Stationary noise: if noise does not depend on time, only difference of times, it is diagonal in fourier space

$$\langle |F(k)|^2 \rangle = FT(corr(x))$$

examples: white noise, random walk

Matched filter $A(x-\delta)^T N^{-1} A(x-\delta)\, m = A^T(x-\delta) N^{-1} d$

if N is stationary, we can do this for all possible δ via FT.

white noise : $\langle n_i \rangle^2 = N$

$\langle n_i n_j \rangle = 0$

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random walk :

$\langle (x_i - x_j)^2 \rangle = c|i-j|$

$N = \langle n_i n_j \rangle$

$\langle (x_i - x_j)^2 \rangle = c|i-j|$

$\langle x_i^2 - 2x_i x_j + x_j^2 \rangle = c|i-j|$

$\langle x_i^2 \rangle - 2\langle x_i x_j \rangle + \langle x_j^2 \rangle = c|i-j|$

for stationary, $Var(x_i)$ is independent of i. Random walks starting from 0 is not stationary because $Var(x_i) = ci$

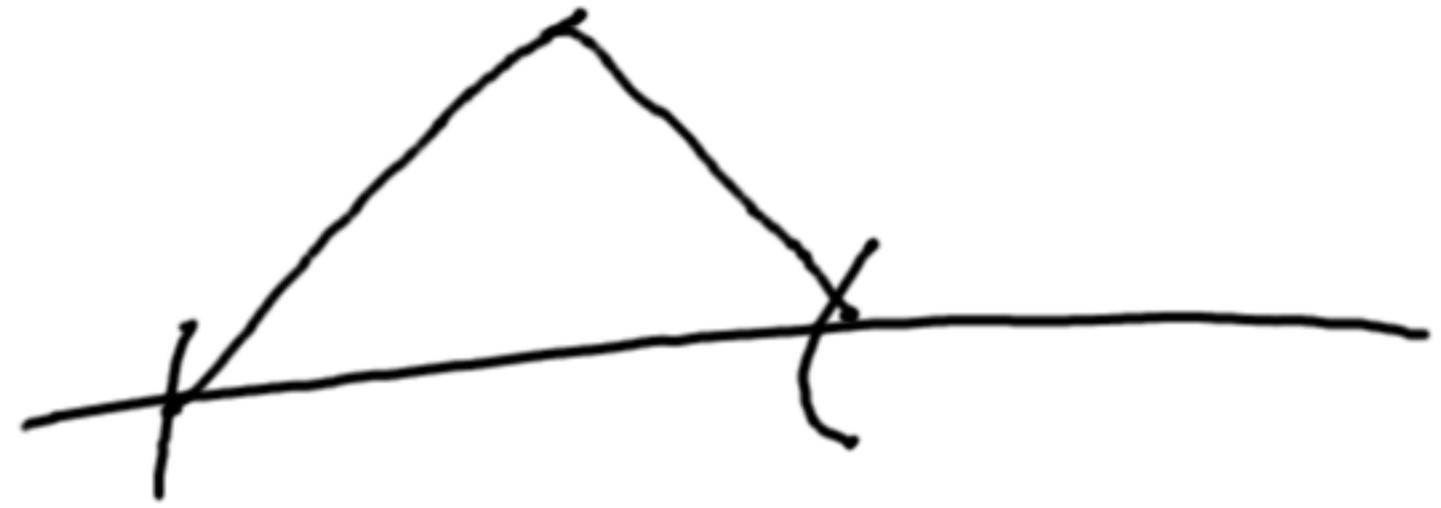
we can take $\langle x_i^2 \rangle = \langle x_j^2 \rangle$,

$\langle x_i^2 \rangle = \langle x_j^2 \rangle = N$

$N - 2\langle x_i x_j \rangle + N = c|i-j|$

$\Rightarrow \langle x_i x_j \rangle = N - \frac{c}{2}|i-j|$

we probably want $N > |i-j|c$

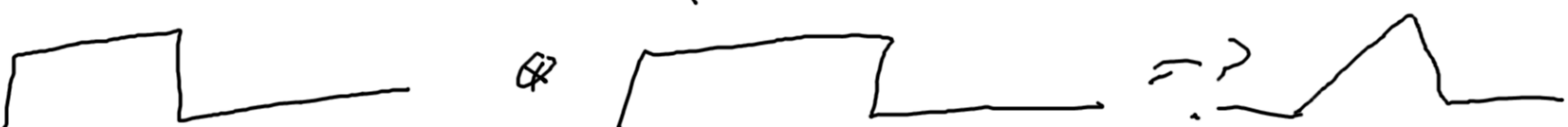
$N \sim \frac{c|i-j|}{2} \Rightarrow$ 

how could we generate
a random walk in fourier space?

$\langle |f(k)|^2 \rangle = FT(g) = FT(\wedge)$ for random walk:

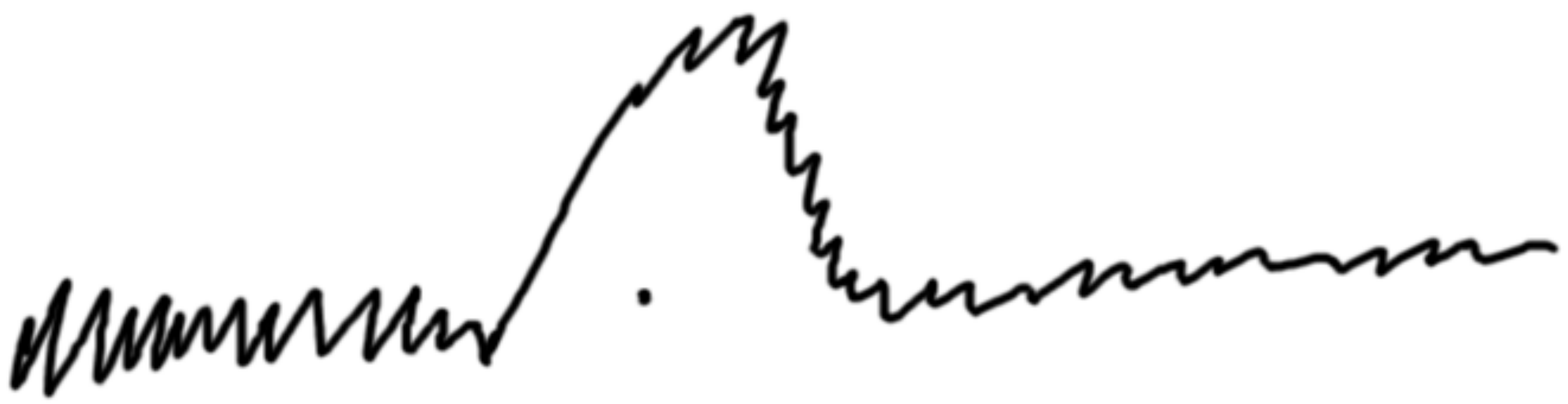
$$FT(\text{boxcar}) = \int_0^a e^{-ikx}\,dx = \frac{1}{-ik} e^{-ikx}\Big|_0^a$$

$$= \frac{i}{k}\left(e^{-ika} - 1\right) = -\frac{i}{k}\left(1 - e^{-ika}\right)$$



$\sim \frac{1}{k^2}$ $\quad \langle |FT(k)|^2 \rangle = \frac{1}{k^2}$, $FT(k) \sim \frac{1}{k}$ random #

Matched filter: there might be a signal in data
where is it?



best-fit amplitude as function of template shift

$$A^T N^{-1} A m = A^T N^{-1} d$$

$$\Rightarrow A(t-\delta)^T N^{-1} A(t-\delta) m = A^T(t-\delta) N^{-1} d$$

if we assume N is stationary

how does $A(t-\delta)^T N^{-1} A(t-\delta)$ depend on δ?

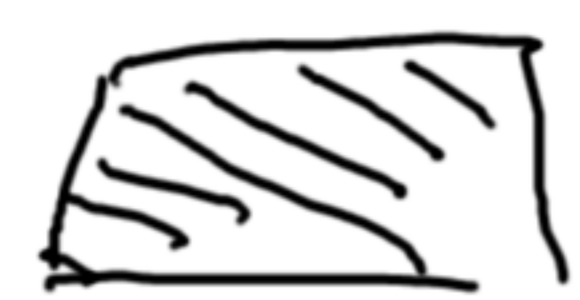
$t' = t - \delta$

$\Rightarrow A(t') N(t'+\delta) A(t') m$

$N(t'+\delta) = ?$ if stationary

$\Rightarrow N(t'+\delta) = N(t')$

(or $N_{ij} = f(i-j)$)



$A^T N^{-1} A$ independent of δ

so it's just a # (for 1 param)

$A^t(t-\delta)\, N^{-1} d(t)$

$\underbrace{(N^{-1} A(t-\delta))}^T d(t)$

$= NA^T \cdot d(t)$

$= \sum_t NA(t+\delta)\, d(t)$ = how do we do this as function of δ?

$= \text{Corr}(NA, d)$ $\quad NA = N^{-1}A$

$= \text{Corr}(A, Nd)$ $\quad Nd = N^{-1}d$