$$\chi^2 = (d-Am)^T N^{-1} (d-Am)$$

$Q: N = QQ^T \Rightarrow N^{-1} = (QQ^T)^{-1} = Q^{T-1} Q^{-1}$

$(d-Am)^T Q^{T-1} Q^{-1} (d-Am)$

$= (Q^{-1}(d-Am))^T (Q^{-1}(d-Am))$

$\Rightarrow \tilde{d} = Q^{-1} d$

$\tilde{A} = Q^{-1} A$

$(\tilde{d} - \tilde{A}m)^T (\tilde{d} - \tilde{A}m)$

replace data/model with new versions

that have $N = I$

$Q \Rightarrow \text{ifT}\,\sigma\ \text{fT}$

$N^{-1} = \text{ifT}\,\sigma^{-2} \cdot \text{fT} \quad \text{ifT}^T = \text{fT}$

$Q = \text{ifT}\,\sigma\,\text{fT}$

$Q^T = \text{ifT}\,\sigma\,\text{fT}$

Q is symmetric in this case

$d \Rightarrow$ dat fT, divide by $\sqrt{N}$ (σ = std dev)

$\Rightarrow$ transform back

d scaled by frequency space noise

fT operator can be a matrix

$y = \text{fT}(x) \Rightarrow y = Ax$ for special A that is like fT

$$\chi^2 = (d - Am)^T \, \mathrm{ift} \, \sigma^{-2} \, \mathrm{FT} \, (d - Am)$$

$$= (\mathrm{FT}(d - Am))^T \sigma^{-2} (\mathrm{FT}(d - Am))$$

$$= (\tilde{d} - \tilde{A}m)^T \sigma^{-2} (\tilde{d} - \tilde{A}m)$$

$$= \left(\frac{\tilde{d}}{\sigma} - \frac{\tilde{A}}{\sigma} m\right)^T \left(\frac{\tilde{d}}{\sigma} - \frac{\tilde{A}}{\sigma} m\right)$$

$$\chi^2 \text{ contribution} = \left(\frac{\tilde{A}}{\sigma}\right)^2$$

$$\tilde{d} = \mathrm{FT}(d)$$

$$\tilde{A} = \mathrm{FT}(A)$$

$$\mathrm{FT} = \frac{1}{\sqrt{N}} \sum f(x) e^{-2\pi i kx/N}$$

$$\mathrm{ifT} = \frac{1}{\sqrt{N}} \sum f(k) e^{2\pi i kx/N}$$

= np.fft.fft(norm='ortho')

---

usual norm, $\mathrm{ift} = \frac{1}{N} (\mathrm{FT})^T$

N = points

$= m = (A^T N^{-1} A)^{-1} (A^T N^{-1} m)$

$Var(m) = (A^T N^{-1} A)^{-1}$

$\langle m m^T \rangle$

$\sigma_m = \sqrt{diag (A^T N^{-1} A)^{-1}}$, 1 param $= \frac{1}{\sqrt{A^T N^{-1} A}}$

prewhitened case, $\sigma_m = \frac{1}{\sqrt{A^T A}}$

$N \to \frac{n(n+1)}{2}$ distinct dof's

$Q = n^2 \approx$ lots of ways to pick Q given N

$QQ^T = N$. cholesky $= LL^T = N$, $Q = cholesky(N)$

Q symmetric $\Rightarrow V \Lambda^{\frac{1}{2}} V^T \Rightarrow Q^T Q = N$

Stationary noise: N can be factored in $(FT)^\dagger \hat{N} (FT)$ — eigendecomp of N

matrix — diagonal

$$f(k) = \sum f(x)\, e^{-2\pi i k x/N}$$

$$= \begin{pmatrix} e^{-2\pi i k x/N} \\ e^{-2\pi i k' x/N} \end{pmatrix}^T \begin{pmatrix} f \end{pmatrix}$$

pdf(radioactive decay) = $e^{-t/\tau}$ ⟸ mean lifetime

prob(decay between t & t+dt) = $dt\, e^{-t/\tau}$

simulate radioactive decay

prob(decay) sometime = 1 $\Rightarrow \int_0^\infty e^{-t/\tau} = \tau e^{-t/\tau}\Big|_0^\infty = \tau$

$\Rightarrow \text{pdf} = \frac{1}{\tau} e^{-t/\tau}$

CDF = prob. particle has decayed by time T

$= \int_0^t \text{pdf}(t)\,dt = \text{CDF} = \int_0^t \tau e^{-t/\tau} \frac{q}{\tau} = e^{-t/\tau}\Big|_0^t$

$= 1 - e^{-t/\tau}$

1.

I know prob in CDF is uniform on [0, 1]

$$CDF = 1 - e^{-t/\tau} = \text{random \# } r$$

$$1 - e^{-t/\tau} = r \Rightarrow 1 - r = e^{-t/\tau}$$

$$-t/\tau = \ln(1-r)$$

$$t = -\tau \ln(1-r)$$

if $CDF = 1 - x^{q+1} = r$

$$x^{q+1} = 1 - r$$

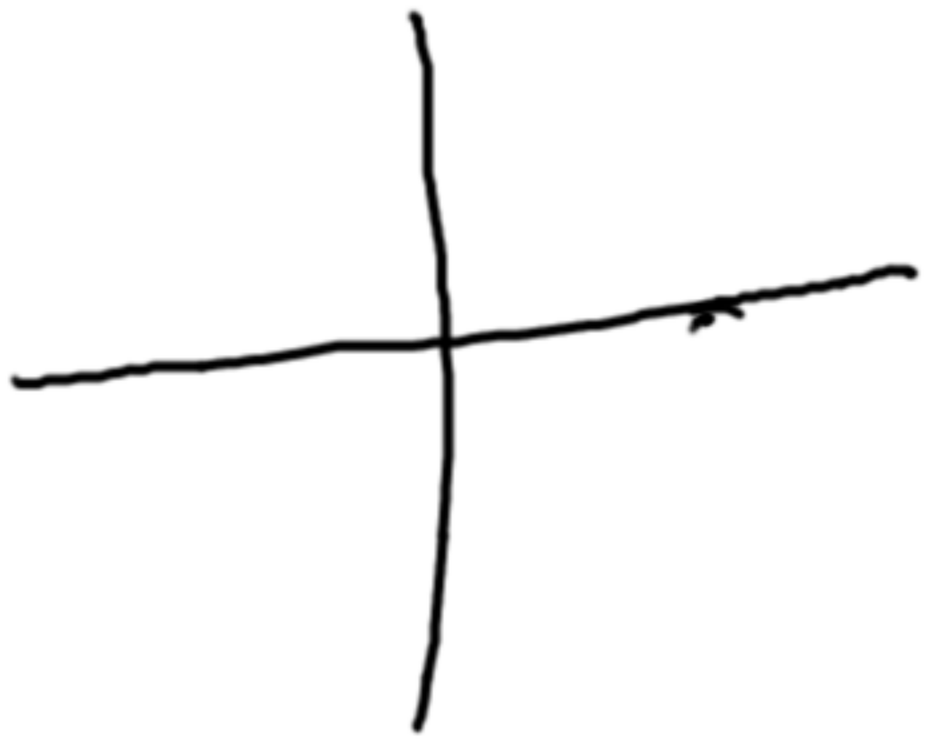
$$x = (1-r)^{1/(1+q)}$$

$$p(x) = x^q. \qquad \text{usually } q < 0$$

$$x \geq 1$$

$$CDF = \int_1^{\infty} x^q = \frac{x^{q+1}}{q+1}\Big|_1^{\infty} = 0 - \frac{1}{q+1} \quad \text{if } q < -1$$

$$\Rightarrow PDF = -(1+q)x^q \Rightarrow CDF = 1 - x^{q+1}$$



$$pdf(x) = e^{-x^2/2}$$

$$pdf(y) = e^{-y^2/2}$$

$$pdf(x,y) = pdf(x)\cdot pdf(y) = e^{-\frac{x^2+y^2}{2}} = e^{-r^2/2}$$

$$cdf(r) = \int e^{-r^2/2}\, r\,dr \Rightarrow \begin{array}{l} u = r^2/2 \\ du = \frac{2r}{2} dr = r\,dr \end{array}$$

$$\Rightarrow \int e^{-u}\,du = -e^{-u}\Big|_0^r = 1 - e^{-r} = rand$$

$$e^{-r} = 1 - rand$$

$$r = -\ln(1-rand)$$

$$\Rightarrow x \Rightarrow (x,y) \Rightarrow r(x,y)$$

Box-Muller transform