n bits, m 0's $n-m$ 1's

How many unique orderings of m 0's, $n-m$ 1's?

$$= \binom{n}{m} = \frac{n!}{(n-m)!\, m!} \approx \frac{\left(\frac{n}{e}\right)^n}{\left(\frac{n-m}{e}\right)^{n-m}\left(\frac{m}{e}\right)^m} \qquad n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

drop e's

$$\frac{n^n}{(n-m)^{n-m}\, m^m} = \frac{1}{\left(1-\frac{m}{n}\right)^{n-m}\left(\frac{m}{n}\right)^m}$$

$$\log_2(\#\text{ways}) \approx -(n-m)\log\left(1-\frac{m}{n}\right) - m\log\left(\frac{m}{n}\right)$$

$$\frac{\log_2(\#\text{ways})}{n\text{-length}} = -\left(1-\frac{m}{n}\right)\log\left(1-\frac{m}{n}\right) - \frac{m}{n}\log\left(\frac{m}{n}\right) \to -(p_1)\log(p_1) - p_0\log(p_0)$$

$$p_0 = \frac{m}{n},\quad p_1 = \frac{n-m}{n}$$

for 2 things $\frac{\log(\#\text{ possible messages})}{\text{length}} = -P_0 \log(P_0) - P_1 \log P_1$

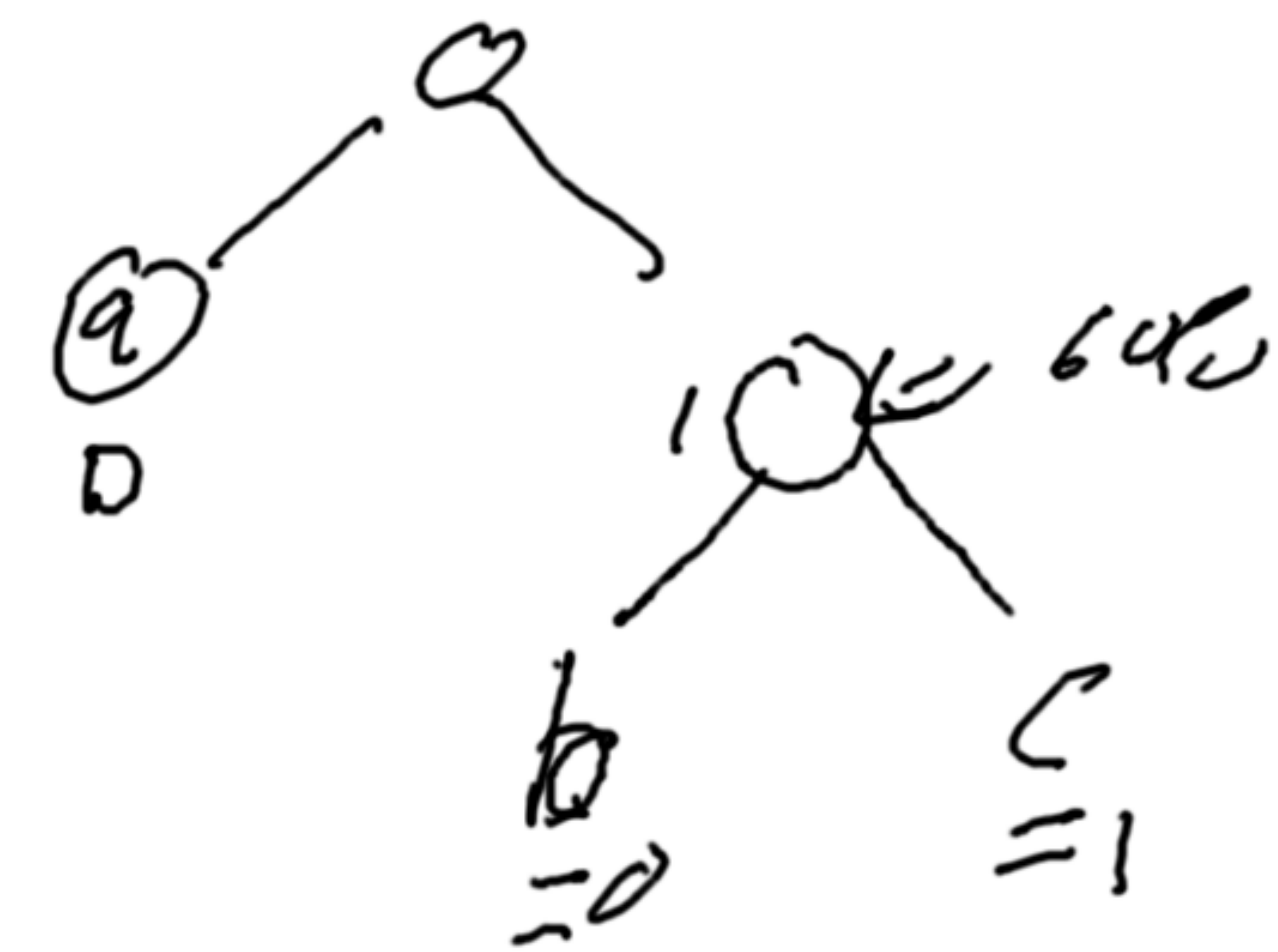
for many things $\Rightarrow$ $-\sum P_i \log_2(P_i) = S$, Shannon Entropy

sort messages, can tell you which with # of length $\log_2(\#\text{ messages})$

So # of bits I need per "thing" is S.

3 letters : a = 40%
b = 35%
c = 25%

a = 50%
b = 25%
c = 25%



0 = a

10 = b

11 = c

\# bits is 1 40%

2 60%

so $.4 \cdot 1 + .6 \cdot 2 = 1.6$ bits/letter

$-\sum (.4, .35, .25) \times \log_2 (.4, .35, .25) = 1.55$

so in this case Huffman did $\frac{1.55}{1.6} \approx 1.03$

within 3% of ideal

$$\begin{pmatrix} d \\ \hline ? \end{pmatrix}^T (N^{-1}) \begin{pmatrix} d \\ \hline ? \end{pmatrix} = \chi^2$$

most likely value for that point is the one that minimizes χ^2

$$\frac{\partial}{\partial \text{point}} (\chi^2) = (0\ 0\ 0\ 0 \ldots 0\ 1)\,(N^{-1}) \begin{pmatrix} d \\ \vdots \\ \hline ? \end{pmatrix} = 0$$

$$(N^{-1}_{\text{last row}}) \begin{pmatrix} d \\ \vdots \\ \hline ? \end{pmatrix} = 0 \qquad \sum N^{-1}_{(-,\, n-1)} \cdot d\begin{pmatrix} 1 \\ \vdots \\ n-1 \end{pmatrix}$$

$$+ N^{-1}_{(\text{end},\,\text{end})} \cdot d_{\text{last}} = 0$$

$$\boxed{d_{\text{est}} = -\sum w_i d_i, \quad w_i = \frac{N^{-1}_{-1,\, i}}{N^{-1}_{-1,-1}}}$$

$N = ?$ we know $N_{ij} = \frac{1}{n}\sum \langle d_i d_j \rangle$

if we assume stationarity

$N_{11} = \langle d_i^2 \rangle$, $N_{01} = \langle d_i d_{i+1} \rangle \cdots$

 $\Rightarrow$ Toeplitz