$DFT(f(x)) = ?$ $\quad F(k) = \int f(x) e^{ikx} dx$ analytic expression

$$F(k)_{\text{on comp.}} = \sum_{0}^{N-1} f(x) e^{-2\pi i kx/N}$$

$$\text{IFT} \quad f(x) = \frac{1}{N} \sum_{0}^{N-1} F(k) e^{2\pi i kx/N}$$

$DFT(f(x+d))$ d: constant

$$\sum f(x+d) e^{-2\pi i kx/N}$$

$$x' = x+d$$

$$\Rightarrow \sum f(x') e^{-2\pi i k(x'-d)/N}$$

$$= \sum e^{2\pi i kd} f(x) e^{-2\pi i kx/N}$$

$$\Rightarrow e^{2\pi i kd} F(x)$$

$DFT(f(-x))$ if $f(x)$ is real?

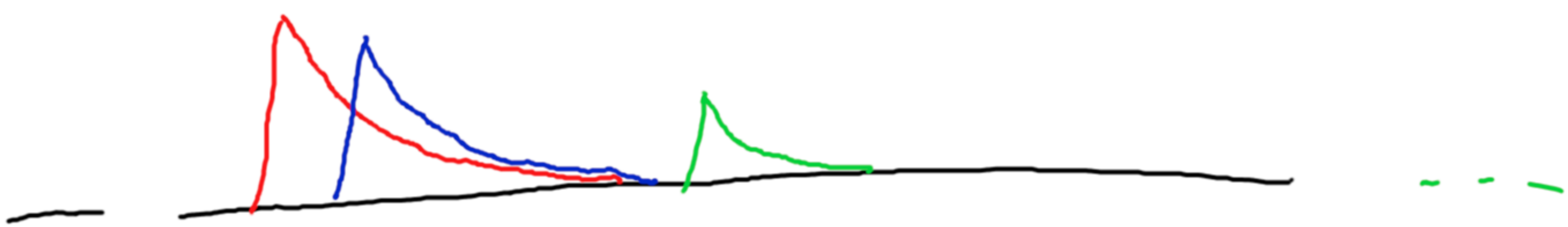
$$\Rightarrow F^*(k) \quad \left(F(k) = \sum f(-x) e^{-2\pi i kx}\right.$$

$$\Rightarrow f(x) e^{2\pi i kx}$$

$$= F^*(x)$$

have detector gets hit w/ particle, T increases

Newton's law of cooling $T(t) = T(t_0) e^{-q(t-t_0)}$ $\quad \frac{dT}{dt} = -kT$



$$\Rightarrow T(t) = \int H(t') e^{-(t-t')k}$$

$T(t) \Rightarrow$

$$= \int \underbrace{H(t')}_{0 \text{ for } t<t'} e^{(t'-t)C}$$

= convolution

$$h(y) = \sum_x f(x)\, g(y-x) \quad \Rightarrow h = g \circledast f$$

$$f(x) = \underline{IFT(F(k))} \qquad g(y-x) = \underline{IFT(G(\ldots}) \quad {}_{y-x)}$$

$$FT(y-x) = FT(x-y) = FT(x)\, e^{-2\pi i k y}$$

$f(y-x)$

$$= \overline{FT}(x+(-y)) = e^{\frac{2\pi i k y}{N}}\, \overline{FT}(x) = \overline{F}(k)\, e^{2\pi i k y / N}$$

$f(x)$

$$h(y) = \sum_x \frac{\sum_k F(k)\, e^{\frac{2\pi i k x}{N}}}{N} \; \frac{\sum \overline{G}(k')\, e^{2\pi i k' y}\, e^{2\pi i k' x / N}}{N}$$

$$= \frac{1}{N^2} \sum_k \sum_{k'} F(k)\, \overline{G}(k')\, e^{2\pi i k' y} \underbrace{\sum_x e^{2\pi i \frac{(k+k')x}{N}}}_{} = N\delta(k+k')$$

$$\Rightarrow k' = -k$$

$$\frac{1}{N^2}\sum_k F(k)\, G(-k)\, e^{-2\pi i k t/N}\, N$$

$$\frac{1}{N}\sum_h F(h)\, G(h)\, e^{2\pi i h t/N}$$

$$\Rightarrow \text{ifft}\left(F(h)\, G(h)\right)$$

$$h = f \otimes g$$

$$H = F \cdot G$$

integral at each t is done by simple product in Fourier space

$$F(k) = \sum_{0}^{N-1} f(x)\, e^{-i2\pi kx/N} \quad \Rightarrow \text{do for each } k$$

$\simeq N^2$ multiplies, exponentials ... —

10^8 data points = how long would DFT take?

$\simeq 10^{16}$ operations $\simeq 10^{17}$ flops ~ clock speed × few ×? # of cores

my laptop on good day 10^{11} $2.5 \times 10^9 \cdot 8 \cdot 8$

$10^{17}/10^{11} = 10^6$ = 2 weeks

$$F(k) = \sum f(x)\, e^{-2\pi i k x/N}$$

$$= \sum_{x \text{ even}} f(x)\, e^{-2\pi i k x/N} + \sum_{\text{odd}} f(x)\, e^{-2\pi i k x/N}$$

$x = 2x$ $\qquad$ $x = 2x+1$

$$e^{-2\pi i k (2x+1)/N}$$

$$= \sum f_e\, e^{-2\pi i k x/(N/2)} + \underbrace{\sum f_o\, e^{-2\pi i k x/(N/2)}}$$

$$= \underbrace{e^{-\frac{2\pi i k}{N}}}_{\text{twiddle factors}} \sum f_{odd}\, e$$

$FFT(N) \Rightarrow 2\,FT(N/2) \Rightarrow 4\,FT(N/4) \Rightarrow 8\,FT(N/8)$

$FT(1^{\#}) = \#$ $\Rightarrow \log_2(N)$

$\log_2(N)$ steps $\Rightarrow$ N data points

total work $= N \cdot \log_2(N)$

$10^8 = 2^{27} = 10^{10}$ op.

Powers of 3, thirds of data,

third = 1, first group, $e^{-2\pi i k/N}$

$n = 0^{N/3}$

third group $e^{\frac{-2\pi i\, 2k}{2}}$

$n_0 = N/3$

Twid $e^{-2\pi i k/N}$

twid$(N/2 \to N) = -$ twid$(0 \to N/2)$

$k < \frac{N}{2}$ $e^{-2\pi i k/N}$

$k > N/2$: $\Rightarrow$ $k \to k + N/2$

$\Rightarrow$ $= e^{-2\pi i (k' + N/2)/N}$

$k = k' + N/2$ $= e^{-2\pi i N/2 /N} e^{-2\pi i k'/N}$

$= e^{-i\pi} e^{-2\pi i k'/N}$

$= -1 \Rightarrow -e^{2\pi i}$