Assignment 3 CMPUT 272 Winter 2012 Posted: February 8
Due before 10:00am, Wednesday March 7
in box labeled "CMPUT 272 Winter 2012" (opposite room CSC 145)

Write solutions in your own words. Include an acknowledgment statement citing (i) all reference materials (other than the textbook and handouts) which you used and (ii) persons with whom you have discussed your solution.

The following information must appear in the top right corner of the first page: CMPUT 272, Assignment 3, *your name*, and the *id* of your lecture section (B1, EB1, B2, or EB2). Please staple the pages together.

1. Prove or disprove that for all $x, y \in \mathbb{R}$ and $n \in \mathbb{N}$:
   (a) $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$;
   (b) $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$;
   (c) $n = \lfloor \frac{n}{2} \rfloor + \lceil \frac{n}{2} \rceil$.

2. Use the Euclidean algorithm to compute
   (a) $\gcd(7020, 652)$;
   (b) a representation of $\gcd(7020, 652)$ as a linear combination of 7020 and 652;
   (c) all common positive divisors of 7020 and 652;
   (d) $\gcd(3510, 652, 544)$.

3. Compute $\phi(n)$ for the following n:
   (a) $n = p$ for some $p \in \mathbb{P}$;
   (b) $n = p^k$ for some $p \in \mathbb{P}$ and $k \in \mathbb{N}^+$;
   (c) $n = p^k q^\ell$ for some $p, q \in \mathbb{P}$ and $k, \ell \in \mathbb{N}^+$.

4. Let X, Y, Z be subsets of the universal set U. Prove or disprove the following:
   (a) $X \cap (X \cup Y) = X$;
   (b) $X - Y = X \cap \sim Y$;
   (c) $(X \cup Y) \times Z = (X \times Z) \cup (Y \times Z)$;
   (d) $(X - Y) \cap (Y - Z) = \emptyset$;
   (e) $(X \cap Y) = \emptyset \Rightarrow (X \times Y) = \emptyset$.

5. Someone tried to state an important property of the mod function and stated

$$\forall m, n, a, b \in \mathbb{Z}, \ (m \bmod d = a) \wedge (n \bmod d = b) \ \Rightarrow \ (m + n) \bmod d = a + b$$

   Is this formula true? Why not? Fix the above formula by minimum amount of editing such that it becomes true and prove it.

6. Determine the following values by using the binomial theorem (textbook p. 54):
   (a) $\displaystyle\sum_{i=0}^{n} \binom{n}{i}$;
   (b) $\displaystyle\sum_{i=0}^{n} \binom{n}{i} (-1)^i$.

7. A student council consists of 15 students.

   (a) In how many ways can a committee of six be selected from the membership of the council?

   (b) Two council members have the same major and are not permitted to serve together on a committee. How many ways can a committee of six be selected from the membership of the council?

   (c) Two council members insist on serving together; if they can't they won't serve at all. How many ways can a committee of six be selected from the council members?

   (d) Suppose the council consists of three freshmen, four sophomores, three juniors, and five seniors.

      i. How many committees of eight contain two representative from each class?

      ii. How many committees of five contain at least one senior and one freshman?

8. Consider the following program.

   $n \in \mathbb{N}$;
   $s, i, p \in \mathbb{N}$;
   $s,\ i,\ p\ := 0,\ 0,\ 2$;
   while $(i < n)$ **do**
      $s,\ i,\ p\ := s + p,\ i + 1,\ p + p$
   endwhile
   POSTCONDITIONS: $s = ?$

   (a) Find a loop-variant for the **while**-loop and prove that the loop terminates.

   (b) Guess the postconditions giving the value of s as a function of n expressed in a *closed form.*

   (c) Find an invariant for the loop that helps in verifying the postconditions when the loop exits.

   (d) Prove that the invariant from (c) is true upon entering the loop.

   (e) Prove that if the invariant from (c) is true and the loop does not exit then the invariant will stay true. Use the same notations as in the handout *Variants and Invariants.*

   (f) Prove that the postconditions guessed in (b) hold when the loop terminates.

9. Study "Section *2 Euclid's Algorithm*" from the handout *Variants and Invariants.* Solve the problem stated in the subsection "*2.5 Extended Euclid's algorithm*", p. 13.