Assignment 2 CMPUT 272 Winter 2012 Posted: January 25
Due before 10:00am, Wednesday February 15
in box labeled "CMPUT 272 Winter 2012" (opposite room CSC 145)

Write solutions in your own words. Include an acknowledgment statement citing (i) all reference materials (other than the textbook and handouts) which you used and (ii) persons with whom you have discussed your solution.

The following information must appear in the top right corner of the first page: CMPUT 272, Assignment 2, *your name*, and the *id* of your lecture section (B1, EB1, B2, or EB2). Please staple the pages together.

1. From Assignment 1, a Boolean function is called *symmetric* if it assigns the same value to all permutations of input values. E.g., a ternary symmetric Boolean function f satisfies $f(0,0,1) = f(0,1,0) = f(1,0,0)$ and $f(1,1,0) = f(1,0,1) = f(0,1,1)$.

   Define $1 > 0$ (if you interpreted them as elements, now they are numbers). A symmetric Boolean function is called *non-decreasing* if $f(x) \geq f(y)$ for two domain elements $x, y \in \times^n\{0,1\}$ whenever "the number of 1's in x" is greater than or equal to "the number of 1's in y". How many non-decreasing symmetric functions with n variables exist? Justify your claims.

   (*Note*: The assumption of symmetric is not needed for the counting — the answer is the same with or without the assumption.)

2. Let the domain of m, n be $\mathcal{Z}$. Determine the truth value of "$\forall m \forall n, (m^2 - n^2 > 0) \Rightarrow (m > n)$".

   Write the negation, converse, inverse, and contrapositive of this implication, and determine their truth values.

3. Write the negation, converse, inverse, and contrapositive of Goldbach's conjecture. Determine their truth values, or state the reason(s) you are not (or nobody is) able to.

4. Using logical notation state that

   > Assuming the Goldbach conjecture one can represent every number bigger than 12 as a sum of four primes none of which is used more than two times.

   An argument for this fact will be discussed in Lab exercise 3.

5. Let the domain of x, y, z be $\mathcal{R}$. Negate and simplify the following statements:

   (a) $\forall x \forall y, (x > y) \Rightarrow (x - y > 0)$

   (b) $\forall x \forall y, (x < y) \Rightarrow (\exists z, (x < z < y))$

   (c) $\forall x \forall y, (|x| = |y|) \Rightarrow (x = \pm y)$

6. Prove that if $n \in \mathcal{Z}^+$ is composite, then there exists a prime p such that $p \mid n$ and $p \leq \sqrt{n}$.

7. Find all $n \in \mathcal{Z}^+$ where $n \mid (5n + 18)$.

8. Let $m, n \in \mathcal{Z}^+$. Prove that if $m^2 \mid n^2$ then $m \mid n$.

   Is it true that if $m^2 \mid n^3$ then $m \mid n$? If your answer is yes, prove it; otherwise, give a counterexample.

9. Given $n \in \mathcal{Z}^+$, when does it have exactly (a) two positive divisors? (b) three positive divisors? (c) four positive divisors? (d) five positive divisors?

10. Study "Section *2 Euclid's Algorithm*" from the handout *Variants and Invariants*. Solve the problem stated in the subsection "*2.4 A variation on Euclid's algorithm*", p. 12.