COMP 2611, DATA STRUCTURES
LECTURE 19

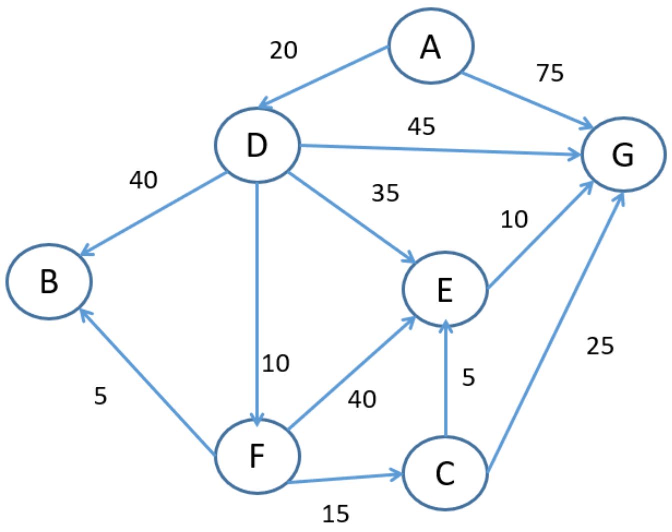
GRAPHS

- **Dijkstra’s Shortest Path Algorithm**
- **Minimum-cost Spanning Tree**

Dijkstra's Shortest Path Algorithm

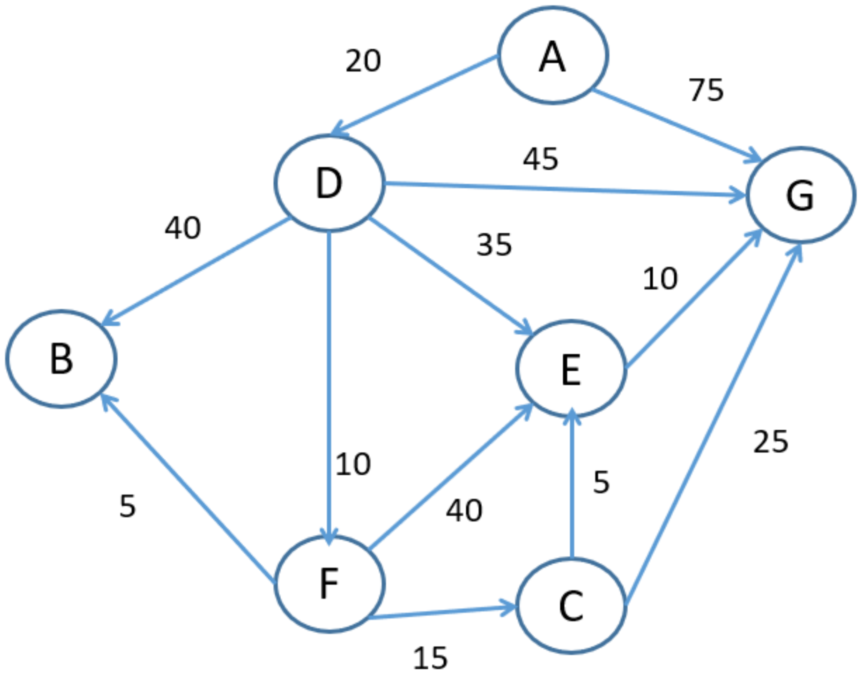
- Dijkstra's algorithm can be used to find the shortest path from a source vertex (e.g., *A*) to every other vertex in the graph.

- The algorithm assumes that the edge weights are non-negative.



vertex	A	B	C	D	E	F	G
parent	nil	nil	nil	nil	nil	nil	nil
cost	0	∞	∞	∞	∞	∞	∞

- V.cost holds the current cost of a path from *A* to a vertex *V*
- V.parent holds the parent of *V* on the current shortest path from *A* to *V*.
- A min-priority queue, Q, will hold vertices based on their current cost. Initially, all the verticles will be placed on *Q*. Vertex *A*, with a cost of 0, will be at the top.

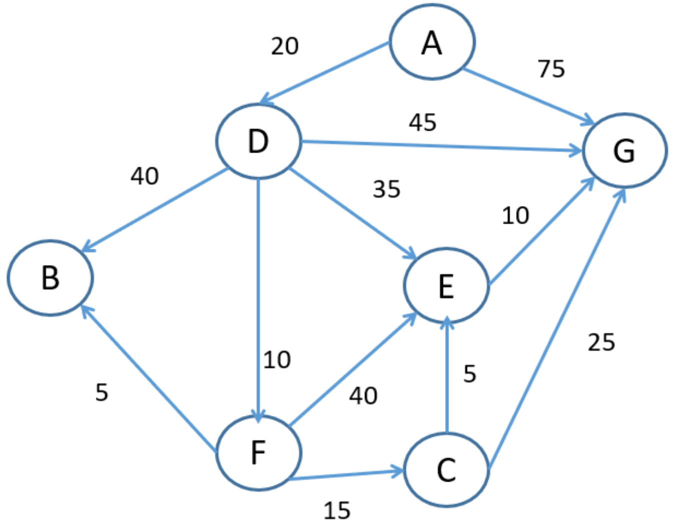


vertex	A	B	C	D	E	F	G
parent	nil	nil	nil	nil	nil	nil	nil
cost	0	∞	∞	∞	∞	∞	∞

Min-Priority Queue

A	

- Take A off the queue.
- Consider all the edges leaving A. We will process them in alphabetical order. The edge (A, D) gives us a path to D at a cost of 20. This is lower than the current cost to D (∞) so we update the cost to 20, set the parent of D to A.
- The edge (A, G) gives us a path to G at a cost of 75. This is lower than the current cost to G (∞) so we update the cost to 75, set the parent of G to A.



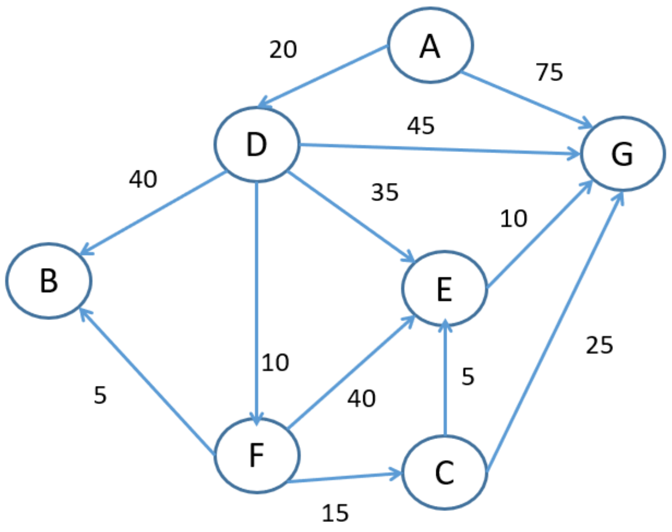
vertex	A	B	C	D	E	F	G
parent	nil	nil	nil	nil	nil	nil	nil
cost	0	∞	∞	∞	∞	∞	∞

vertex	A	B	C	D	E	F	G
parent	nil	nil	nil	A	nil	nil	A
cost	0	∞	∞	20	∞	∞	75

Min-Priority Queue

D	

- Take D off the queue.
- Consider all the edges leaving D. The edge (D, B) gives us a path to B at a cost of 20 + 40 = 60 (cost to D + weight of (D, B)). This is lower than the current cost to B (∞) so we update the cost to 60, set the parent of B to D.
- The edge (D, E) gives us a path to E at a cost of 20 + 35 = 55 (cost to D + weight of (D, E)). This is lower than the current cost to E (∞) so we update the cost to 55, set the parent of E to D.



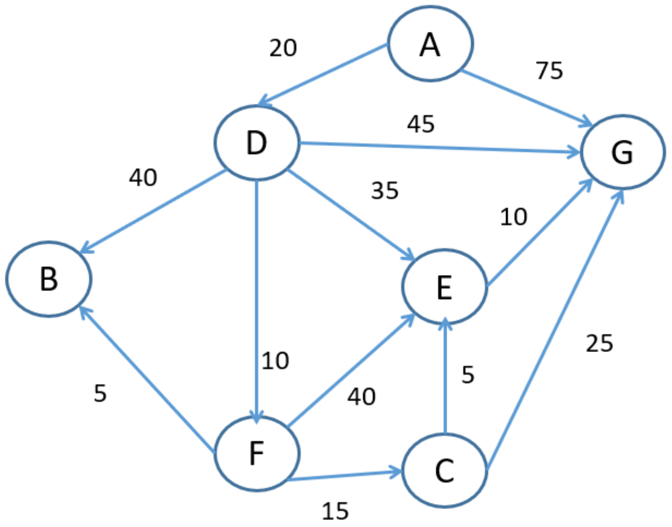
vertex	A	B	C	D	E	F	G
parent	nil	nil	nil	A	nil	nil	A
cost	0	∞	∞	20	∞	∞	75

vertex	A	B	C	D	E	F	G
parent	nil	D	nil	A	D	nil	A
cost	0	60	∞	20	55	∞	75

- (Still processing D)
- The edge (D, F) gives us a path to F at a cost of 20 + 10 = 30 (cost to D + weight of (D, F)). This is lower than the current cost to F (∞) so we update the cost to 30, set the parent of F to D.
- The edge (D, G) gives us a path to G at a cost of 20 + 45 = 65 (cost to D + weight of (D, G)). This is lower than the current cost to G (75) so we update the cost to 65, set the parent of G to D.

Min-Priority Queue

D	

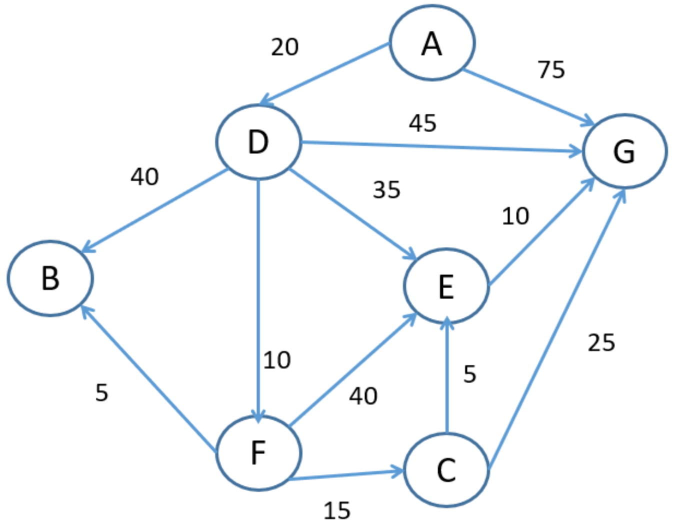


vertex	A	B	C	D	E	F	G
parent	nil	D	nil	A	D	nil	A
cost	0	60	∞	20	55	∞	75

vertex	A	B	C	D	E	F	G
parent	nil	D	nil	A	D	D	D
cost	0	60	∞	20	55	30	65

- Take F off the queue. Consider all the edges leaving F.
- The edge (F, B) gives us a path to B at a cost of 30 + 5 = 35 (cost to F + weight of (F, B)). This is lower than the current cost to B (60) so we update the cost to 35, set the parent of B to F.
- The edge (F, C) gives us a path to C at a cost of 30 + 15 = 45 (cost to F + weight of (F, C)). This is lower than the current cost to C (∞) so we update the cost to 45, set the parent of C to F.

Min-Priority Queue



vertex	A	B	C	D	E	F	G
parent	nil	D	nil	A	D	D	D
cost	0	60	∞	20	55	30	65

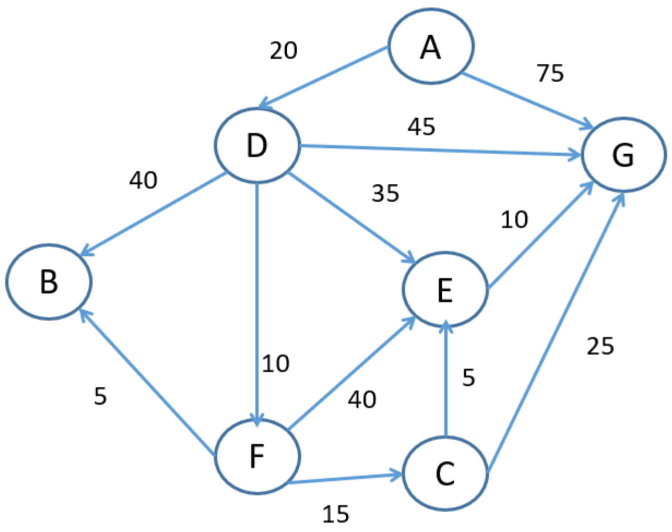
vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	D	D	D
cost	0	35	45	20	55	30	65

Min-Priority Queue

F	

- (Still processing F)

- The edge (F, E) gives us a path to E at a cost of 30 + 40 = 70 (cost to F + weight of (F, E)). This is higher than the current cost to E (55) so we leave E as it is.



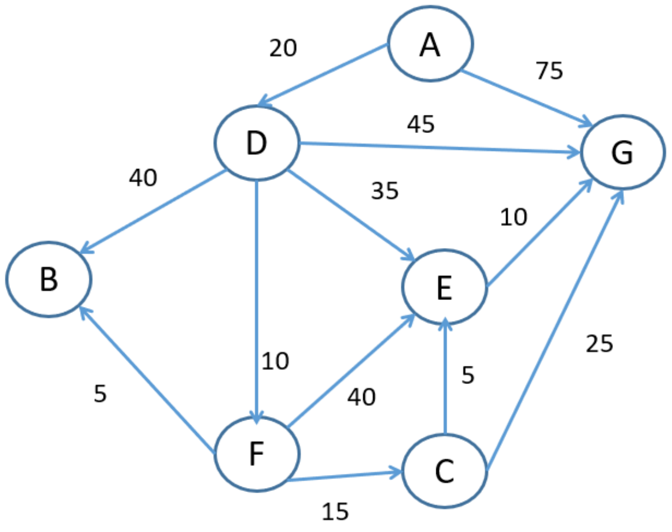
vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	D	D	D
cost	0	35	45	20	55	30	65

vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	D	D	D
cost	0	35	45	20	55	30	65

Min-Priority Queue

B	

- Take B off the queue. Consider all the edges leaving B.
- There are no edges leaving B. So, we take the next element off the queue.



vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	D	D	D
cost	0	35	45	20	55	30	65

vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	D	D	D
cost	0	35	45	20	55	30	65

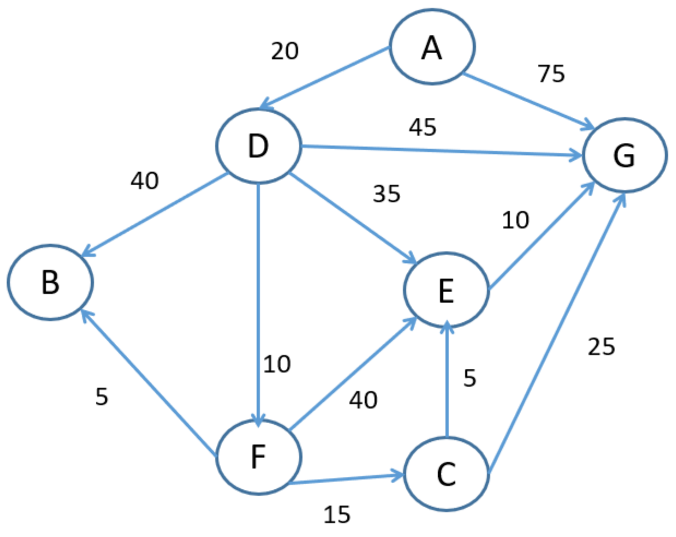
Min-Priority Queue

C

- Take C off the queue. Consider all the edges leaving C.

- The edge (C, E) gives us a path to E at a cost of 45 + 5 = 50 (cost to C + weight of (C, E). This is lower than the current cost to E (55) so we update the cost to 50, set the parent of E to C.

- The edge (C, G) gives us a path to G at a cost of 45 + 25 = 70 (cost to C + weight of (C, G). This is higher than the current cost to G (65) so we leave G as it is.



vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	D	D	D
cost	0	35	45	20	55	30	65

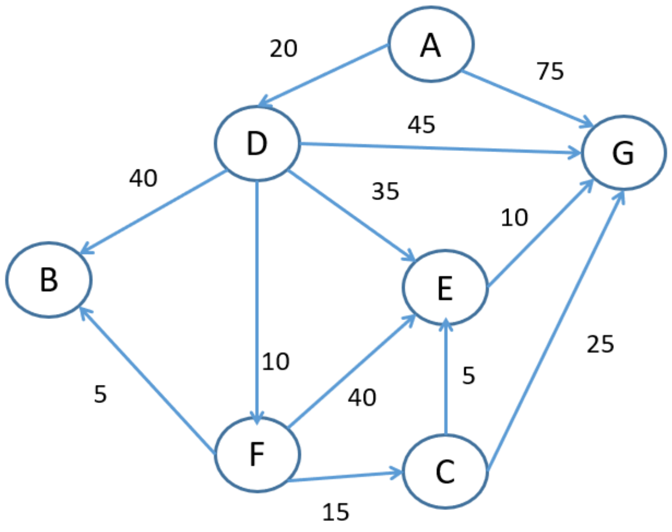
vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	C	D	D
cost	0	35	45	20	50	30	65

Min-Priority Queue

E	

- Take E off the queue. Consider all the edges leaving E.

- The edge (E, G) gives us a path to G at a cost of 50 + 10 = 60 (cost to E + weight of (E, G). This is lower than the current cost to G (65) so we update the cost to 60, set the parent of G to E.



vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	C	D	D
cost	0	35	45	20	50	30	65

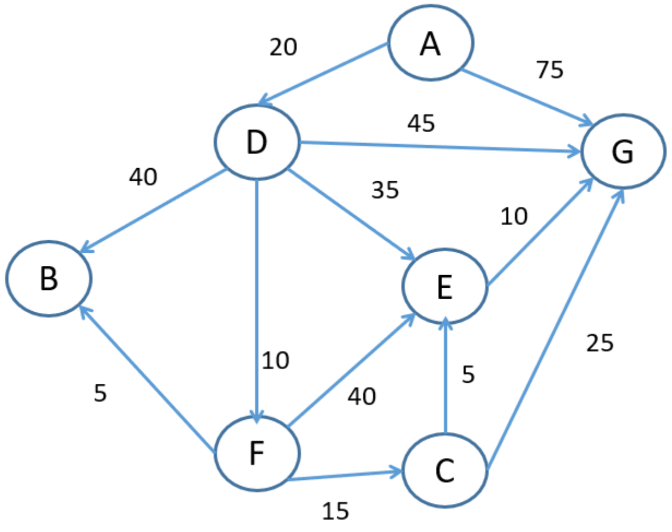
vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	C	D	E
cost	0	35	45	20	50	30	60

Min-Priority Queue

G	

- Take G off the queue. Consider all the edges leaving G.
- There are no edges leaving G.
- The queue is now empty, so the algorithm terminates. The results are:

Cost to B: 35, Path: A → D → F → B
Cost to C: 45, Path: A → D → F → C
Cost to D: 20, Path: A → D
Cost to E: 50, Path: A → D → F → C → E
Cost to F: 30, Path: A → D → F
Cost to G: 60, Path: A → D → F → C → E → G

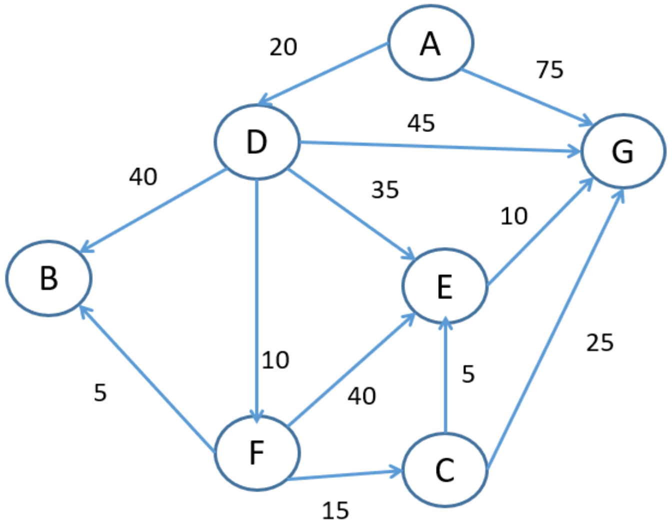


vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	C	D	E
cost	0	35	45	20	50	30	60

vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	C	D	E
cost	0	35	45	20	50	30	60

- Given table, how to get paths?

```
Cost to B: 35, Path: A → D → F → B
Cost to C: 45, Path: A → D → F → C
Cost to D: 20, Path: A → D
Cost to E: 50, Path: A → D → F → C → E
Cost to F: 30, Path: A → D → F
Cost to G: 60, Path: A → D → F → C → E → G
```

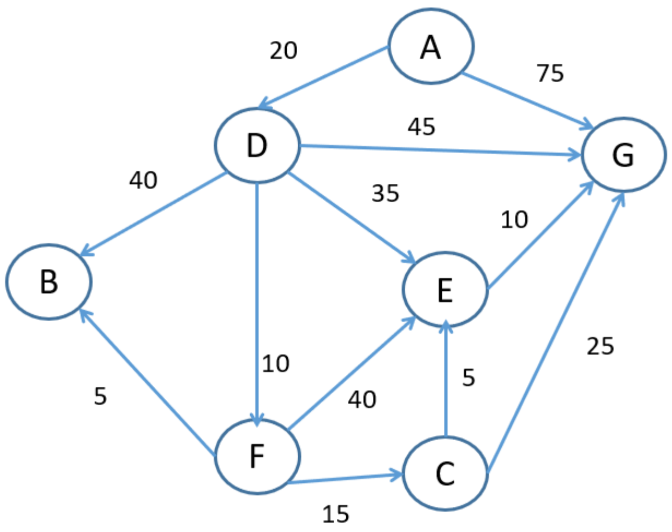


vertex	A	B	C	D	E	F	G
parent	nil	F	F	A	C	D	E
cost	0	35	45	20	50	30	60

Dijkstra's Shortest Path Algorithm

- Dijkstra's Algorithm is an example of a *single-source shortest path algorithm* which finds the shortest path from a source node to a destination node.

- There are others such as the Bellman-Ford Algorithm.

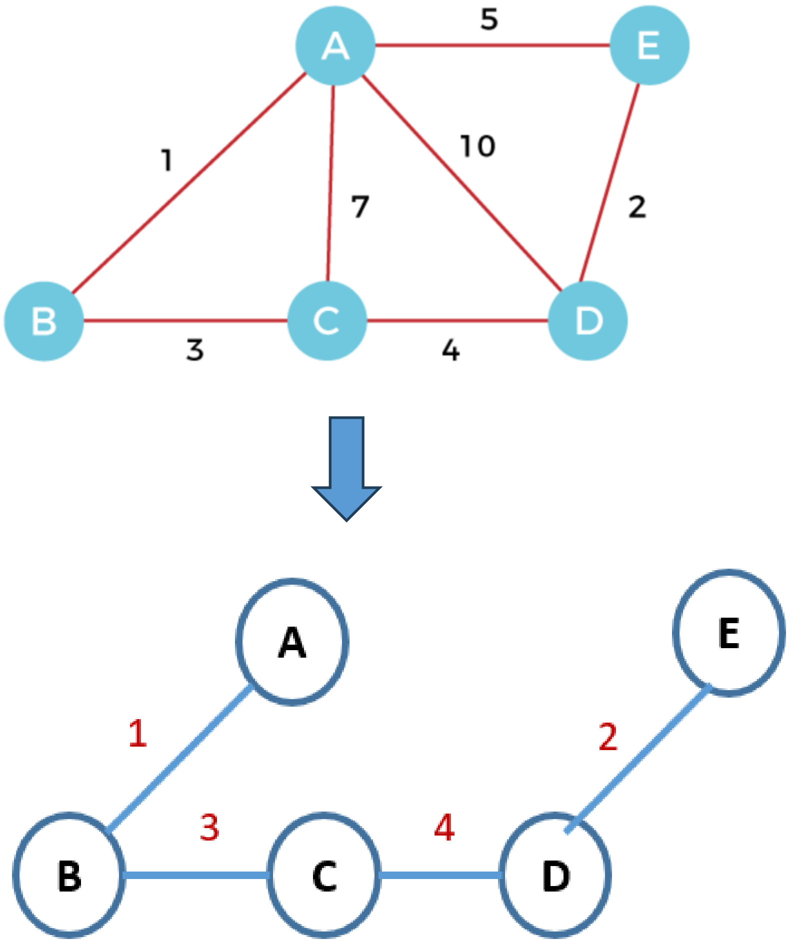


Minimum-Cost Spanning Tree

- A minimum-cost spanning tree (MST) is a subset of the edges of a connected, undirected, edge-weighted graph that connects all the vertices together without any cycles and with the minimum possible total edge weight.

- It is a way of finding the most economical way to connect a set of vertices.

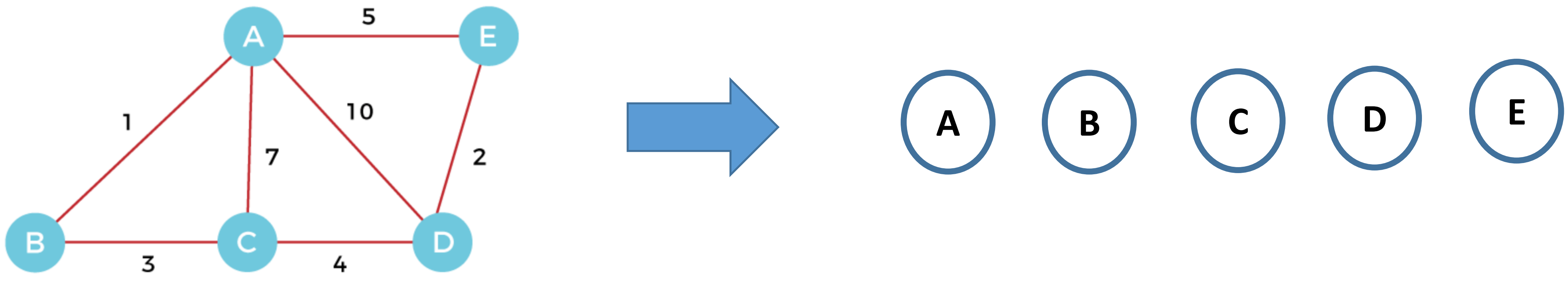
- A minimum-cost spanning tree has precisely n-1 edges, where n is the number of vertices in the graph.



Find a Minimum-Cost Spanning Tree: Kruskal's Algorithm

- For each *v* in *V*, create a tree consisting of *v* only.
- Sort the edges of *E* by non-decreasing weight.
- For each edge (*u*, *v*) in *E*, if *u* and *v* belong to different trees, connect them with the edge (*u*, *v*).

```
(A, B, 1)
(D, E, 2)
(B, C, 3)
(C, D, 4)
(A, E, 5)
(A, C, 7)
(A, D, 10)
```



Find a Minimum-Cost Spanning Tree: Kruskal’s Algorithm

~~(A, B, 1)~~
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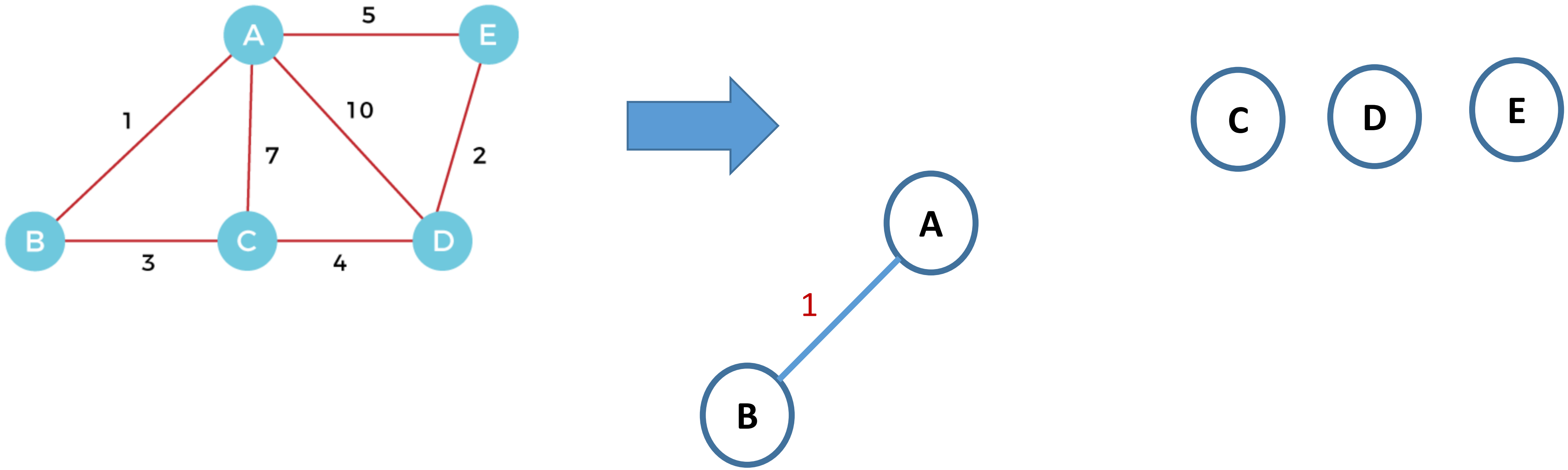
(A, C, 7)
(A, D, 10)



Find a Minimum-Cost Spanning Tree: Kruskal's Algorithm

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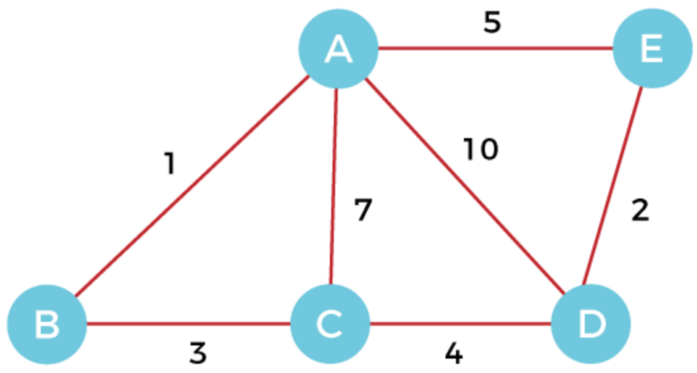

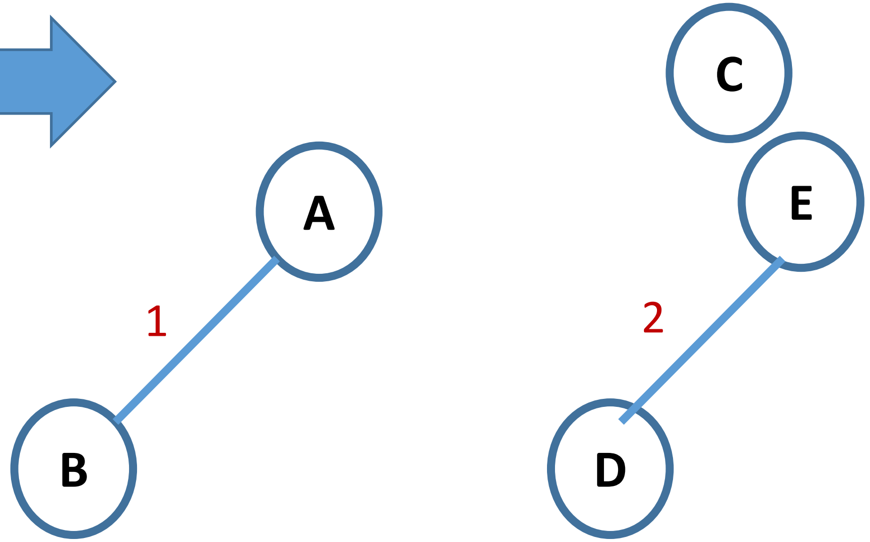

Find a Minimum-Cost Spanning Tree: Kruskal's Algorithm

~~(A, B, 1)~~
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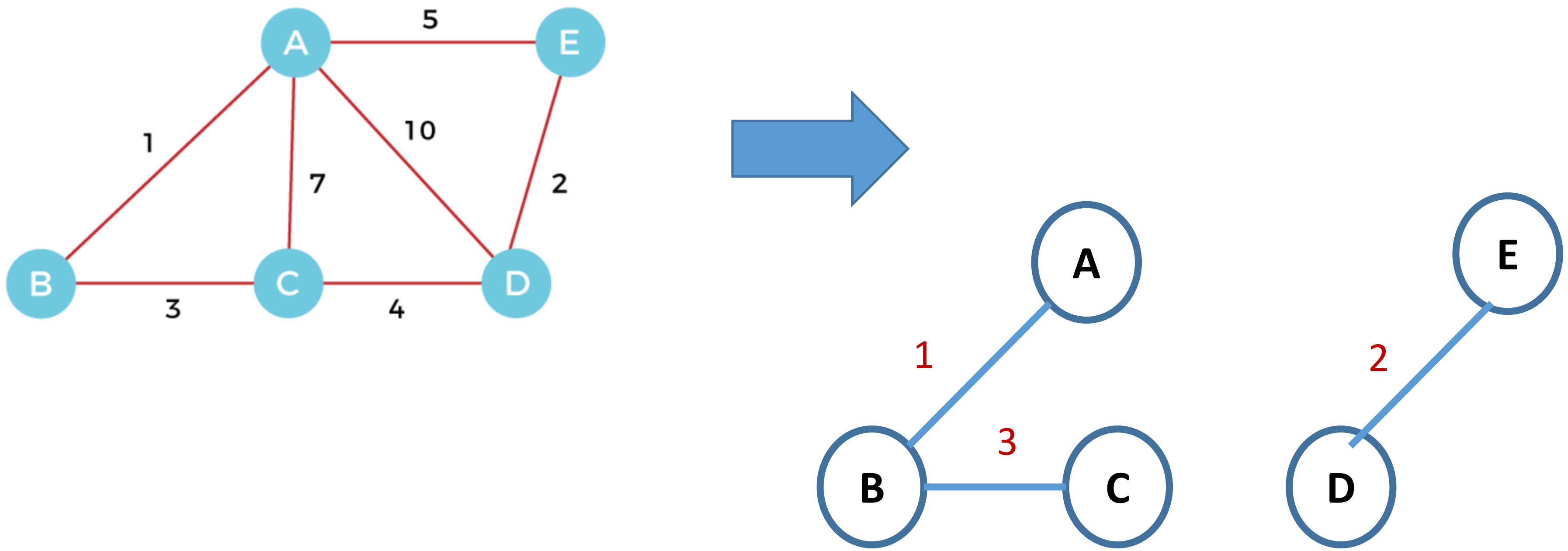
(C, D, 4)
(A, E, 5)
(A, C, 7)
(A, D, 10)



Find a Minimum-Cost Spanning Tree:
Kruskal's Algorithm

~~(A, B, 1)~~
~~(D, E, 2)~~
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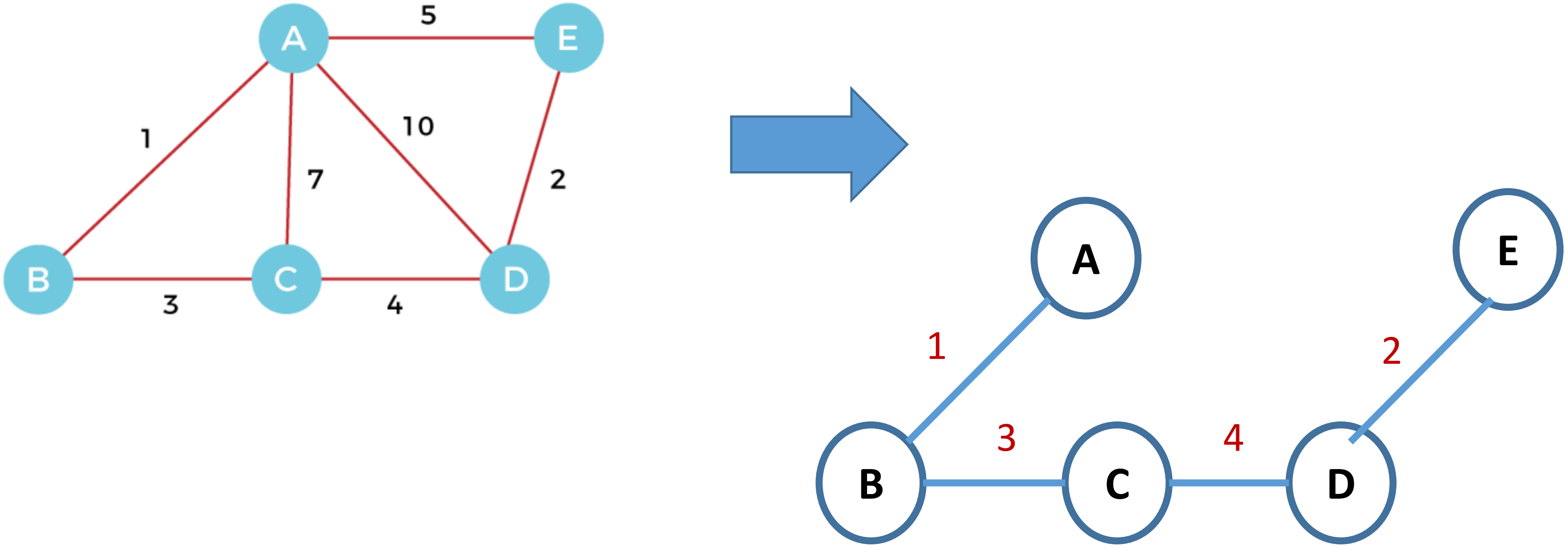
~~(C, D, 4)~~
(A, E, 5)
(A, C, 7)
(A, D, 10)



Find a Minimum-Cost Spanning Tree: Kruskal's Algorithm

~~(A, B, 1)~~
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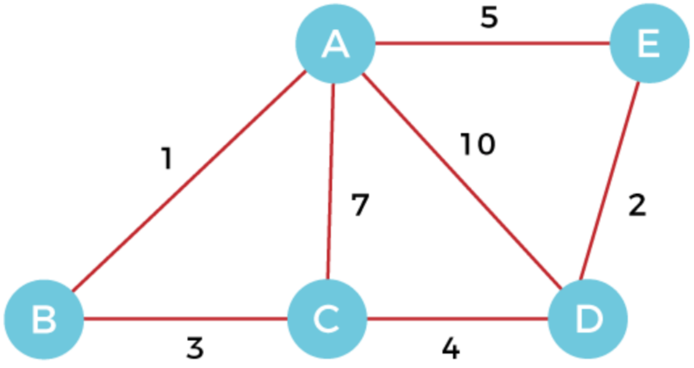

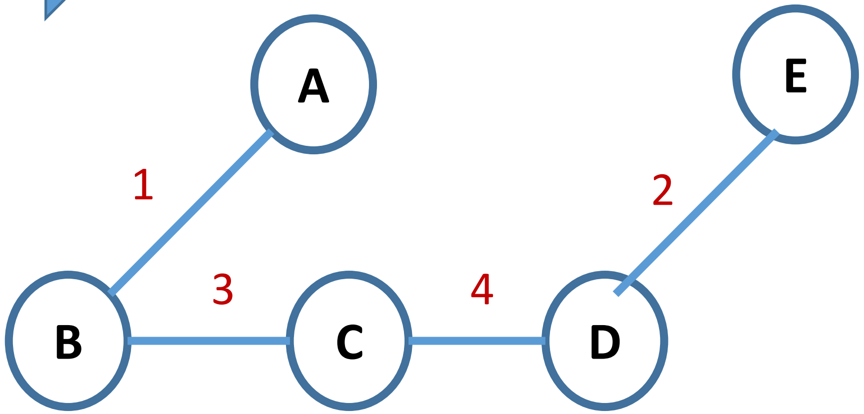

Find a Minimum-Cost Spanning Tree: Kruskal's Algorithm

~~(A, B, 1)~~
~~(D, E, 2)~~
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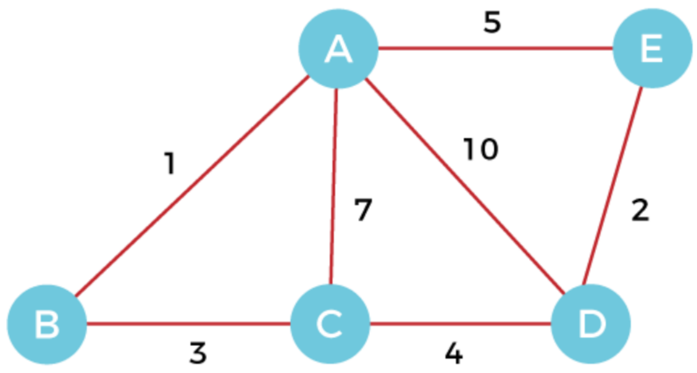
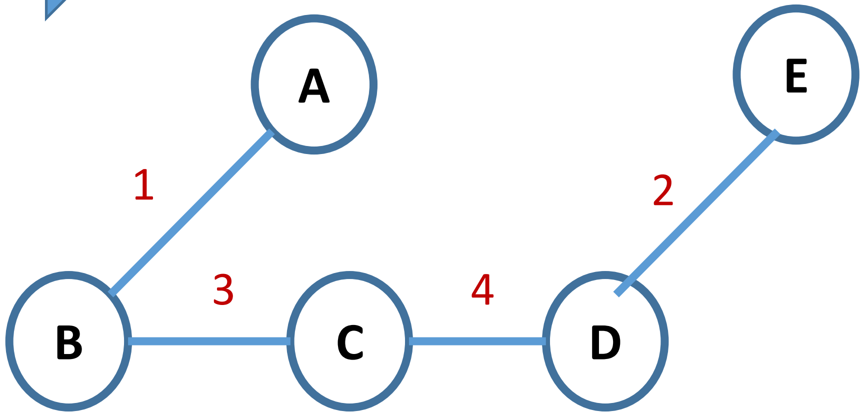
(A, D, 10)





Find a Minimum-Cost Spanning Tree: Kruskal's Algorithm

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~~(D, E, 2)~~
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~~(A, E, 5)~~
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~~(A, D, 10)~~