190F Foundations of Data Science

Fall 2018

Lecture 25

Center and Spread

Center and Spread

Questions

- How can we quantify natural concepts like “center” and “variability”?

- Why do many of the empirical distributions that we generate come out bell shaped?

- How is sample size related to the accuracy of an estimate?

Average

The Average (or Mean)

Data: 2, 3, 3, 9 **Average = (2+3+3+9)/4 = 4.25**

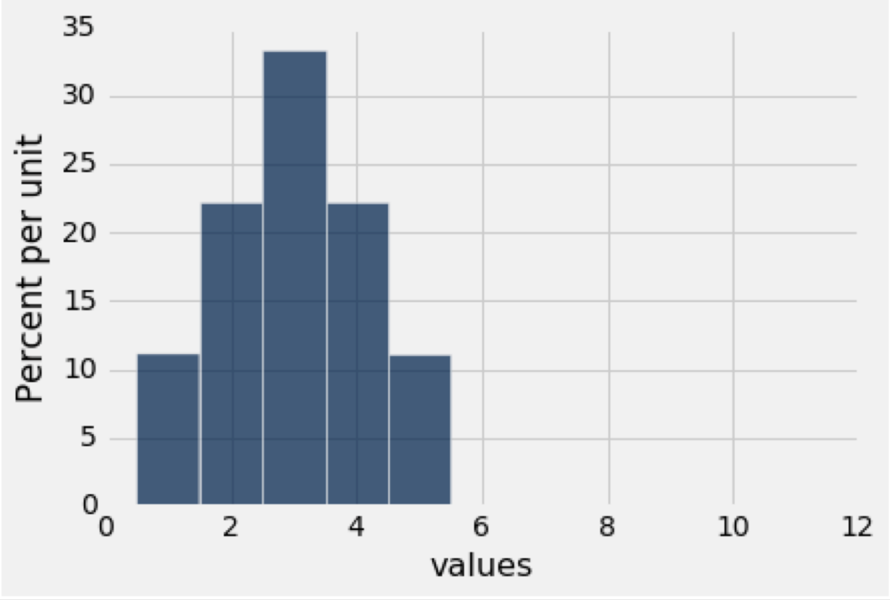
- Need not be a value in the collection
- Need not be an integer even if the data are integers
- Somewhere between min and max, but not necessarily halfway in between
- Same units as the data
- Smoothing operator: collect all the contributions in one big pot, then split evenly

(Demo)

Discussion Question

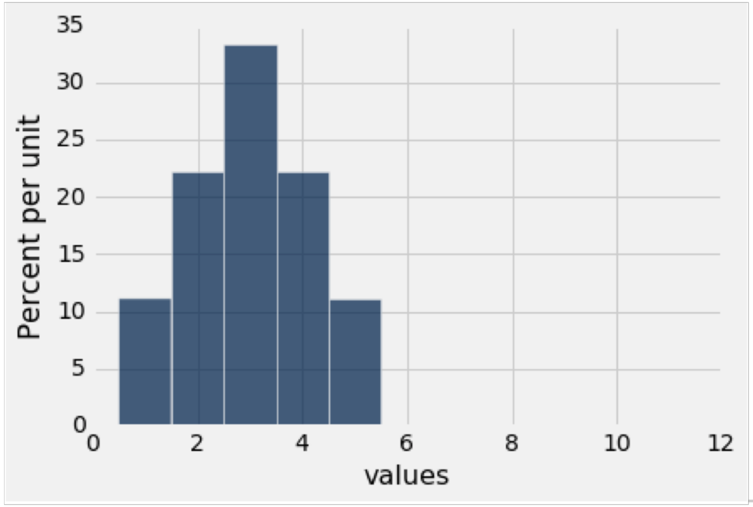
Create a data set that has this histogram. (You can do it with a short list of whole numbers.)

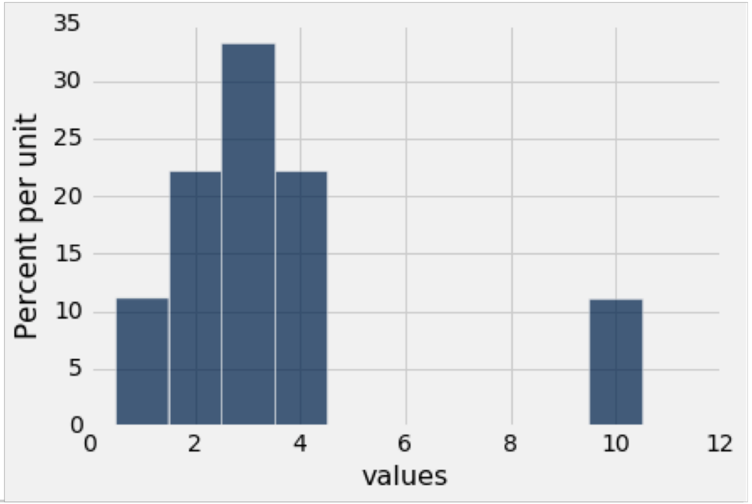
What are its median and mean?



Discussion Question

Are the medians of these two distributions the same or different? Are the means the same or different? If you say “different,” then say which one is bigger.





Comparing Mean and Median

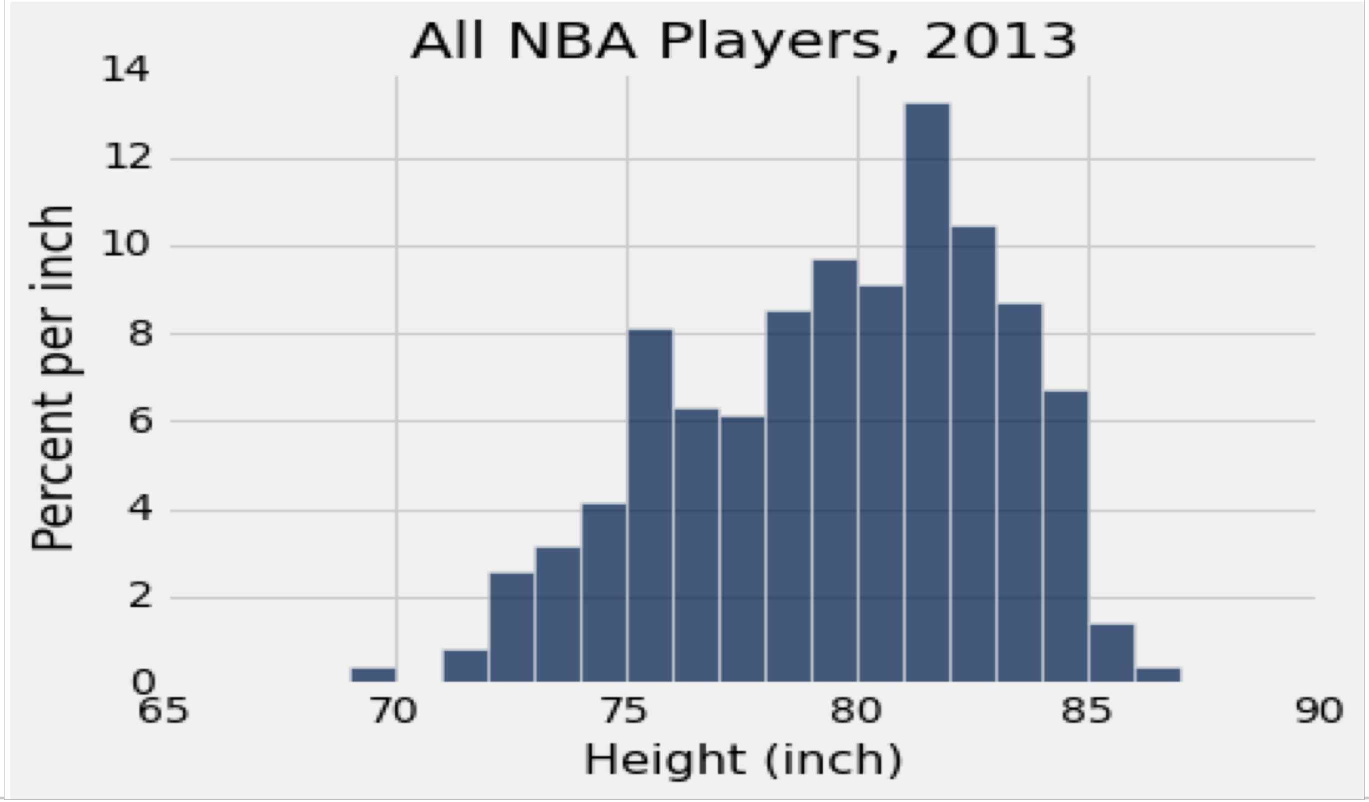
- **Mean:** Balance point of the histogram
- **Median:** Half-way point of data; half the area of histogram is on either side of median
- If the distribution is symmetric about a value, then that value is both the average and the median.
- If the histogram is skewed, then the mean is pulled away from the median in the direction of the tail.

Discussion Question

Which is bigger?

(a) mean

(b) median



Standard Deviation

Defining Variability

Plan A: “biggest value - smallest value”

- Doesn’t tell us much about the shape of the distribution

Plan B:

- Measure variability around the mean
- Need to figure out a way to quantify this

(Demo)

How Far from the Average?

- Standard deviation (SD) measures roughly how far the data are from their average

- SD = root mean square of deviations from average

  5 4 3 2 1

- SD has the same units as the data

Why Use the SD?

There are two main reasons.

- **The first reason:**

No matter what the shape of the distribution, the bulk of the data are in the range “average ± a few SDs”

- **The second reason:**

Coming up in the next lecture.

Chebyshev's Inequality

How Big are Most of the Values?

No matter what the shape of the distribution,
the bulk of the data are in the range “average ± a few SDs”

Chebyshev’s Inequality
No matter what the shape of the distribution,
the proportion of values in the range “average ± *z* SDs” is

at least $1 - 1/z^2$

Chebyshev's Bounds

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888…%)
average ± 4 SDs	at least 1 - 1/16 (93.75%)
average ± 5 SDs	at least 1 - 1/25 (96%)

No matter what the distribution looks like

Standard Units

Standard Units

- How many SDs above average?
- ***z* = (value - average)/SD**
  - Negative z: value below average
  - Positive z: value above average
  - z = 0: value equal to average
- When values are in standard units: average = 0, SD = 1
- Chebyshev: At least 96% of the values of *z* are between -5 and 5

(Demo)

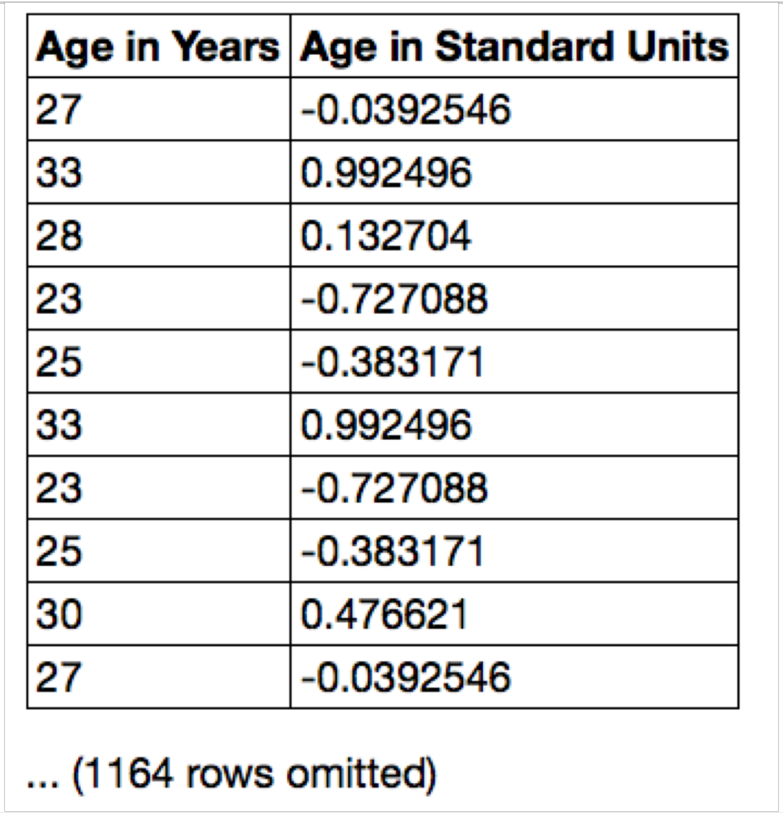
Discussion Question

Find whole numbers that are close to:

(a) the average age

(a) the SD of the ages

(Demo)



Age in Years	Age in Standard Units
27	-0.0392546
33	0.992496
28	0.132704
23	-0.727088
25	-0.383171
33	0.992496
23	-0.727088
25	-0.383171
30	0.476621
27	-0.0392546

... (1164 rows omitted)

The SD and the Histogram

- Usually, it's not easy to estimate the SD by looking at a histogram.

- But if the histogram has a bell shape, then you can.

The SD and Bell-Shaped Curves

If a histogram is bell-shaped, then

- the average is at the center
- the SD is the distance between the average and the points of inflection on either side

(Demo)

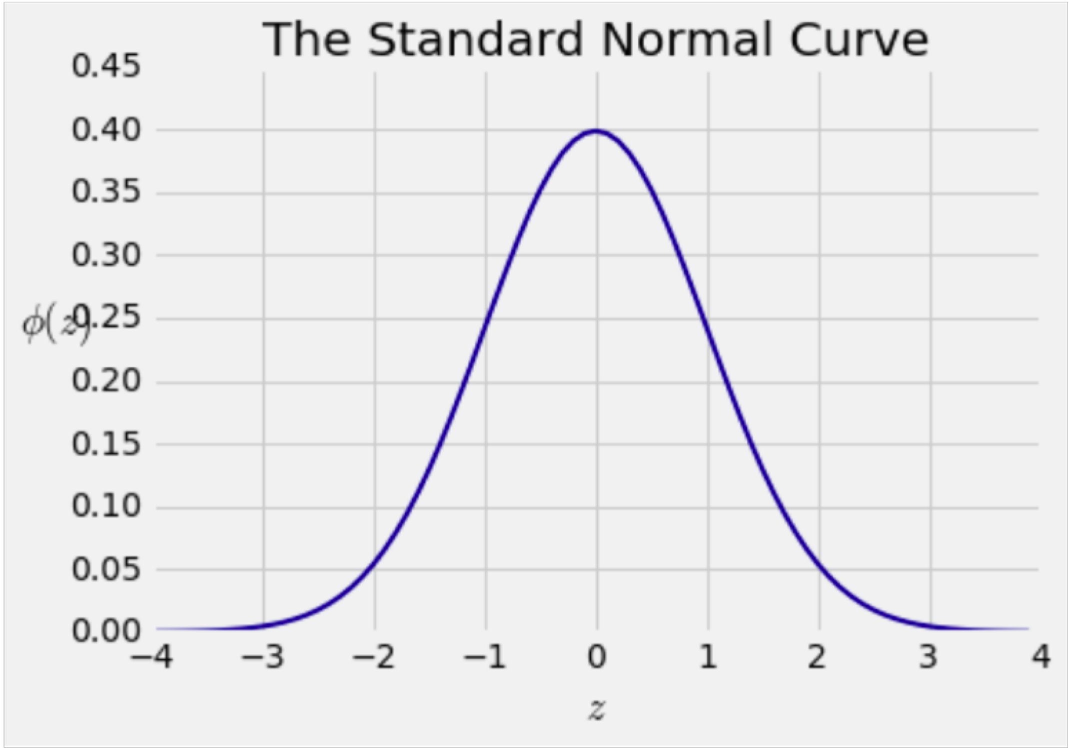
The Normal Distribution

The Standard Normal Curve

A beautiful formula that we won’t use at all:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

Bell Curve


The Standard Normal Curve
$\phi(z)$
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−4
−3
−2
−1
0
1
2
3
4
z

Normal Proportions

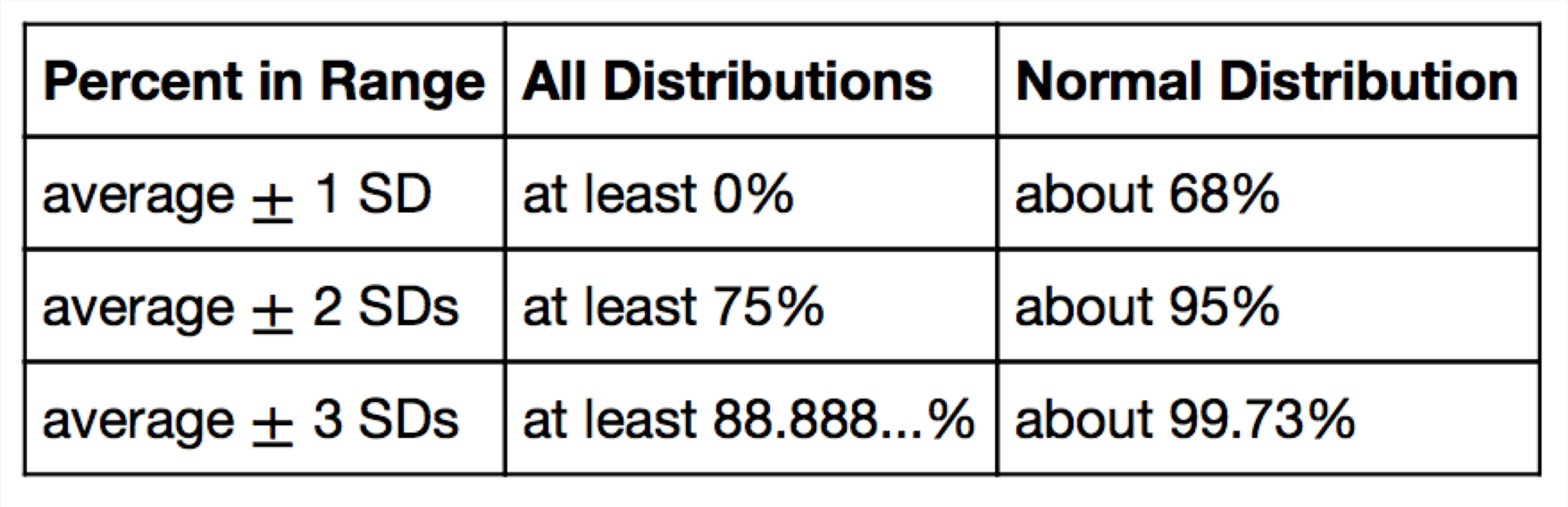
How Big are Most of the Values?

No matter what the shape of the distribution,
the bulk of the data are in the range “average ± a few SDs”

If a histogram is bell-shaped, then

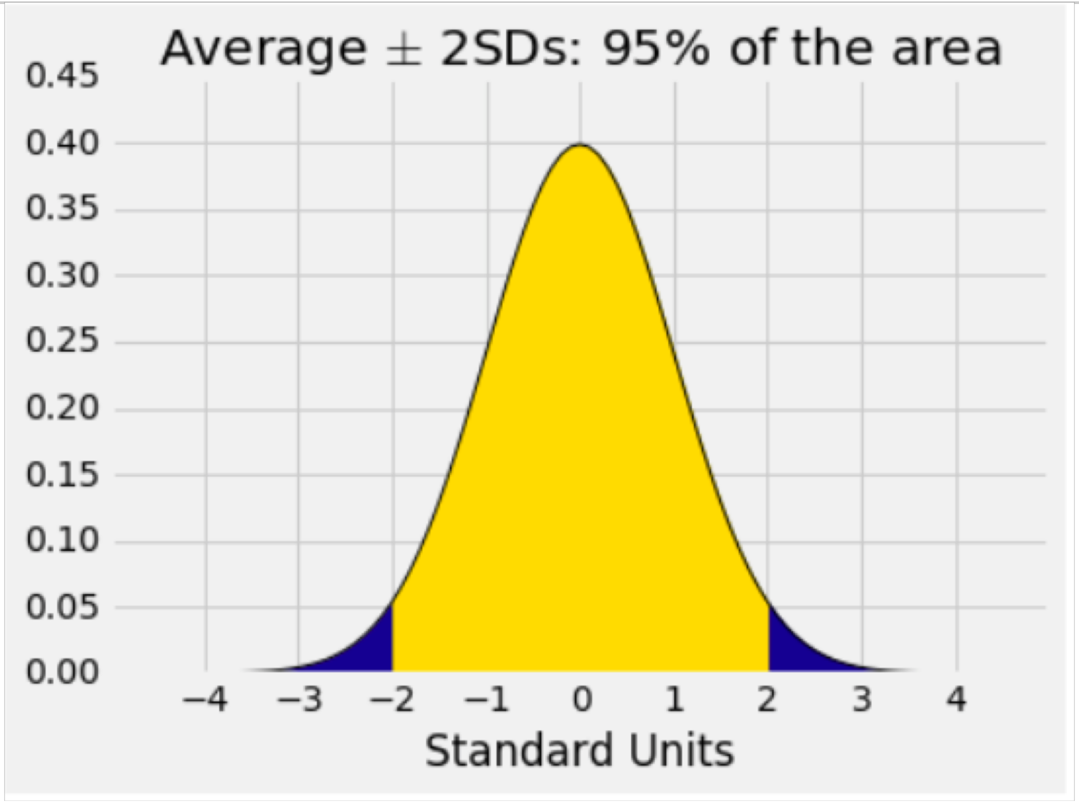
- Almost all of the data are in the range “average ± 3 SDs”

Bounds and Normal Approximations



Percent in Range	All Distributions	Normal Distribution
average ± 1 SD	at least 0%	about 68%
average ± 2 SDs	at least 75%	about 95%
average ± 3 SDs	at least 88.888...%	about 99.73%

A “Central” Area



Central Limit Theorem

Second Reason for Using the SD

If the sample is

- large, and
- drawn at random with replacement,

Then, *regardless of the distribution of the population,*

the probability distribution of the sample sum (or of the sample average) is roughly normal

(Demo)