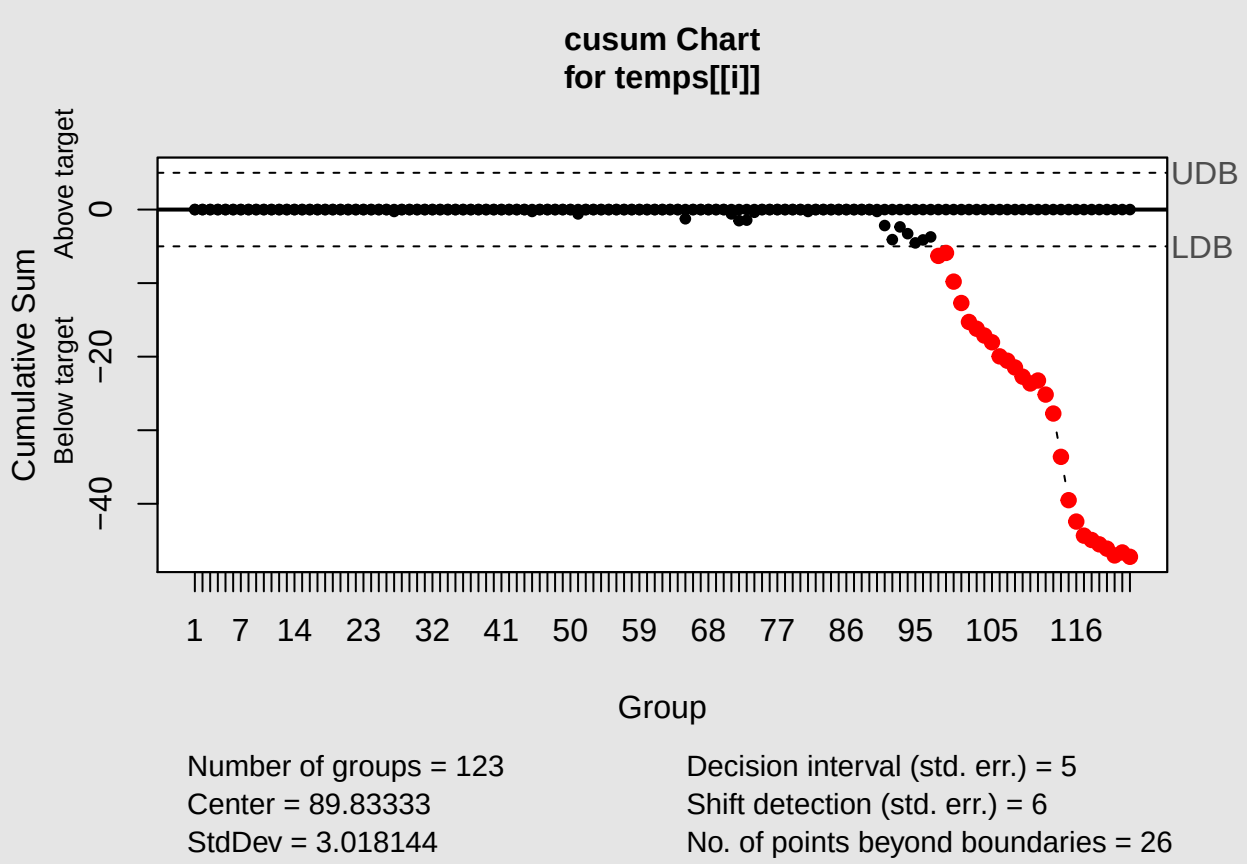

```
# Determine the summer mean and standard deviation value we want to use
# We just used the month of July as a base average summer temperature with no change
avg_summer <- rep(0,nrow(temps))
sd_summer <- rep(0,nrow(temps))
for (i in 1:nrow(temps)){
  avg_summer[i] <- mean(temps[[i]][1:30])
  sd_summer[i] <- sd(temps[[i]][1:30])
}

# Use the cusum function
CUSUMmodels <- vector(mode="list", length=nrow(temps))
CUSUMviolations <- vector(mode="list", length=nrow(temps))
```

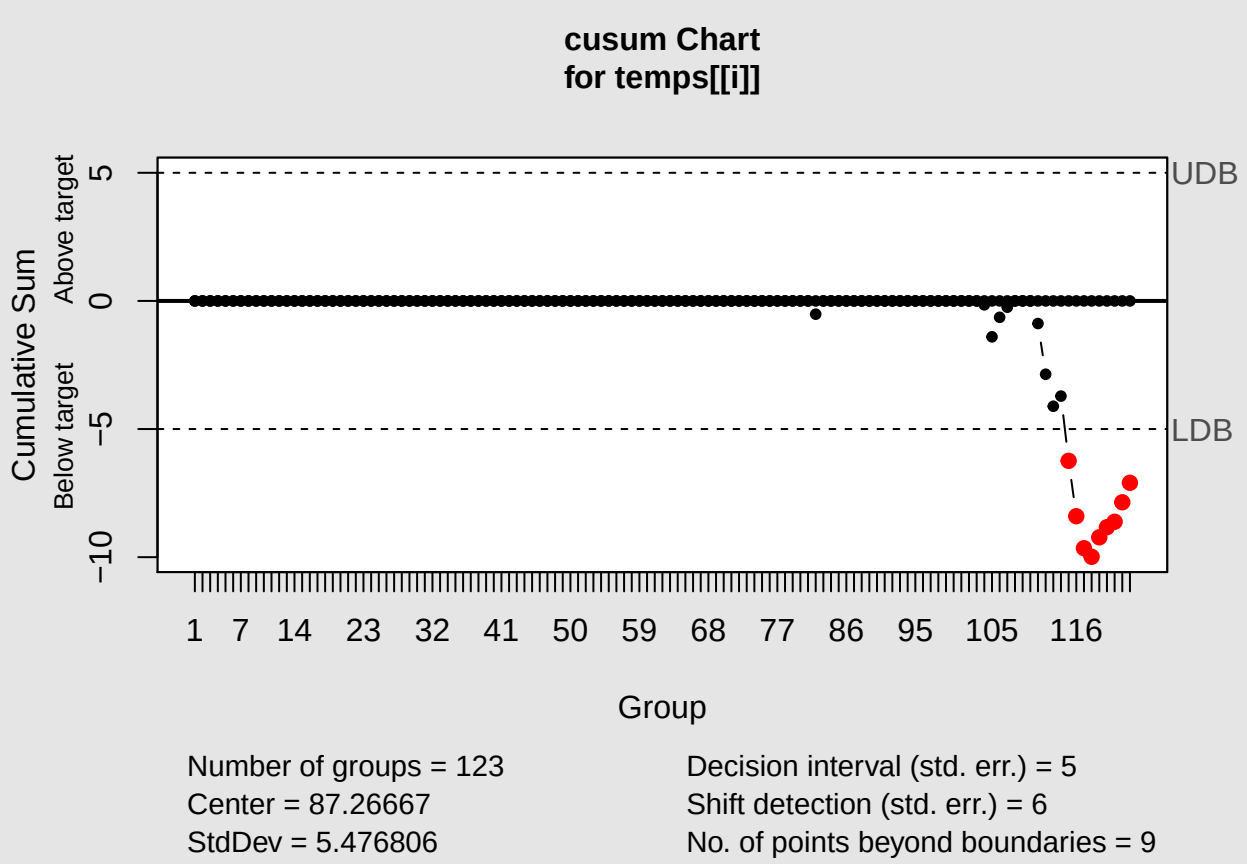
Loop through each year and run the CUSUM function center is the "target" summer temperature each year std.dev is the standard deviation of the summer temperature each year set decision.interval as the upper and lower bound in standard deviations/errors from 0 change set se.shift as the amount of shift to detect (which corresponds to 2*C in terms of standard deviations/errors)

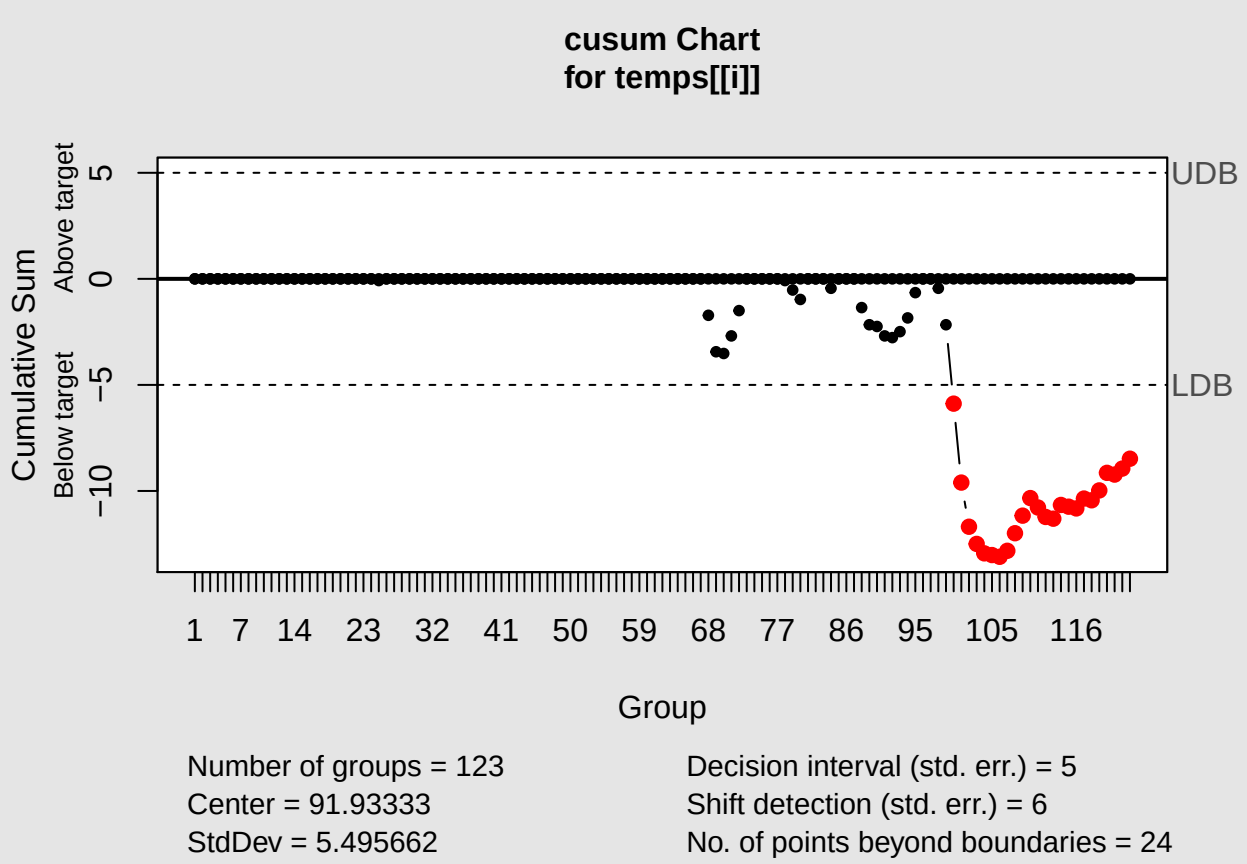
```
DI = 5 #out of control when past 5 standard deviations/errors
SS = 6 #equals 6/2 = 3 standard deviations/errors shift
for (i in 1:nrow(temps)){
  CUSUMmodels[[i]] <- cusum(temps[[i]], center=avg_summer[i], std.dev = sd_summer[i], decision.interval=
  CUSUMviolations[[i]] <- CUSUMmodels[[i]]$violations$lower
}
```

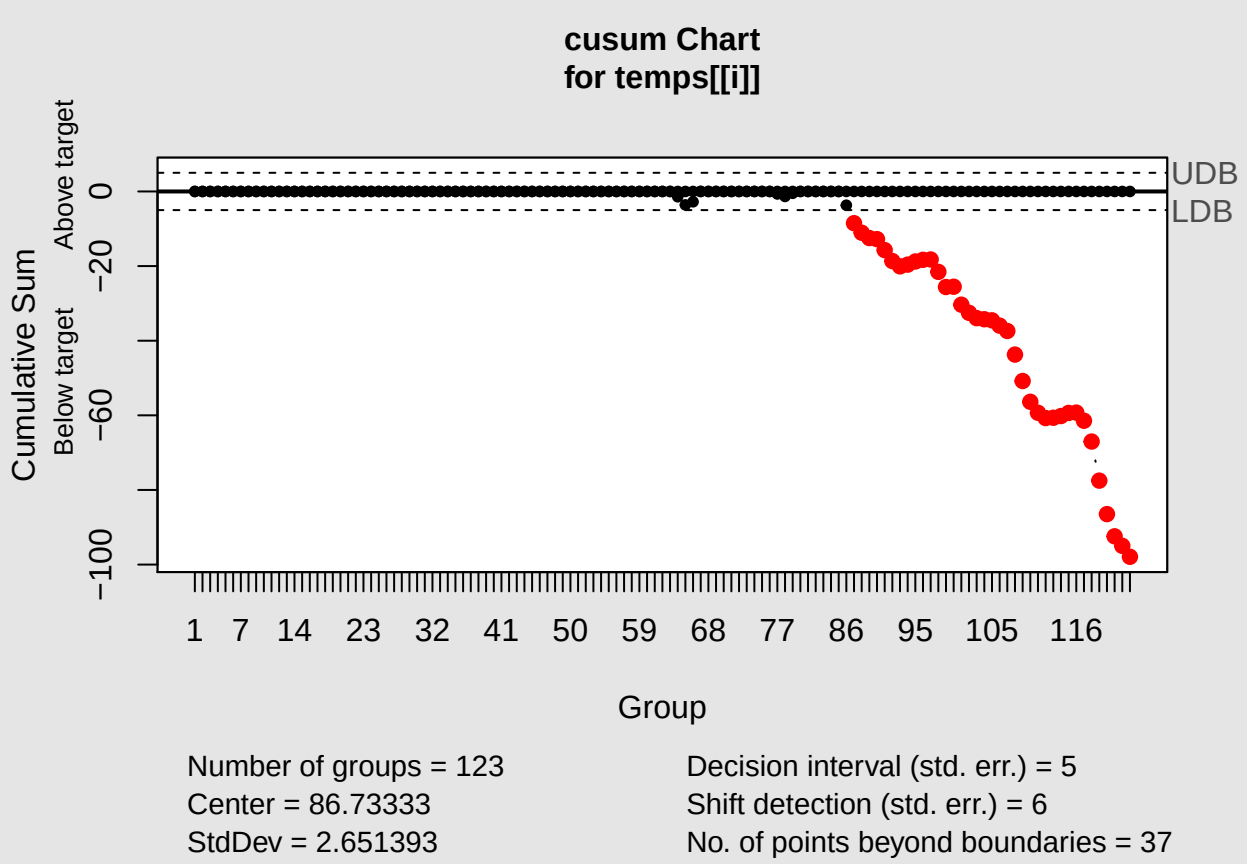

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
5
0
−5
−15
−25
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 91.33333
StdDev = 4.922386
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 31

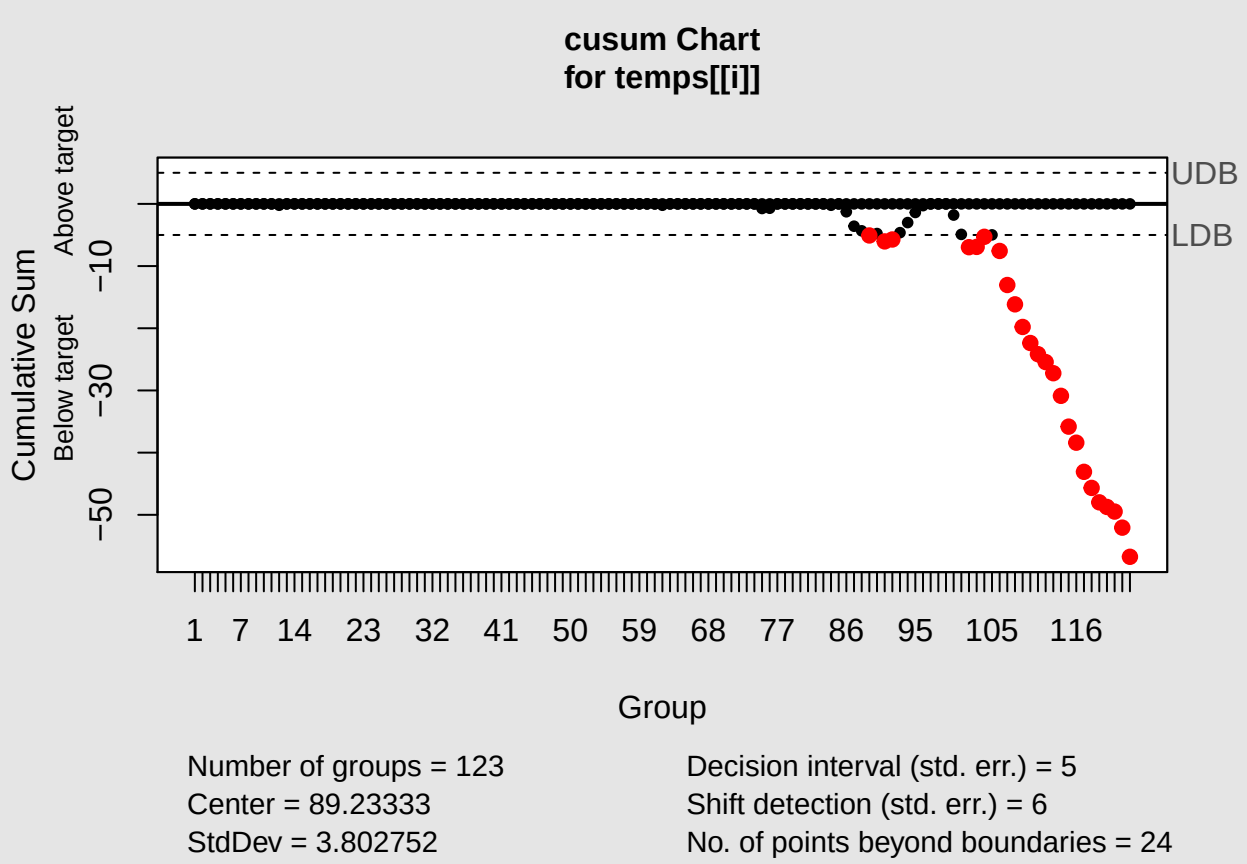

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−20
−40
−60
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 87.76667
StdDev = 3.460948
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 32

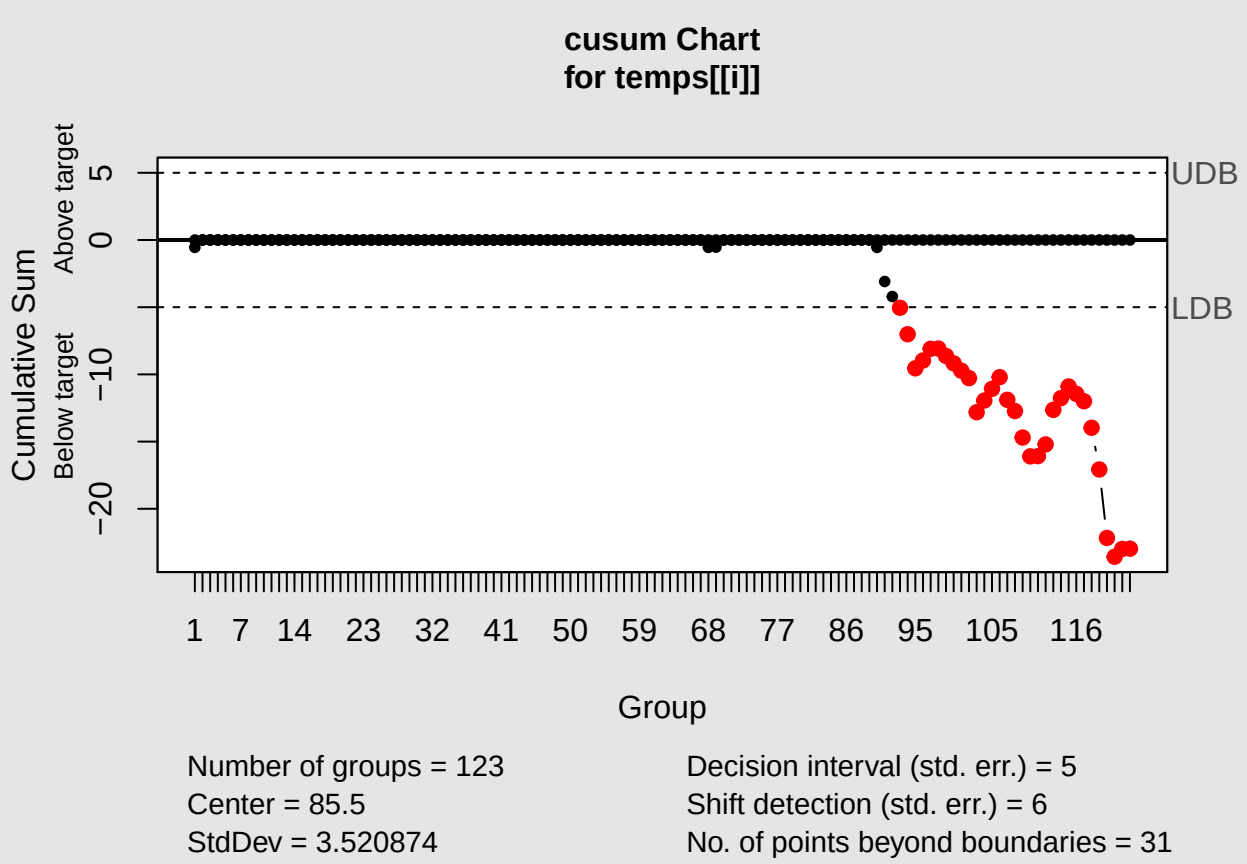

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−20
−40
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 89.83333
StdDev = 3.018144
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 26

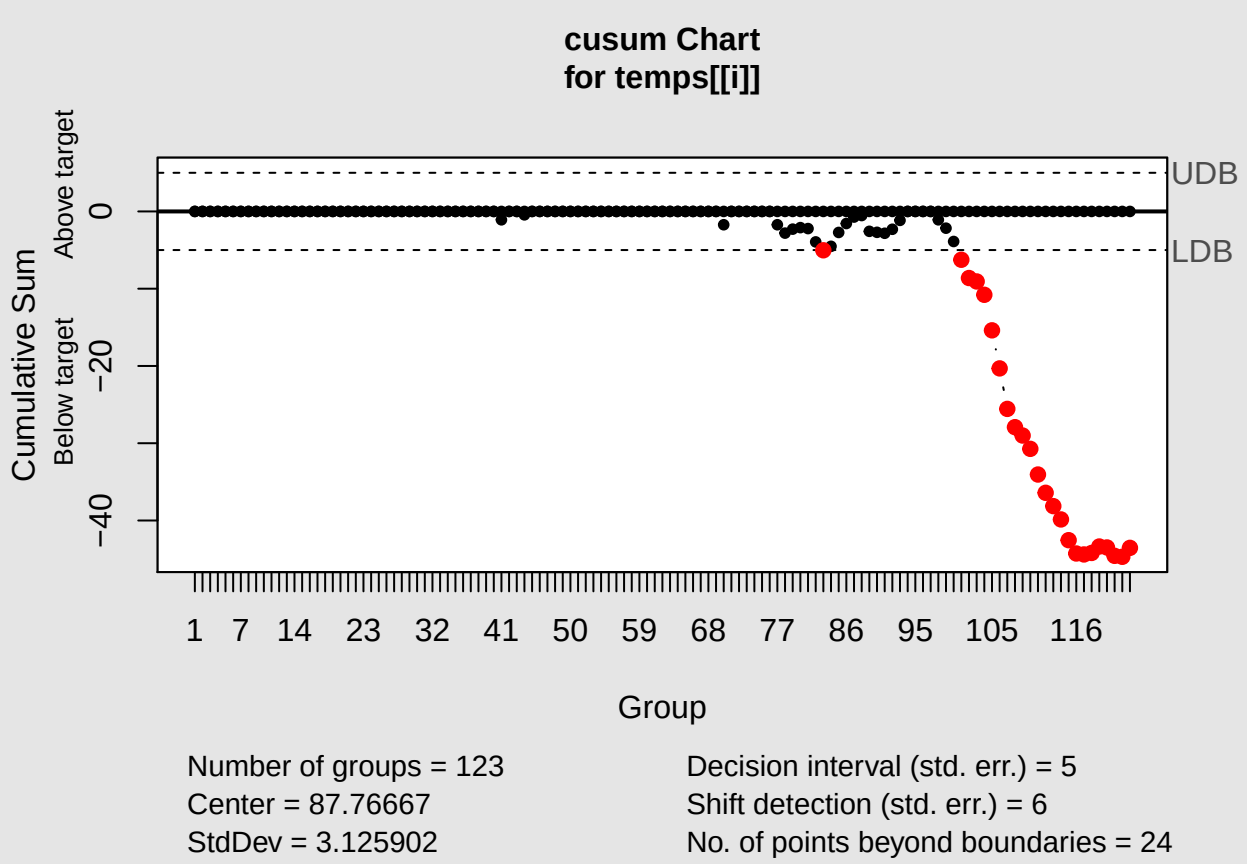

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
5
0
−5
−10
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 87.26667
StdDev = 5.476806
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 9

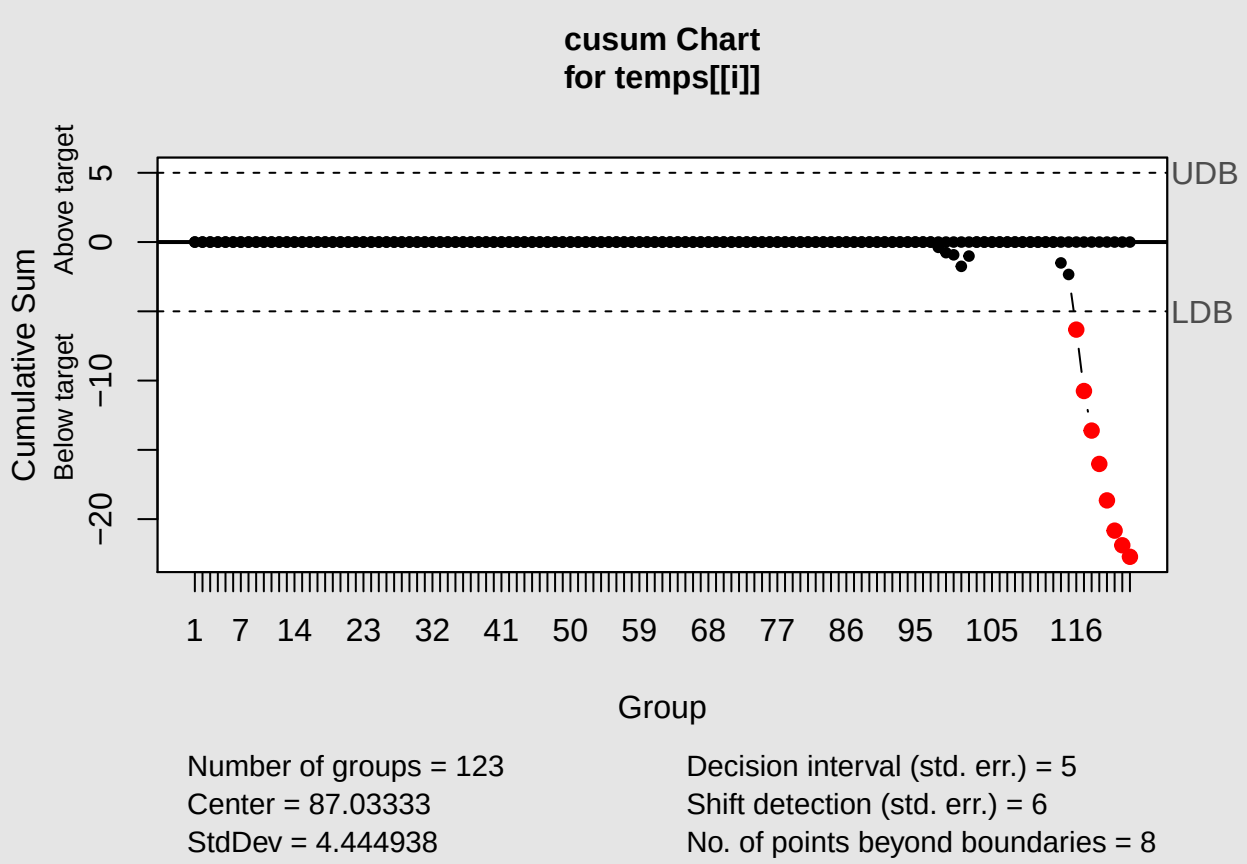

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
5
0
−5
−10
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 91.93333
StdDev = 5.495662
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 24

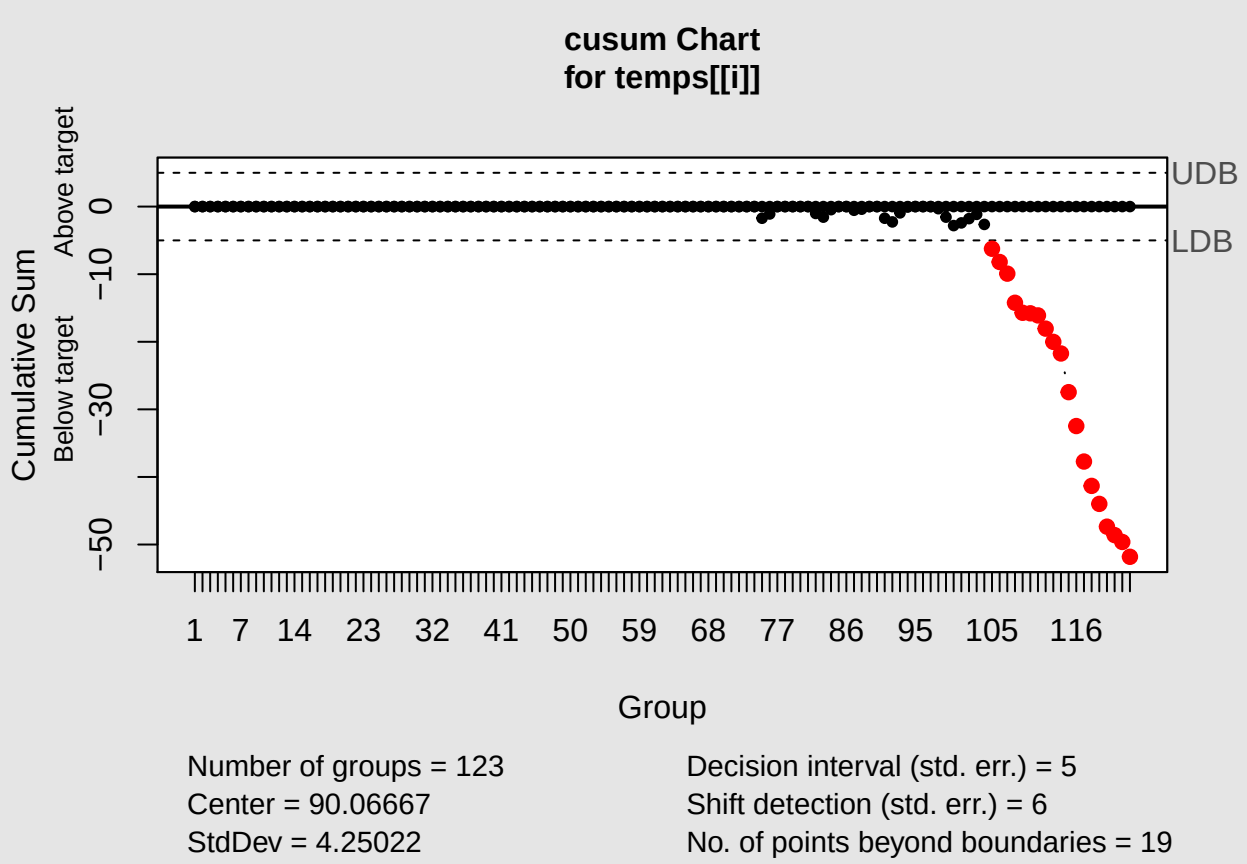

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−20
−60
−100
UDB
LDB
1 7 14 23 32 41 50 59 68 77 86 95 105 116
Group
Number of groups = 123
Center = 86.73333
StdDev = 2.651393
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 37

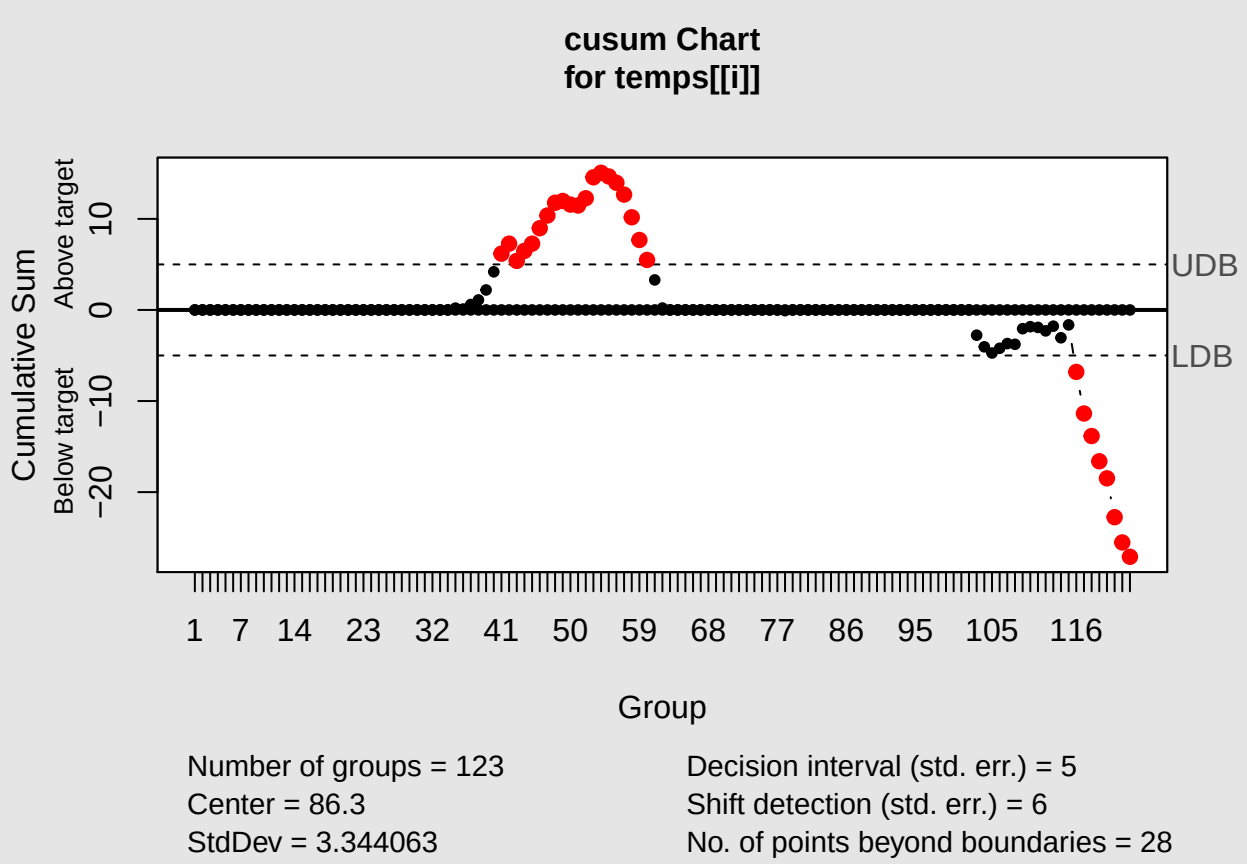

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
−10
−30
−50
UDB
LDB
1 7 14 23 32 41 50 59 68 77 86 95 105 116
Group
Number of groups = 123
Center = 89.23333
StdDev = 3.802752
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 24

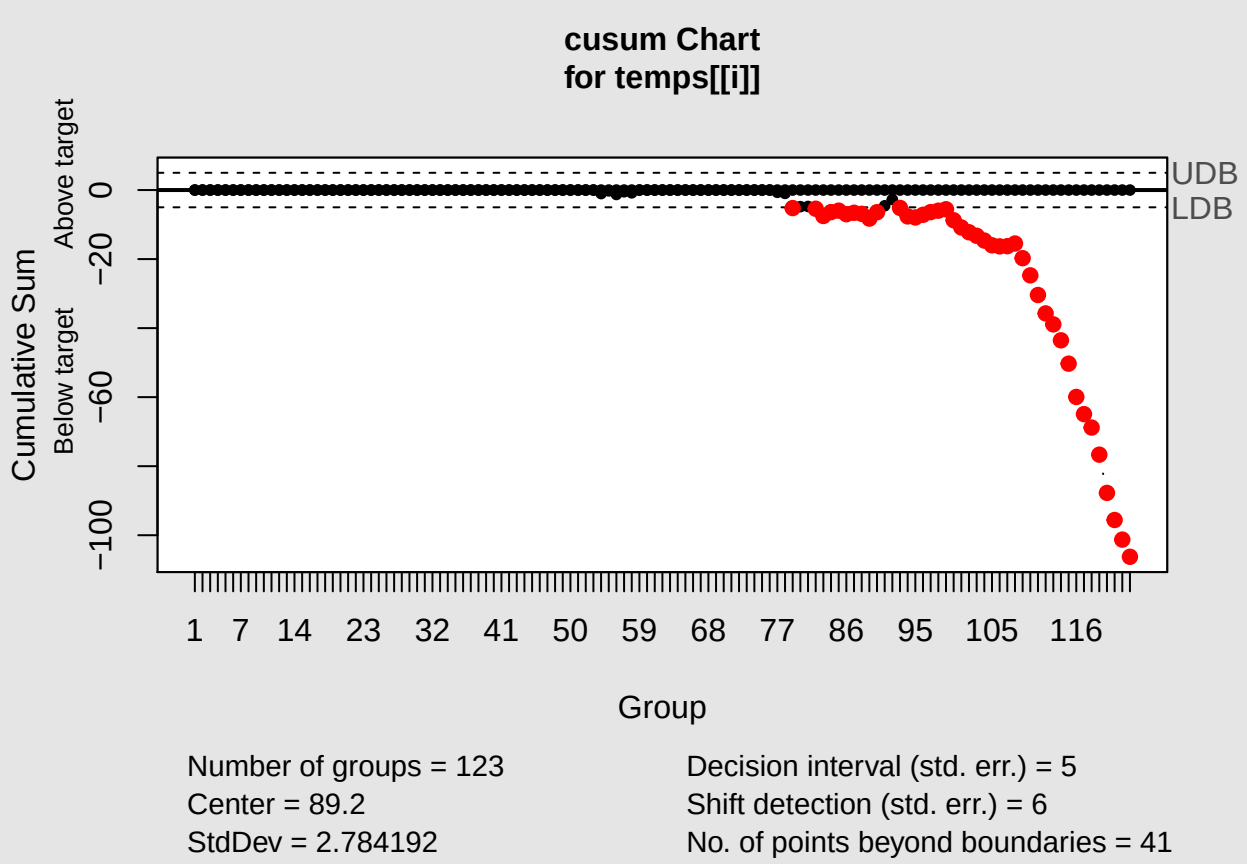

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
5
0
−10
−20
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 85.5
StdDev = 3.520874
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 31

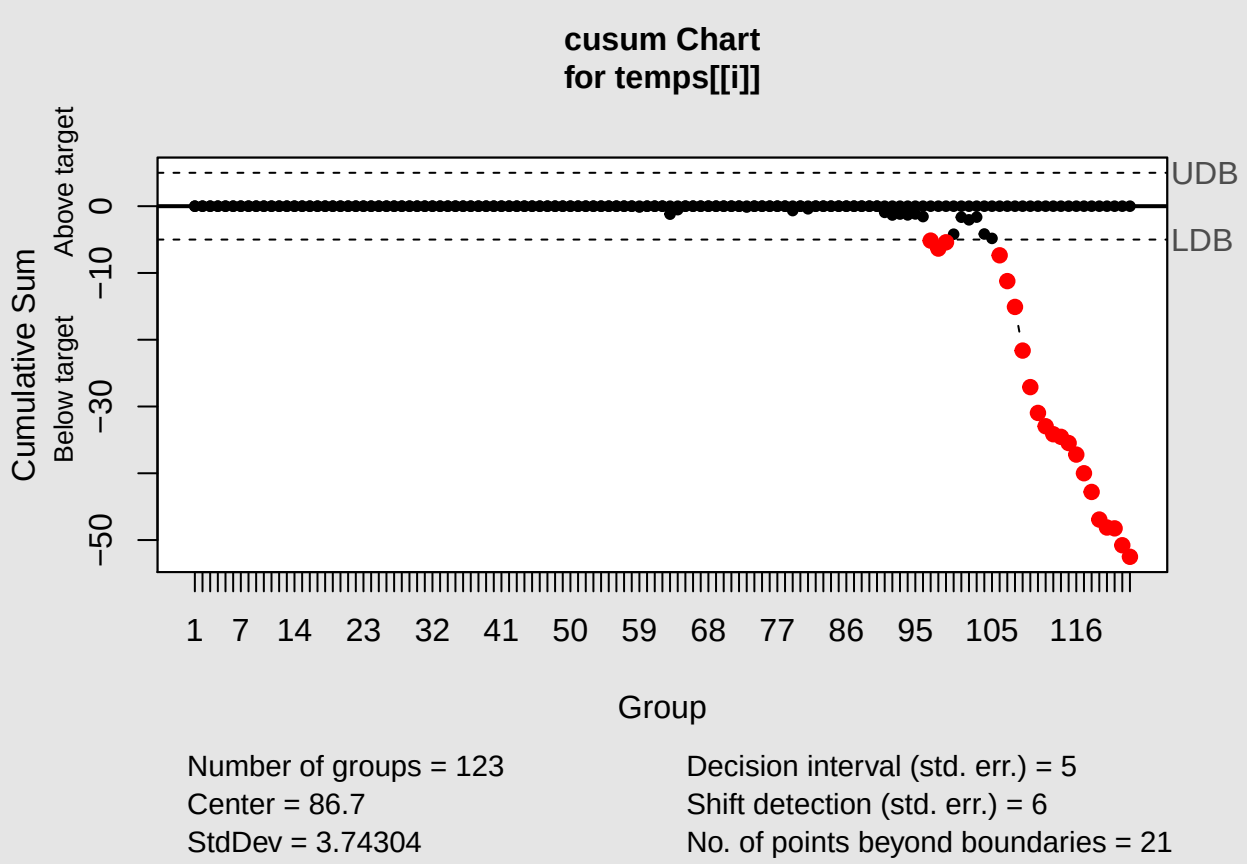

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−20
−40
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 87.76667
StdDev = 3.125902
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 24

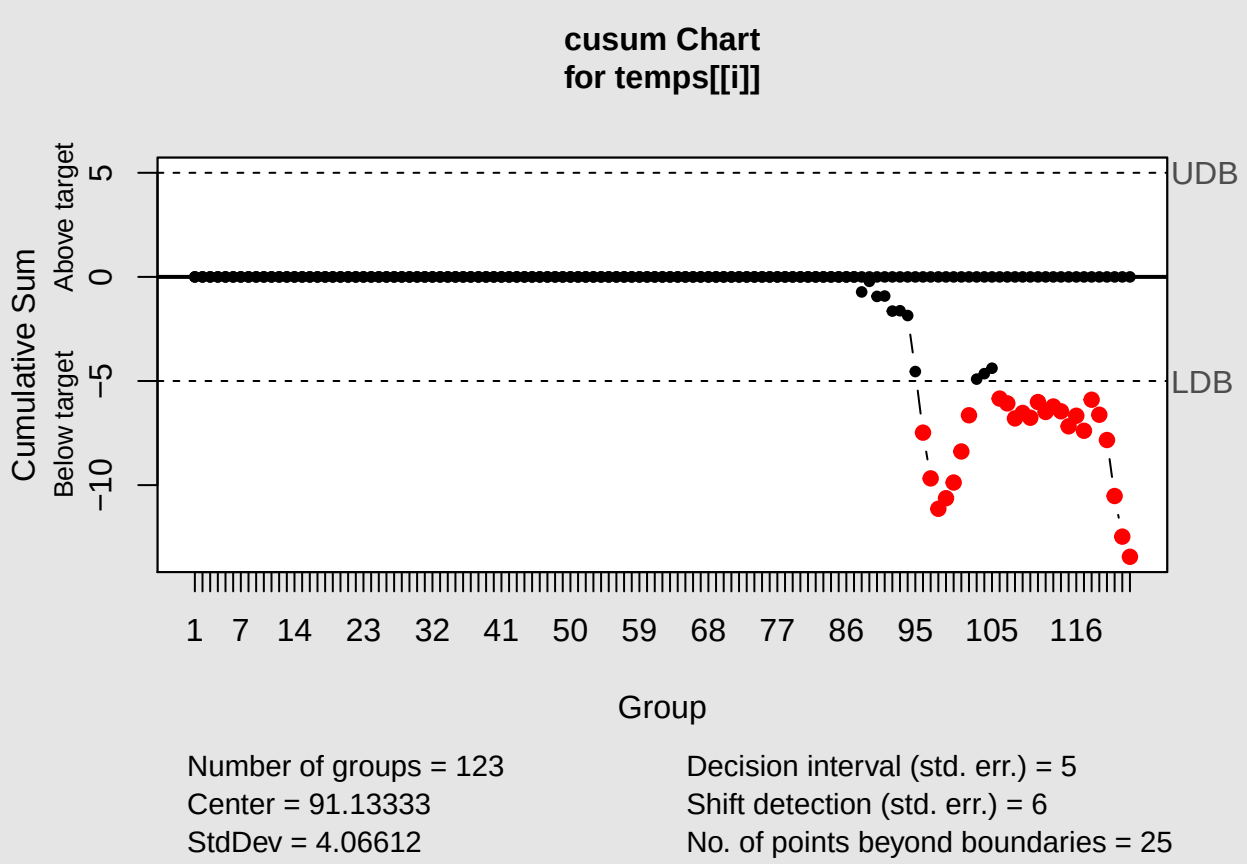

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
5
0
−10
−20
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 87.03333
StdDev = 4.444938
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 8

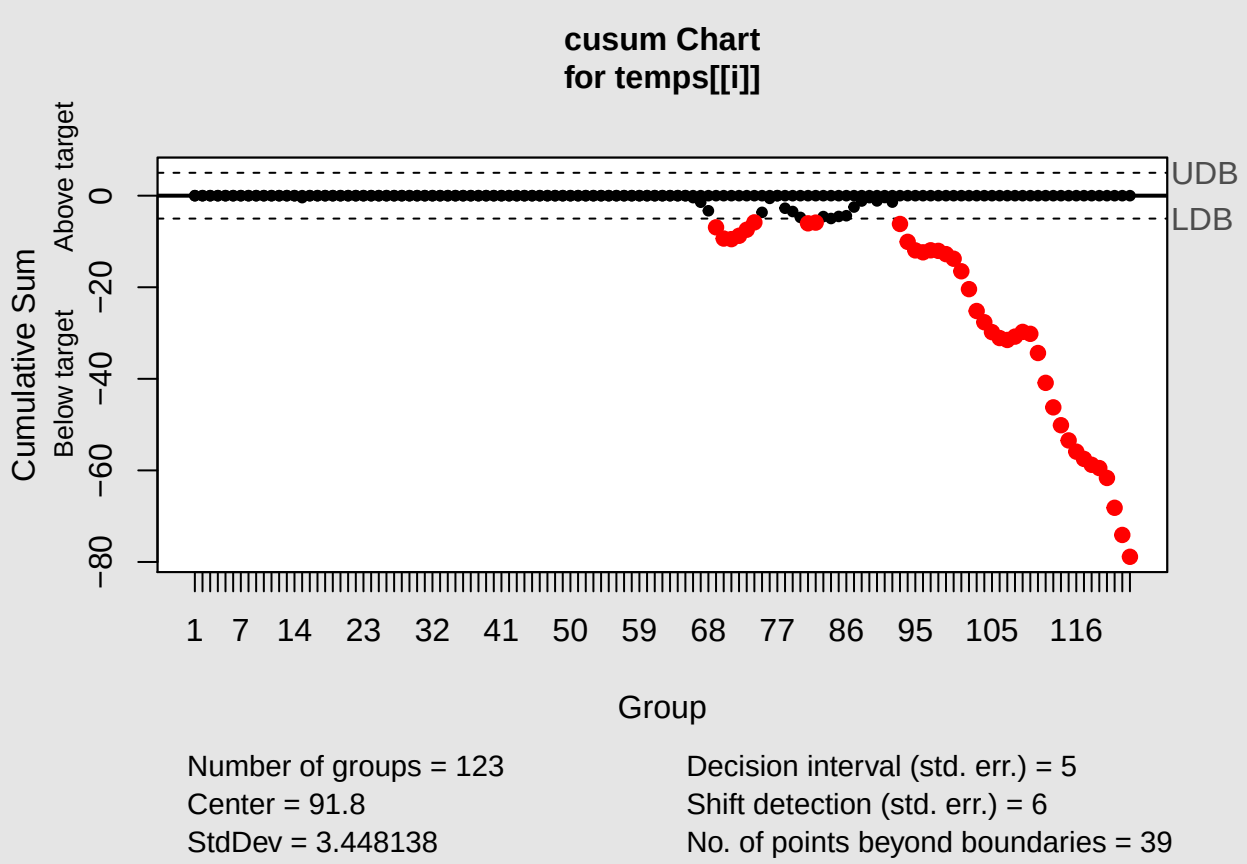

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−10
−30
−50
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 90.06667
StdDev = 4.25022
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 19

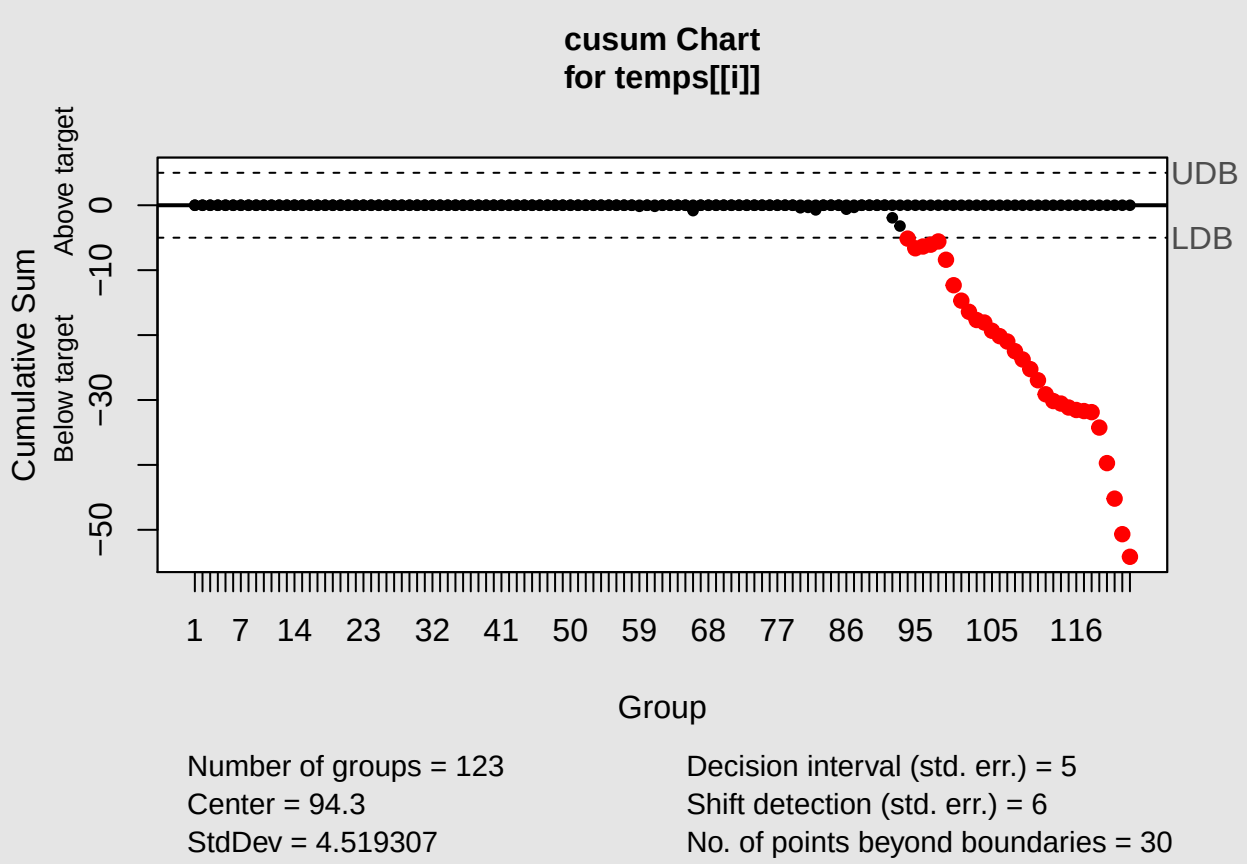

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
10
0
−10
−20
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 86.3
StdDev = 3.344063
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 28

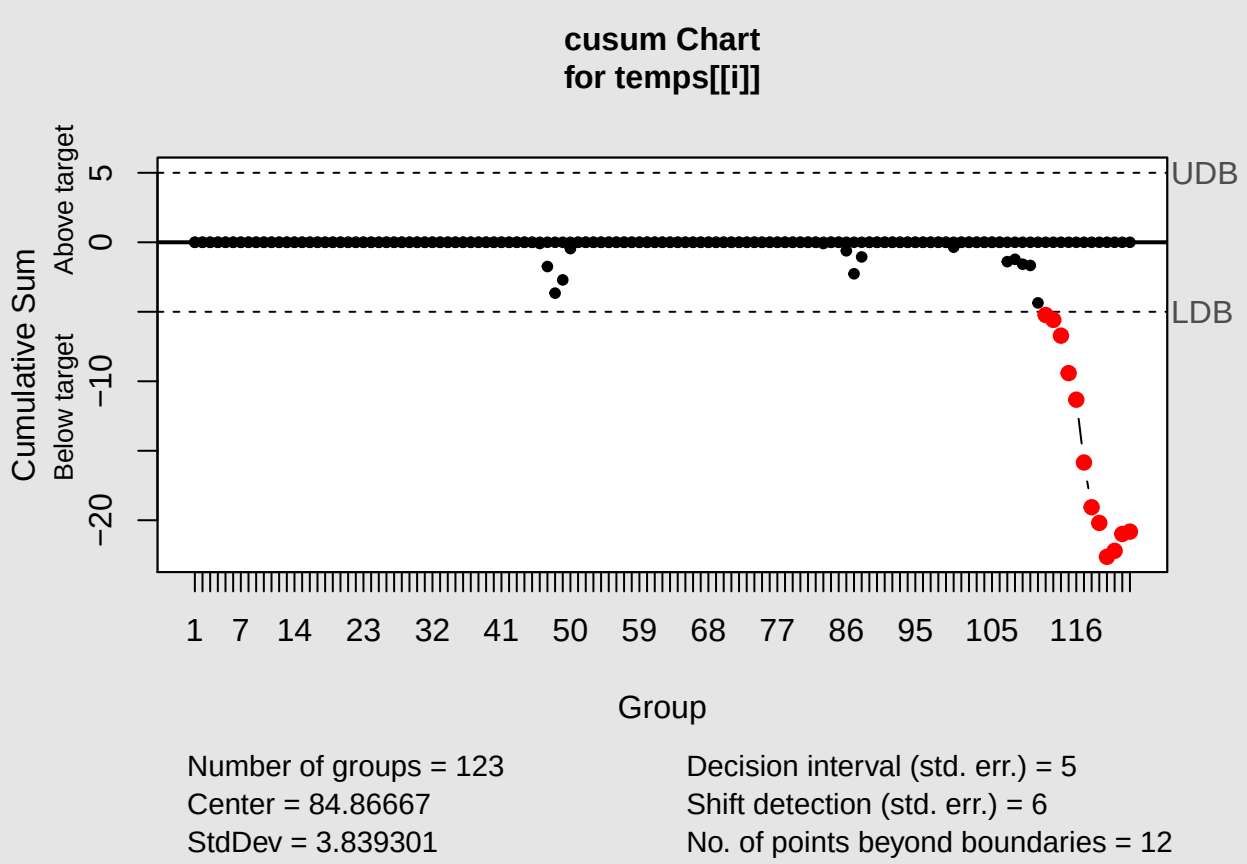

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−20
−60
−100
UDB
LDB
1 7 14 23 32 41 50 59 68 77 86 95 105 116
Group
Number of groups = 123
Center = 89.2
StdDev = 2.784192
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 41

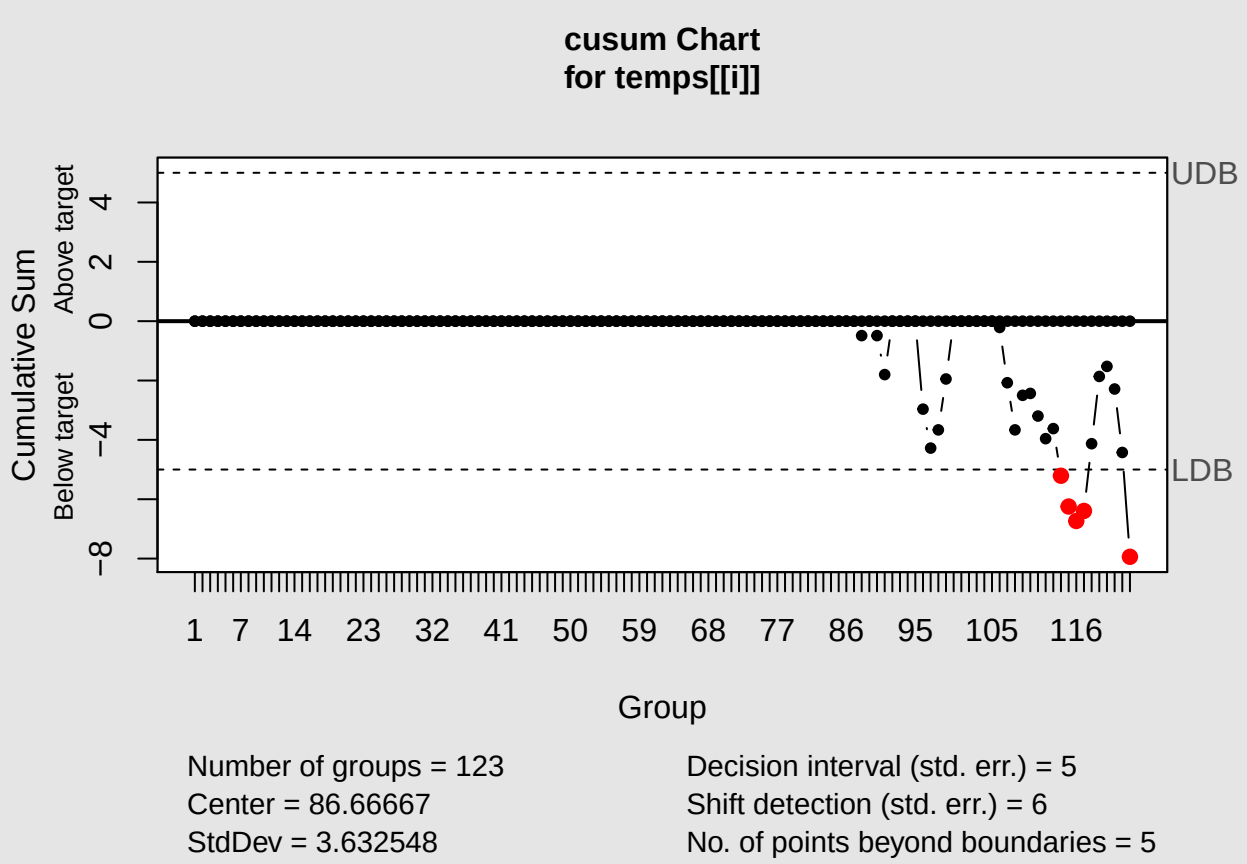

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−10
−30
−50
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 86.7
StdDev = 3.74304
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 21

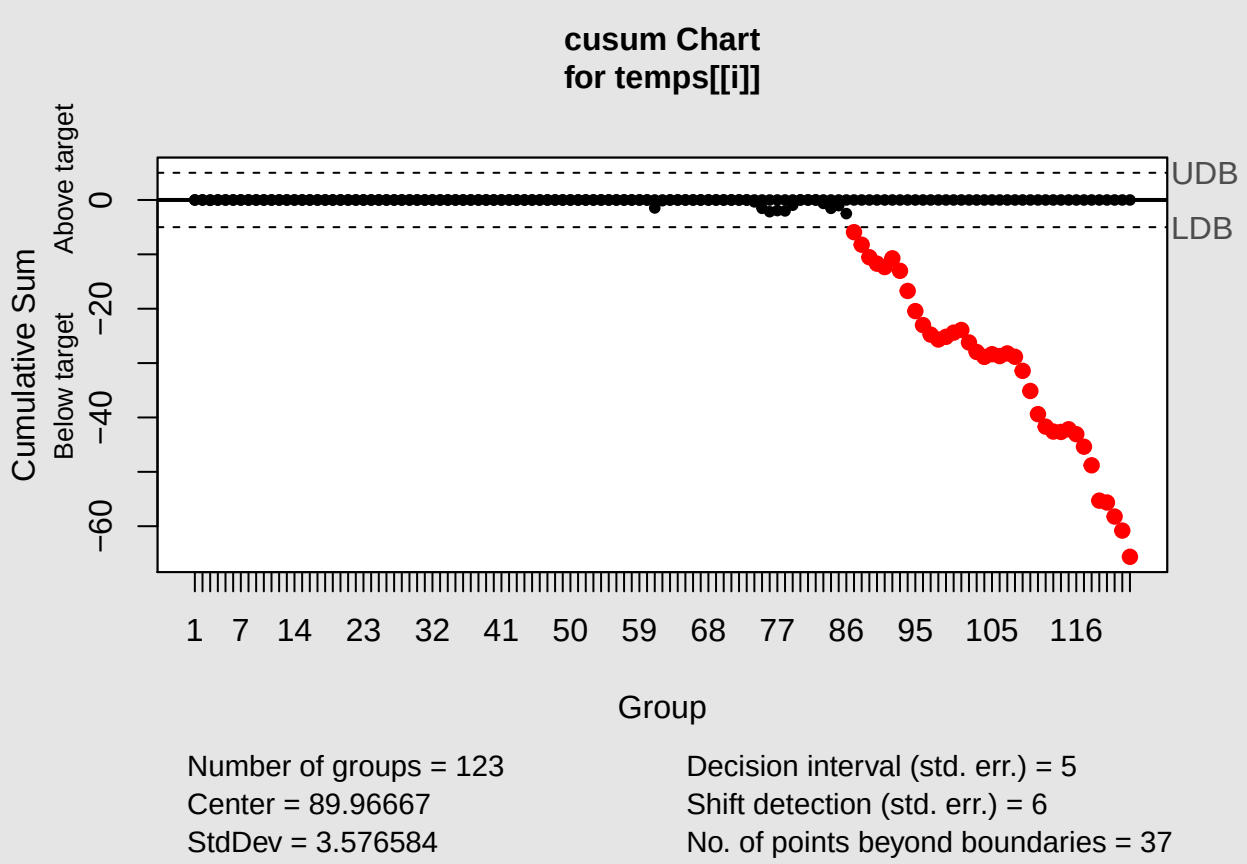

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
5
0
−5
−10
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 91.13333
StdDev = 4.06612
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 25


cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−20
−40
−60
−80
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 91.8
StdDev = 3.448138
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 39

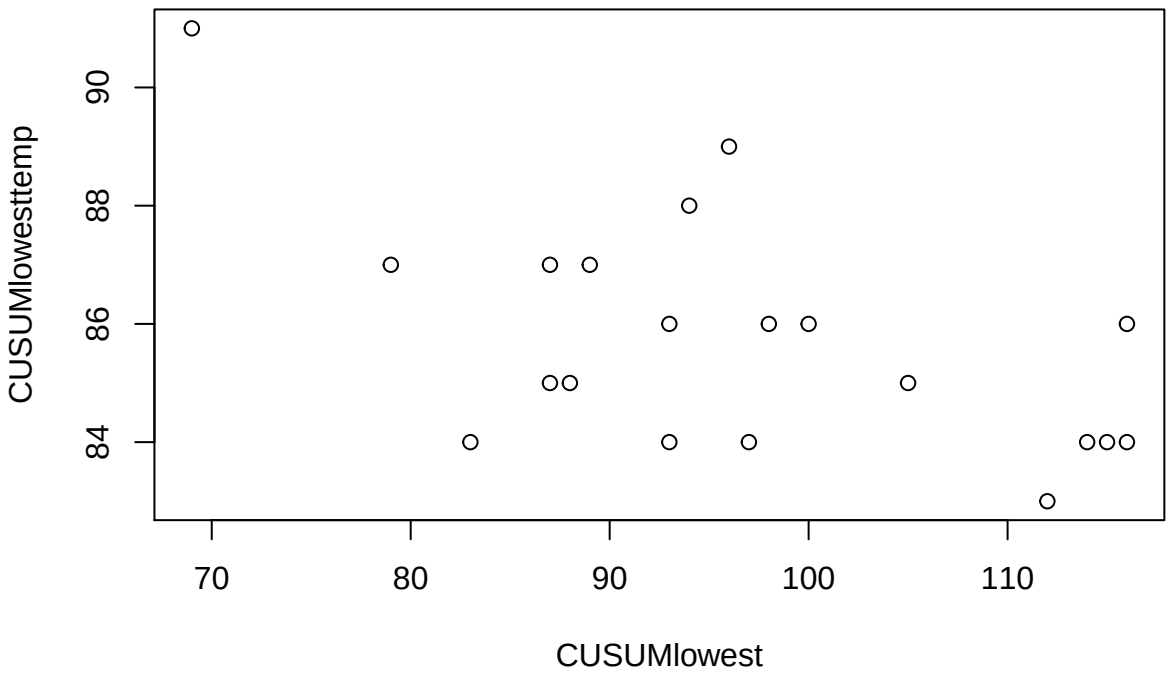

cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
0
−10
−30
−50
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 94.3
StdDev = 4.519307
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 30


cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
5
0
−10
−20
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 84.86667
StdDev = 3.839301
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 12


cusum Chart
for temps[[i]]
Cumulative Sum
Above target
Below target
4
2
0
−4
−8
UDB
LDB
1
7
14
23
32
41
50
59
68
77
86
95
105
116
Group
Number of groups = 123
Center = 86.66667
StdDev = 3.632548
Decision interval (std. err.) = 5
Shift detection (std. err.) = 6
No. of points beyond boundaries = 5

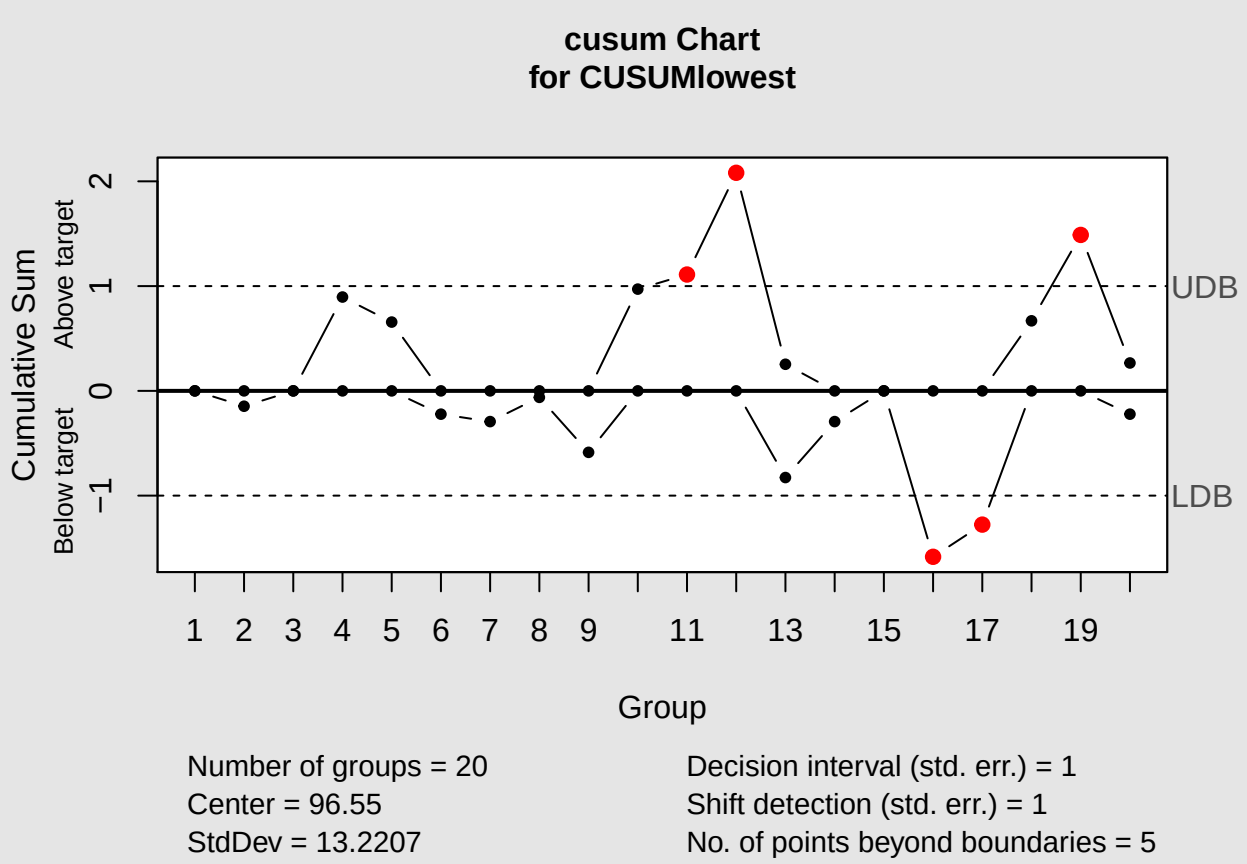



```
#concerned with the lower violations since significant drops in temperature indicates transfer from sum

CUSUMlowest <- rep(0, nrow(temps))
CUSUMlowesttemp <- rep(0, nrow(temps))
for (i in 1:nrow(temps)){
  CUSUMlowest[i] <- min(CUSUMviolations[[i]])
  CUSUMlowesttemp[i] <- as.integer(mean(temps[[i]][1:CUSUMlowest[i]]))
}

#From the plot, an expected trend of a decrease in temperature as the day number get larger

plot(CUSUMlowest,CUSUMlowesttemp)
```




Take the unofficial end of summer day and temperature data and use CUSUM analysis to determine if the unofficial end of summer days and summer climate are changing

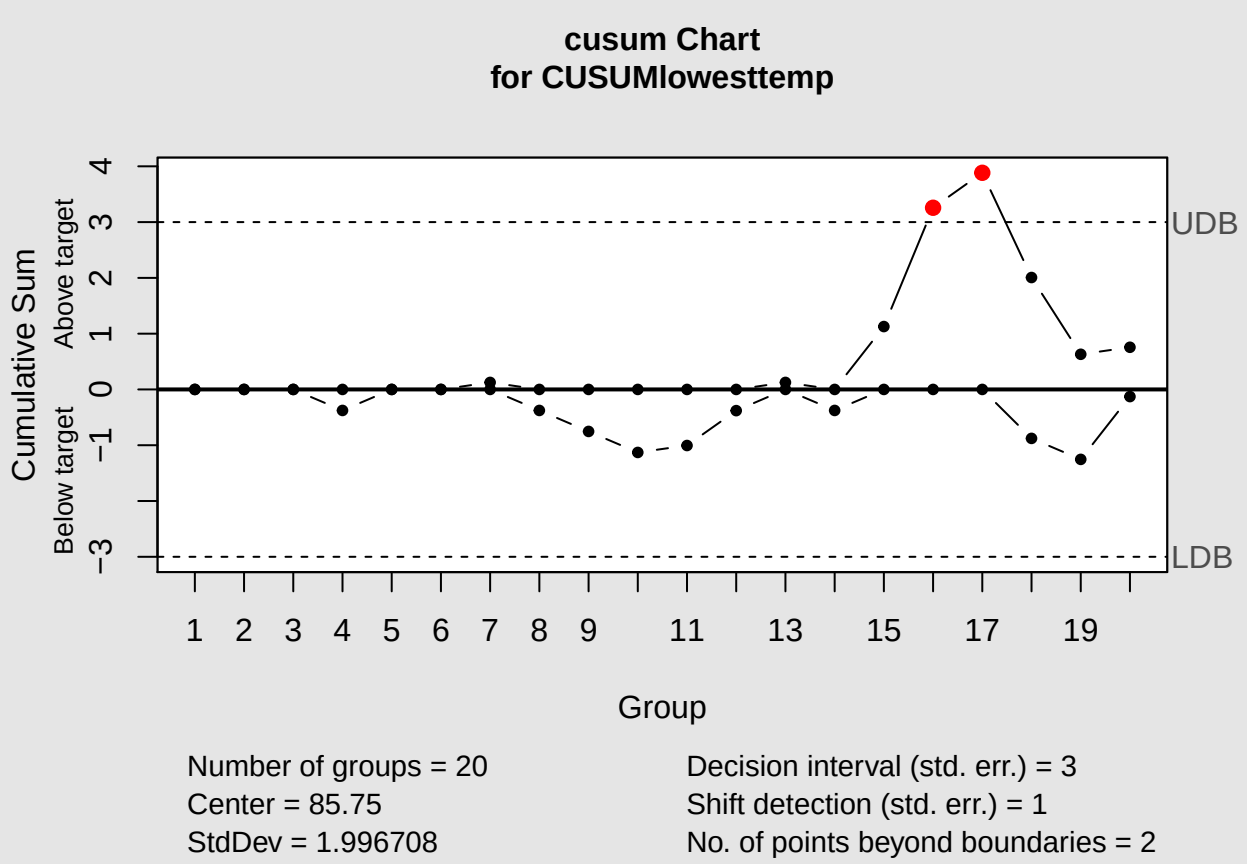
```
# Determine the mean and standard deviation value, here used the averages of the CUSUMlowest and CUSUMl
avg_day <- mean(CUSUMlowest)
sd_day <- sd(CUSUMlowest)
avg_temp <- mean(CUSUMlowesttemp)
sd_temp <- sd(CUSUMlowesttemp)

# Run the CUSUM function on unofficial end of summer day
DI = 1 #out of control when past 1 standard deviations/errors
SS = 1 #equals 1/2 = 1/2 standard deviations/errors shift

CUSUMmodel_day <- cusum(CUSUMlowest, center=avg_day, std.dev = sd_day, decision.interval=DI, se.shift=SS
```




```
CUSUMviolations_day <- CUSUMmodel_day$violations

#There are a few years where the unofficial end of summer changed significantly from the mean day
```

Run the CUSUM function on unofficial end of summer day average summer temperature (summer climate)

```
DI = 3 #out of control when past 3 standard deviations/errors = about 6 degrees
SS = 1 #equals 1/2 = 1/2 standard deviations/errors shift = about 1 degree

CUSUMmodel_temp <- cusum(CUSUMlowesttemp, center=avg_temp, std.dev = sd_temp, decision.interval=DI, se.
```




```
CUSUMviolations_temp <- CUSUMmodel_temp$violations

#There are a few years where the summer climate drops significantly from the mean summer climate
#These years align with the years where the unofficial end of summer day also change significantly (yea
```