Missing Data

The breast cancer data set breast-cancer-wisconsin.data.txt from http://archive.ics.uci.edu/ml/machine-learning-databases/breast-cancer-wisconsin/ (description at http://archive.ics.uci.edu/ml/datasets/Breast+Cancer+Wisconsin+%28Original%29) has missing values. 1. Use the mean/mode imputation method to impute values for the missing data. 2. Use regression to impute values for the missing data 3. Use regression with perturbation to impute values for the missing data. 4. use classification models, SVM and KNN build to impute missing value.

```
# Find the missing data
rm(list = ls())
set.seed(1)
data <- read.table("breast-cancer-wisconsin.txt", stringsAsFactors = FALSE, header = FALSE, sep = ",")
for (i in 2:11) {
  print(paste0("V",i))
  print(table(data[,i]))
}
## [1] "V2"
##
          2
               3
                   4
                        5
                            6
                                 7
                                      8
                                          9
                                              10
                  80 130
                                23
                                     46
                                         14
                                              69
## 145
        50 108
                           34
##
   [1]
        "V3"
##
##
     1
          2
               3
                   4
                        5
                            6
                                 7
                                      8
                                              10
##
   384
         45
              52
                  40
                       30
                           27
                                19
                                     29
                                              67
   [1]
        "V4"
##
          2
                                 7
##
               3
                   4
                        5
                            6
                                      8
                                          9
                                              10
     1
         59
   353
              56
                  44
                       34
                           30
                                30
                                     28
                                          7
                                              58
##
   [1]
        "V5"
##
##
          2
               3
                   4
                        5
                            6
                                 7
                                      8
                                          9
                                              10
     1
                  33
                       23
                           22
                                13
   407
         58
              58
                                     25
                                          5
                                              55
        "V6"
##
   [1]
##
##
          2
               3
                   4
                        5
                            6
                                 7
                                      8
                                          9
                                              10
     1
                  48
##
    47 386
             72
                       39
                           41
                                12
                                     21
                                              31
   [1]
        "V7"
##
##
             10
                            4
                                                   9
##
          1
                   2
                        3
                                 5
                                      6
                                          7
                                               8
    16 402 132
                  30
                       28
                           19
                                30
        "V8"
##
   [1]
##
##
          2
               3
                        5
                            6
                                 7
                                      8
                                          9
                                              10
                   4
## 152 166 165
                  40
                           10
                                73
                                     28
                       34
                                         11
```

[1] "V9"

```
##
##
        2
           3
               4
                      6
                           7
                              8
                                  9 10
    1
                  5
## 443 36 44
             18 19 22
## [1] "V10"
##
        2
                       6
                           7
                              8 10
            3
               4
                   5
    1
## 579 35 33 12
                       3
## [1] "V11"
##
##
    2
        4
## 458 241
data[which(data$V7 == "?"),]
##
           V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
## 24 1057013 8
                 4
                   5
                            ?
                              7
                      1
                         2
## 41 1096800 6
                 6
                    6
                       9
                         6
                                          2
## 140 1183246 1
                 1
                    1
                       1
                         1
                                          2
                    3
                       1
                                         2
## 146 1184840 1
                 1
                    2
                         3
## 159 1193683 1
                      1
## 165 1197510 5
                       1
                          2 ?
                               3 1
                                         2
                 1
                    1
                          2
                            ?
                                         2
## 236 1241232 3
                 1
                    4
                       1
                               3
## 250 169356 3 1
                    1
                       1
                         2 ?
                                         2
## 276
       432809 3 1
                       1
                         2
                            ?
## 293
       563649 8 8
                    8
                       1
                               6 10
                                          4
## 295
       606140 1
                    1
                       1
                          2
                               2 1
                                          2
                 1
                    3
                      1 2 ? 2 3
## 298
       61634 5
                 4
## 316 704168 4
                 6
                    5
                         2 ?
                               3 1
                                          2
## 322 733639 3
                 1
                    1
                       1
                               2 1
                                          2
## 412 1238464 1 1 1 1 1
                            ?
## 618 1057067 1 1 1 1 1 ? 1 1
nrow(data[which(data$V7 == "?"),])/nrow(data)
## [1] 0.02288984
missing <- which(data$V7 == "?", arr.ind = TRUE)
missing
```

[1] 24 41 140 146 159 165 236 250 276 293 295 298 316 322 412 618

Mean/Mode Imputation

```
getmode <- function(v) {
  uniqv <- unique(v)
  uniqv[which.max(tabulate(match(v, uniqv)))]
}
# Find the mode of V7.
mode_V7 <- as.numeric(getmode(data[-missing,"V7"]))
mode_V7</pre>
```

```
# Impute V7 for observations with missing data for V7 to mode_V7.
data_mode_imp <- data
data_mode_imp[missing,]$V7 <- mode_V7
data_mode_imp$V7 <- as.integer(data_mode_imp$V7)</pre>
```

Regression Imputation

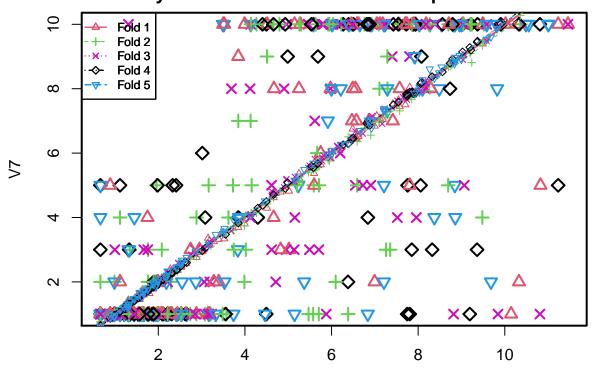
```
data_modified <- data[-missing,2:10]</pre>
data_modified$V7 <- as.integer(data_modified$V7)</pre>
# Generate linear model using all other factors as predictors
model \leftarrow lm(V7\sim V2+V3+V4+V5+V6+V8+V9+V10, data = data_modified)
summary(model)
##
## Call:
## lm(formula = V7 ~ V2 + V3 + V4 + V5 + V6 + V8 + V9 + V10, data = data_modified)
## Residuals:
##
      Min
               1Q Median
                                3Q
                                       Max
## -9.7316 -0.9426 -0.3002 0.6725 8.6998
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.616652  0.194975  -3.163  0.00163 **
## V2
               0.230156  0.041691  5.521  4.83e-08 ***
## V3
              -0.067980 0.076170 -0.892 0.37246
                          0.073420 4.637 4.25e-06 ***
## V4
               0.340442
## V5
               0.339705
                          0.045919 7.398 4.13e-13 ***
## V6
               0.090392
                          0.062541 1.445 0.14883
## V8
               0.320577
                          0.059047 5.429 7.91e-08 ***
## V9
               0.007293
                          0.044486 0.164 0.86983
## V10
              -0.075230
                          0.059331 -1.268 0.20524
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 2.274 on 674 degrees of freedom
## Multiple R-squared: 0.615, Adjusted R-squared: 0.6104
## F-statistic: 134.6 on 8 and 674 DF, p-value: < 2.2e-16
# use stepwise regression for variable selection.
step(model,show=FALSE)
## Start: AIC=1131.43
## V7 ~ V2 + V3 + V4 + V5 + V6 + V8 + V9 + V10
##
##
         Df Sum of Sq
                         RSS
                                AIC
```

```
## - V9
          1
               0.139 3486.8 1129.5
## - V3
                4.120 3490.8 1130.2
          1
## - V10
                8.317 3495.0 1131.0
                      3486.6 1131.4
## <none>
## - V6
          1
               10.806 3497.5 1131.5
## - V4
              111.227 3597.9 1150.9
          1
## - V8
              152.482 3639.1 1158.7
          1
## - V2
              157.657 3644.3 1159.6
          1
## - V5
          1
              283.119 3769.8 1182.8
##
## Step: AIC=1129.45
## V7 ~ V2 + V3 + V4 + V5 + V6 + V8 + V10
##
         Df Sum of Sq
                        RSS
## - V3
                4.028 3490.8 1128.2
          1
## - V10
          1
                8.179 3495.0 1129.0
## <none>
                      3486.8 1129.5
## - V6
              11.211 3498.0 1129.7
## - V4
             114.768 3601.6 1149.6
          1
## - V2
          1
              158.696 3645.5 1157.8
## - V8
          1
              160.776 3647.6 1158.2
## - V5
          1
              285.902 3772.7 1181.3
##
## Step: AIC=1128.24
## V7 ~ V2 + V4 + V5 + V6 + V8 + V10
##
         Df Sum of Sq
                        RSS
## - V6
                8.606 3499.4 1127.9
          1
## - V10
                8.889 3499.7 1128.0
          1
## <none>
                      3490.8 1128.2
## - V4
          1
              153.078 3643.9 1155.6
## - V2
          1
              155.308 3646.1 1156.0
## - V8
          1
              157.123 3647.9 1156.3
## - V5
              282.133 3772.9 1179.3
          1
##
## Step: AIC=1127.92
## V7 ~ V2 + V4 + V5 + V8 + V10
##
         Df Sum of Sq
                       RSS
## - V10
             5.562 3505.0 1127.0
## <none>
                      3499.4 1127.9
## - V2
              159.594 3659.0 1156.4
          1
## - V8
              169.954 3669.4 1158.3
          1
## - V4
              206.785 3706.2 1165.1
         1
## - V5
          1
              295.807 3795.2 1181.3
##
## Step: AIC=1127.01
## V7 ~ V2 + V4 + V5 + V8
##
##
         Df Sum of Sq
                       RSS
## <none>
                      3505.0 1127.0
## - V2
              155.70 3660.7 1154.7
## - V8
          1
             172.42 3677.4 1157.8
## - V4
          1
              201.22 3706.2 1163.1
```

```
## - V5 1 290.68 3795.7 1179.4
##
## Call:
## lm(formula = V7 ~ V2 + V4 + V5 + V8, data = data_modified)
## Coefficients:
## (Intercept)
                        V2
                                                  V5
                                                               ٧8
      -0.5360
                    0.2262
                                 0.3173
                                              0.3323
                                                           0.3238
##
# Generate the linear model that stepwise regression recommends.
model2 <- lm(V7~V2+V4+V5+V8, data = data_modified)</pre>
summary(model2)
##
## Call:
## lm(formula = V7 ~ V2 + V4 + V5 + V8, data = data_modified)
## Residuals:
##
      Min
               1Q Median
                               3Q
## -9.8115 -0.9531 -0.3111 0.6678 8.6889
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.53601
                          0.17514 -3.060 0.0023 **
## V2
               0.22617
                          0.04121 5.488 5.75e-08 ***
## V4
                          0.05086 6.239 7.76e-10 ***
               0.31729
## V5
               0.33227
                          0.04431
                                    7.499 2.03e-13 ***
                          0.05606 5.775 1.17e-08 ***
## V8
               0.32378
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 2.274 on 678 degrees of freedom
## Multiple R-squared: 0.6129, Adjusted R-squared: 0.6107
## F-statistic: 268.4 on 4 and 678 DF, p-value: < 2.2e-16
# Use cross-validation to test how good this model really is.
library(DAAG)
## Warning: package 'DAAG' was built under R version 4.0.2
## Loading required package: lattice
model_cv <- cv.lm(data_modified, model2, m=5)</pre>
## Analysis of Variance Table
##
## Response: V7
##
             Df Sum Sq Mean Sq F value Pr(>F)
## V2
             1 3185
                          3185
                                616.2 < 2e-16 ***
```

```
## V4
               1
                   1683
                           1683
                                  325.5 < 2e-16 ***
## V5
               1
                    510
                            510
                                   98.6 < 2e-16 ***
## V8
                    172
                            172
                                   33.4 1.2e-08 ***
                   3505
                              5
## Residuals 678
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
## Warning in cv.lm(data modified, model2, m = 5):
##
  As there is >1 explanatory variable, cross-validation
  predicted values for a fold are not a linear function
## of corresponding overall predicted values. Lines that
## are shown for the different folds are approximate
```

Small symbols show cross-validation predicted values



Predicted (fit to all data)

```
##
## fold 1
## Observations in test set: 136
                                 12
                                       21
                                             22
                                                    29
                                                                37
               4.663 10.0184 1.213 6.62 6.575
## Predicted
                                                1.213
                                                       1.213 10.15 5.02 7.572
## cvpred
               4.619 10.0582 1.255 6.52 6.465 1.255
## V7
               4.000 10.0000 1.000 10.00 7.000 1.000
                                                       1.000
                                                             1.00 3.00 8.000
## CV residual -0.619 -0.0582 -0.255
                                    3.48 0.535 -0.255 -0.255 -9.07 -1.82 0.553
##
                                                74
                                                       80
                  56
                        57
                              59
                                    64
                                           67
                                                            81
                                                                   82
## Predicted
               5.598 5.750
                            3.50 3.39
                                       1.990
                                              7.7 1.213
                                                          3.15
               5.369 5.638 3.53 3.32 1.982 7.5 1.255 3.35 1.954 4.75
## cvpred
```

```
## CV residual -0.369 0.362 6.47 -1.32 -0.982 2.5 -0.255 -2.35 -0.954 3.25
                  89
                         91
                             104
                                     109
                                           118
                                                123
                                                       125
                                                             127
                            4.82
                                 0.9873 7.54
                                               7.18
               1.990
                     1.311
                                                     7.415 6.59 4.51 0.663
## Predicted
## cvpred
               1.982
                     1.425
                            4.79
                                  1.0696 7.64 7.05
                                                     7.524
                                                            6.57 4.29 0.715
               1.000 1.000 3.00 1.0000 10.00 10.00 7.000 10.00 10.00 1.000
## V7
## CV residual -0.982 -0.425 -1.79 -0.0696 2.36 2.95 -0.524
                                                            3.43 5.71 0.285
                              145
                        138
                                                               173
##
                131
                                    156
                                          160
                                                 166
                                                        169
                                                                      179 187
## Predicted
               2.53 1.1158
                            1.213
                                   5.15
                                         7.94 1.9896 1.763
                                                            0.9873
                                                                    1.990 6.49
                                        7.90 1.9819 1.796
## cvpred
               2.46 1.0861
                            1.255
                                  5.10
                                                            1.0696
                                                                   1.982 6.75
## V7
               1.00 1.0000 1.000 10.00 10.00 2.0000 1.000 1.0000 1.000 8.00
## CV residual -1.46 -0.0861 -0.255
                                  4.90
                                         2.10 0.0181 -0.796 -0.0696 -0.982 1.25
                 194
                        195
                             197
                                    200
                                          202
                                                210
                                                     215
                                                            221
                                                                    226
                                                                          231
                     1.763 6.470
                                                                 0.9873 6.864
## Predicted
               1.311
                                  1.440
                                         8.18
                                              2.22 11.46
                                                          1.643
               1.425 1.796 6.425
                                  1.441
                                         8.12
                                              2.17 11.44
                                                          1.752
## cvpred
                                                                 1.0696 6.883
## V7
               1.000 1.000 7.000 1.000 10.00 1.00 10.00 1.000
                                                                 1.0000 7.000
## CV residual -0.425 -0.796 0.575 -0.441
                                        1.88 -1.17 -1.44 -0.752 -0.0696 0.117
                238 248 255
                                259
                                      272
                                             275
                                                    281
                                                          302
                                                                 303
               6.99 3.85 5.98 1.763 2.22
                                          1.763
                                                 1.763
                                                        1.311 9.491
## Predicted
                                                                     1.311
## cvpred
               6.84 3.70 5.82 1.796 2.17
                                           1.796
                                                 1.796
                                                        1.425 9.392
## V7
               2.00 9.00 8.00 1.000 1.00 1.000
                                                 1.000 1.000 10.000 1.000
## CV residual -4.84 5.30 2.18 -0.796 -1.17 -0.796 -0.796 -0.425 0.608 -0.425
##
                 318
                      324
                            327 331
                                       341 350
                                                  354
                                                        358
                                                               364
                                                                     376
               8.290
                     7.12
                           3.99 6.55
                                      5.28 5.25
                                                7.71 9.039 2.9499 0.663
## Predicted
## cvpred
               8.361 7.09 3.81 6.43 5.16 5.36 7.82 9.021 2.9251 0.715
               8.000 10.00 10.00 8.00 10.00 8.00 10.00 10.000 3.0000 1.000
## CV residual -0.361 2.91 6.19 1.57 4.84 2.64
                                                2.18 0.979 0.0749 0.285
                                            397
                  377
                          378
                               381
                                     388
                                                  403
                                                         408
                                                               414
                                                                     417
               0.9873 0.9873 0.663 3.18 1.763
## Predicted
                                                2.53
                                                      0.9873
                                                              2.53
                                                                   7.74
## cvpred
               1.0696 1.0696 0.715 3.14 1.796
                                                2.46
                                                      1.0696
                                                              2.49 7.72
## V7
               1.0000 1.0000 1.000 1.00 1.000
                                                1.00
                                                      1.0000
                                                              1.00 10.00
## CV residual -0.0696 -0.0696 0.285 -2.14 -0.796 -1.46 -0.0696 -1.49
                                                                    2.28
##
                  418
                        421
                             426
                                     429
                                           439
                                                 447
                                                        455
                                                              459
                                                                     473
                                                                            477
               0.9873 2.745 11.23
                                  0.9873 2.64 0.663 0.8897
                                                                         1.659
## Predicted
                                                            1.885
                                                                  1.794
               1.0696 2.774 11.25
                                  1.0696 2.57 0.715 0.9004 1.781
## cvpred
                                                                   1.643
## V7
               1.0000 3.000 10.00 1.0000 1.00 1.000 1.000 1.000 1.000
                                                                         1.000
## CV residual -0.0696 0.226 -1.25 -0.0696 -1.57 0.285 0.0996 -0.781 -0.643 -0.596
##
                482
                      492
                            498
                                   500
                                         501
                                                502
                                                     511
                                                           524
                                                                  528
                                                                         533
               2.88 7.05
                          1.342
                                 1.666
                                        2.44
                                              1.666 0.663
                                                          7.89
                                                                1.990
                                                                      1.311
## Predicted
## cvpred
               2.76 7.16 1.272 1.627 2.35
                                             1.627 0.715 7.81
                                                               1.982
## V7
               1.00 10.00 1.000 1.000 1.00 1.000 1.000 10.00 1.000 1.000
## CV residual -1.76 2.84 -0.272 -0.627 -1.35 -0.627 0.285 2.19 -0.982 -0.425
                536
                     537
                            539
                                   543
                                          544
                                                 551
                                                       552
                                                             556
                                                                    563
## Predicted
               2.28
                    2.22
                         1.666
                                 1.568
                                       1.666
                                              1.440
                                                    1.311
                                                            2.31
                                                                 1.311 10.33
## cvpred
               2.40 2.17
                          1.627
                                 1.458
                                       1.627
                                              1.441 1.425 2.34 1.425 10.51
## V7
               1.00 1.00 1.000 1.000
                                       1.000 1.000 1.000 1.00 1.000 10.00
## CV residual -1.40 -1.17 -0.627 -0.458 -0.627 -0.441 -0.425 -1.34 -0.425 -0.51
##
                567
                      569
                                        580
                                               590
                                                      597
                           570
                                 571
                                                           616
                                                                  617 626 628
## Predicted
               2.08 3.52 10.82
                                6.46
                                     1.311 1.568 1.983
                                                          2.30
                                                               1.440 1.75 0.89
## cvpred
               2.12 3.34 10.79
                                6.45
                                      1.425
                                            1.458
                                                   1.951
                                                          2.27
                                                                1.441 1.73 0.90
               1.00 10.00 5.00 10.00 1.000 1.000 1.000 1.00
                                                               1.000 4.00 5.00
## V7
## CV residual -1.12 6.66 -5.79 3.55 -0.425 -0.458 -0.951 -1.27 -0.441 2.27 4.10
                                   642
##
                636
                      638
                            641
                                         649
                                                663 665
                                                          673
                                                                676
                                                                       677
## Predicted
               2.07 3.28 2.007 1.440 10.33 1.622 2.1 1.54 2.29 1.305
```

```
## cvpred
              2.06 3.25 1.926 1.441 10.51 1.717 2.1 1.61 2.24 1.393
## V7
              ## CV residual -1.06 -1.25 -0.926 -0.441 -8.51 -0.717 -1.1 -0.61 -1.24 -0.393
               679
                                      696
                   682
                           688
                                695
                                            699
## Predicted
             0.663 8.71 1.440 1.116 0.8897
                                           7.81
             0.715 8.74 1.441 1.086 0.9004 8.04
## cvpred
             1.000 10.00 1.000 2.000 1.0000 5.00
## CV residual 0.285 1.26 -0.441 0.914 0.0996 -3.04
##
## Sum of squares = 675
                        Mean square = 4.96
                                            n = 136
##
## fold 2
## Observations in test set: 137
                 3
                     16
                            17
                                26
                                     27
                                          40
                                                53
                                                     54
                                                          62
                                                                66
                                                                     73
             1.763 5.58 1.666 3.85 1.44 4.13 5.589 7.11 0.987
## Predicted
                                                             3.99 3.57
## cvpred
             1.731 5.54 1.635 3.81 1.44 4.06 5.466 6.99 1.048 3.74
             2.000 1.00 1.000 7.00 1.00 7.00 5.000 8.00 2.000 2.00 1.00
## V7
## CV residual 0.269 -4.54 -0.635 3.19 -0.44 2.94 -0.466 1.01 0.952 -1.74 -2.53
                77
                    79
                          83
                              85
                                                99 102
                                    92
                                           93
                                                          111 112 115
## Predicted
              1.94 1.76 2.22 7.29 1.448 1.990 5.667 3.16 2.291 4.51 2.08
## cvpred
              2.09 1.73 2.12 7.30 1.478 1.927 5.733 3.28 2.253 4.42 2.08
## V7
              1.00 3.00 1.00 9.00 1.000 1.000 6.000 5.00 2.000 9.00 3.00
## CV residual -1.09 1.27 -1.12 1.70 -0.478 -0.927 0.267 1.72 -0.253 4.58 0.92
                                  143
                                       150
                                            152 154
                                                      172
               120
                     139
                           141
                                                            176
                                                                 178
                                                                       182
## Predicted
             1.763 1.983 1.116 5.710 7.09 4.27 1.34 1.31 6.76 6.38 0.663
## cvpred
             1.731 1.984 1.148 5.582 6.96 4.14 1.34 1.34 6.78 6.50 0.757
## V7
             ## CV residual 0.269 -0.984 -0.148 -0.582 3.04 5.86 1.66 -0.34 3.22 -5.50 0.243
                                                      220
               183
                   198
                          199
                               204
                                    208
                                                218
                                                           222
                                                                225
                                           211
## Predicted
              2.44 3.21 0.663 2.22 1.31 10.359 1.31 3.08
                                                          6.91 7.57 7.294
## cvpred
              2.32 3.11 0.757 2.12
                                   1.34 10.276 1.34 3.02 6.56 7.31 7.188
## V7
              1.00 1.00 1.000 1.00 1.00 10.000 1.00 1.00 10.00 10.00 8.000
## CV residual -1.32 -2.11 0.243 -1.12 -0.34 -0.276 -0.34 -2.02
                                                         3.44
                                                               2.69 0.812
               233
                    235
                          237
                               242
                                     243 244
                                               254
                                                      261
                                                           265
                                                                267
                                                                      268
## Predicted
              5.47
                   2.08 6.22
                             2.43
                                   1.537 1.96
                                              6.92
                                                   9.206
                                                          7.26
                                                                5.92
                                                                    4.66
              5.40 2.08 6.16 2.39 1.535 1.92 6.95 9.096 7.11 5.80 4.62
## cvpred
## V7
              1.00 1.00 10.00 1.00 1.000 5.00 10.00 10.000 3.00 10.00 10.00
## CV residual -4.40 -1.08 3.84 -1.39 -0.535 3.08 3.05 0.904 -4.11 4.20 5.38
##
              271 274
                        279
                              282
                                     287
                                          288 289
                                                    290
                                                         291
                                                               292
                                                                    294
              4.8 3.62 1.31 1.870 9.516
                                        1.44 3.72 6.45 0.663 1.31 5.64
## Predicted
              4.8 3.56 1.34 1.865 9.502 1.44 3.60 6.47 0.757 1.34 5.51
## cvpred
             10.0 4.00 1.00 1.000 10.000 1.00 5.00 10.00 1.000 1.00 10.00
## CV residual 5.2 0.44 -0.34 -0.865 0.498 -0.44 1.40 3.53 0.243 -0.34 4.49
##
              297
                   299
                         306
                              312
                                   317
                                         319
                                               323
                                                       328
                                                            333
                                                                 344
                                                                       349
             4.16 2.25 5.30 0.663 4.14 1.31 1.763 0.9873 2.54 0.663 5.71
## Predicted
             4.12 2.13 5.14 0.757 4.14 1.34 1.731
                                                   1.0481 2.51 0.757 5.75
## cvpred
## V7
             5.00 1.00 10.00 1.000 10.00 1.00 1.000 1.000 1.000 1.00 1.00
## CV residual 0.88 -1.13 4.86 0.243 5.86 -0.34 -0.731 -0.0481 -1.51 0.243 -4.75
               351
                    353
                          362
                                366
                                     368
                                            371
                                                 374
                                                        395
                                                              398 404
## Predicted
              2.23 4.02 6.04 1.213 9.05 1.983
                                                2.22 1.622 1.342 1.12
              2.20 4.08 5.98 1.244 8.99 1.984
## cvpred
                                                2.16 1.745 1.344 1.15
## V7
              ## CV residual -1.20 -1.08 4.02 -0.244 1.01 -0.984 -1.16 -0.745 -0.344 2.85
##
               413
                   416
                         428
                              432
                                     433
                                          441 442
                                                     444
                                                          450
                                                                452
```

```
## Predicted
               9.48 3.74 6.10 2.88 1.892 9.24 2.88 0.663 8.70 1.57 7.85
               9.35 3.73 6.06 2.78 1.831 8.81 2.88 0.757 8.68 1.54 7.67
## cvpred
## V7
               4.00 3.00 2.00 1.00 1.000 10.00 4.00 2.000 10.00 1.00 10.00
## CV residual -5.35 -0.73 -4.06 -1.78 -0.831 1.19 1.12 1.243 1.32 -0.54 2.33
                471
                      475
                           481
                                  495
                                        505
                                                518
                                                       519
                                                            525
                                                                  531
               1.44 1.57
                          1.57 5.200 0.663 0.9873
                                                    1.765
                                                           1.44 5.26 1.892
## Predicted
                          1.54 5.151 0.757
                                            1.0481
## cvpred
               1.44 1.54
                                                    1.827
                                                           1.44 5.09 1.831
               1.00 1.00 1.00 5.000 1.000 1.000 1.000 1.00 10.00 2.000
## V7
## CV residual -0.44 -0.54 -0.54 -0.151 0.243 -0.0481 -0.827 -0.44
                                                                 4.91 0.169
##
                 549
                       550
                             553
                                  557
                                        558
                                               560
                                                     564
                                                             574
                                                                   577
                                                                           579
## Predicted
               1.116
                     6.59
                           2.74
                                 2.54
                                       2.23
                                             1.892
                                                   1.44
                                                         0.9873
                                                                 1.892 0.9873
                     6.36
                           2.70
                                 2.51
                                       2.20
                                             1.831
## cvpred
               1.148
                                                    1.44
                                                          1.0481
                                                                 1.831 1.0481
## V7
               1.000 5.00 1.00
                                1.00 1.00 1.000
                                                    1.00
                                                          1.0000 1.000 1.0000
## CV residual -0.148 -1.36 -1.70 -1.51 -1.20 -0.831 -0.44 -0.0481 -0.831 -0.0481
##
                593
                      600
                            601
                                  607
                                        611
                                               615
                                                     624
                                                           633
                                                                 640
                                                                      644
## Predicted
               5.95
                     2.52
                          1.44
                                1.674
                                       8.27
                                             1.213 0.663 0.663
                                                               2.23 0.663
                                                                           3.48
               5.76 2.59 1.44 1.674 7.94
                                            1.244 0.757 0.757
## cvpred
                                                               2.20 0.757 3.52
## V7
              10.00 1.00 1.00 1.000 10.00 1.000 1.000 1.000 1.00 1.00 1.00
## CV residual 4.24 -1.59 -0.44 -0.674 2.06 -0.244 0.243 0.243 -1.20 0.243 -2.52
                 662
                        664
                              666
                                   670
                                         683
                                               685
                                                      691
                                                            697
                     1.622 0.663 8.69
                                                    1.328
## Predicted
               1.990
                                       2.22 0.663
                                                          7.35
                     1.745 0.757 8.74 2.12 0.757
## cvpred
               1.927
                                                    1.417
               1.000 1.000 1.000 5.00 1.00 1.000 1.000 3.00
## V7
## CV residual -0.927 -0.745 0.243 -3.74 -1.12 0.243 -0.417 -4.38
##
## Sum of squares = 672
                         Mean square = 4.9
                                              n = 137
##
## fold 3
## Observations in test set: 137
                  7
                        10
                              11 15
                                         23
                                                30
                                                      32
                                                            42
                                                                 45
                                                                       46
                                                                            50
## Predicted
               1.31 1.666 1.311 7.8
                                     1.440 1.298
                                                    1.54
                                                          4.95
                                                               8.82 0.987 4.90
## cvpred
               1.21 1.702 1.211 7.9
                                      1.433 1.172
                                                   1.48
                                                         5.18 8.89 0.894 4.94
## V7
              10.00 1.000 1.000 9.0 1.000 1.000 1.00 3.00 1.00 1.000 8.00
## CV residual 8.79 -0.702 -0.211 1.1 -0.433 -0.172 -0.48 -2.18 -7.89 0.106 3.06
##
                 61
                      63
                            65
                                69
                                      71
                                            78
                                                  84
                                                          86
                                                                88
                                                                     94
                                                                            97
               5.14 5.98 0.987 7.40 2.53 2.22 3.06 4.1260 5.98 0.987
## Predicted
                                                                         1.222
## cvpred
               5.12 6.04 0.894 7.42 2.57 2.30 3.02 4.0896 5.84 0.894
## V7
               3.00 8.00 1.000 9.00 1.00 1.00 2.00 4.0000 10.00 1.000 1.000
## CV residual -2.12 1.96 0.106 1.58 -1.57 -1.30 -1.02 -0.0896
                                                            4.16 0.106 -0.179
##
                101
                      105
                           106
                                 108
                                       124
                                              136 147
                                                         149
                                                                151
                                                                     162
              4.616 10.81 4.62 7.17 4.13 2.216 3.69
                                                       3.08
## Predicted
                                                             1.311
                                                                    1.99 0.996
              4.823 10.88 4.71 6.91 4.11 2.288 3.59
                                                       3.05
                                                             1.211
                                                                    2.02 0.910
## cvpred
              5.000 1.00 3.00 10.00 10.00
                                           2.000 8.00 1.00
## V7
                                                             1.000
                                                                    1.00 3.000
## CV residual 0.177 -9.88 -1.71 3.09
                                     5.89 -0.288 4.41 -2.05 -0.211 -1.02 2.090
                167
                      175
                           184
                                 185
                                       186
                                             201
                                                    203
                                                          207
                                                                219
                                                                      240
                                                              7.96
               6.45 6.13
                          8.06
                                6.13
                                      1.54
                                            7.65 1.311
                                                         6.55
## Predicted
                                                                    4.96
                                                                          3.19
## cvpred
               6.39 6.23 7.96 6.21
                                      1.48
                                            7.74 1.211
                                                        6.67
                                                              7.91
                                                                    5.19
                                                                          3.23
## V7
              10.00 10.00 10.00 10.00
                                      1.00 10.00 1.000 5.00 4.00 10.00 2.00
## CV residual 3.61 3.77 2.04
                                3.79 -0.48 2.26 -0.211 -1.67 -3.91 4.81 -1.23
                 245
                       249 260
                                 269
                                        270
                                              296
                                                     307
                                                           315
                                                                 325
                                                                       330
               1.311 1.99 4.12 7.52
                                     1.311 7.74 1.311 0.987 1.311 7.22
## Predicted
## cvpred
               1.211 2.02 4.07 7.63 1.211 7.66 1.211 0.894 1.211 7.38
## V7
               1.000 1.00 8.00 4.00 1.000 10.00 1.000 1.000 1.000 10.00
## CV residual -0.211 -1.02 3.93 -3.63 -0.211 2.34 -0.211 0.106 -0.211 2.62
```

```
##
                334
                      335
                            337
                                  340
                                        346
                                              357
                                                    359 360 363
               5.79 5.46 6.04 5.82 0.663 2.850
                                                  5.15 5.61 1.76 9.583
## Predicted
                                                                          1.298
               5.75 5.54
## cvpred
                           6.07 5.93 0.578 2.882
                                                  5.26 5.82 1.73 9.483
## V7
              10.00 10.00 10.00 10.00 1.000 3.000 4.00 7.00 3.00 10.000 1.000
## CV residual
              4.25
                     4.46
                           3.93 4.07 0.422 0.118 -1.26 1.18 1.27
                                                                   0.517 - 0.172
##
                 373
                       375
                             380
                                   382
                                         384
                                               385
                                                     387
                                                            389
                                                                   391
                                                                         415
                            3.17 6.94 0.890 0.890 6.21
## Predicted
               1.983
                     1.76
                                                         1.213
                                                                1.320
## cvpred
               1.999
                      1.73
                            3.18 7.15 0.847 0.847 6.24
                                                         1.164
                                                                 1.226
                                                                        5.86
## V7
               1.000 1.00 1.00 10.00 1.000 1.000 10.00 1.000 1.000 10.00
## CV residual -0.999 -0.73 -2.18 2.85 0.153 0.153
                                                   3.76 -0.164 -0.226
                                                                       4.14
                427
                      437
                            446
                                   448
                                         449
                                               451
                                                     461
                                                           463
                                                                  464
                                                                        472
               3.15 5.88 0.890
                                 1.568 0.663
                                              2.33
                                                    2.23
                                                          2.46
## Predicted
                                                                1.342
                                                                       2.46
## cvpred
               3.14 5.99 0.847
                                1.655 0.578
                                              2.37
                                                   2.32
                                                         2.59
                                                                1.386
                                                                       2.59
                                                               1.000 1.00
## V7
               1.00 1.00 1.000 1.000 1.000 1.00 1.00
                                                         1.00
## CV residual -2.14 -4.99 0.153 -0.655 0.422 -1.37 -1.32 -1.59 -0.386 -1.59
##
                 479
                       480
                             484
                                    487
                                          489
                                                491
                                                      494
                                                            497
                                                                  507
                                                                         512
               1.568 8.18
                                  1.440
                                         5.71 0.663
                                                     9.03 0.663
                                                                 9.06
## Predicted
                            8.39
                                                                      1.892
## cvpred
               1.655
                     8.05 8.40
                                  1.433
                                         5.63 0.578 8.90 0.578
                                                                 9.07
               1.000 10.00 10.00 1.000
                                        3.00 1.000 10.00 1.000 5.00 1.000
## V7
## CV residual -0.655
                     1.95 1.60 -0.433 -2.63 0.422
                                                     1.10 0.422 -4.07 -0.972
##
                 513
                        514
                              521
                                     522
                                           523
                                                 529
                                                        530
                                                               534
                                                                      542
                                                                             546
               1.568
                     1.440 0.663
                                  1.342
                                          6.91
                                                2.76
                                                     1.666
                                                            1.440 1.116
## Predicted
                     1.433 0.578 1.386
                                          7.08
                                                2.85
                                                     1.702 1.433 1.117
## cvpred
               1.655
                                                                           1.972
               1.000 1.000 1.000 1.000
                                          5.00
                                               1.00 1.000 1.000 1.000 1.000
## V7
## CV residual -0.655 -0.433 0.422 -0.386 -2.08 -1.85 -0.702 -0.433 -0.117 -0.972
                548
                       555
                             561 568
                                        585
                                               588
                                                     598
                                                            603
                                                                  605
              0.890 1.116
                            2.22 1.67
                                       3.23
                                             1.892
                                                   2.85
                                                         1.666 6.48 8.162
## Predicted
## cvpred
              0.847
                    1.117
                            2.29 1.70
                                       3.31
                                             1.972 2.88 1.702 6.49 8.237
              1.000 1.000
                            1.00 3.00
                                      1.00 1.000 1.00 1.000 10.00 8.000
## V7
## CV residual 0.153 -0.117 -1.29 1.30 -2.31 -0.972 -1.88 -0.702 3.51 -0.237
##
                 614
                       622
                               627
                                      630
                                            634
                                                  637
                                                         639
                                                                650
                                                                       655
                                                                              656
## Predicted
               1.213
                      4.71
                            6.2002
                                   1.342
                                           5.49
                                                9.84
                                                      1.342
                                                              1.440
                                                                     1.763
                                                                           1.440
## cvpred
               1.164
                      4.76
                            6.0967
                                    1.386 5.62 9.95
                                                      1.386
                                                              1.433
                                                                     1.749
                                                                            1.433
                                                                     1.000
## V7
               1.000 2.00
                            6.0000 1.000 3.00 1.00 1.000 1.000
                                                                            1.000
## CV residual -0.164 -2.76 -0.0967 -0.386 -2.62 -8.95 -0.386 -0.433 -0.749 -0.433
                                  672
                                        675
                                                          686
                                                                690
                659
                      660
                            661
                                              681
                                                    684
                                                                      692
                                                                             693
## Predicted
               8.18 0.663 0.987 2.10 0.987 11.46 0.663 0.663 0.663
                                                                     6.72
               8.16 0.578 0.894 2.08 0.894 11.51 0.578 0.578 0.578
## cvpred
                                                                     6.60
                                                                           1.117
## V7
              10.00 1.000 1.000 1.00 1.000 10.00 1.000 1.000 1.000 5.00 1.000
## CV residual 1.84 0.422 0.106 -1.08 0.106 -1.51 0.422 0.422 0.422 -1.60 -0.117
                 694
## Predicted
               1.440
## cvpred
               1.433
## V7
               1.000
## CV residual -0.433
##
                                                n = 137
## Sum of squares = 835
                          Mean square = 6.09
##
## fold 4
## Observations in test set: 137
                       5
                              8
                                   20
                                                35
##
                 2
                                         34
                                                      39
                                                           47
                                                                 48
                                                                       52
                                                                             58
## Predicted
               4.5 2.65 1.855
                                2.44
                                       1.87
                                             1.757 6.47 4.99 0.987 3.847
                                                                           4.49
## cvpred
               4.5 2.60 1.847
                                 2.38
                                       1.83 1.746 6.44 5.08 0.963 3.827
                                                                           4.50
              10.0 1.00 1.000 1.00 1.00 1.000 10.00 9.00 1.000 4.000 1.00
## V7
```

```
## CV residual 5.5 -1.60 -0.847 -1.38 -0.83 -0.746 3.56 3.92 0.037 0.173 -3.50
##
                 70
                       72
                              75
                                    76
                                           90
                                                  95
                                                      103 110
                                                                 116
## Predicted
               1.311
                     6.38
                           4.298 1.9521
                                        1.546
                                              1.537 2.31 5.69 0.663 0.663
                    6.29 4.292 1.9488 1.509 1.504 2.29 5.68 0.642 0.642
## cvpred
               1.284
               1.000
                    2.00 4.000 2.0000
                                        1.000 1.000 1.00 9.00 5.000 3.000
## CV residual -0.284 -4.29 -0.292 0.0512 -0.509 -0.504 -1.29 3.32 4.358 2.358
                122
                       132
                             133
                                   155
                                         163
                                               168
                                                     170
                                                             171
                                                                    177
                           7.43 0.663
## Predicted
              1.9896
                     1.537
                                      1.763 9.19 0.9958 1.1158 1.537 1.311
## cvpred
              1.9439
                     1.504 7.50 0.642
                                      1.724 9.28 0.9675
                                                         1.0814 1.504 1.284
## V7
              2.0000 1.000 10.00 1.000 1.000 1.000 1.0000 1.0000 1.000
## CV residual 0.0561 -0.504
                            2.50 0.358 -0.724 -8.28 0.0325 -0.0814 -0.504 -0.284
                                                      217 223
##
                       193
                              206
                                    212
                                           213
                                                 216
                                                                 227
                 188
## Predicted
               9.219 1.892 9.025
                                  8.736 1.311
                                               7.76 0.987 2.33
                                                               7.87
                                                                     8.06
## cvpred
               9.196 1.842 9.144 8.791 1.284 7.82 0.963 2.27
                                                                7.91
## V7
              10.000 1.000 10.000 8.000 1.000 5.00 1.000 5.00 10.00 5.00
## CV residual 0.804 -0.842 0.856 -0.791 -0.284 -2.82 0.037 2.73 2.09 -3.16
##
                     239
                            247
                                 252
                                       253
                                             263
                                                  264
                                                        273
                234
                                                              283
                                                                    300
                                                                         305
## Predicted
               6.27 8.076 9.371
                                7.94 4.41 7.72 7.94
                                                      4.66
                                                             5.58
                                                                   6.39
                                                                        6.50
               6.26\ 8.192\ 9.477\ 7.87\ 4.37\ 7.83\ 7.87\ 4.71\ 5.55\ 6.30\ 6.46
## cvpred
## V7
              10.00 9.000 10.000 10.00 10.00 10.00 10.00 10.00 10.00 10.00 10.00
## CV residual 3.74 0.808 0.523 2.13 5.63 2.17 2.13 5.29
                                                             4.45 3.70 3.54
                309 310
                           311
                                 321
                                      332
                                            336
                                                   338
                                                        339
                                                               342
                                                                     343
               7.85 2.41
                        1.213 4.93
                                     2.22 0.663 1.311 0.987 1.311 0.890 7.89
## Predicted
               7.90 2.37 1.183 4.90
                                     2.16 0.642 1.284 0.963
                                                             1.284 0.862 7.84
## cvpred
## V7
               3.00 5.00 1.000 10.00 1.00 1.000 1.000 1.000 1.000 1.000 10.00
## CV residual -4.90 2.63 -0.183 5.10 -1.16 0.358 -0.284 0.037 -0.284 0.138 2.16
               347
                     348
                            352
                                 355
                                        356
                                                361
                                                      365
                                                             372
                                                                   379
                                                                        386
               2.22 0.663 1.537 0.987 1.666 9.9068 1.537
## Predicted
                                                          1.298
                                                                 2.44
                                                                       1.77
               2.19 0.642 1.504 0.963 1.623 10.0105 1.504 1.328 2.38
                                                                      1.75
## cvpred
## V7
               1.00 1.000 1.000 1.000 1.000 10.0000 1.000 1.000 1.00
## CV residual -1.19 0.358 -0.504 0.037 -0.623 -0.0105 -0.504 -0.328 -1.38 -0.75
##
                390
                     392
                            393
                                  394
                                         402
                                                405
                                                     406
                                                            407
                                                                   410
                                                                        411
## Predicted
              1.892 7.54
                         1.440 0.663
                                      1.1158
                                             1.328 0.987
                                                         1.983
                                                                1.757 0.987
              1.842 7.58
                         1.403 0.642 1.0814 1.293 0.963 1.966 1.746 0.963
## cvpred
              2.000 10.00 1.000 1.000
                                     1.0000 1.000 1.000 1.000 1.000 1.000
## CV residual 0.158 2.42 -0.403 0.358 -0.0814 -0.293 0.037 -0.966 -0.746 0.037
                420
                     423
                           431
                                434
                                      436
                                             438 443
                                                       445
                                                              453 456
              0.890 2.62 0.987 2.10 6.85 1.342 1.33 3.55 1.780 3.02 8.56
## Predicted
              0.862 2.63 0.963
                               2.08
                                     6.99
                                           1.301 1.29
                                                      3.47
                                                           1.733 2.96
## cvpred
## V7
              ## CV residual 0.138 -1.63 0.037 -1.08 3.01 -0.301 1.71 -2.47 -0.733 3.04 1.48
##
                458 462
                           465
                                 474
                                       483
                                              485
                                                    488
                                                         490
                                                                496
                                                                      503
               9.36 1.12 1.342 1.342 11.23 1.885 10.81 3.083 1.440
## Predicted
                                                                    1.998
## cvpred
               9.45 1.08 1.301 1.301 11.31 1.864 10.89 3.048 1.403 1.948
## V7
               3.00 5.00 1.000 1.000 5.00 1.000 10.00 4.000 1.000 1.000
## CV residual -6.45 3.92 -0.301 -0.301 -6.31 -0.864 -0.89 0.952 -0.403 -0.948
                504
                        506
                               509
                                    516
                                          520
                                                 526
                                                       527
                                                              535
                                                                    538
                                                                         540
               1.990 1.1158 1.568 6.88 6.82 1.448
## Predicted
                                                     1.342
                                                           1.213 2.53
                                                                       2.12
## cvpred
               1.944 1.0814 1.521 6.86 6.93 1.407 1.301 1.183 2.51
                                                                        2.06
## V7
               1.000 1.0000 1.000 10.00 10.00 1.000 1.000 1.000 1.00
## CV residual -0.944 -0.0814 -0.521 3.14 3.07 -0.407 -0.301 -0.183 -1.51 -1.06
              554
                     559
                          562
                                572
                                       573
                                             578
                                                  581
                                                          584
                                                                586
## Predicted
             1.98 1.213 2.22 8.80 1.440 0.987 2.21 1.1158 0.663 8.32
              1.97 1.183 2.16 8.92 1.403 0.963 2.19 1.0814 0.642 8.37
## cvpred
```

```
5.00 1.000 1.00 10.00 1.000 1.000 1.00 1.000 1.000 3.00
591
                     595
                            599
                                  604
                                         609
                                               632
                                                      646
                                                            667
                                                                1.763 1.885
               7.81 5.54
                          1.440
                                 7.75 10.328
                                             1.892
                                                    1.440
                                                           2.22
## Predicted
## cvpred
               7.90 5.60
                         1.403
                                 7.77 10.433
                                             1.842
                                                    1.403
                                                           2.19
                                                                1.724 1.864
               1.00 10.00 1.000 1.00 10.000 1.000 1.000 1.00 1.000 1.000
## V7
## CV residual -6.90 4.40 -0.403 -6.77 -0.433 -0.842 -0.403 -1.19 -0.724 -0.864
##
                 678
                       689
                             698
## Predicted
               1.568
                     1.342
                            6.84
## cvpred
               1.521
                     1.301
                            6.89
## V7
               1.000 1.000 4.00
## CV residual -0.521 -0.301 -2.89
## Sum of squares = 779
                         Mean square = 5.68
                                              n = 137
##
## fold 5
## Observations in test set: 136
                       9
                            13
                                 14
                                       18
                                             19
                                                  25
                                                         28
                                                                31
                                                                     33
                                                                           38
                 1
                                    1.99 7.24
               2.22 0.890 3.84 1.31
                                                      1.892
## Predicted
                                                1.31
                                                           1.440
                                                                   7.21
                                                                         3.74
## cvpred
               2.33 0.883 3.91 1.30 2.07 7.34
                                                1.30
                                                      1.993
                                                             1.478
                                                                   7.32
                                                                         3.94
## V7
               1.00 1.000 3.00 3.00 1.00 10.00
                                                1.00 1.000
                                                            1.000 5.00
                                                                        1.00
## CV residual -1.33 0.117 -0.91 1.70 -1.07 2.66 -0.30 -0.993 -0.478 -2.32 -2.94
                                             96
##
                 43
                            49
                                  60
                                        68
                                                   98
                                                              107
                       44
                                                        100
                                                                   113
                                                                          114
                                                                  8.27
               6.92 5.15
                          2.65
                                5.37
                                      3.49
                                           1.31
                                                 2.22
                                                       8.54
                                                             8.85
## Predicted
                                                                        8.486
## cvpred
                                                2.33
               6.78 5.16 2.76
                               5.49 3.50
                                           1.30
                                                      8.67
                                                             8.86 8.61
              10.00 1.00 1.00 2.00 10.00 1.00 1.00 10.00 10.00
## CV residual 3.22 -4.16 -1.76 -3.49 6.50 -0.30 -1.33
                                                      1.33
                                                             1.14
                                                                  1.39 - 0.492
                117
                      121
                             126
                                    128
                                           134
                                                 135
                                                        137
                                                              142
                                                                   144
                                                                         148
               3.53 1.961 0.9873 1.763
                                              1.440
                                                      1.666 0.890 0.663 0.987
## Predicted
                                        1.440
               3.66 1.921 0.9625 1.815
                                        1.478 1.478
                                                      1.735 0.883 0.625 0.962
## cvpred
## V7
               2.00 1.000 1.0000 1.000
                                        1.000 1.000
                                                      1.000 1.000 5.000 2.000
## CV residual -1.66 -0.921 0.0375 -0.815 -0.478 -0.478 -0.735 0.117 4.375 1.038
##
                153
                     157
                            158
                                  161
                                         174
                                              180
                                                     189
                                                            190
                                                                  191
                                                                       192
               8.85 1.30
                         1.537
                                6.89 10.554
## Predicted
                                             3.51 8.101
                                                         1.946
                                                                9.82
                                                                     8.84
               8.97
                    1.24
                          1.557 6.99 10.538
                                             3.57
                                                   8.276
                                                         1.856
                                                                9.87
## cvpred
               5.00 1.00 1.000 10.00 10.000 10.00 8.000 1.000 8.00 10.00
## V7
## CV residual -3.97 -0.24 -0.557 3.01 -0.538 6.43 -0.276 -0.856 -1.87
##
                196
                     205
                           209
                                  214 224
                                            229
                                                  230
                                                         246
                                                               251
                                                                     256
                                                                          257
               1.99 1.31
                          1.31 10.488 6.21
                                           1.31 8.81
                                                       2.548 0.981
                                                                   4.13
## Predicted
               2.07 1.30 1.30 10.557 6.27 1.30 8.83 2.673 0.903 4.06
## cvpred
                                                                        1.14
               1.00 1.00 1.00 10.000 8.00 1.00 10.00 2.000 1.000 10.00 1.00
## V7
## CV residual -1.07 -0.30 -0.30 -0.557 1.73 -0.30 1.17 -0.673 0.097
                                                                   5.94 -0.14
                 258
                      262
                            266
                                   277
                                          278 280
                                                    284
                                                          285
                                                                286
                                                                     301
                                                                           308
## Predicted
               1.440 9.00
                           3.17
                                1.440 0.9873 5.91 5.58 6.93 11.01
                                                                    8.37
                                                                         1.31
## cvpred
               1.478 8.91
                           3.16 1.478 0.9625 5.97 5.68 7.04 11.05 8.33 1.30
               1.000 10.00 1.00 1.000 1.0000 7.00 10.00 10.00 10.00 4.00
## V7
## CV residual -0.478 1.09 -2.16 -0.478 0.0375 1.03 4.32 2.96 -1.05 -4.33 -0.30
##
                                   326
                                                                     399
                313
                     314
                            320
                                            329
                                                  370
                                                        383
                                                               396
                         5.233
## Predicted
               6.84 0.663
                                1.757
                                       3.86109
                                                1.622 2.41
                                                            1.440
                                                                  1.440
## cvpred
               7.02 0.625
                          5.285
                                 1.756
                                       4.00182
                                                1.518
                                                       2.44
                                                             1.478
                                1.000
                                       4.00000 1.000 1.00 1.000 1.000
## V7
               1.00 1.000 5.000
## CV residual -6.02 0.375 -0.285 -0.756 -0.00182 -0.518 -1.44 -0.478 -0.478
##
                 400 401
                            409
                                   419
                                          422
                                               424
                                                     425
                                                           430 435
## Predicted
               1.298 7.92 2.187 2.865 9.815 2.53 1.12 1.21 6.00 1.568
```

```
## cvpred
               1.181 7.84 2.178 2.951 9.865 2.55 1.14 1.22 5.96 1.655
## V7
               1.000 9.00 2.000 2.000 10.000 1.00 1.00 1.00 8.00 1.000
## CV residual -0.181 1.16 -0.178 -0.951 0.135 -1.55 -0.14 -0.22 2.04 -0.655
                460
                      466
                           467
                                 468
                                        469
                                              470
                                                   476
                                                          478 486
                                                                      493
## Predicted
               2.20
                    8.86
                         7.21
                               6.65
                                      1.342
                                            1.30
                                                 1.12
                                                       1.342 1.33
               2.21 8.97 7.38 6.70 1.398 1.24
                                                 1.14 1.398 1.31
## cvpred
               ## V7
## CV residual -1.21 -4.97 2.62 3.30 -0.398 -0.24 -0.14 -0.398 1.69 -0.735
##
                 499
                       508
                            510
                                  515
                                        517
                                              532
                                                   545
                                                          547
                                                                565
                                                                      575
                                                                            576
               1.666 0.663 0.890 8.95 0.663
                                            1.98 2.18
                                                               1.99
## Predicted
                                                       9.583
                                                                    7.21
                                                                          2.53
## cvpred
               1.735 0.625 0.883 9.03 0.625
                                            2.01 2.12 9.526
                                                               2.07 7.32 2.61
               ## CV residual -0.735 3.375 0.117 0.97 0.375 -1.01 -1.12 0.474 -1.07 -5.32 -1.61
##
                582
                      583
                           587
                                 592
                                        594
                                               596
                                                     602
                                                           608
                                                                  610
               8.03 6.01 11.01
                               6.40 1.885 1.892 0.9873 0.663
                                                               1.568
## Predicted
                                                                       9.68
## cvpred
               7.90 6.08 11.05 6.29
                                     1.933
                                            1.993 0.9625 0.625
                                                                1.655
                                                                       9.61
## V7
              10.00 10.00 10.00 10.00 1.000 1.000 1.000
                                                                       2.00
                                                               1.000
## CV residual 2.10
                    3.92 - 1.05
                               3.71 -0.933 -0.993 0.0375 0.375 -0.655 -7.61
                613
                       619
                             620
                                    621
                                          623
                                               625
                                                     629
                                                           631
                                                                 635
                                                                        643
                           1.892
## Predicted
              11.01 1.666
                                  1.440
                                         3.33
                                              2.22 0.890
                                                          2.43
                                                                1.12
                                                                     1.440
## cvpred
              11.05 1.735 1.993 1.478
                                         3.47 2.34 0.883
                                                          2.47
                                                               1.14
## V7
              10.00 1.000 1.000 1.000
                                        1.00 1.00 1.000
                                                         1.00
                                                               1.00
## CV residual -1.05 -0.735 -0.993 -0.478 -2.47 -1.34 0.117 -1.47 -0.14 -0.478
                645
                      647
                            648 651
                                        652
                                               653
                                                     654
                                                            657
                                                                  669
                                                                        671
## Predicted
              0.890 0.981 1.328 1.45
                                     1.652 1.892
                                                   1.666
                                                          1.892
                                                                4.46 7.288
## cvpred
              0.883 0.903 1.311 1.48
                                     1.649 1.993 1.735
                                                          1.993
                                                                4.51 7.234
## V7
              1.000 1.000 1.000 4.00 1.000 1.000 1.000
                                                          1.000 1.00 8.000
## CV residual 0.117 0.097 -0.311 2.52 -0.649 -0.993 -0.735 -0.993 -3.51 0.766
                680
                      687
## Predicted
              0.890 0.663
## cvpred
              0.883 0.625
## V7
              1.000 1.000
## CV residual 0.117 0.375
##
## Sum of squares = 591
                         Mean square = 4.35
                                              n = 136
## Overall (Sum over all 136 folds)
## ms
## 5.2
SST <- sum((as.numeric(data[-missing,]$V7) - mean(as.numeric(data[-missing,]$V7)))^2)
R2_cv <- 1 - attr(model_cv, "ms")*nrow(data[-missing,])/SST
# Get predictions for missing V7 values.
V7_hat <- predict(model2, newdata = data[missing,])</pre>
# Impute V7 for observations with missing data for V7 to predicted values with this linear model.
data_reg_imp <- data
data_reg_imp[missing,]$V7 <- V7_hat</pre>
data_reg_imp$V7 <- as.numeric(data_reg_imp$V7)</pre>
# Round the V7_hat values since the originals are all integer
data reg imp[missing,]$V7 <- round(V7 hat)</pre>
data_reg_imp$V7 <- as.integer(data_reg_imp$V7)</pre>
```

```
# Make sure no V7 values are outside of the orignal range.
data_reg_imp$V7[data_reg_imp$V7 > 10] <- 10
data_reg_imp$V7[data_reg_imp$V7 < 1] <- 1
```

Regression with Perturbation Imputation

```
set.seed(1)
# Perturb the predictions for missing V7 values with a random normal distribbtion
V7_hat_pert <- rnorm(nrow(data[missing,]), V7_hat, sd(V7_hat))</pre>
V7_hat_pert
## [1] 4.078 8.386 -0.855 5.138 1.707 0.407 3.790 3.391 3.343 5.413
## [11] 4.320 3.386 3.875 -3.118 3.467 0.564
# Notice that we get some negative values when we perturb the predicted values.
data_reg_pert_imp <- data
data_reg_pert_imp[missing,]$V7 <- V7_hat_pert</pre>
data_reg_pert_imp$V7 <- as.numeric(data_reg_pert_imp$V7)</pre>
# Round the V7_hat_pert values to integers.
data_reg_pert_imp[missing,]$V7 <- round(V7_hat_pert)</pre>
data_reg_pert_imp$V7 <- as.integer(data_reg_pert_imp$V7)</pre>
# Make sure no V7 values are outside of the orignal range.
data_reg_pert_imp$V7[data_reg_pert_imp$V7 > 10] <- 10</pre>
data_reg_pert_imp$V7[data_reg_pert_imp$V7 < 1] <- 1</pre>
```

Comparing Results of Classification Models

```
set.seed(1)
# split data
training <- sample(nrow(data), size = floor(nrow(data) * 0.7))
validation <- setdiff(1:nrow(data), training)
# KNN Models
library(kknn)

## Warning: package 'kknn' was built under R version 4.0.2

acc_knn <- rep(0,25)
for (k in 1:5) {</pre>
```

```
knn_model <- kknn(V11~V2+V3+V4+V5+V6+V7+V8+V9+V10, data_mode_imp[training,], data_mode_imp[validation
  # Compare models using validation set.
  pred <- as.integer(fitted(knn_model)+0.5) # round off to 2 or 4</pre>
 acc_knn[k] = sum(pred == data_mode_imp[validation,]$V11) / nrow(data_mode_imp[validation,])
# Data with regression imputation
for (k in 1:5) {
  # Fit k-nearest-neighbor model using training set, validate on test set.
  knn_model <- kknn(V11~V2+V3+V4+V5+V6+V7+V8+V9+V10, data_reg_imp[training,], data_reg_imp[validation,]
  # Compare models using validation set.
 pred <- as.integer(fitted(knn_model)+0.5) # round off to 2 or 4</pre>
 acc_knn[k+5] = sum(pred == data_reg_imp[validation,]$V11) / nrow(data_reg_imp[validation,])
# Data with regression with perturbation imputation
for (k in 1:5) {
  # Fit k-nearest-neighbor model using training set, validate on test set.
 knn_model <- kknn(V11~V2+V3+V4+V5+V6+V7+V8+V9+V10, data_reg_pert_imp[training,], data_reg_pert_imp[va
  # Compare models using validation set.
 pred <- as.integer(fitted(knn_model)+0.5) # round off to 2 or 4</pre>
 acc_knn[k+10] = sum(pred == data_reg_pert_imp[validation,] $V11) / nrow(data_reg_pert_imp[validation,]
# Data without missing variables
# Check how many of the missing indices fall into the training set
length(intersect(missing, training))/length(missing)
## [1] 0.625
# Since this is relatively close to 70% and since the number of observations
# with missing values is so small, we can simply take out the missing indices
# from the training and validation sets.
training no missing <- setdiff(training, intersect(missing, training))</pre>
validation_no_missing <- setdiff(validation, intersect(missing, validation))</pre>
```

```
# Replace missing data with 0's so that V7 can be read as type integer and
# skip over missing values in modeling.
data no missing <- data
\label{eq:data_no_missing} $$V7[data$V7 == "?"] <- 0$
data_no_missing$V7 <- as.integer(data_no_missing$V7)</pre>
for (k in 1:5) {
  # Fit k-nearest-neighbor model using training set, validate on test set.
  knn_model <- kknn(V11~V2+V3+V4+V5+V6+V7+V8+V9+V10, data_no_missing[training_no_missing,],
                    data_no_missing[validation_no_missing,], k=k)
  # Compare models using validation set.
  pred <- as.integer(fitted(knn_model)+0.5) # round off to 2 or 4</pre>
 acc_knn[k+15] = sum(pred == data_no_missing[validation_no_missing,]$V11) /
    nrow(data_no_missing[validation_no_missing,])
# Add a binary variable to the original data to indicate if an
# observation has a missing V7 value.
data_binary <- data
data_binary$V12[data$V7 == "?"] <- 0
data_binary$V12[data$V7 != "?"] <- 1</pre>
# Create interaction factor for V7 and V12.
data_binary$V13[data$V7 == "?"] <- 0</pre>
data_binary$V13[data$V7 != "?"] <- as.integer(data[-missing,]$V7)</pre>
# Use the interaction factor in the modeling.
for (k in 1:5) {
  # Fit k-nearest-neighbor model using training set, validate on test set.
 knn_model <- kknn(V11~V2+V3+V4+V5+V6+V8+V9+V10+V13, data_binary[training,], data_binary[validation,],
  # Compare models using validation set.
 pred <- as.integer(fitted(knn_model)+0.5) # round off to 2 or 4</pre>
  acc_knn[k+20] = sum(pred == data_binary[validation,]$V11) / nrow(data_binary[validation,])
acc_knn
## [1] 0.943 0.943 0.900 0.900 0.900 0.948 0.948 0.905 0.905 0.905 0.948 0.948
```

[13] 0.900 0.900 0.900 0.951 0.951 0.912 0.912 0.912 0.948 0.948 0.900 0.900

```
## [25] 0.900
```

```
# SVM Models
library(kernlab)
acc_svm < -rep(0,30)
amounts \leftarrow c(0.0001, 0.001, 0.01, 0.1, 1, 10)
# Data with mode imputation
for (i in 1:6) {
  # Fit model using training set.
  model_svm <- ksvm(as.matrix(data_mode_imp[training,2:10]),</pre>
                       as.factor(data_mode_imp[training,11]),
                       type = "C-svc", # Use C-classification method
                       kernel = "vanilladot", # Use simple linear kernel
                       C = amounts[i])
  # Compare models using validation set.
  pred <- predict(model svm, data mode imp[validation,2:10])</pre>
 acc_svm[i] = sum(pred == data_mode_imp[validation,11]) / nrow(data_mode_imp[validation,])
## Setting default kernel parameters
# Data with regression imputation
for (i in 1:6) {
  # Fit model using training set.
  model_svm <- ksvm(as.matrix(data_reg_imp[training,2:10]),</pre>
                    as.factor(data_reg_imp[training,11]),
                    type = "C-svc", # Use C-classification method
                    kernel = "vanilladot", # Use simple linear kernel
                    C = amounts[i])
  # Compare models using validation set.
  pred <- predict(model_svm, data_reg_imp[validation,2:10])</pre>
  acc_svm[i+6] = sum(pred == data_reg_imp[validation,11]) / nrow(data_reg_imp[validation,])
## Setting default kernel parameters
## Setting default kernel parameters
```

```
## Setting default kernel parameters
# Data with regression with perturbation imputation
for (i in 1:6) {
  # Fit model using training set.
  model_svm <- ksvm(as.matrix(data_reg_pert_imp[training,2:10]),</pre>
                    as.factor(data_reg_pert_imp[training,11]),
                    type = "C-svc", # Use C-classification method
                    kernel = "vanilladot", # Use simple linear kernel
                    C = amounts[i])
  # Compare models using validation set.
 pred <- predict(model_svm, data_reg_pert_imp[validation,2:10])</pre>
  acc_svm[i+12] = sum(pred == data_reg_pert_imp[validation,11]) / nrow(data_reg_pert_imp[validation,])
## Setting default kernel parameters
# Data without missing variables
for (i in 1:6) {
  # Fit model using training set.
 model_svm <- ksvm(as.matrix(data_no_missing[training_no_missing,2:10]),</pre>
                    as.factor(data no missing[training no missing,11]),
                    type = "C-svc", # Use C-classification method
                    kernel = "vanilladot", # Use simple linear kernel
                    C = amounts[i])
  # Compare models using validation set.
  pred <- predict(model_svm, data_no_missing[validation_no_missing,2:10])</pre>
  acc_svm[i+18] = sum(pred == data_no_missing[validation_no_missing,11]) /
    nrow(data[validation_no_missing,])
## Setting default kernel parameters
## Setting default kernel parameters
## Setting default kernel parameters
```

Setting default kernel parameters

```
## Setting default kernel parameters
## Setting default kernel parameters
# Data with binary variable to indicate if an observation
# has a missing V7 value. Use the interaction factor for modeling
for (i in 1:6) {
  # Fit model using training set.
 model_svm <- ksvm(as.matrix(data_binary[training,c(2:6,8:10,13)]),</pre>
                    as.factor(data_binary[training,11]),
                    type = "C-svc", # Use C-classification method
                    kernel = "vanilladot", # Use simple linear kernel
                    C = amounts[i])
  # Compare models using validation set.
  pred <- predict(model_svm, data_binary[validation,c(2:6,8:10,13)])</pre>
  acc_svm[i+24] = sum(pred == data_binary[validation,11]) / nrow(data_binary[validation,])
## Setting default kernel parameters
acc_svm
## [1] 0.657 0.948 0.971 0.957 0.957 0.957 0.657 0.948 0.971 0.957 0.957 0.957
## [13] 0.657 0.948 0.971 0.957 0.957 0.957 0.652 0.951 0.975 0.961 0.956 0.956
## [25] 0.657 0.948 0.971 0.957 0.957 0.957
```