Practice Set 7 (Breadth First Search)

Data Structure (CS 223)

Q1: Answer the following questions for an undirected graph with N vertices and M edges:

- What is complexity of the BFS algorithm?

  Answer: $O(N + M)$

- How many times is each edge inspected (i.e., the vertex on either end is looked at) during the BFS algorithm on an undirected graph?

  Answer: Twice (once each when the vertex on either end is dequeued).

- What is the maximum number of times a vertex is enqueued?

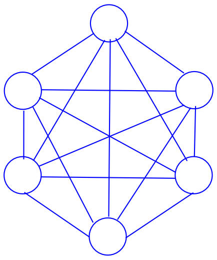
  Answer: Once (when the first edge incident on the vertex is relaxed).

- While executing BFS, other than the start vertex, how many times is each vertex looked at?

  Answer: As many times as the number of edges incident on the vertex.

- At any point, what is the maximum number of vertices on the queue? Draw a graph on 6 vertices for which this maximum number is realized, irrespective of the starting node.

  Answer: $N - 1$, where N is the number of vertices.



Q2: Draw an undirected graph on 6 nodes such that at any point the queue contains at most 2 vertices, irrespective of where BFS starts.

Answer: $v_0 - v_1 - v_2 - v_3 - v_4 - v_5$

Q3: Describe how you can use BFS to determine whether or not an undirected graph is connected.

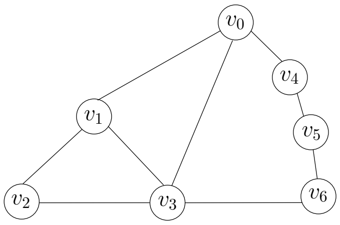
Answer: Start BFS at an arbitrary vertex. Upon termination, if there exists a vertex v for which $level(v) = \infty$, then the graph is not connected, otherwise it is connected.

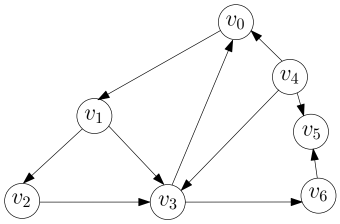
Q4: Describe how you can use BFS to determine whether or not v is reachable from u.

Answer: Start BFS at u. Then a vertex v is reachable if and only if $level(v) \neq \infty$ upon termination.

Q5: Starting from node v_1, illustrate the BFS algorithm on the following graphs. Show:

- the queue, and the level array at each stage
- the final BFS tree.





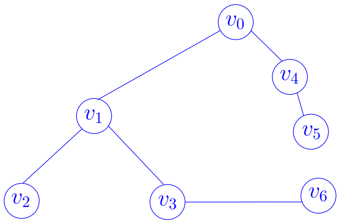
BFS Undirected

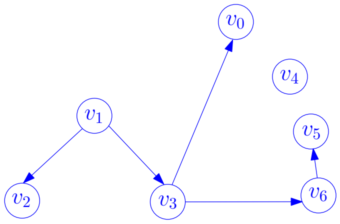
	level array							queue
	v_0	v_1	v_2	v_3	v_4	v_5	v_6	
At Start:	∞	0	∞	∞	∞	∞	∞	$[v_1]$
Dequeue (Relax v_1)	1	0	1	1	∞	∞	∞	$[v_0, v_2, v_3]$
Dequeue (Relax v_0)	1	0	1	1	2	∞	∞	$[v_2, v_3, v_4]$
Dequeue (Relax v_2)	1	0	1	1	2	∞	∞	$[v_3, v_4]$
Dequeue (Relax v_3)	1	0	1	1	2	∞	2	$[v_4, v_6]$
Dequeue (Relax v_4)	1	0	1	1	2	3	2	$[v_6, v_5]$
Dequeue (Relax v_6)	1	0	1	1	2	3	2	$[v_5]$
Dequeue (Relax v_5)	1	0	1	1	2	3	2	[ ]

BFS Directed

	level array							queue
	v_0	v_1	v_2	v_3	v_4	v_5	v_6	
At Start:	∞	0	∞	∞	∞	∞	∞	$[v_1]$
Dequeue (Relax v_1)	∞	0	1	1	∞	∞	∞	$[v_2, v_3]$
Dequeue (Relax v_2)	∞	0	1	1	∞	∞	∞	$[v_3]$
Dequeue (Relax v_3)	2	0	1	1	∞	∞	2	$[v_0, v_6]$
Dequeue (Relax v_0)	2	0	1	1	∞	∞	2	$[v_6]$
Dequeue (Relax v_6)	2	0	1	1	∞	3	2	$[v_5]$
Dequeue (Relax v_5)	2	0	1	1	∞	3	2	[ ]

BFS Trees


v_0
v_1
v_2
v_3
v_4
v_5
v_6


v_0
v_1
v_2
v_3
v_4
v_5
v_6