Assignment 1 CMPUT 272 Winter 2012 Posted: January 9
Due before 10:00am, Wednesday February 1
in box labeled "CMPUT 272 Winter 2012" (opposite room CSC 145)

Write solutions in your own words. Include an acknowledgment statement citing (i) all reference materials (other than the textbook and handouts) which you used and (ii) persons with whom you have discussed your solution.

The following information must appear in the top right corner of the first page: CMPUT 272, Assignment 1, *your name*, and the *id* of your lecture section (B1, EB1, B2, or EB2). Please staple the pages together.

1. Use truth tables to prove the De Morgan's rules for Boolean functions stated on page 6 of the textbook. Using these results, without using truth tables, prove that $\sim(p \wedge q \wedge r) = \sim p \vee \sim q \vee \sim r$ and its dual $\sim(p \vee q \vee r) = \sim p \wedge \sim q \wedge \sim r$.

2. Consider the Boolean function f of three variables (named here p, q, and r) given by the following table.

p	q	r	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Express function f: (a) in disjunctive normal form, (b) in conjunctive normal form, (c) using only Boolean operators $\sim$ and $\vee$, and (d) using only Boolean operators $\sim$ and $\wedge$.

Simplify your answers the best you can.

3. A Boolean function is called *symmetric* if it assigns the same value to all permutations of input values. E.g., a ternary symmetric Boolean function f satisfies $f(0,0,1) = f(0,1,0) = f(1,0,0)$ and $f(1,1,0) = f(1,0,1) = f(0,1,1)$.
   (a) Enumerate all Boolean functions of two variables using tabular form. How many of them are symmetric? Justify your answer.
   (b) How many of ternary Boolean functions are symmetric? Justify your answer.
   (c) Determine the number of symmetric Boolean functions of n variables. Justify your answer.

4. Convert each of the following unsigned integers into base 2, 8, 10 and 16 (subscripts represent the base each number is written in). Show the details of your conversion.

   (a) 10010101_2 (b) 3775_8 (c) 1023_{10} (d) FEDC_{16}.

5. Using base 16, express the 16-bit 2's complement (that is, $2^{16} - x$) for each number (that is, x) from the previous question.

   (a) 10010101_2 (b) 3775_8 (c) 1023_{10} (d) FEDC_{16}.

6. *Sheffer stroke* is a binary Boolean operator denoted by $p \uparrow q$ and defined to be equivalent to $\sim(p \wedge q)$. This operator corresponds to the NAND gate used in circuit design. Define the following three Boolean functions using only the $\uparrow$ operator.

   (a) $\sim p$ (b) $p \vee q$ (c) $p \wedge q$.

   Remark: This proves that we can express any Boolean function using only Sheffer stroke.

7. Write negations, converses, inverses and contrapositives of the following statements in English. Try to make your answers easy to read.

   (a) If P is a triangle, then P is a polygon;

   (b) If Jim is Ann's father, then Tom is Ann's uncle and Mary is her aunt and Susan is her cousin.

8. Rewrite the following statements using only $\sim$ and $\vee$, and using only $\sim$ and $\wedge$:

   (a) $(\sim p \wedge q) \Rightarrow (r \vee \sim q)$

   (b) $\Big((p \Rightarrow (q \Rightarrow r)) \Leftrightarrow ((p \wedge q) \Rightarrow r)\Big) \wedge \sim p \wedge \sim q \wedge \sim r$

   Simplify your answers the best you can.

9. There are 10 baskets of balls, 10 balls in each basket. The balls are indistinguishable unless weighed on a scale which tells their exact weight. It is known that among these 10 baskets, 9 of them contain balls weighing 1 pound each. One of the baskets contains balls weighing 0.9 pound each. Design a strategy to use the scale only once to determine which basket contains lighter balls. Justify your answer.

10. Study "*Section 1: The Coffee Can Problem*" from the handout *Variants and Invariants*. Answer Parts 1, 2, and 3 in the subsection "*1.5 Various coffee cans*", p. 7–8, for all three variations A, B, and C. Try to be brief in answering Part 3.