scaling. Other, more mundane, possibilities include the trapped ion mode, a nearly-forgotten instability mechanism which is particularly relevant to collisionless plasmas.

2.3 18th

2.3.1 Residual Zonal Flows in Noncircular Plasmas—10:17 Seville

In this section, we evaluate the residual zonal flow level in a shaped tokamak plasma. To study the total susceptibility specifically, the Miller model is adopted for the local equilibrium. Except for three parameters ϵ, κ and δ determining the flux shape, and q, s, α for local equilibrium solution, Miller model requires three other parameters s_κ, s_δ, and Δ' for the poloidal field, which cannot be assigned *a priori* values. Based on experience numerical equilirbia, volume average values are taken in Miller's original paper [8]. It must be underlined that without specified s_κ, s_δ, and Δ', Miller model can only be used to fit experimental data.

The magnetic drift velocity in low-β plasma is

$$\boldsymbol{v}_d = \frac{v_\parallel^2 + \mu B}{\omega_c}\hat{b} \times \frac{\nabla B}{B} = -v_\parallel \hat{b} \times \nabla(\frac{v_\parallel}{\omega_c}). \tag{2.1}$$

The rapid spatioal variation of the axisymmetric potential across the magnetic field is assumed to be contained in an eikonal factor

$$\phi = \phi_k e^{iS(\psi)}, \tag{2.2}$$

the gyrokinetic equation can thus be expressed as

$$[\partial_t + v_\parallel \nabla_\parallel + i\omega_d]\delta H = \frac{e_j F_0}{T_j}\frac{\partial J_0 \delta\phi}{\partial t} + S_0 F_0, \tag{2.3}$$

where the magnetic drift frequency is

$$\begin{aligned}
\omega_d &= -iS' v_\parallel \hat{b} \times \nabla(\frac{v_\parallel}{\omega_c}) \cdot \nabla r, \qquad (2.4)\\
&= -iS' v_\parallel \hat{b} \times \nabla\theta \cdot \nabla r \partial_\theta(\frac{v_\parallel}{\omega_c})\\
&= -iS' v_\parallel \frac{1}{B}[\nabla\zeta \times \nabla\psi + I\nabla\zeta] \times \nabla\theta \cdot \nabla\psi \frac{dr}{d\psi}\partial_\theta(\frac{v_\parallel}{\omega_c})\\
&= iv_\parallel \frac{1}{J_\psi B}\frac{dr}{d\psi}\partial_\theta(\frac{IS' v_\parallel}{\omega_c})\\
&= iv_\parallel \frac{1}{J_\psi B}\partial_\theta(\frac{qS'}{rB_{unit}}\frac{Iv_\parallel}{\omega_c})\\
&= iv_\parallel \frac{1}{qR_0 G_\theta}\partial_\theta(\frac{qS' I}{rB_{unit}}\frac{v_\parallel}{\omega_c})\\
&\equiv iv_\parallel \nabla_\parallel(\frac{dS}{d\psi}\frac{Iv_\parallel}{\omega_c})\\
&\equiv iv_\parallel \nabla_\parallel Q,
\end{aligned}$$

which is in agreement with [9]. Then, taking $Q \to 0$ for electrons, the total susceptibility can be written as

$$\chi = 1 - \frac{1}{n_0}\langle F_0 J_0 e^{iQ}\overline{e^{-iQ} J_0}\rangle. \tag{2.5}$$

Here, $J_0 = J_0(\frac{k_r v_\perp \nabla r}{\omega_{ci}})$ with $k_r = S'$,

$$\overline{(\cdots)} = \frac{\oint (\cdots) \frac{dl}{v_\parallel}}{\oint \frac{dl}{v_\parallel}} = \frac{\oint (\cdots) \frac{B}{B_p} \frac{\partial l_p}{\partial \theta} \frac{d\theta}{v_\parallel}}{\oint \frac{B}{B_p} \frac{\partial l_p}{\partial \theta} \frac{d\theta}{v_\parallel}} \tag{2.6}$$

is the bounce or transit averaging, where [9]

$$dl = \frac{B}{B_p} dl_p = \frac{B}{B_p} \frac{\partial l_p}{\partial \theta} d\theta, \tag{2.7}$$

and

$$\langle \cdots \rangle = \frac{\oint (\cdots) \frac{dl_p}{B_p}}{\oint \frac{dl_p}{B_p}} = \frac{\oint (\cdots) d\theta \frac{G_\theta}{B}}{\oint d\theta \frac{G_\theta}{B}}, \tag{2.8}$$

denotes the flux-surface average.

2.3.1.1 Long wavelength limit

In the long wavelength limit, the residual zonal flow level in a tokamak with a general cross section is given by [9, 10]

$$\begin{aligned} \chi &= 1 + \frac{m_i}{n_0 T_i} \langle \frac{\omega_{ci}^2}{k_\perp^2} \rangle \langle \int d^3 \boldsymbol{v} F_0 Q [Q - \overline{Q}] \rangle \\ &= 1 + \frac{m_i (q I k_r)^2}{(r B_{unit})^2 n_0 T_i} \langle \frac{B^2}{k_\perp^2} \rangle \langle \int d^3 \boldsymbol{v} F_0 \frac{v_\parallel}{B} [\frac{v_\parallel}{B} - \overline{\frac{v_\parallel}{B}}] \rangle, \end{aligned} \tag{2.9}$$

where $k_\perp = \nabla S(\psi) = \nabla r \partial_r S = \nabla r S' \equiv \nabla r k_r$ and $\frac{dS}{d\psi} = \frac{q}{r B_{unit}} \frac{dS}{dr}$. In deriving Eq. (2.9), we have assumed $k_\perp^2 \rho_i^2 \ll 1$ and specified the Miller equilibrium model. Therefore,

$$\langle \frac{k_\perp^2}{B^2} \rangle = k_r^2 \langle \frac{(\nabla r)^2}{B^2} \rangle. \tag{2.10}$$

Assuming a Maxwellian distribution for ions,

$$F_0 = \frac{n_0}{(\pi v_{ti}^2)^{3/2}} e^{-\frac{m_i E}{T_i}}, \tag{2.11}$$

we obtain

$$\langle \int d^3 \boldsymbol{v} F_0 (\frac{v_\parallel}{B})^2 \rangle = \frac{P_\parallel}{m_i} \langle \frac{1}{B^2} \rangle = \frac{n_0 T_i}{m_i} \langle \frac{1}{B^2} \rangle. \tag{2.12}$$

To evalue the velocity integrals, we introduce the pitch angle variable

$$\lambda = \frac{\mu B_{unit}}{E}, \tag{2.13}$$

with $\mu = \frac{v_\perp^2}{2B}$ and $E = \frac{v^2}{2}$, the magnetic moment and kinetic energy of ions. It is clear that the parallel velocity is

$$v_\parallel = \sqrt{2(E - \mu B)} = \sqrt{2E(1 - \lambda \frac{B}{B_{unit}})}, \tag{2.14}$$

thus particles with $E > \mu B_{max}$ ($0 < \lambda < \frac{B_{unit}}{B_{max}}$) pass freely along the field line while those with $E < \mu B_{max}$ ($\frac{B_{unit}}{B_{max}} < \lambda < \frac{B_{unit}}{B_{min}}$) will be reflected from the high magnetic field region and become trapped. The velocity integration needed is defined in terms of

$$\int d^3\boldsymbol{v} = \pi \sum_\sigma \frac{B}{B_{unit}} \int_0^{+\infty} dE \int_0^{\frac{B_{unit}}{B}} d\lambda (\frac{2E}{1-\lambda\frac{B}{B_{unit}}})^{1/2}. \tag{2.15}$$

Noting that $\overline{\frac{v_\parallel}{B}} = 0$ for trapped particles, we can reach

$$\langle \int d^3\boldsymbol{v} F_0 \frac{v_\parallel}{B}\overline{\frac{v_\parallel}{B}} \rangle \tag{2.16}$$

$$\begin{aligned}
&= \frac{\pi}{\oint d\theta \frac{G_\theta}{B}} \sum_\sigma \oint d\theta \frac{G_\theta}{B}\frac{B}{B_{unit}} \iint F_0 (\frac{2E}{1-\lambda\frac{B}{B_{unit}}})^{1/2} \frac{v_\parallel}{B}\overline{\frac{v_\parallel}{B}} dEd\lambda \\
&= \frac{\pi}{\oint d\theta \frac{G_\theta}{B}} \sum_\sigma \oint d\theta \frac{G_\theta}{B}\frac{B}{B_{unit}} \iint F_0 (\frac{2E}{1-\lambda\frac{B}{B_{unit}}})^{1/2} \frac{v_\parallel}{B}[\frac{\oint \frac{\partial l_p}{\partial\theta}\frac{d\theta}{B_p}}{\oint \frac{B}{B_p}\frac{\partial l_p}{\partial\theta}\frac{d\theta}{v_\parallel}}] dEd\lambda \\
&= \frac{2\pi}{\oint d\theta \frac{G_\theta}{B}} \oint d\theta \frac{G_\theta}{B}\frac{B}{B_{unit}} \iint F_0 (\frac{2E}{1-\lambda\frac{B}{B_{unit}}})^{1/2} \frac{2E\sqrt{(1-\lambda\frac{B}{B_{unit}})}}{B}[\frac{\oint \frac{\partial l_p}{\partial\theta}\frac{d\theta}{B_p}}{\oint \frac{B}{B_p}\frac{\partial l_p}{\partial\theta}\frac{d\theta}{\sqrt{(1-\lambda\frac{B}{B_{unit}})}}}] dEd\lambda \\
&= \frac{3n_0T_i}{2m_i\oint d\theta \frac{G_\theta}{B}} \oint d\theta \frac{G_\theta}{B}\frac{1}{B_{unit}} \int_0^{\lambda_c} [\frac{\oint \frac{\partial l_p}{\partial\theta}\frac{d\theta}{B_p}}{\oint \frac{B}{B_p}\frac{\partial l_p}{\partial\theta}\frac{d\theta}{\sqrt{(1-\lambda\frac{B}{B_{unit}})}}}] d\lambda,
\end{aligned} \tag{2.17}$$

where B_{unit} is a reference magnetic field and $\lambda_c = B_{unit}/B_{max}$ sets the trapping-passing boundary. Furthermore, noting that

$$\frac{G_\theta}{B} = \frac{1}{qR_0B_p}\frac{\partial l}{\partial\theta}, \tag{2.18}$$

and, for passing particles

$$\oint d\theta \frac{G_\theta}{B} = \oint d\theta \frac{1}{qR_0B_p}\frac{\partial l}{\partial\theta}, \tag{2.19}$$

we have

$$\langle \int d^3\boldsymbol{v} F_0 \frac{v_\parallel}{B}\overline{\frac{v_\parallel}{B}} \rangle \tag{2.20}$$

$$= \frac{3n_0T_iqR_0}{2m_i} \oint d\theta \frac{G_\theta}{B}\frac{1}{B_{unit}} \int_0^{\lambda_c} \frac{d\lambda}{\oint \frac{B}{B_p}\frac{\partial l_p}{\partial\theta}\frac{d\theta}{\sqrt{(1-\lambda\frac{B}{B_{unit}})}}}.$$

Therefore, the dielectric susceptibility is given by

$$\begin{aligned}
\chi &= 1 + \frac{m_i(qIk_r)^2}{(rB_{unit})^2 n_0 T_i}\langle\frac{B^2}{k_\perp^2}\rangle\langle\int d^3\boldsymbol{v}F_0\frac{v_\parallel}{B}[\frac{v_\parallel}{B} - \overline{\frac{v_\parallel}{B}}]\rangle \qquad (2.21) \\
&= 1 + \frac{(qI)^2}{(rB_{unit})^2}\langle\frac{B^2}{(\nabla r)^2}\rangle[\langle\frac{1}{B^2}\rangle - \frac{3qR_0}{2}\oint d\theta\frac{G_\theta}{B}\frac{1}{B_{unit}}\int_0^{\lambda_c}\frac{d\lambda}{\oint \frac{B}{B_p}\frac{\partial l_p}{\partial\theta}\frac{d\theta}{\sqrt{(1-\lambda\frac{B}{B_{unit}})}}}] \\
&= 1 + \frac{(qI)^2}{(rB_{unit})^2}\langle\frac{B^2}{(\nabla r)^2}\rangle[\frac{\oint d\theta\frac{G_\theta}{B^3}}{\oint d\theta\frac{G_\theta}{B}} - \frac{3qR_0}{2}\oint d\theta\frac{G_\theta}{B}\frac{1}{B_{unit}}\int_0^{\lambda_c}\frac{d\lambda}{\oint \frac{B}{B_p}\frac{\partial l_p}{\partial\theta}\frac{d\theta}{\sqrt{(1-\lambda\frac{B}{B_{unit}})}}}]
\end{aligned} \tag{2.22}$$

In Fig. (2.1), we compare our numerical results with the total susceptibility derived in [11]. We find very good agreement.

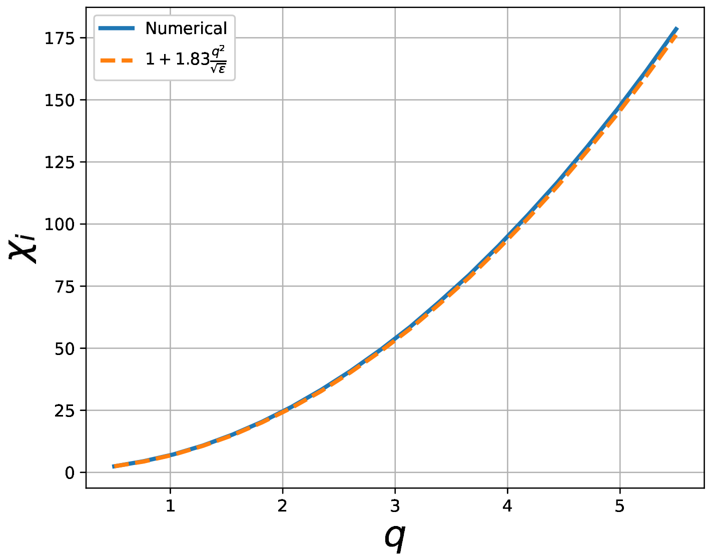

Figure 2.1: Total susceptibility for $\epsilon = 0.1$, $s = 1.0$, $\alpha_{mhd} = 0$, $\kappa = 1$, $\delta = 0$, $\Delta' = 0$, $s_\delta = 0$ and $s_\kappa = 0$.