$\nabla x = f(x) - f(x-dx)$ for advection

or $f(x+dx) - f(x)$

$$\nabla x = \frac{f(x+dx) - f(x-dx)}{2}$$

terra incognita

$f(t,x) = e^{ikx}$

$f(t+dt, dx) = e^{ikx}\left(1 - \alpha\left(1 - e^{-ikx}\right)\right)$ for upwind derivative

upwind = using myself + cell flows into me

switch to downwind $\Rightarrow e^{ikx}\left(1 + \alpha\left(1 - e^{+ikx}\right)\right)$

$\Rightarrow$ unconditionally unstable

Centered deriv $= \frac{1}{2}$ (upwind + downwind)

<u>only use upwind derivatives!</u>

$p = nkT$

$E = a n k T$ $= \frac{3}{2}$ for ideal gas

$p = qB$, $F = \frac{p}{q}$

$\gamma = 1 + \frac{1}{a}$ $\left(= \frac{5}{3} \text{ for } a = \frac{3}{2}\right)$

$\gamma a = a + 1$ $= \frac{7}{5}$ for $a = \frac{5}{2}$

$\gamma - 1 = \frac{1}{a}$

$E = (\gamma - 1) p$

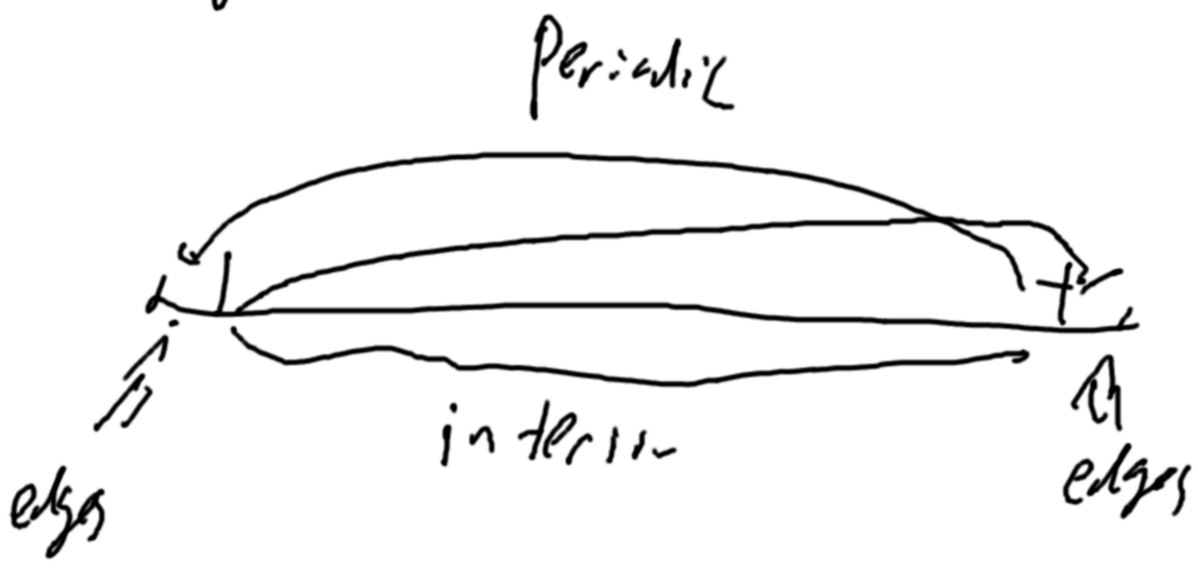
$a = \frac{1}{\gamma - 1}$

$E = aP$

$P = \frac{E}{\gamma - 1}$

$\frac{1}{a} = \gamma - 1$

$= \frac{P}{\gamma - 1}$, $P = (\gamma - 1) E$



smooth at boundary
but <u>not</u> periodic

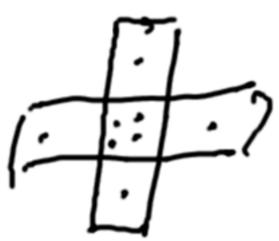
$\nabla^2 V = 0$ $\Rightarrow$ transform to code?

$\Rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \dots = 0$

$\frac{\partial^2 V}{\partial x^2} = ?$ $\quad \frac{\partial V}{\partial x_r} = \frac{f(x+dx) - f(x)}{dx}$ $\quad \frac{\partial V}{\partial x_l} = \frac{f(x) - f(x-dx)}{dx}$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{f_{right} - f_{left}}{dx} = \frac{\frac{f(x+dx)-f(x)}{dx} - \frac{f(x)-f(x-dx)}{dx}}{dx}$$

$$= \frac{f(x+dx) - 2f(x) + f(x-dx)}{dx^2}$$



$\Rightarrow$ in 2d, $\nabla^2 V = V_{up} + V_{down} + V_{left} + V_{right} - 4V_0 = 0$

$\Rightarrow$ if $\rho = 0 \Rightarrow V_0 = \frac{V_a + V_b + V_c + V_d}{4} = \overline{V_n}$

$\nabla^2 v = 0$, then $v_j = \overline{v}_{\text{circle centered on } v_j}$

simple algorithm: replace all points by average of neighbors, stop when things "settle down"

$\Rightarrow$ method of relaxation

in practice, only replace interior points by average, boundary cells have fixed v