$V = \frac{1}{r}$, $F = \frac{1}{r^2}$

uniform ρ : $V = ?$

$F(r<R) =$

$m = \rho r^3$

$G = \rho \frac{r^3}{r^2} \sim \rho r$

outside $F = \frac{GM}{R^2}$

$F_{in} = \frac{GMr}{R^3}$

$F_{out} = \frac{GM}{r^2}$

$a \to 0$ as $r \to 0$

Good! nothing goes crazy!

Softening: replace force law with one that

goes to $\frac{1}{r^2}$ for large r

goes to 0 for $r \to 0$

$$f = \frac{\vec{r}}{|r^3|} \Rightarrow \frac{\vec{r}}{r^3 + \epsilon}$$

$$\Rightarrow \frac{\vec{r}}{|r^3|}, \; r \geq R, \quad \frac{\vec{r}}{R^3} \; r < R$$

$$a \sim \frac{\vec{r}}{|r^3|} \Big/ \frac{\vec{r}}{|R^3|}$$ = max where?

max when $r = R$

$$a_{max} = \frac{1}{R^2}$$

$V_{max} = ?$

$V(r > R) \sim \frac{1}{r} \Rightarrow \frac{1}{R}$ at edge

$\delta V(r < R) \sim \frac{1}{2}R$ at edge to center

$V \sim \frac{1}{2}v^2 \Leftarrow \frac{3}{2}R$

a_{max}, V_{max}

$t \sim V_{max}/a_{max}$

$r \sim .01$

$a_{max} \sim \frac{1}{r^2} \sim 10^{-4}$

$v_{max} \sim \frac{1}{r^{1/2}}$

$t \sim \frac{v_{max}}{a_{max}} \sim r^{-3/2} \sim 10^{-3}$

Virial theorem: relation between U, T, P set by force law

for $f = \frac{1}{r^2} \Rightarrow T = -\frac{1}{2} P$

$P = P_0 = U,$
$T = 0$

$\Rightarrow$ let collapse

in equilibrium

$T_e + P_e = T_{i} + P_i = U$

$-\frac{1}{2} P_e = -P_{in}$

$P_{eq} \sim 2 P_{in}$

$P \sim \frac{3}{r}$

$\Rightarrow R_{eq.} \simeq \frac{1}{2} R_{in}$

how much work does it take to calculate forces? every particle sees every other particle. so n particles : n ops to get force on 1

$\sim$ force on n particles $\sim n^2$. $\sim \frac{n^3}{2}$

10^9 particles : how long to do forces? $\frac{10^{18}}{2} \sim$ 1 ns / square root

$\sim \frac{1}{2} \times 10^9$ sec / core $\sim$ 15 years

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what is grav potential everywhere in space?

$\rho(r) \Rightarrow$ pot? $V(r) = \int \frac{\rho(r')}{|r-r'|} d^3r'$

$g = \frac{1}{r} \Rightarrow \int \rho(r') g(r-r') d^3r' \Rightarrow \rho \otimes \frac{1}{r}$

10^9 particles $\Rightarrow$ 10^9 gridcells

how many ops to calculate V?

$n \log_2(n) \cdot 2 \cdot 4 \simeq 3 \times 10^{10}$ $\approx$ few seconds (if RAM)

gone from 15 yrs to get forces to 10 seconds to get potential!

forces come from $\vec{\nabla} V$

 ⇐ grid ρ

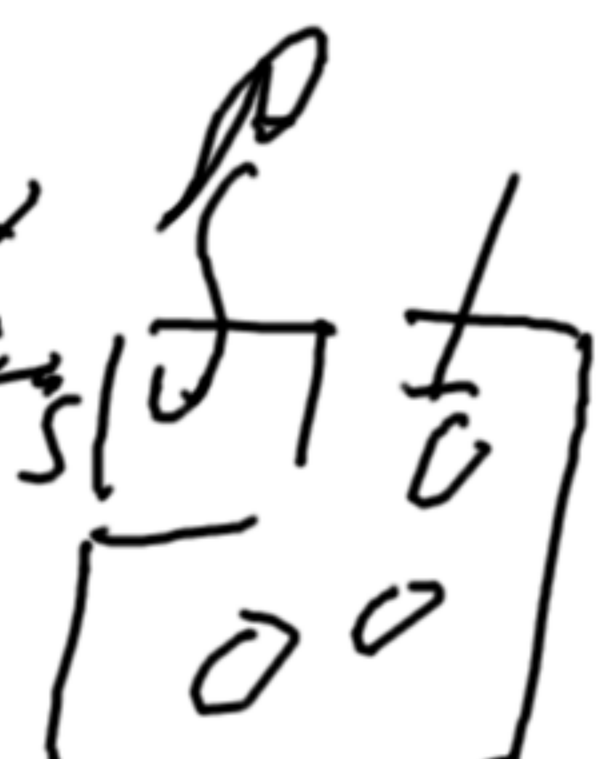
 ⇐ 1/r

n.p. collaps

ρ ⇒ no macroscopic collapse, so dense regions will collapse.

FFT's wrap around so particles on 1 edge feel other edge strongly
⇒ periodic boundary conditions

No wrap convolution
⇒ non-periodic Boundary conditions



Steps to do particle-mesh (particles with forces derived from a mesh)

grid particles onto 2/3D-grid to get ρ

an important test is 1 particle - does it stay put?

$FFT(\rho) \times FFT(\frac{1}{r})$

$IFFT(\rho \cdot \frac{1}{r}) \Rightarrow V$

$\nabla P \Rightarrow f$, leapfrog step

$\frac{1}{r} \Rightarrow$ softened $\frac{1}{r}$

Force a particle exerts on self?
want this $= 0$