Volatility of US Bond Yields

Build a model to study the nature of volatility in the case of US government bond yields.

```
# Load the packages
library(xts)
library(readr)
# Load the data
yc_raw <- read_csv("datasets/FED-SVENY.csv")
# Convert the data into xts format
yc_all <- as.xts(x = yc_raw[, -1], order.by = yc_raw$Date)

# Show only the 1st, 5th, 10th, 20th and 30th columns
yc_all_tail <- tail(yc_all[, c(1, 5, 10, 20, 30)])
yc_all_tail
```

```
FALSE            SVENY01 SVENY05 SVENY10 SVENY20 SVENY30
FALSE 2019-03-22  2.4222  2.2613  2.4553  2.7661  3.0178
FALSE 2019-03-25  2.3901  2.2281  2.4449  2.7610  3.0216
FALSE 2019-03-26  2.3811  2.2016  2.4249  2.7508  3.0138
FALSE 2019-03-27  2.3560  2.1931  2.4020  2.7092  2.9785
FALSE 2019-03-28  2.3601  2.2137  2.4058  2.6907  2.9605
FALSE 2019-03-29  2.3719  2.2398  2.4143  2.6939  2.9538
```

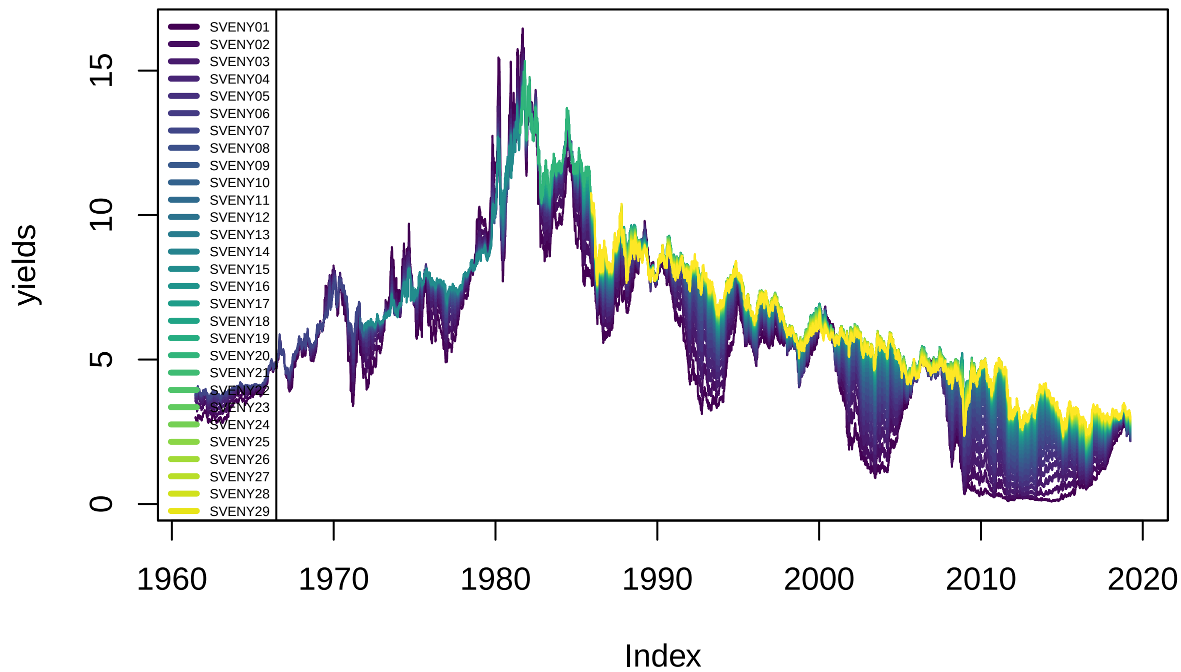
Visualize the yields over time

```
library(viridis)

# Define plot arguments
yields  <- yc_all
plot.type  <- "single"
plot.palette <- viridis(30)
asset.names <- colnames(yc_all)

# Plot the time series
plot.zoo(x = yields, plot.type = plot.type, col = plot.palette)

# Add the legend
legend(x = "topleft", legend = asset.names,
       col = plot.palette, cex = 0.45, lwd = 3)
```



Calculate the changes in the yield levels

```
# Differentiate the time series
ycc_all <- diff.xts(yc_all)

# Show the tail of the 1st, 5th, 10th, 20th and 30th columns
ycc_all_tail <- tail(ycc_all[, c(1, 5, 10, 20, 30)])
ycc_all_tail
```

```
FALSE            SVENY01 SVENY05 SVENY10 SVENY20 SVENY30
FALSE 2019-03-22 -0.0412 -0.1039 -0.0878 -0.0924 -0.0864
FALSE 2019-03-25 -0.0321 -0.0332 -0.0104 -0.0051  0.0038
FALSE 2019-03-26 -0.0090 -0.0265 -0.0200 -0.0102 -0.0078
FALSE 2019-03-27 -0.0251 -0.0085 -0.0229 -0.0416 -0.0353
FALSE 2019-03-28  0.0041  0.0206  0.0038 -0.0185 -0.0180
FALSE 2019-03-29  0.0118  0.0261  0.0085  0.0032 -0.0067
```

properties

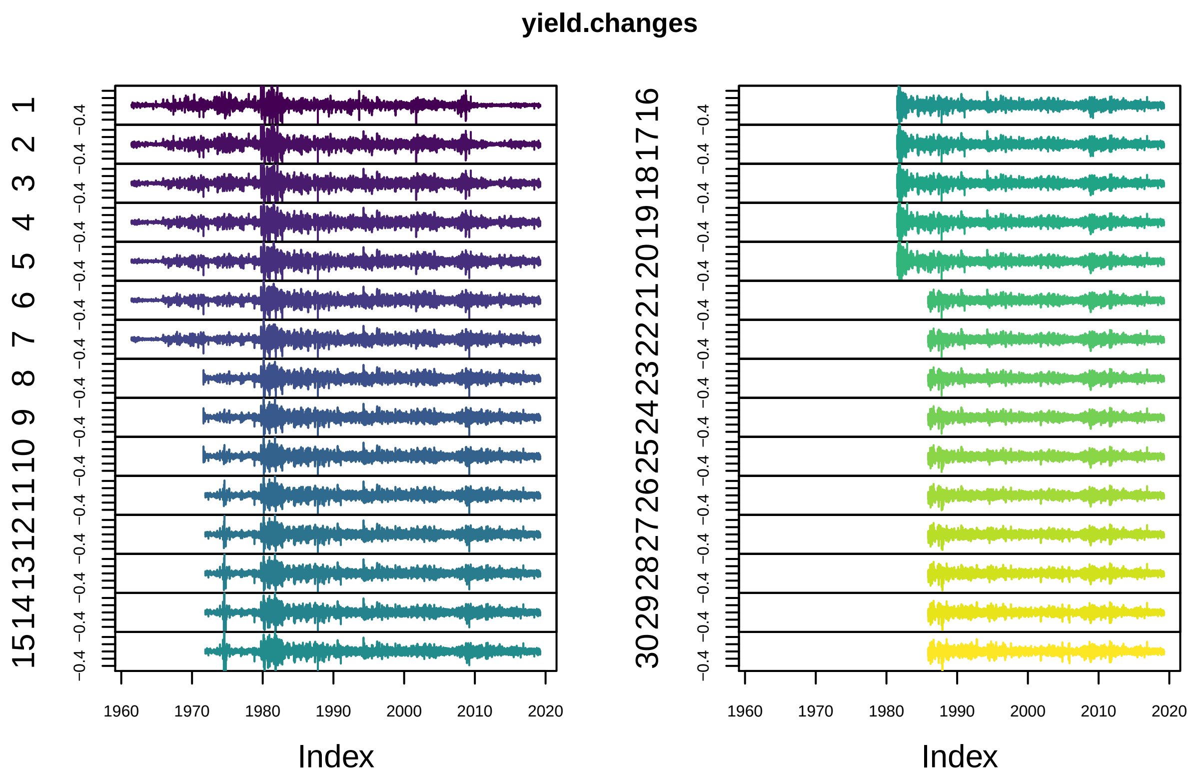
The direction (positive or negative) of a return is mostly independent of the previous day's return.

The magnitude of the return is similar to the previous day's return. That means, i

```
# Define the plot parameters
yield.changes <- ycc_all
```

```r
plot.type <- "multiple"


# Plot the differtianted time series
plot.zoo(x = yield.changes, plot.type = plot.type,
     ylim = c(-0.5, 0.5), cex.axis = 0.7,
     ylab = 1:30, col = plot.palette)
```



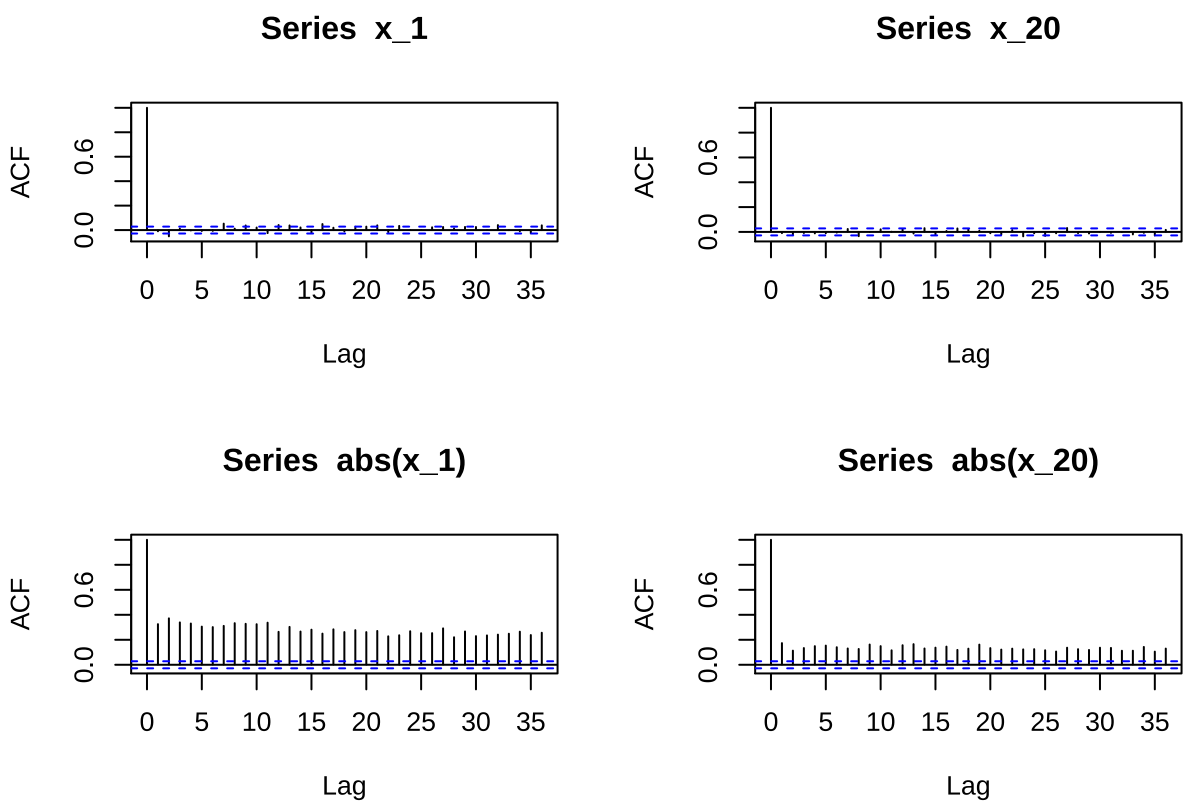
#Statistics

```r
# Filter for changes in and after 2000
ycc <- ycc_all["2000/", ]

# Save the 1-year and 20-year maturity yield changes into separate variables
x_1 <- ycc[, "SVENY01"]
x_20 <- ycc[, "SVENY20"]

# Plot the autocorrelations of the yield changes
par(mfrow=c(2,2))
acf_1 <- acf(x_1)
acf_20 <- acf(x_20)

# Plot the autocorrelations of the absolute changes of yields
acf_abs_1 <- acf(abs(x_1))
acf_abs_20 <- acf(abs(x_20))
```



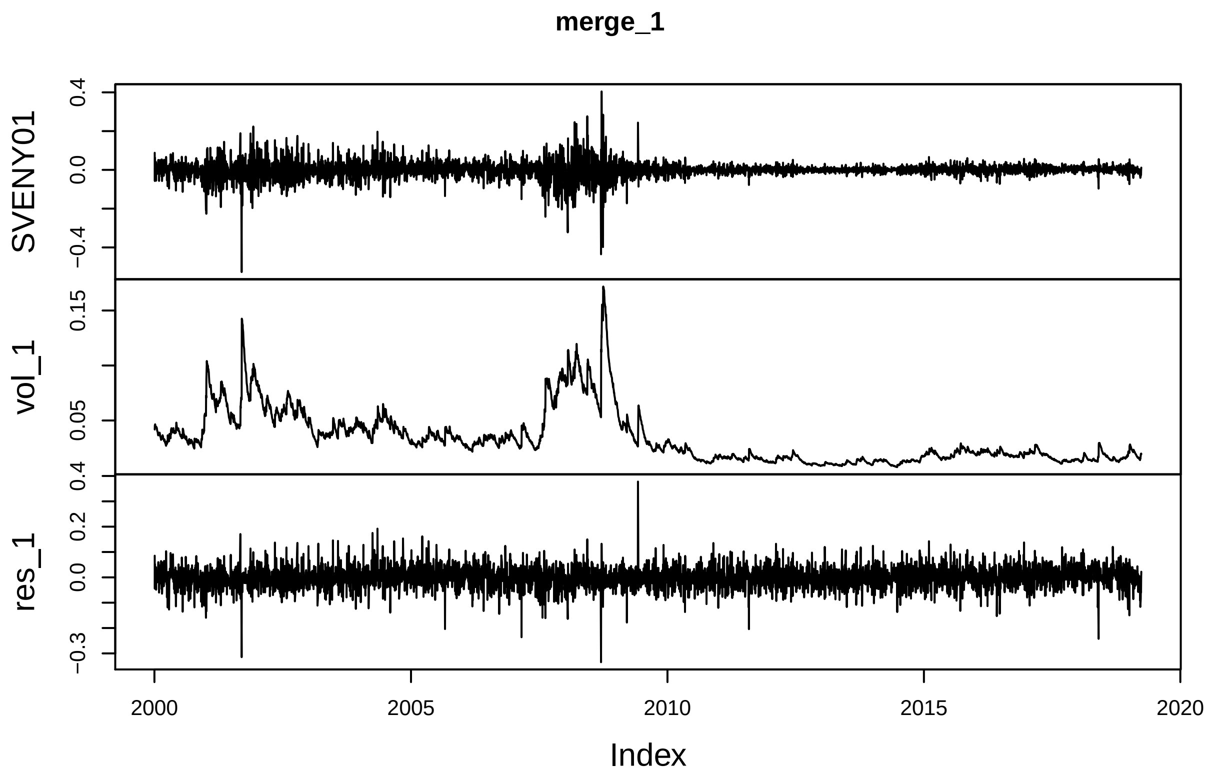
Generalized AutoRegressive Conditional Heteroskedasticity (GARCH)

```
library(rugarch)
# Specify the GARCH model with the skewed t-distribution
spec <- ugarchspec(distribution.model = "sstd")

# Fit the model
fit_1 <- ugarchfit(x_1, spec = spec)

# Save the volatilities and the rescaled residuals
vol_1 <- sigma(fit_1)
res_1 <- scale(residuals(fit_1, standardize = TRUE)) * sd(x_1) + mean(x_1)

# Plot the yield changes with the estimated volatilities and residuals
merge_1 <- merge.xts(x_1, vol_1, res_1)
plot.zoo(merge_1)
```

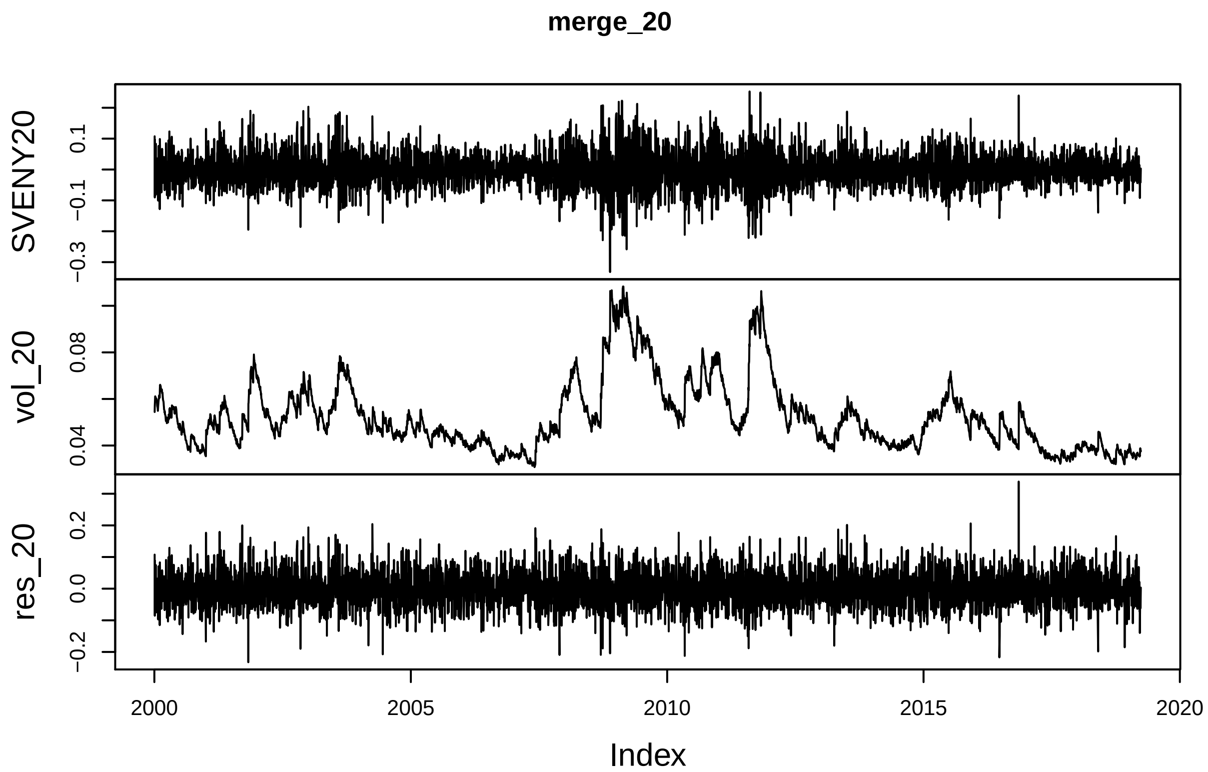


Fitting the 20-year maturity

```
# Fit the model
fit_20 <- ugarchfit(x_20, spec = spec)

# Save the volatilities
vol_20 <- sigma(fit_20)
res_20 <- scale(residuals(fit_20, standardize = TRUE)) * sd(x_20) + mean(x_20)

# Plot the yield changes with the estimated volatilities and residuals
merge_20 <- merge.xts(x_20, vol_20, res_20)
plot.zoo(merge_20)
```



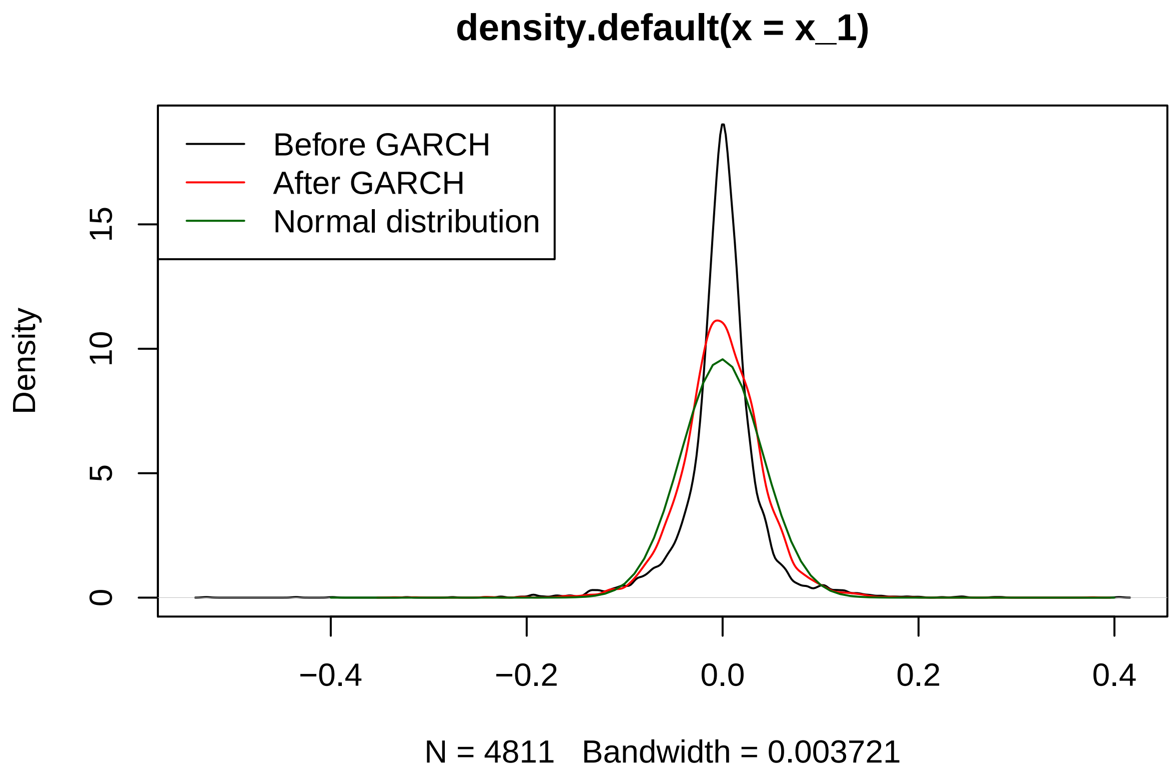
Distributions

```r
# Calculate the kernel density for the 1-year maturity and residuals
density_x_1 <- density(x_1)
density_res_1 <- density(res_1)

# Plot the density digaram for the 1-year maturity and residuals
plot(density_x_1)
lines(density_res_1, col = "red")

# Add the normal distribution to the plot
norm_dist <- dnorm(seq(-0.4, 0.4, by = .01), mean = mean(x_1), sd = sd(x_1))
lines(seq(-0.4, 0.4, by = .01),
      norm_dist,
      col = "darkgreen"
     )

# Add legend
legend <- c("Before GARCH", "After GARCH", "Normal distribution")
legend("topleft", legend = legend,
       col = c("black", "red", "darkgreen"), lty=c(1,1))
```



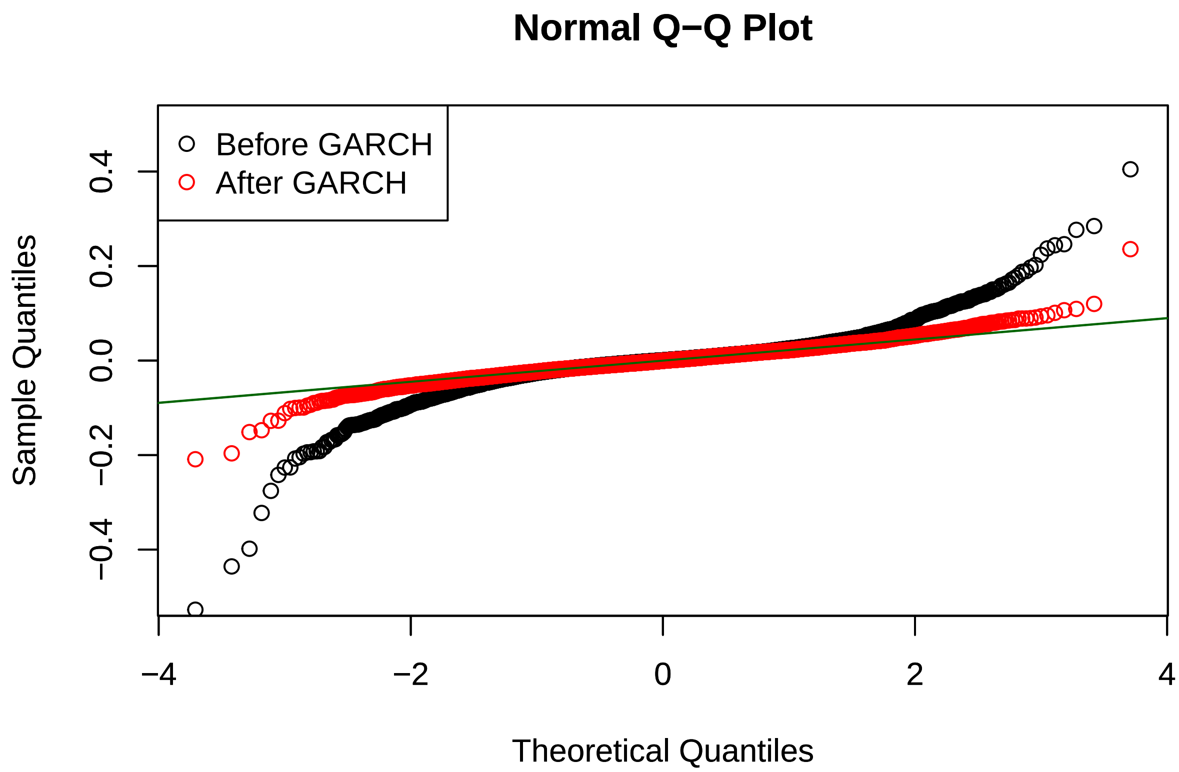
QQ plot

```r
# Define plot data: the 1-year maturity yield changes and the residuals
data_orig <- x_1
data_res <- res_1

# Define the benchmark distribution (qnorm)
distribution <- qnorm

# Make the Q-Q plot of original data with the line of normal distribution
qqnorm(data_orig, ylim = c(-0.5, 0.5))
qqline(data_orig, distribution = distribution, col = "darkgreen")

# Make the Q-Q plot of GARCH residuals with the line of normal distribution
par(new=TRUE)
qqnorm(data_res * 0.623695122815242, col = "red", ylim = c(-0.5, 0.5))
qqline(data_res * 0.623695122815242, distribution = distribution, col = "darkgreen")
legend("topleft", c("Before GARCH", "After GARCH"), col = c("black", "red"), pch=c(1,1))
```



GARCH revealed how volatility changed over time. The model bring the residuals closer to normal distribution