Time Series Analysis of Proportion of Issued License Plates in Shanghai

CY

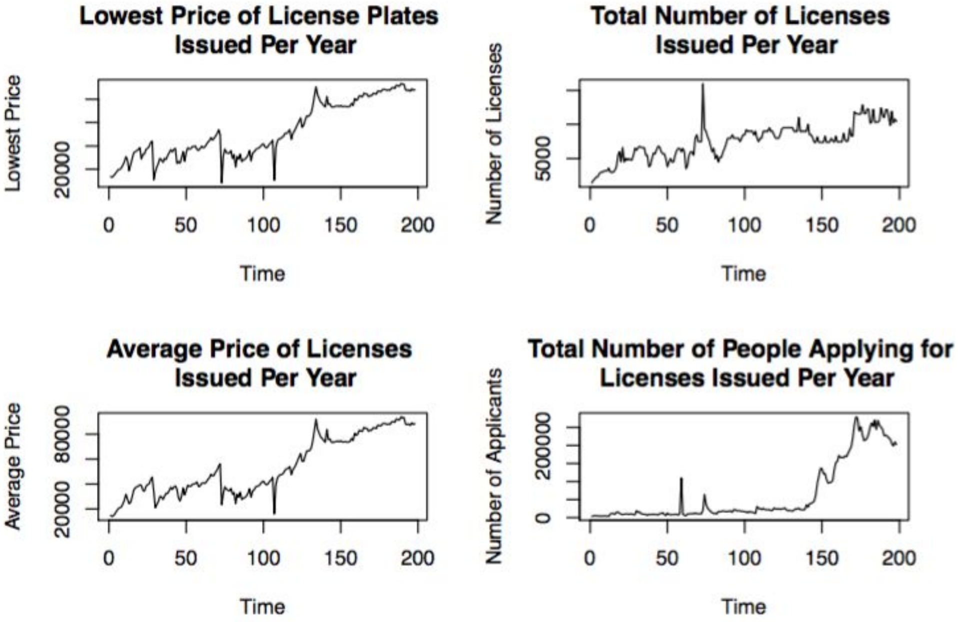
Introduction

- Monthly data from January of 2002 to 2018
- Sell a limited number of license plates to fossil-fuel car buyers
- Forecast the monthly proportion of license plates issued to the number of applicants up until the year 2020
- License plate in Shanghai is referred to as "the most expensive piece of metal in the world"
- Average price is about $13,000
- Government’s attempt to combat Shanghai’s air pollution problem

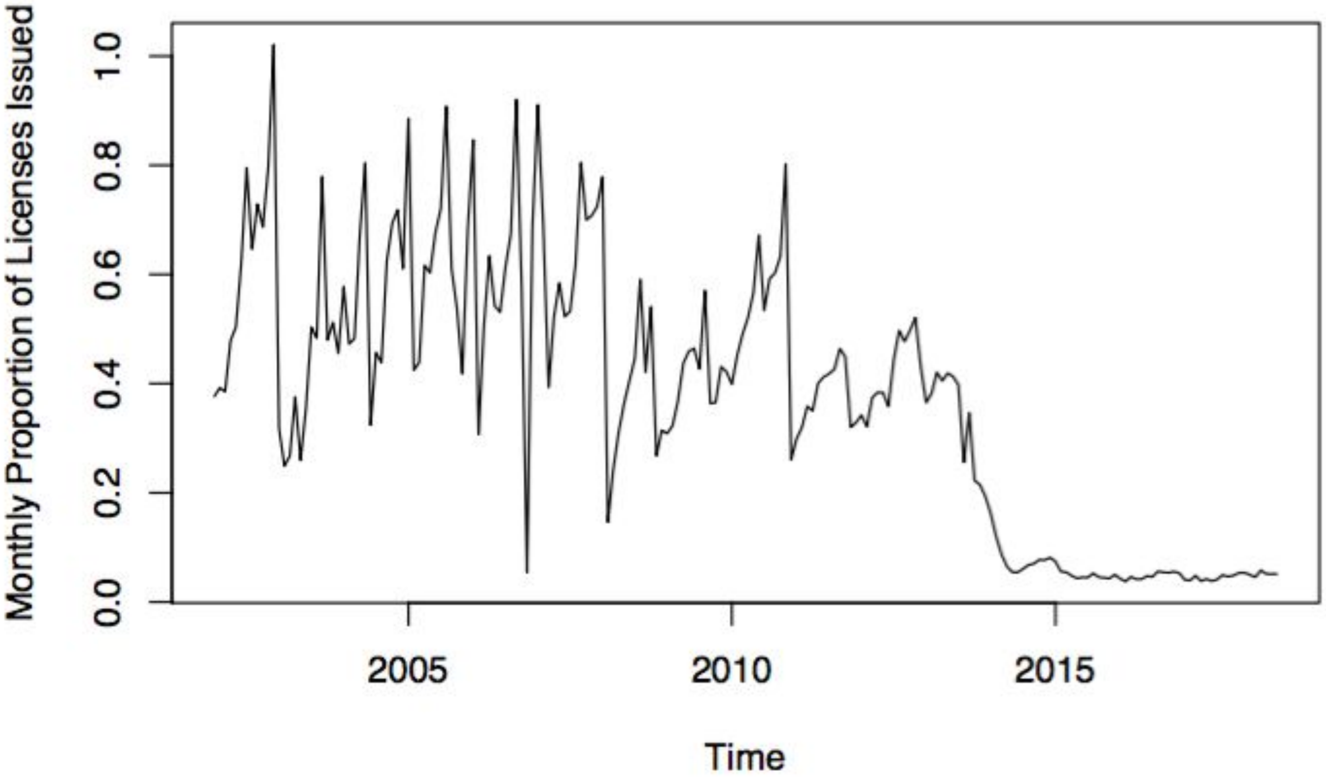
Initial Analysis

We convert the data into a time series and plot each of the four variables:

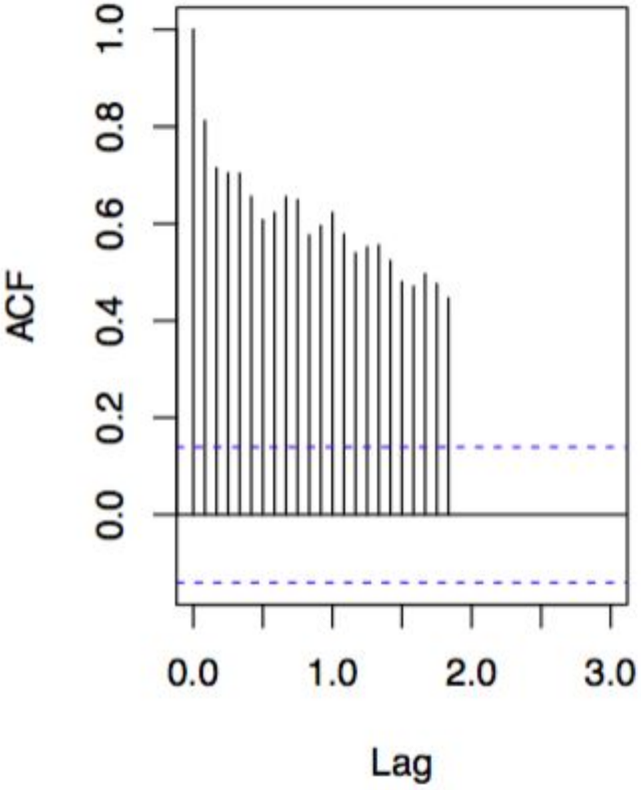
1. Lowest price
2. Total number of license plates issued
3. Average price
4. Total number of applicants

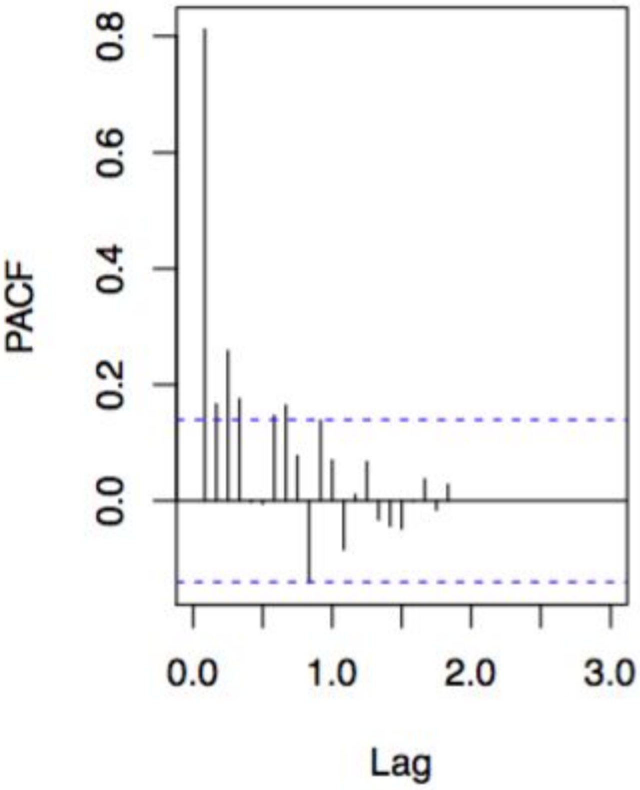


Monthly Proportion of Licenses Issued in Shanghai (2002-2018)



ACF and PACF of Proportion of Shanghai-Issued License Plates

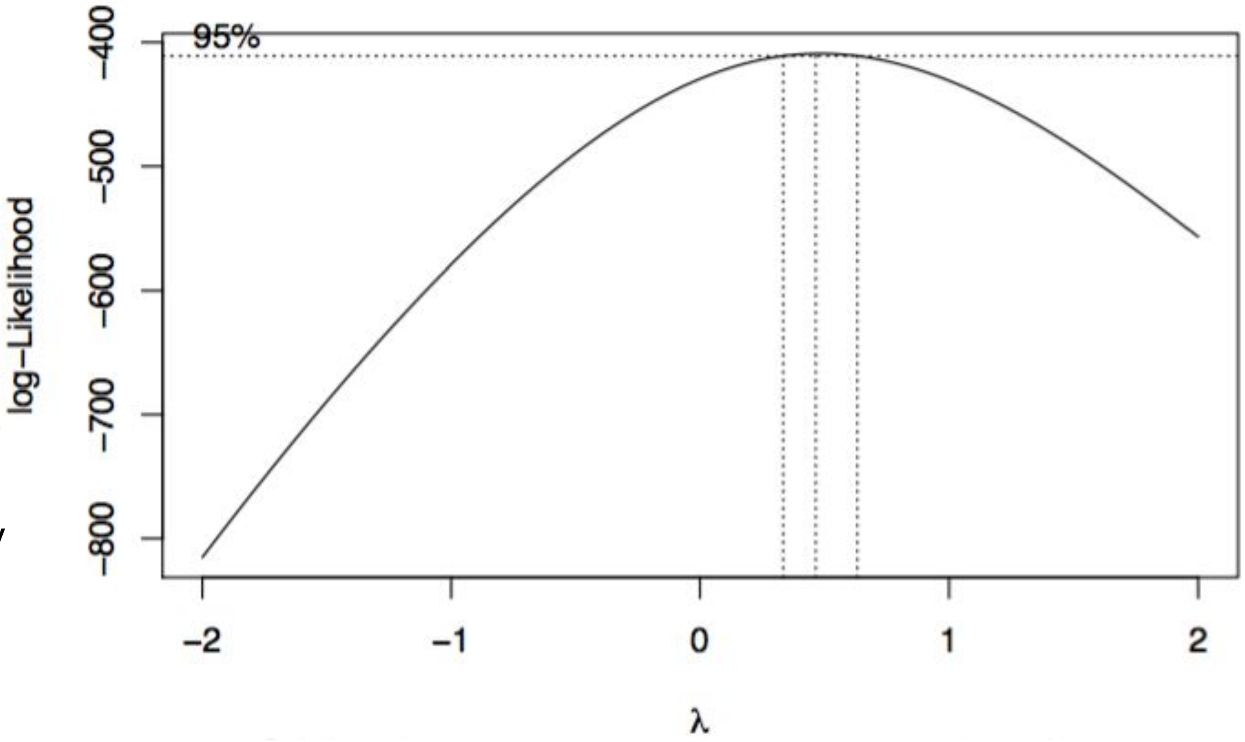




Transformations

Box-Cox

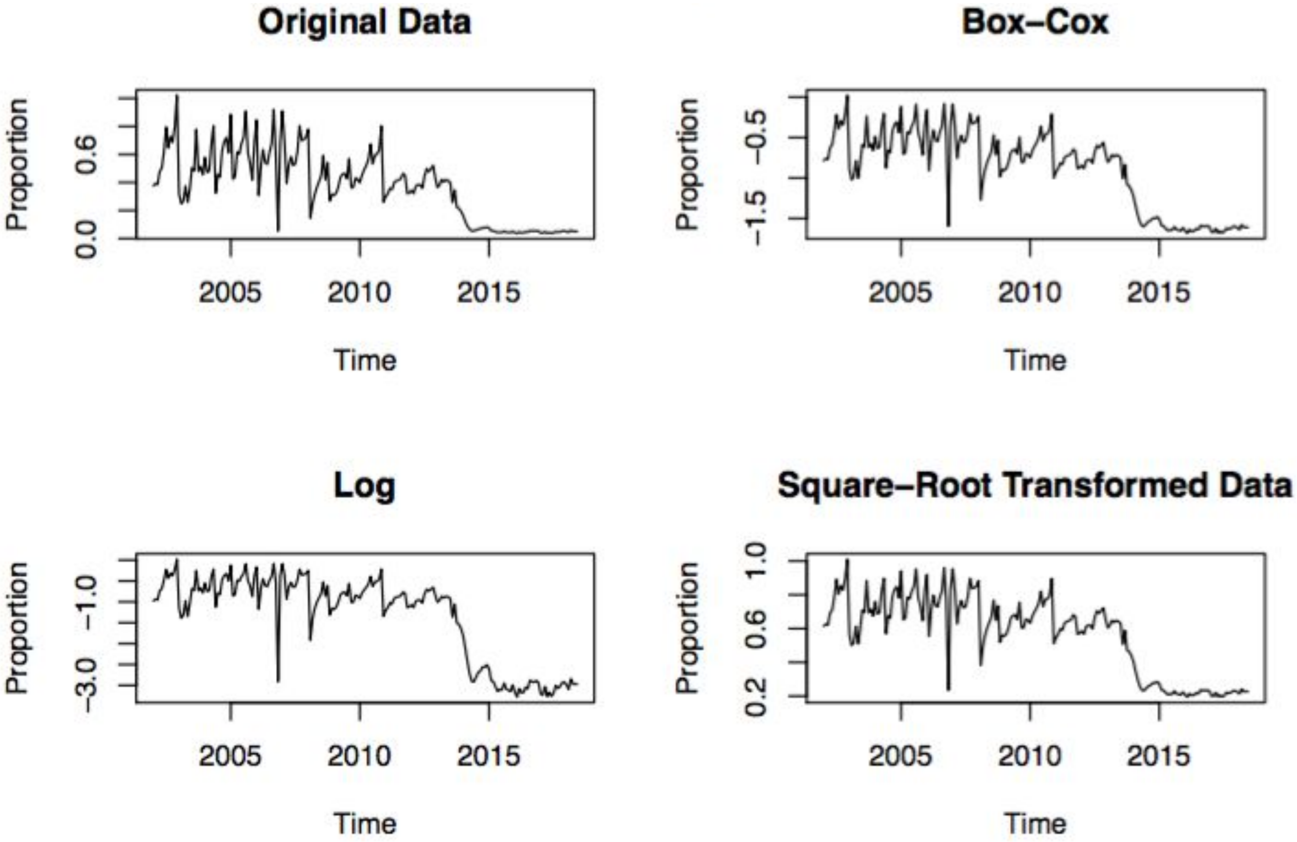
- Initial time series was not stationary and due to heteroscedasticity, violated our constant error of variance assumption
- Variance of error changes over time
- λ = 0.46 which is relatively close to 0.5
- Indicates square-root transformation performs best



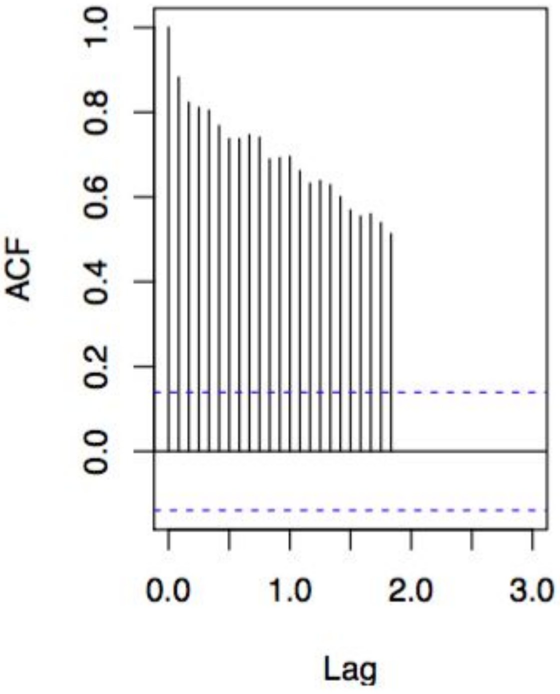
Transformations

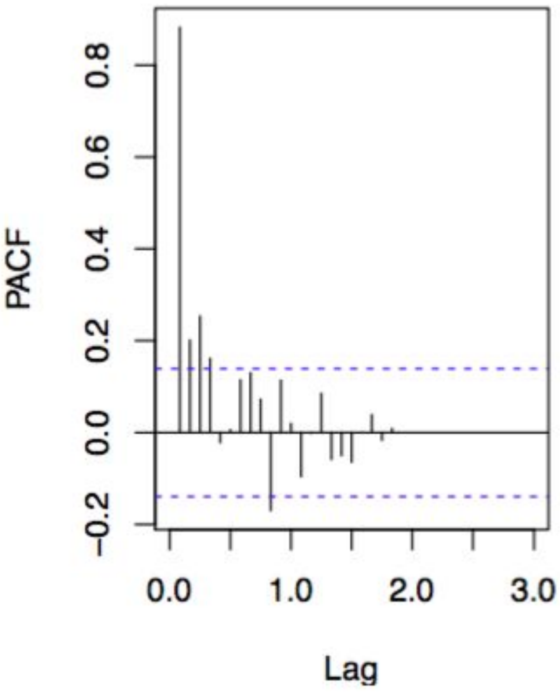
	Variance
Original Time Series	0.0609
Box-Cox Transformation	0.2393
Log Transformation	1.0672
Square-Root Transformation	0.0540

- Find the variances of each transformation to determine the best fit for our model
- Square-root transformation gives the smallest variance value



ACF and PACF of Square-Root Transformed Time Series

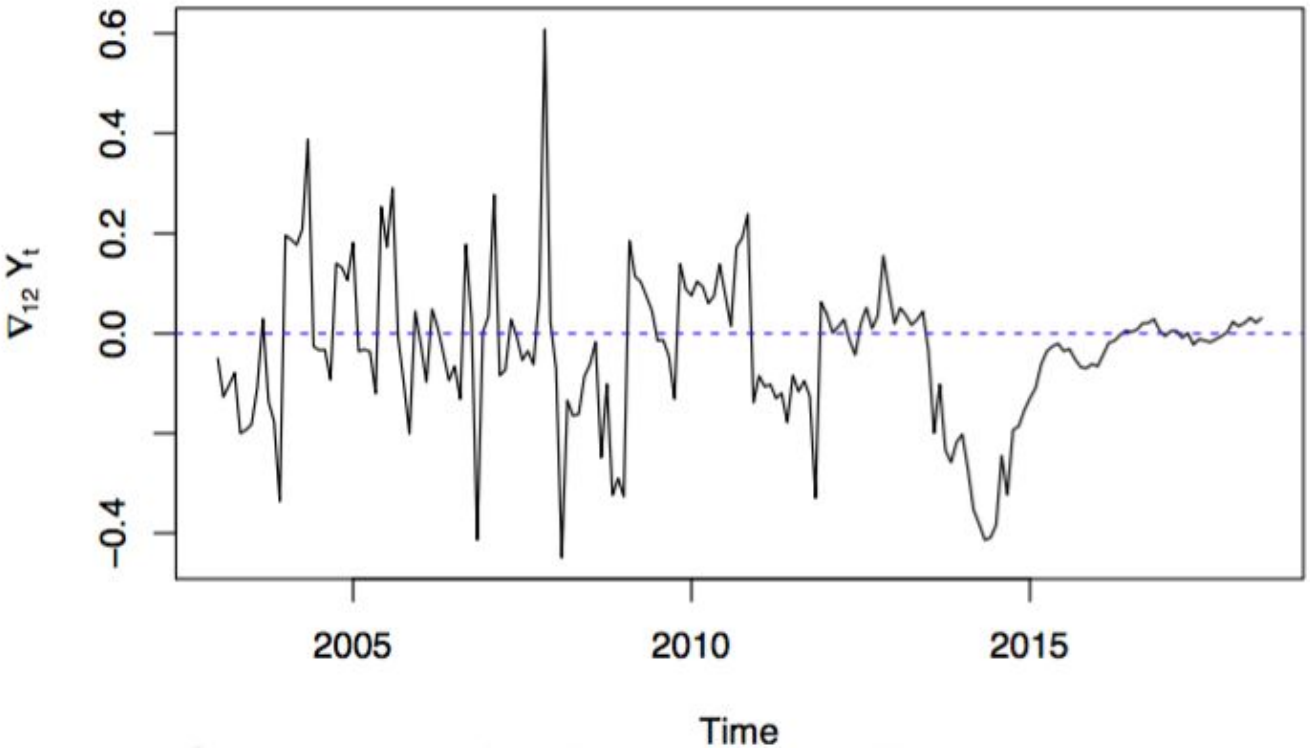




- ACF is still tailing off
- PACF cuts off around lag 0.8
- Supports our initial assumption that the series follows an AR(p) model

Differencing to Remove Seasonality and Trend

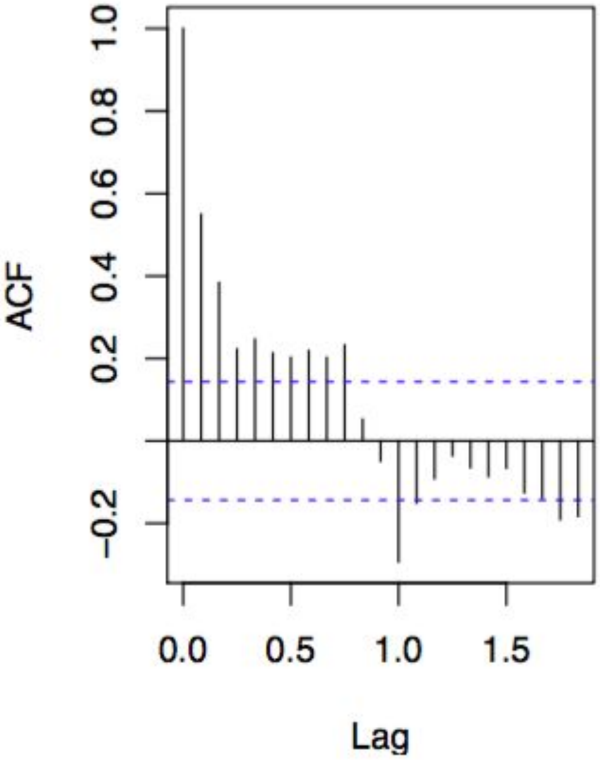
Removing seasonality

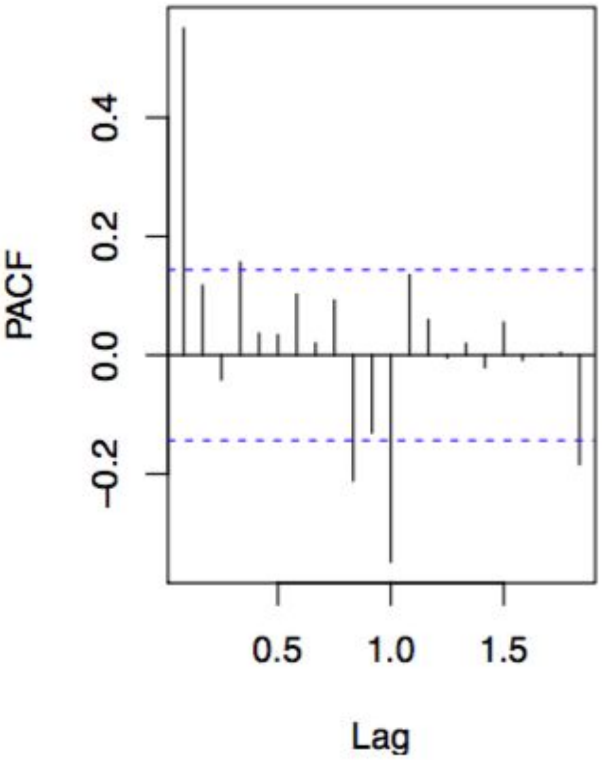


- Data still does not look stationary after applying square-root transformation, so we apply differencing
- Difference once at lag 12 to remove seasonality
- De-seasonalized data fluctuates around the mean = 0 line

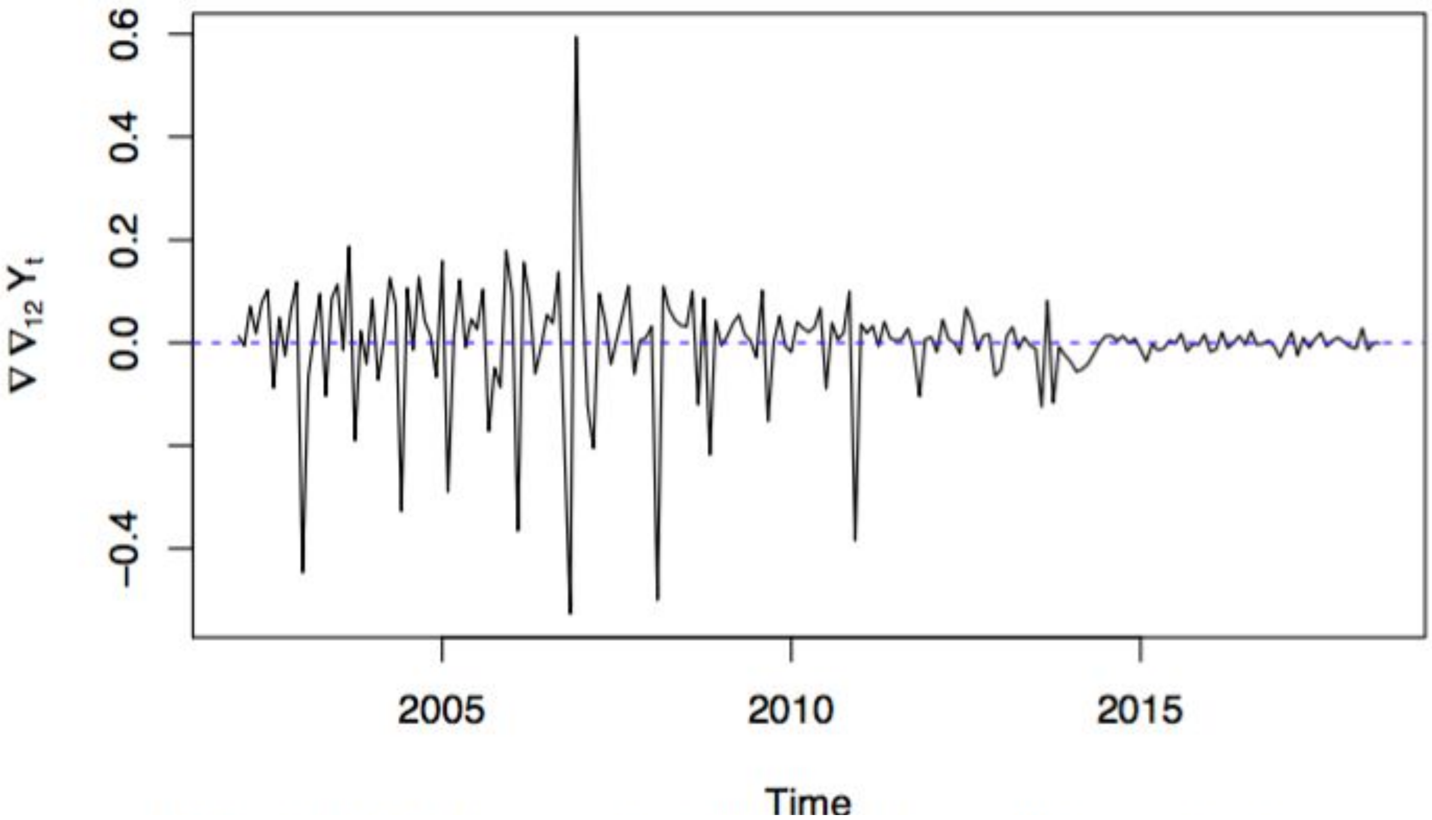
ACF and PACF plots of de-seasonalized data

- ACF slowly begins to decay
- PACF oscillates between the bounds



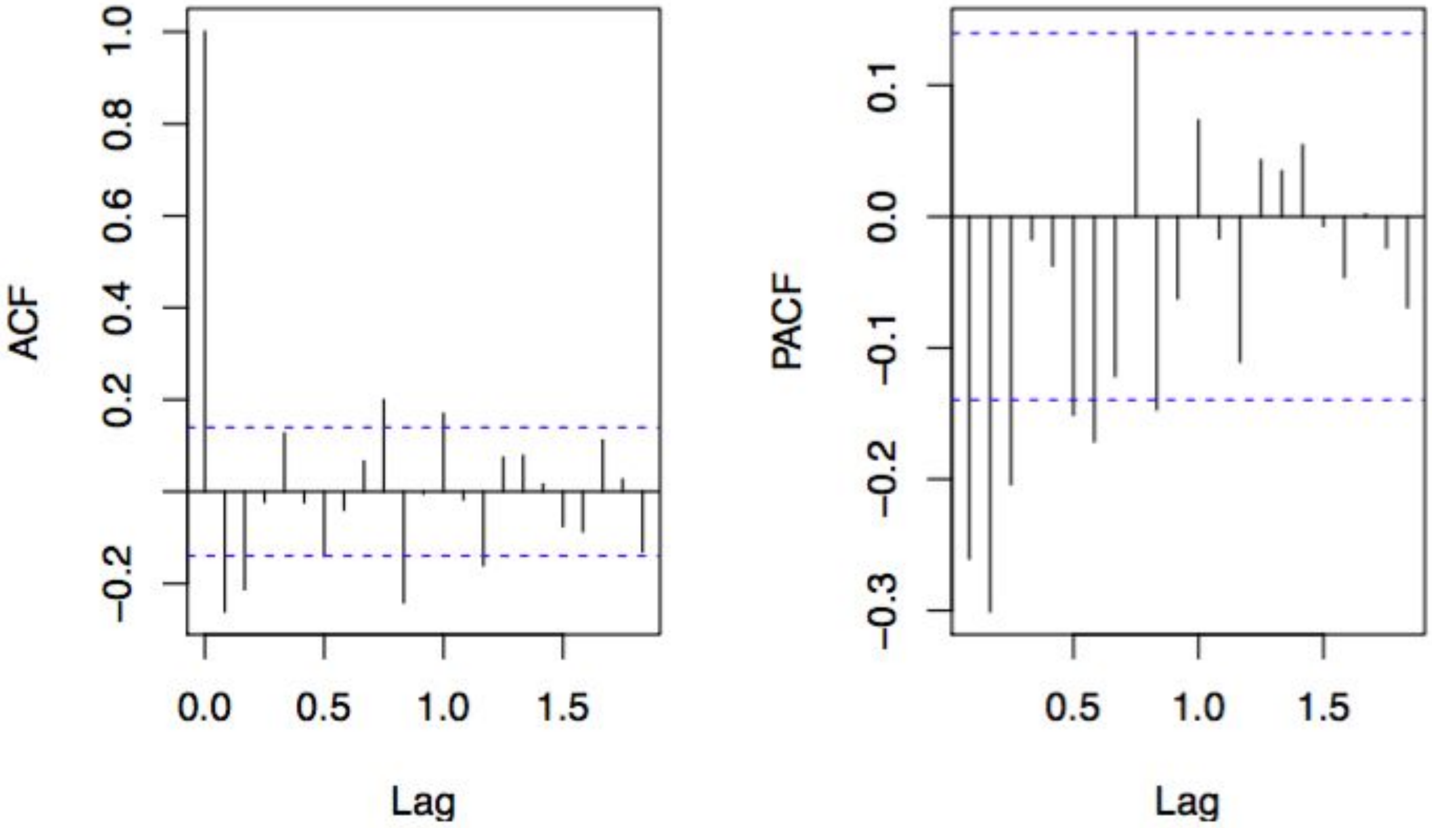


Removing trend and seasonality



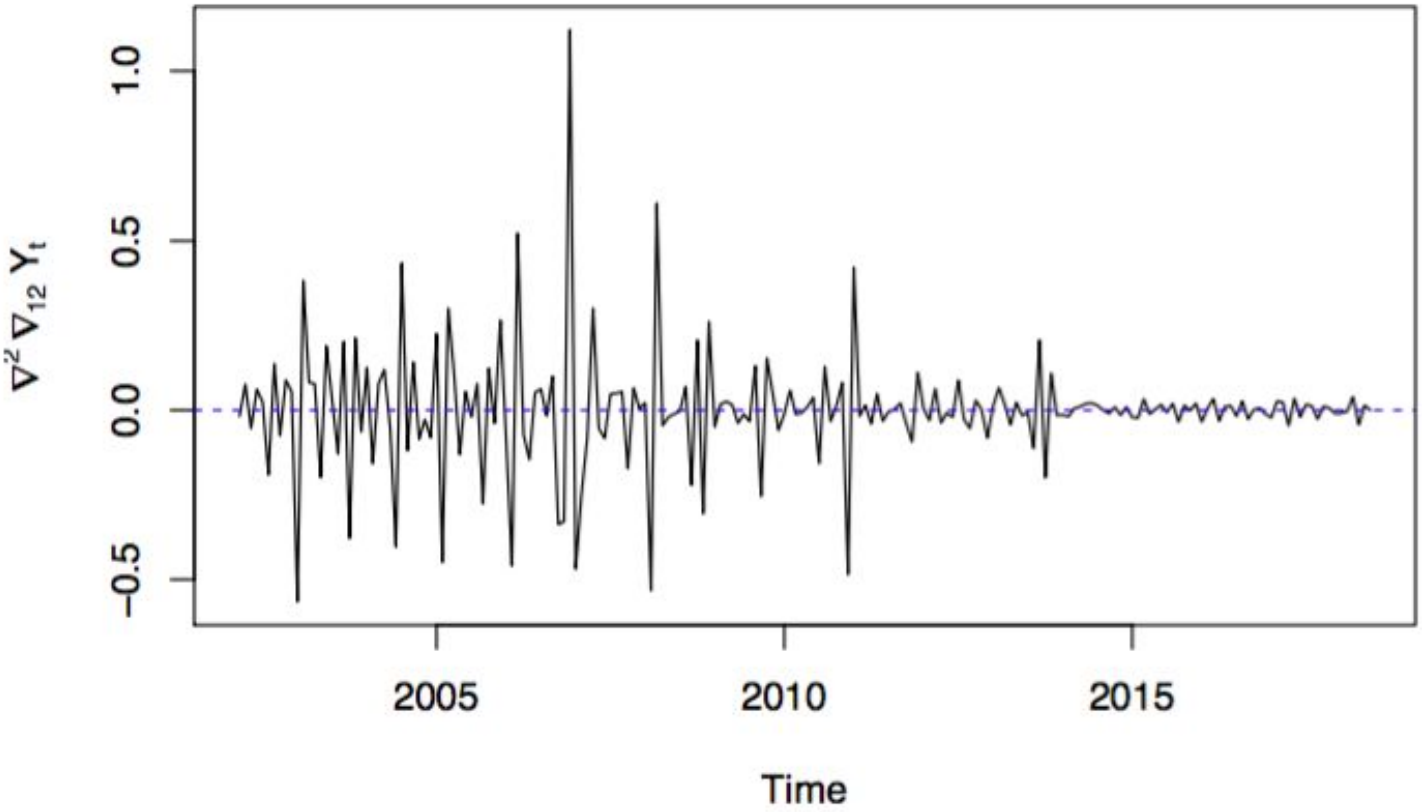
- Difference again at lag 1 to remove trend
- Plot fluctuates very closely around the mean = 0 line
- Stationary

ACF and PACF of de-trended and de-seasonalized data



- ACF oscillates between the bounds
- PACF seems to cut off around lag 0.1

Overdifferencing



- Variance increases from 0.012 to 0.031 when we difference a 2nd time at lag 1
- Indicates overdifferencing
- Only difference at lag 1 once

Parameter Estimation using Yule-Walker

- Preliminary estimation using Yule-Walker gives an AR(10) model

```
Call:
ar(x = shanghai_prop.diff1, method = "yule-walker")

Coefficients:
      1        2        3        4        5        6        7        8
-0.4189  -0.4095  -0.2802  -0.1567  -0.2030  -0.2471  -0.2010  -0.1161
      9       10
 0.0761  -0.1469

Order selected 10  sigma^2 estimated as  0.009353
```

Fitting an ARMA process

- Use auto.arima() function to find the estimated model is a ARIMA(1,0,1)

```
Series: shanghai_prop.diff1
ARIMA(1,0,1) with zero mean

Coefficients:
         ar1      ma1
      0.3560  -0.8184
s.e.  0.0962   0.0570

sigma^2 estimated as 0.009847:  log likelihood=176.33
AIC=-346.65   AICc=-346.53   BIC=-336.8
```

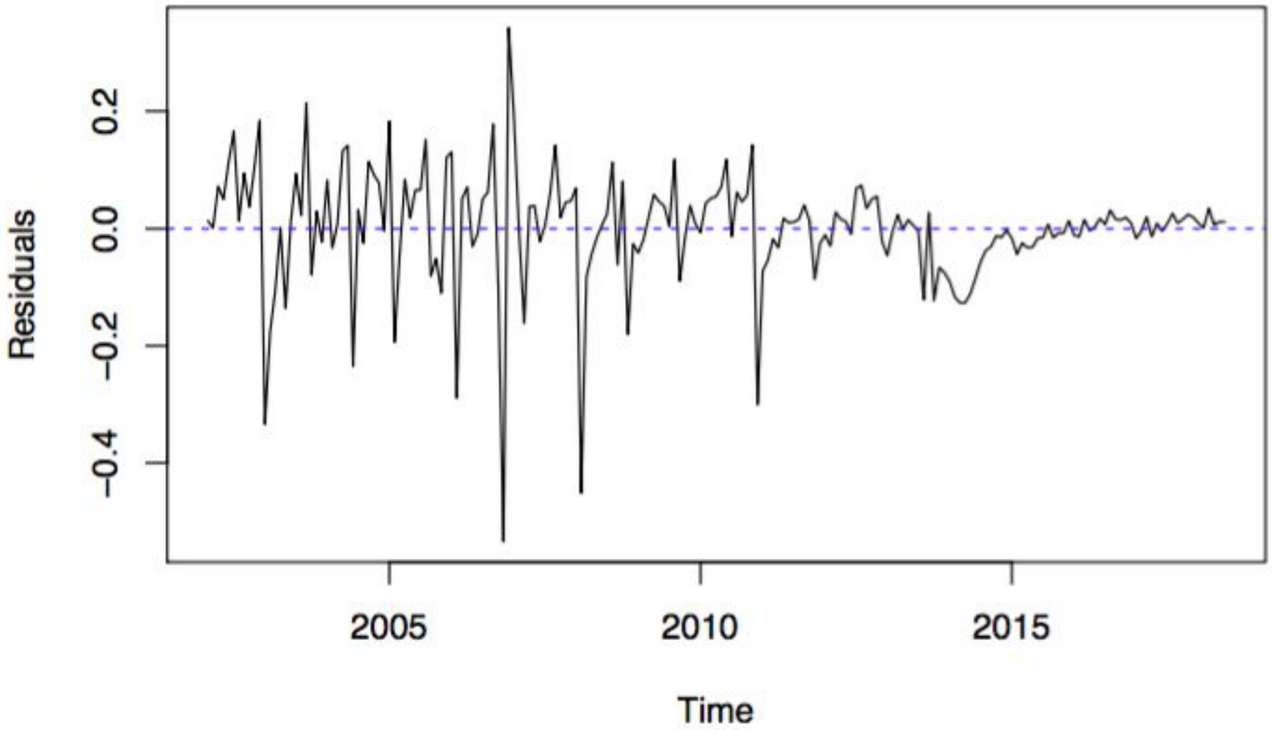
Checking for best model fit

- We run further AIC tests and find our best model using estimated orders of (p,q)
- Test each of the possible ARMA(p,q) parameter values to see which process gives us the smallest value of AIC using a for-loop
- ARMA(1,1) gives us the lowest AIC value of -346.0987, so it is the best model for our data

	AIC
AR(1)	-317.5312
MA(1)	-336.1856
ARMA(1,1)	-346.0987

Plotting Residuals of ARMA(1,1)

- Residuals seem to oscillate about the line at error 0



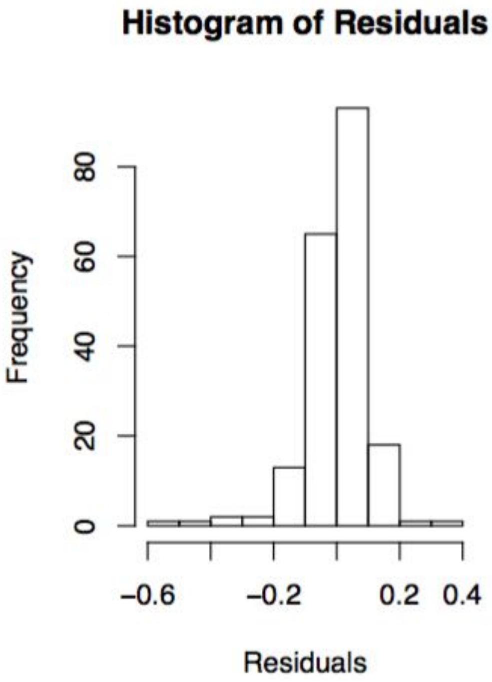
Diagnostic Checking of Residuals

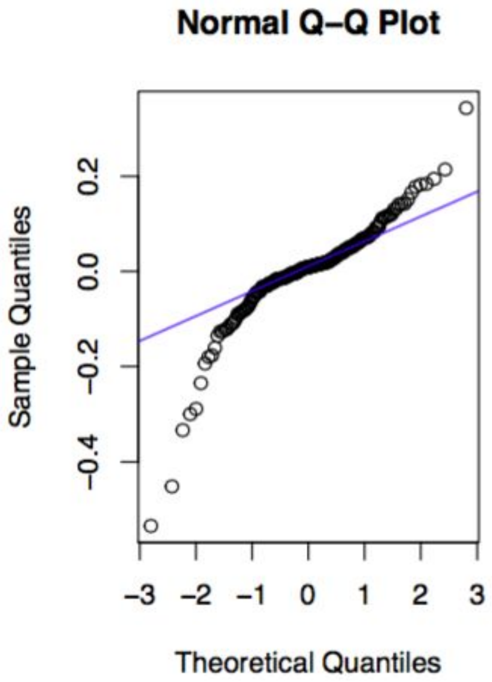
```
        Shapiro-Wilk normality test

data:  err
W = 0.86508, p-value = 3.169e-12

        Box-Ljung test

data:  err
X-squared = 0.080816, df = 1, p-value = 0.7762

        Box-Pierce test

data:  err
X-squared = 0.079598, df = 1, p-value = 0.7778
```

- Check for normality of errors, if the residuals are not heteroskedastic and have constant variance, and if the residuals are serially correlated
- ARMA(1,1) passes the other 2 tests but does not pass the Shapiro-Wilk test of normality of errors
- Ljung-Box test (✔️)
  - constant variance
  - residuals are not heteroskedastic
  - residuals are random
- Box-Pierce test (✔️)
  - residuals are serially correlated

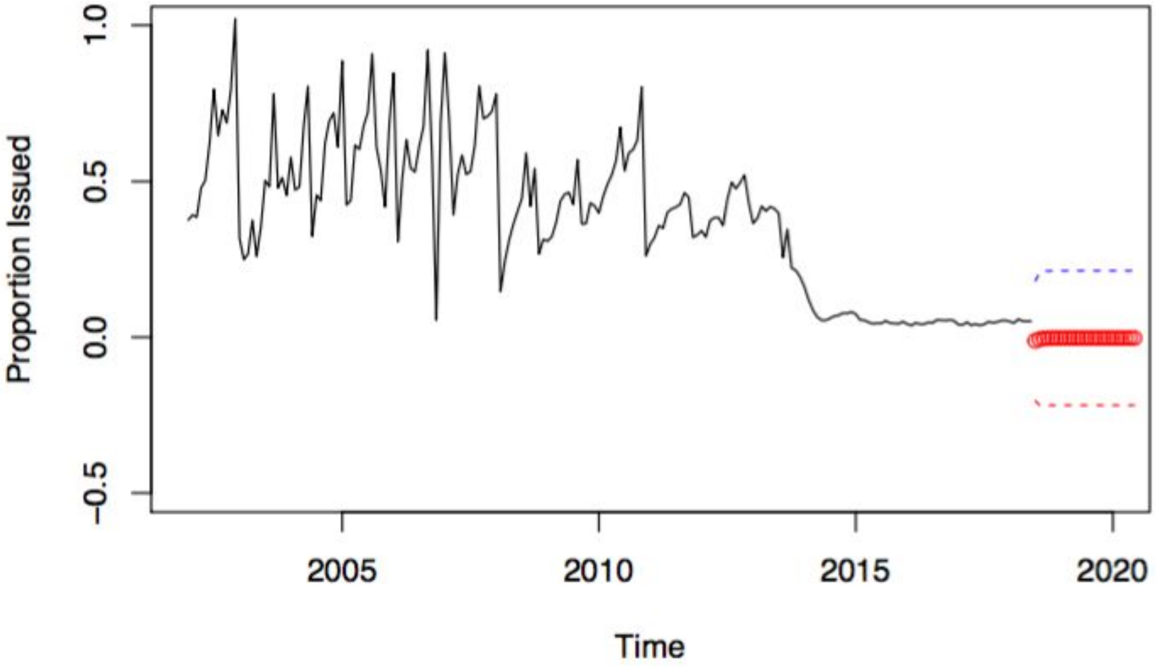
Histogram and QQ-Plot

- Histogram shows data is normally distributed
- QQ-plot shows errors follow diagonal line so they are normally distributed





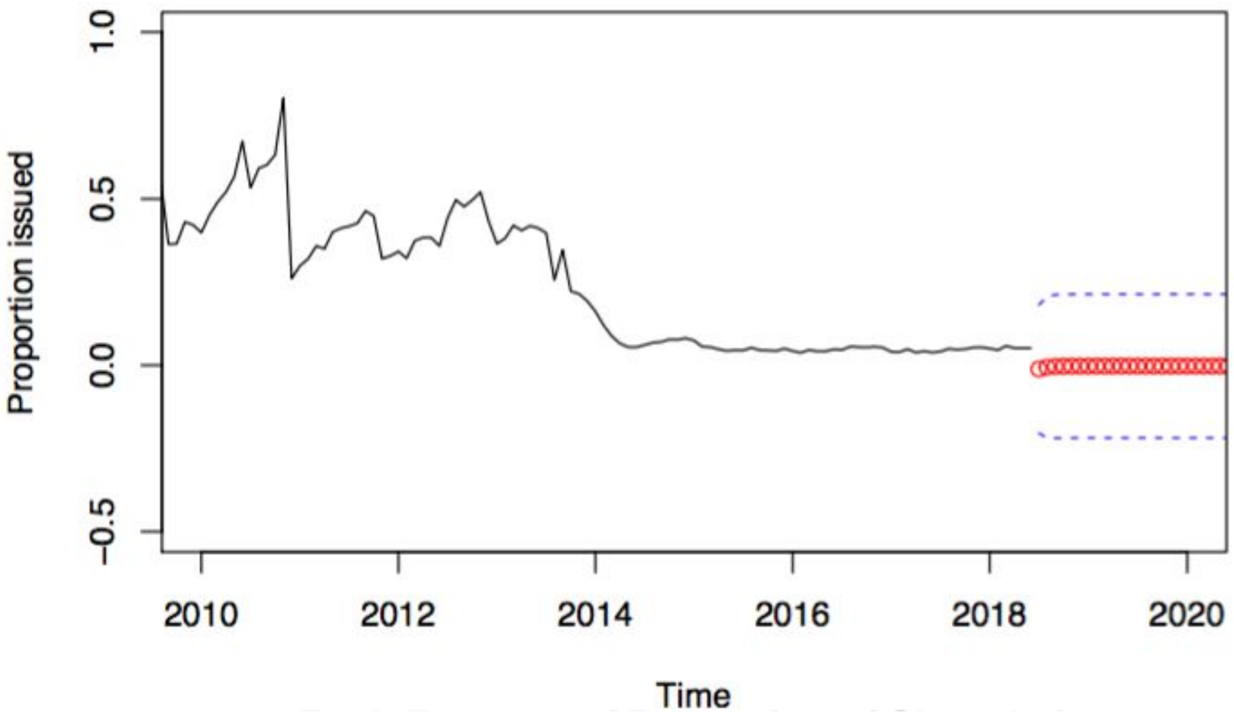
Forecasting



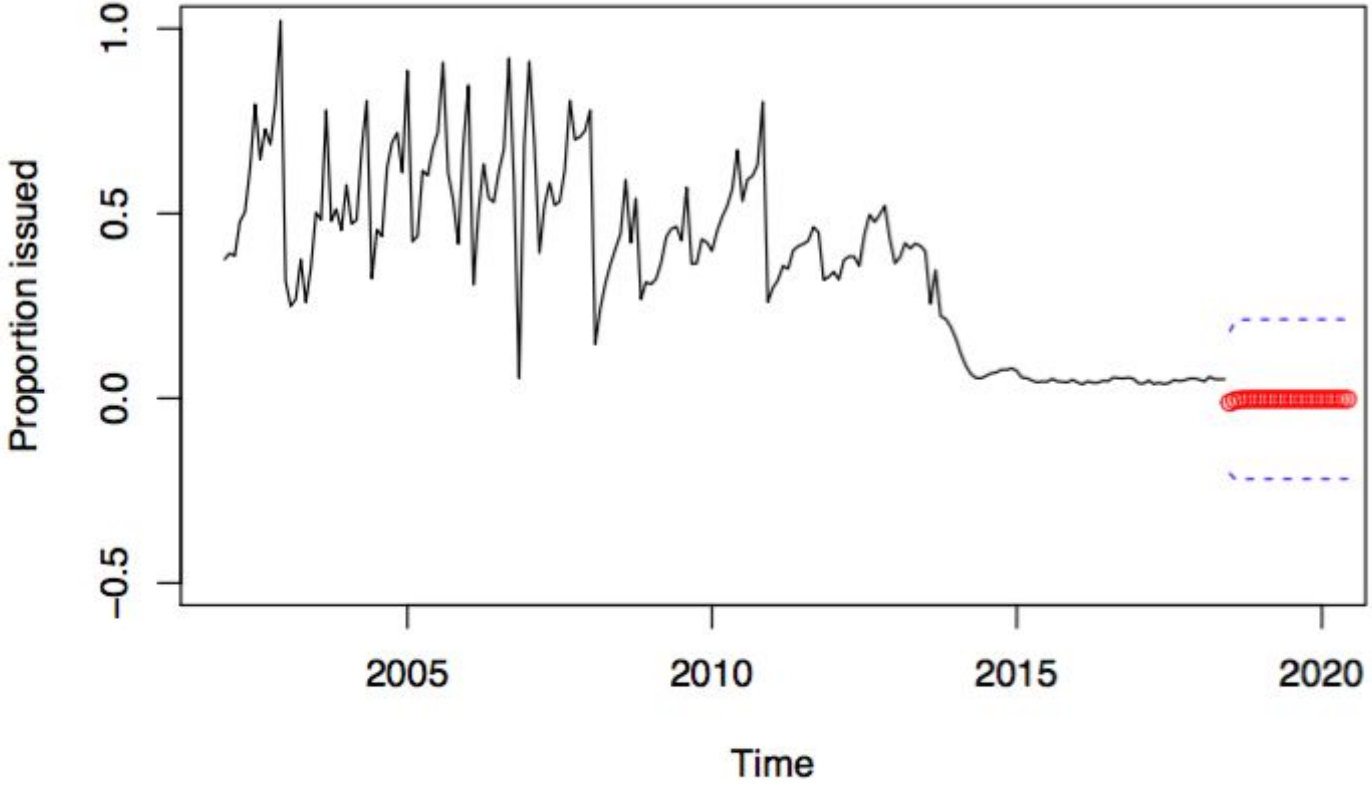
- We predict the proportion of license plates issued for the next 2 years using our ARMA(1,1) model
- Plot upper and lower bounds to calculate a 95% confidence interval for the predicted values

Back-Transformation

- We obtain back forecasted values for the proportion for the next 2 years
- Our predicted values stay consistent for the next 2 years (2018-2020) so our proportion remains relatively consistent



Back Forecast of Proportion of Shanghai-issued license plates from 2002-2020



Conclusion

- Used monthly data to analyze the proportion of Shanghai-issued license plates per month to total number of applicants from 2002 to 2018
- Concluded ARMA(1,1) was the best model using Yule-Walker method, for-loop to compare AIC values, and auto.arima() function
- Forecasted values for the next 2 years (2018 to 2020)
- Predicted values show the proportion will stay relatively consistent as time goes on
- Consistent trend as number of license plates issued and number of applicants continue to fluctuate

The Future of Shanghai

- If a certain number of people apply for a license plate per month, then the Shanghai government attempts to regulate the number of license plates by proportionally reducing the number available at auction
- Goal is to contain or reduce pollution, as forecasted for the next 2 years

Thank you!