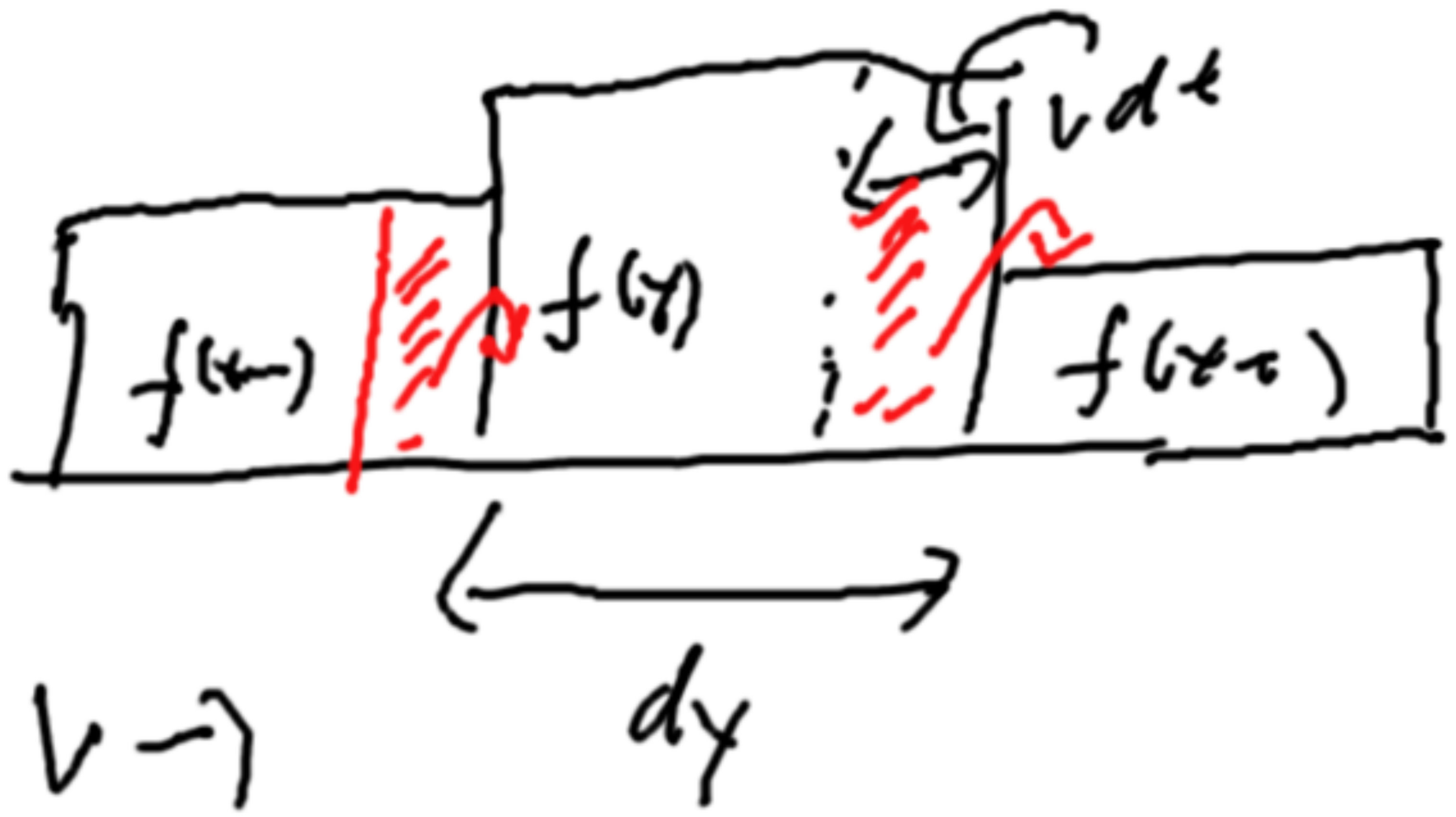

come back after dt time
what is the new density?

new = old − what left + what came in

in time dt, everything moved $v\,dt$ to right
so fraction that left is $\frac{v\,dt}{dx}$

so $f(x, t+dt) = f(x,t) - v\frac{dt}{dx}f(x,t) + v\frac{dt}{dx}f(x-dx,t)$

$v\frac{dt}{dx} = \alpha$

flowing out on right $= \rho_r v_r$

flowing in on left is $\rho_l v_l$

$$\frac{d\rho}{dt} = -\frac{\partial}{\partial x}(\rho v)$$

or in 3d: $\dfrac{\partial \rho}{\partial t} = -\nabla \cdot (\rho v)$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0}$$ $\Rightarrow$ conservation of ρ

advection: v is constant

so solving $\dfrac{\partial \rho}{\partial t} + v\, \nabla \cdot \rho = 0$

in 1-d: $\dfrac{\partial \rho}{\partial t} + v \dfrac{\partial \rho}{\partial x} = 0$

$$f_{x,t+dt} = f_{x,t}(1-\alpha) + \alpha f_{x-dx,t}$$

$f(x) = e^{ikx}$

$$f_{t+dt} = e^{ikx}\cdot(1-\alpha) + \alpha e^{ik(x-dx)}$$

$$= (1-\alpha)e^{ikx} + \alpha e^{-ikdx}e^{ikx}$$

$$= e^{ikx}\left(1-\alpha+\alpha e^{-ikdx}\right)$$

$$= e^{ikx}\underbrace{\left(1-\alpha\left(\underline{1-e^{-ikdx}}\right)\right)}$$

if $|\ | > 1$, solution will grow exponentially

$1-e^{-ikdx}$ bounded by 0 (at $k=0, \ldots$)

2 (at $kdx = \pi, 3\pi \ldots$)

so real part always < 1

if real part < -1

also be unstable

take inside $= 2$ (largest limit)

$1-\alpha(2) < -1$ we have trouble

$2\alpha < -2$

$\alpha > 1$ in trouble

for our solver, $\theta > 1$ we will be unstable

$\theta = \frac{v\,dt}{dx}$ $\frac{v\,dt}{dx} > 1$ unstable

$$\boxed{\text{courant } \frac{v\,dt}{dx} \leq 1}$$ $\Rightarrow$ CFL condition

or, $dt \leq \frac{dx}{V}$ length of time to cross a cell is dx/V so need $dt <$ cell crossing time

$\frac{v\,dt}{dx} \Rightarrow$ courant ~~#~~

$$f(t+dt) = f(t)\left(1 - \theta\left(1 - e^{-\lambda dt}\right)\right)$$

$$= f(t)\left(1 - 1 + e^{-\lambda dt}\right) \quad \text{if } \theta = 1$$

$$= e^{-\lambda dt} f(t)$$

$$\frac{df}{dt} + v\frac{df}{dx} = 0 \Rightarrow \quad \text{take } \frac{df}{dx} = \frac{f(x) - f(x-1)}{dx}$$

plug in e^{ikx}

$$f_{new} = f_{old} - \frac{\alpha}{2}\left(e^{ik(x+\delta)} - e^{ik(x-\delta)}\right)$$

$$= e^{ikx}\left(1 - \frac{\alpha}{2}\left[e^{ik\delta} - e^{-ik\delta}\right]\right)$$

$$= e^{ikx}\left(1 - \frac{\alpha}{\cancel{2}}\left(i\sin(k\delta)\right)\right)$$

$$= \underbrace{e^{ikx}}_{1}\underbrace{\left[1 - i\alpha\sin(k\delta)\right]}_{\text{absolute value?}}$$