homework-07a

December 11, 2023

1 Homework 7 - Part A

Note that there are two different notebooks for HW assignment 7. This is part A. There will be two different assignments in gradescope for each part. The deadlines are the same for both parts.

1.1 References

• Lectures 24-26 (inclusive).

1.2 Instructions

- Type your name and email in the "Student details" section below.
- Develop the code and generate the figures you need to solve the problems using this notebook.
- For the answers that require a mathematical proof or derivation you should type them using latex. If you have never written latex before and you find it exceedingly difficult, we will likely accept handwritten solutions.
- The total homework points are 100. Please note that the problems are not weighed equally.

```
[]: import numpy as np
     import matplotlib.pyplot as plt
     %matplotlib inline
     import matplotlib_inline
     matplotlib_inline.backend_inline.set_matplotlib_formats('svg')
     import seaborn as sns
     sns.set_context("paper")
     sns.set_style("ticks")
     import scipy
     import scipy.stats as st
     import urllib.request
     import os
     def download(
         url : str,
         local_filename : str = None
     ):
         """Download a file from a url.
         Arguments
```

1.3 Student details

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In this problem, you must use a deep neural network (DNN) to perform a regression task. The dataset we are going to use is the Airfoil Self-Noise Data Set From this reference, the description of the dataset is as follows:

The NASA data set comprises different size NACA 0012 airfoils at various wind tunnel speeds and angles of attack. The span of the airfoil and the observer position were the same in all of the experiments.

Attribute Information: This problem has the following inputs: 1. Frequency, in Hertzs.

2. The angle of attack, in degrees. 3. Chord length, in meters. 4. Free-stream velocity, in meters per second. 5. Suction side displacement thickness, in meters.

The only output is:

6. Scaled sound pressure level in decibels.

You will have to do regression between the inputs and the output using a DNN. Before we start, let's download and load the data.

```
[]: # !curl -0 --insecure "https://archive.ics.uci.edu/ml/

machine-learning-databases/00291/airfoil_self_noise.dat"
```

```
% Total
             % Received % Xferd Average Speed
                                                   Time
                                                           Time
                                                                     Time
                                                                           Current
                                  Dload Upload
                                                   Total
                                                           Spent
                                                                     Left
  0
        0
                   0
                               0
                         0
0100 59984
              0 59984
                          0
                                0
                                     133k
```

The data are in simple text format. Here is how we can load them:

```
[]: data = np.loadtxt("airfoil_self_noise.dat")
data
```

```
[]: array([[8.00000e+02, 0.00000e+00, 3.04800e-01, 7.13000e+01, 2.66337e-03, 1.26201e+02], [1.00000e+03, 0.00000e+00, 3.04800e-01, 7.13000e+01, 2.66337e-03, 1.25201e+02], [1.25000e+03, 0.00000e+00, 3.04800e-01, 7.13000e+01, 2.66337e-03, 1.25951e+02],
```

```
[4.00000e+03, 1.56000e+01, 1.01600e-01, 3.96000e+01, 5.28487e-02, 1.06604e+02], [5.00000e+03, 1.56000e+01, 1.01600e-01, 3.96000e+01, 5.28487e-02, 1.06224e+02], [6.30000e+03, 1.56000e+01, 1.01600e-01, 3.96000e+01, 5.28487e-02, 1.04204e+02]])
```

You may work directly with data, but, for your convenience, I am going to put them also in a nice Pandas DataFrame:

```
[]: import pandas as pd

df = pd.DataFrame(
    data,
    columns=[
        "Frequency",
        "Angle_of_attack",
        "Chord_length",
        "Velocity",
        "Souction_thickness",
        "Sound_pressure",
    ],
)
df
```

[]:	Frequency	Angle_of_attack	Chord_length	Velocity	Suction_thickness	\
0	800.0	0.0	0.3048	71.3	0.002663	
1	1000.0	0.0	0.3048	71.3	0.002663	
2	1250.0	0.0	0.3048	71.3	0.002663	
3	1600.0	0.0	0.3048	71.3	0.002663	
4	2000.0	0.0	0.3048	71.3	0.002663	
•••	•••	•••			•••	
1498	2500.0	15.6	0.1016	39.6	0.052849	
1499	3150.0	15.6	0.1016	39.6	0.052849	
1500	4000.0	15.6	0.1016	39.6	0.052849	
1501	5000.0	15.6	0.1016	39.6	0.052849	
1502	6300.0	15.6	0.1016	39.6	0.052849	

Sound_pressure 0 126.201 1 125.201 2 125.951 3 127.591 4 127.461 1498 110.264 1499 109.254

1500	106.604
1501	106.224
1502	104.204

[1503 rows x 6 columns]

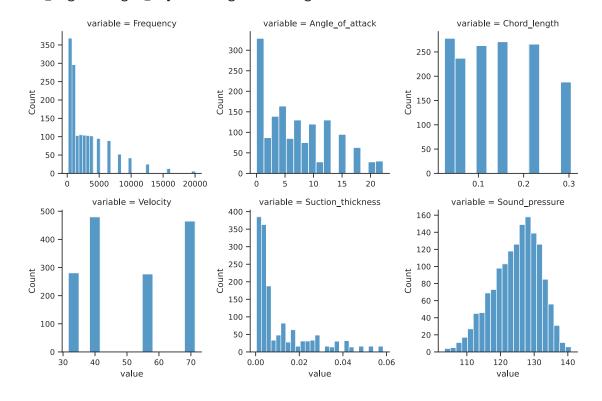
1.3.1 Part A - Analyze the data visually

It is always a good idea to visualize the data before you start doing anything with them.

Part A.I. - Do the histograms of all variables Use as many code segments as you need below to plot the histogram of each variable (all inputs and the output in separate plots) Discuss whether or not you need to standardize the data before moving to regression.

Answer:

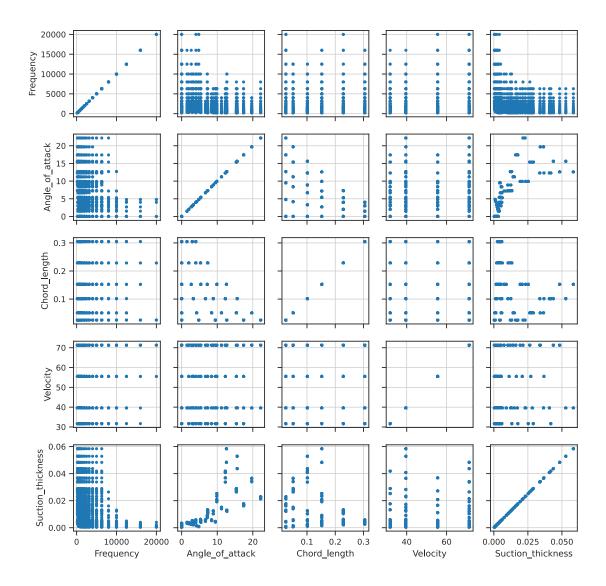
/nix/store/m8m18kk2yg02wjngr2nlvycr8nxcrk32-python3-3.10.12env/lib/python3.10/site-packages/seaborn/axisgrid.py:118: UserWarning: The figure layout has changed to tight self._figure.tight_layout(*args, **kwargs)



Given the wide range of scales present in each of the variables, it is evident that we need to standardize our data before moving onto regression.

Part A.II - Do the scatter plots between all input variables Do the scatter plot between all input variables. This will give you an idea of the range of experimental conditions. Whatever model you build will only be valid inside the domain implicitly defined with your experimental conditions. Are there any holes in the dataset, i.e., places where you have no data?

```
[ ]: n_targets = 1
     n_features = df.shape[1] - n_targets
     fig, ax = plt.subplots(n_features, n_features)
     fig.set_size_inches(10,10)
     for i in range(n_features):
         for j in range(n_features):
             ax[i, j].plot(df.iloc[:, j], df.iloc[:, i], '.')
             ax[i, j].grid()
             if j == 0:
                 ax[i, j].set_ylabel(df.columns[i])
             else:
                 ax[i, j].yaxis.set_ticklabels([])
             if i == n_features - 1:
                 ax[i, j].set_xlabel(df.columns[j])
             else:
                 ax[i, j].xaxis.set_ticklabels([])
```



Most of the data has some kind of "stepping" going on where we tend to not have data in certain ranges, but some are worse than others. For example, Suction Thickness covers its supported range quite well, but velocity has extreme gaps in between each cluster of data points. In fact, velocity only exists in 4 discrete values, shown in the cell below:

[]: df["Velocity"].value_counts()

[]: Velocity

39.6 480 71.3 465 31.7 281 55.5 277

Name: count, dtype: int64

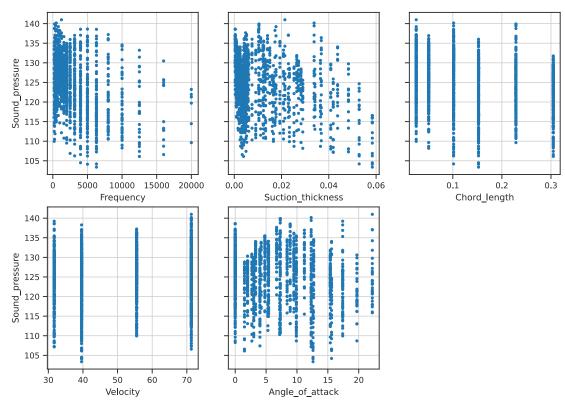
Part A.III - Do the scatter plots between each input and the output Do the scatter plot between each input variable and the output. This will give you an idea of the functional relationship between the two. Do you observe any obvious patterns?

```
[]: n_targets = 1
    n_features = df.shape[1] - n_targets
    fig, ax = plt.subplots(2, 3, sharey=True)
    fig.set_size_inches(10,7)
    for i in range(n_features):
        row = i % 2
        col = i % 3
        ax[row, col].plot(df.iloc[:, i], df.iloc[:, n_features], '.')
        ax[row, col].grid()

        ax[row, col].set_xlabel(df.columns[i])

if col == 0:
        ax[row, col].set_ylabel(df.columns[n_features])

ax[-1, -1].axis("off");
```



There is no obvious pattern that we can discern between any single input and the output... the correlation values for these are probably all very weak

1.3.2 Part B - Use DNN to do regression

Let start by separating inputs and outputs for you:

```
[ ]: X = data[:, :-1]
y = data[:, -1][:, None]
```

Part B.I - Make the loss Use standard torch functionality to create a function that gives you the sum of square error followed by an L2 regularization term for the weights and biases of all network parameters (remember that the L2 regularization is like putting a Gaussian prior on each parameter). Follow the instructions below and fill in the missing code.

Answer:

```
[]: import torch
import torch.nn as nn

# Use standard torch functionality to define a function
# mse_loss(y_obs, y_pred) which gives you the mean of the sum of the square
# of the difference between y_obs and y_pred
# Hint: This is already implemented in PyTorch. You can just reuse it.
mse_loss = nn.MSELoss()
```

Your mse_loss: 1.25
What you should be getting: 1.25

```
[]: # Now, we will create a regularization term for the loss
# I'm just going to give you this one:

def 12_reg_loss(params):

"""

This needs an iterable object of network parameters.

You can get it by doing `net.parameters()`.
```

```
Returns the sum of the squared norms of all parameters.

12_reg = torch.tensor(0.0)
for p in params:
    12_reg += torch.norm(p) ** 2
return 12_reg
```

```
[]: # Finally, let's add the two together to make a mean square error loss
     # plus some weight (which we will call req_weight) times the sum of the squaredu
      ⇔norms
     # of all parameters.
     # I give you the signature and you have to implement the rest of the code:
     def loss_func(y_obs, y_pred, reg_weight, params):
         Parameters:
         y obs
                        The observed outputs
         y_pred -
                        The predicted outputs
         reg_weight - The regularization weight (a positive scalar)
         params
                       An iterable containing the parameters of the network
         Returns the sum of the MSE loss plus reg_weight times the sum of the ⊔
      \hookrightarrow squared norms of
         all parameters.
         11 11 11
         return mse_loss(y_obs, y_pred) + reg_weight * 12_reg_loss(params)
```

The loss without regularization: 1.25

This should be the same as this: 1.25 The loss with regularization: 1.32

Part B.III - Write flexible code to perform regression When training neural networks, you must hand-pick many parameters, from the network structure to the activation functions to the regularization parameters to the details of the stochastic optimization. Instead of mindlessly going through trial and error, it is better to think about the parameters you want to investigate (vary) and write code that allows you to train networks with all different parameter variations repeatedly. In what follows, I will guide you through writing code for training an arbitrary regression network having the flexibility to:

- standardize the inputs and output or not
- experiment with various levels of regularization
- change the learning rate of the stochastic optimization algorithm
- change the batch size of the optimization algorithm
- change the number of epochs (how many times the optimization algorithm does a complete sweep through all the data.

```
[]: # We will start by creating a class that encapsulates a regression
     # network so that we can turn on or off input/output standardization
     # without too much fuss.
     # The class will represent a trained network model.
     # It will "know" whether or not during training we standardized the data.
     # I am not asking you to do anything here, so you can run this code segment
     # or read through it if you want to know the details.
     from sklearn.preprocessing import StandardScaler
     class TrainedModel(object):
         A class that represents a trained network model.
         The main reason I created this class is to encapsulate the standardization
         process in an excellent way.
         Parameters:
                             A network.
         standardized - True if the network expects standardized features and_{\sqcup}
      \hookrightarrow outputs
                             standardized targets. False otherwise.
                             A feature scalar - Ala scikit.learn. Must have
         feature_scaler -
      \hookrightarrow transform()
                             and inverse transform() implemented.
         target_scaler -
                             Similar to feature_scaler but for targets...
```

```
def __init__(
             self, net, standardized=False, feature scaler=None, target_scaler=None
         ):
             self.net = net
             self.standardized = standardized
             self.feature_scaler = feature_scaler
             self.target_scaler = target_scaler
         def __call__(self, X):
             Evaluates the model at X.
             # If not scaled, then the model is just net(X)
             if not self.standardized:
                 return self.net(X)
             # Otherwise:
             # Scale X:
             if self.feature_scaler is not None and self.target_scaler is not None:
                 X_scaled = self.feature_scaler.transform(X)
                 # Evaluate the network output - which is also scaled:
                 y_scaled = self.net(torch.Tensor(X_scaled))
                 # Scale the output back:
                 y = self.target_scaler.inverse_transform(y_scaled.detach().numpy())
                 return y
             else:
                 raise TypeError("If standardized=True, then feature_scaler and_
      →target_scaler must not be None")
[]: # Go through the code that follows and fill in the missing parts
     from sklearn.model_selection import train_test_split
     # We need this for a progress bar:
     from tqdm import tqdm
     def train_net(X, y, net, reg_weight, n_batch, epochs, lr, test_size=0.33,
                   standardize=True):
         11 11 11
         A function that trains a regression neural network using stochastic gradient
         descent and returns the trained network. The loss function being minimized \Box
      \hookrightarrow is
         `loss_func`.
         Arguments:
                        The observed features
         X
                    - The observed targets
         y
```

The batch size you want to use for stochastic optimization

The network you want to fit

net

 n_batch

```
epochs
                  How many times do you want to pass over the training
\hookrightarrow dataset.
  l.r
                   The learning rate for the stochastic optimization algorithm.
                   What fraction of the data should be used for testing.
  test size -
\hookrightarrow (validation).
  standardize -
                  Whether or not you want to standardize the features and
\hookrightarrow the targets.
  11 11 11
  # Split the data
  X_train, X_test, y_train, y_test = train_test_split(X, y,_
# Standardize the data
  if standardize:
       # Build the scalers
      feature_scaler = StandardScaler().fit(X)
      target_scaler = StandardScaler().fit(y)
       # Get scaled versions of the data
      X_train_scaled = feature_scaler.transform(X_train)
      y train scaled = target scaler.transform(y train)
      X_test_scaled = feature_scaler.transform(X_test)
      y_test_scaled = target_scaler.transform(y_test)
  else:
      feature_scaler = None
      target_scaler = None
      X_train_scaled = X_train
      y_train_scaled = y_train
      X_test_scaled = X_test
      y_test_scaled = y_test
  # Turn all the numpy arrays to torch tensors
  X_train_scaled = torch.Tensor(X_train_scaled)
  X_test_scaled = torch.Tensor(X_test_scaled)
  y_train_scaled = torch.Tensor(y_train_scaled)
  y_test_scaled = torch.Tensor(y_test_scaled)
  # This is pytorch magic to enable shuffling of the
  # training data every time we go through them
  train_dataset = torch.utils.data.TensorDataset(X_train_scaled,__
→y_train_scaled)
  train data loader = torch.utils.data.DataLoader(train dataset,
                                                    batch_size=n_batch,
                                                    shuffle=True)
  # Create an Adam optimizing object for the neural network `net`
  # with learning rate `lr`
  optimizer = torch.optim.Adam(params=net.parameters(), lr=lr)
```

```
# This is a place to keep track of the test loss
  test_loss = []
  # Iterate the optimizer.
  # Remember, each time we go through the entire dataset we complete an
→ `epoch`
  # I have wrapped the range around tydm to give you a nice progress bar
  # to look at
  for e in tqdm(range(epochs)):
       # This loop goes over all the shuffled training data
       # That's why the DataLoader class of PyTorch is convenient
      for i, (X_batch, y_batch) in enumerate(train_data_loader):
           # Perform a single optimization step with loss function
           # loss_func(y_batch, y_pred, req_weight, net.parameters())
           # Hint 1: You have defined loss_func() already
           # Hint 2: Consult the hands-on activities for an example
           # Your code here
           optimizer.zero grad()
          y_pred = net(X_batch)
           loss = loss_func(y_batch, y_pred, reg_weight=reg_weight, params=net.
→parameters())
          loss.backward()
           optimizer.step()
           # test_loss.append(loss.item())
           # if i % 1000 == 0:
                print(f"epoch {e}, i {i}: loss = {loss.item():1.2e}")
       # Evaluate the test loss and append it on the list `test_loss`
      y_pred_test = net(X_test_scaled)
      ts_loss = mse_loss(y_test_scaled, y_pred_test)
      test_loss.append(ts_loss.item())
   # Make a TrainedModel
  trained_model = TrainedModel(net, standardized=standardize,
                                feature_scaler=feature_scaler,
                                target_scaler=target_scaler)
   # Make sure that we return properly scaled
```

```
# Return everything we need to analyze the results return trained_model, test_loss, X_train, y_train, X_test, y_test
```

Use this to test your code:

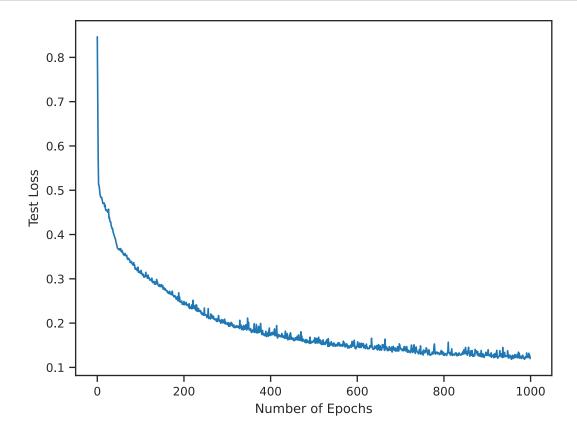
```
[]: # A simple one-layer network with 10 neurons
net = nn.Sequential(nn.Linear(5, 20), nn.Sigmoid(), nn.Linear(20, 1))
epochs = 1000
lr = 0.01
reg_weight = 0
n_batch = 100
model, test_loss, X_train, y_train, X_test, y_test = train_net(
    X, y, net, reg_weight, n_batch, epochs, lr
)
```

```
100% | 1000/1000 [00:14<00:00, 68.32it/s]
```

There are a few more things for you to do here. First, plot the evolution of the test loss as a function of the number of epochs:

```
[]: fig, ax = plt.subplots()

ax.plot(test_loss)
ax.set_xlabel("Number of Epochs")
ax.set_ylabel("Test Loss");
```



Now plot the observations vs predictions plot for the training data:

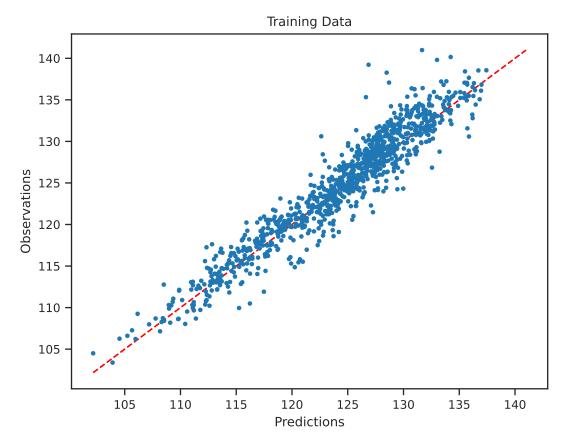
```
[]: fig, ax = plt.subplots()

X_train_tensor = torch.Tensor(X_train)
y_pred_train = model(X_train_tensor)

# Plot the line of perfect fit
y_min = np.min(np.concatenate([y_pred_train, y_train]))
y_max = np.max(np.concatenate([y_pred_train, y_train]))
ax.plot([y_min, y_max], [y_min, y_max], 'r--')

ax.plot(y_pred_train, y_train, '.')

ax.set_title("Training Data")
ax.set_ylabel("Dbservations");
```



And do the observations vs predictions plot for the test data:

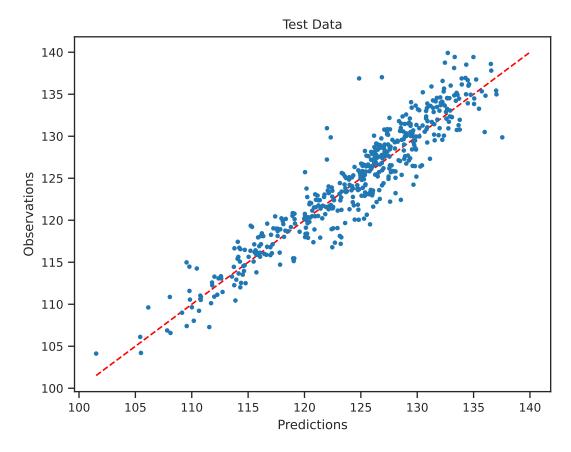
```
fig, ax = plt.subplots()

X_test_tensor = torch.Tensor(X_test)
y_pred_test = model(X_test_tensor)

# Plot the line of perfect fit
y_min = np.min(np.concatenate([y_pred_test, y_test]))
y_max = np.max(np.concatenate([y_pred_test, y_test]))
ax.plot([y_min, y_max], [y_min, y_max], 'r--')

ax.plot(y_pred_test, y_test, '.')

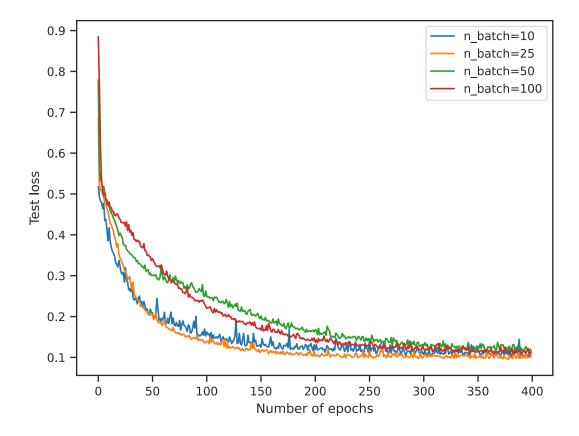
ax.set_title("Test Data")
ax.set_xlabel("Predictions");
```



Part C.I - Investigate the effect of the batch size For the given network, try batch sizes of 10, 25, 50, and 100 for 400 epochs. In the sample plot, show the evolution of the test loss function

for each case. Which batch sizes lead to faster training times and why? Which one would you choose?

```
[]: epochs = 400
     lr = 0.01
     reg weight = 0
     test_losses = []
     models = \Pi
     batches = [10, 25, 50, 100]
     for n_batch in batches:
         print('Training n_batch: {0:d}'.format(n_batch))
         net = nn.Sequential(nn.Linear(5, 20),
                         nn.Sigmoid(),
                         nn.Linear(20, 1))
         model, test_loss, X_train, y_train, X_test, y_test = train_net(
             Х,
             у,
             net,
             reg_weight,
             n_batch,
             epochs,
             lr
         test_losses.append(test_loss)
         models.append(model)
    Training n_batch: 10
    100%
               | 400/400 [00:38<00:00, 10.43it/s]
    Training n_batch: 25
    100%|
               | 400/400 [00:16<00:00, 24.85it/s]
```



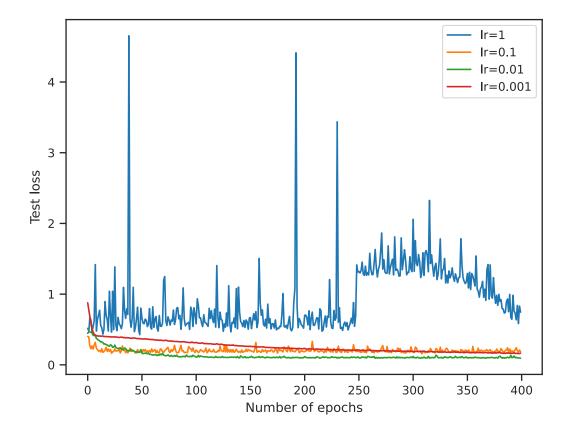
The larger batch sizes take less time to train. This is probably due to a combination of factors. For one, PyTorch is designed to take advantage of vector operations and is good at parallelizing large vector operations (GPUs also perform best this way). When we choose a small batch size, though, we break up all of these large operations into a large number of smaller operations in series. This is slower to train. However, we observe that the model tends to converge faster and to a lower test loss value when the batch size is smaller. We assume that this translates to a more accurate model.

I would choose the batch size of 50 it is a good compromise between speed and accuracy, since it completed in less than a third of the time that 10 did and reached a minimum loss that was basically the same as the batch size of 25.

Part C.II - Investigate the effect of the learning rate Fix the batch size to the best one you identified in Part C.I. For the given network, try learning rates of 1, 0.1, 0.01, and 0.001 for 400 epochs. In the sample plot, show the evolution of the test loss function for each case. Does the algorithm converge for all learning rates? Which learning rate would you choose?

```
[]: epochs = 400
lrs = [1, 0.1, 0.01, 0.001]
reg_weight = 0
test_losses = []
```

```
models = []
     n_batch = 10
     for lr in lrs:
         print('Training lr: {}'.format(lr))
         net = nn.Sequential(nn.Linear(5, 20),
                         nn.Sigmoid(),
                         nn.Linear(20, 1))
         model, test_loss, X_train, y_train, X_test, y_test = train_net(
             Х,
             у,
             net,
             reg_weight,
             n_batch,
             epochs,
             lr
         )
         test_losses.append(test_loss)
         models.append(model)
    Training lr: 1
    100%|
              | 400/400 [00:34<00:00, 11.45it/s]
    Training lr: 0.1
              | 400/400 [00:36<00:00, 10.81it/s]
    100%|
    Training lr: 0.01
    100%|
              | 400/400 [00:36<00:00, 11.05it/s]
    Training lr: 0.001
              | 400/400 [00:37<00:00, 10.76it/s]
    100%|
[]: fig, ax = plt.subplots(dpi=100)
     for tl, lr in zip(test_losses, lrs):
         ax.plot(tl, label="lr={}".format(lr))
     ax.set_xlabel("Number of epochs")
     ax.set_ylabel("Test loss")
     plt.legend(loc="best");
```



The training times were all roughly equal for the different learning rates tested. The algorithm converged for all learning rates except for lr=1, which seemed to begin osciallating out of control as the number of epochs increased. The learning rate of 0.01 probably looks like the best choice here, since it leads to a more stable convergence at a low loss value that is basically equivalent to the value that 0.1 reaches, but 0.1 has more variation in the loss as a function of epoch number. 0.001 converges to a very stable value, but the loss is much higher, so this is too small of a value.

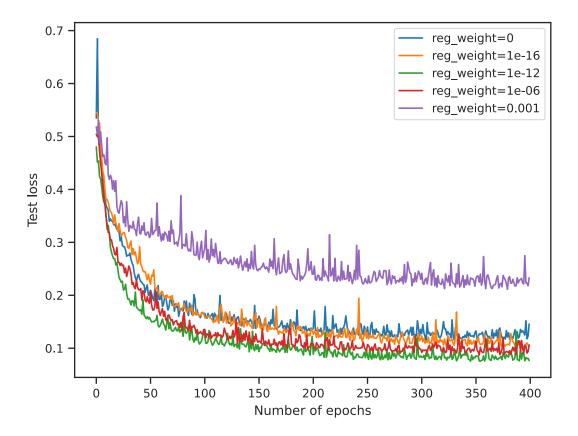
Part C.III - Investigate the effect of the regularization weight Fix the batch size to the value you selected in C.I and the learning rate to the value you selected in C.II. For the given network, try regularization weights of 0, 1e-16, 1e-12, 1e-6, and 1e-3 for 400 epochs. In the sample plot, show the evolution of the test loss function for each case. Which regularization weight seems to be the best and why?

```
[]: epochs = 400
lr = 0.01
reg_weights = [0, 1e-16, 1e-12, 1e-6, 1e-3]
test_losses = []
models = []
for reg_weight in reg_weights:
    print('Training reg_weight: {}'.format(reg_weight))
```

```
nn.Sigmoid(),
                         nn.Linear(20, 1))
         model, test_loss, X_train, y_train, X_test, y_test = train_net(
             Х,
             у,
             net,
             reg_weight,
             n_batch,
             epochs,
             lr
         test_losses.append(test_loss)
         models.append(model)
    Training reg_weight: 0
    100%|
               | 400/400 [00:35<00:00, 11.12it/s]
    Training reg_weight: 1e-16
    100%|
               | 400/400 [00:36<00:00, 10.91it/s]
    Training reg_weight: 1e-12
    100%|
               | 400/400 [00:36<00:00, 10.95it/s]
    Training reg_weight: 1e-06
               | 400/400 [00:37<00:00, 10.78it/s]
    100%|
    Training reg_weight: 0.001
    100%|
               | 400/400 [00:36<00:00, 10.90it/s]
[]: fig, ax = plt.subplots(dpi=100)
     for tl, reg_weight in zip(test_losses, reg_weights):
         ax.plot(tl, label="reg_weight={}".format(reg_weight))
     ax.set_xlabel("Number of epochs")
     ax.set_ylabel("Test loss")
```

net = nn.Sequential(nn.Linear(5, 20),

plt.legend(loc="best");



Once again, each value takes about the same time to train. reg_weight=1e-16 seems to perform the best in this case. It probably strikes the balance between penalizing large weights and thus overfitting without preventing the model from being able to learn. It seems that the model was not at great risk for overfitting in the first place, since the weight of 0 is somewhat comparable to this.

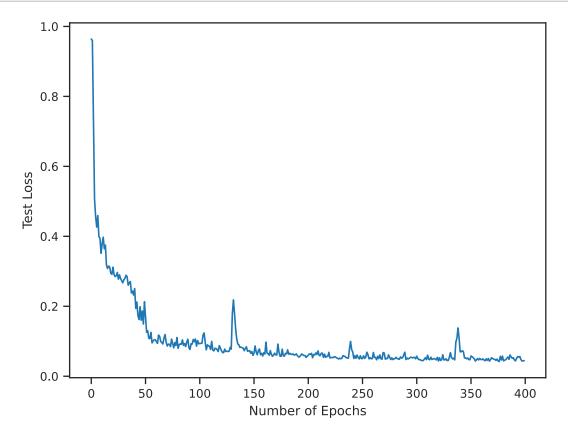
Part D.I - Train a bigger network You have developed some intuition about the parameters involved in training a network. Now, let's train a larger one. In particular, use a 5-layer deep network with 100 neurons per layer. You can use the sigmoid activation function, or you can change it to something else. Make sure you plot: - the evolution of the test loss as a function of the epochs - the observations vs predictions plot for the test data

```
lr = 0.01
reg_weight = 1e-16
n_batch = 50
model, test_loss, X_train, y_train, X_test, y_test = train_net(
    X, y, net, reg_weight, n_batch, epochs, lr
)
```

100%| | 400/400 [00:29<00:00, 13.42it/s]

```
[]: fig, ax = plt.subplots()

ax.plot(test_loss)
ax.set_xlabel("Number of Epochs")
ax.set_ylabel("Test Loss");
```



```
[]: fig, ax = plt.subplots()

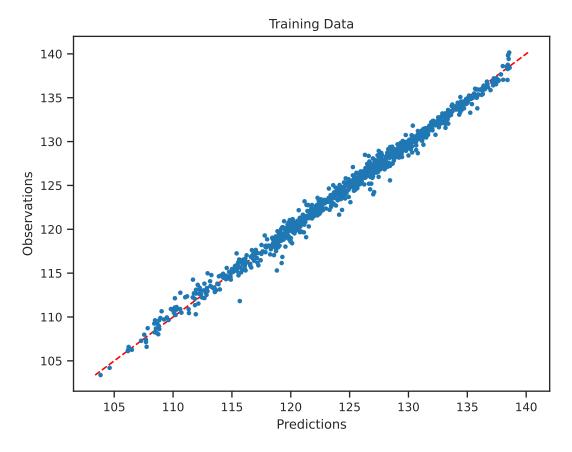
X_train_tensor = torch.Tensor(X_train)
y_pred_train = model(X_train_tensor)

# Plot the line of perfect fit
```

```
y_min = np.min(np.concatenate([y_pred_train, y_train]))
y_max = np.max(np.concatenate([y_pred_train, y_train]))
ax.plot([y_min, y_max], [y_min, y_max], 'r--')

ax.plot(y_pred_train, y_train, '.')

ax.set_title("Training Data")
ax.set_xlabel("Predictions")
ax.set_ylabel("Observations");
```

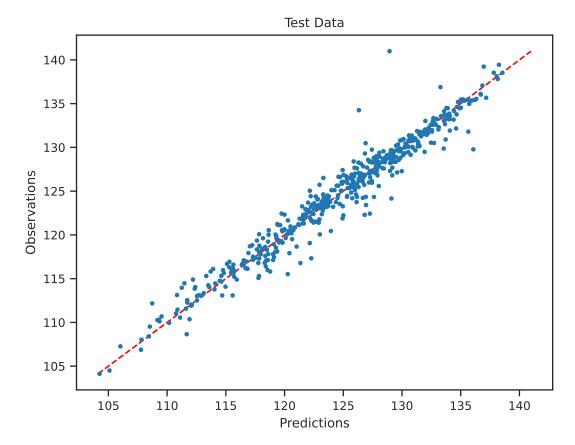


```
[]: fig, ax = plt.subplots()

X_test_tensor = torch.Tensor(X_test)
y_pred_test = model(X_test_tensor)

# Plot the line of perfect fit
y_min = np.min(np.concatenate([y_pred_test, y_test]))
y_max = np.max(np.concatenate([y_pred_test, y_test]))
ax.plot([y_min, y_max], [y_min, y_max], 'r--')
```

```
ax.plot(y_pred_test, y_test, '.')
ax.set_title("Test Data")
ax.set_xlabel("Predictions")
ax.set_ylabel("Observations");
```



Part D.II - Make a prediction Visualize the scaled sound level as a function of the stream velocity for a fixed frequency of 2500 Hz, a chord length of 0.1 m, a suction side displacement thickness of 0.01 m, and an angle of attack of 0, 5, and 10 degrees.

Answer:

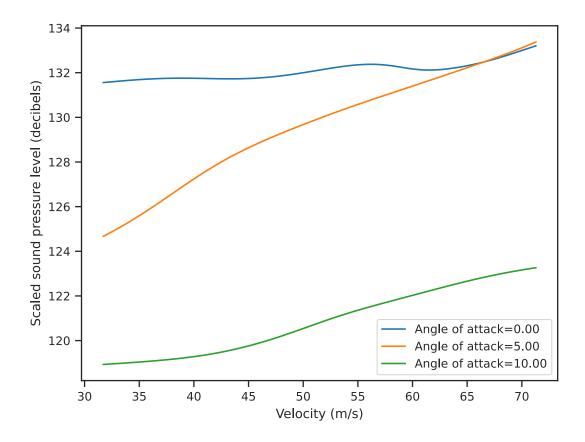
This is just a check for your model. You will have to run the following code segments for the best model you have found.

```
[]: best_model = model

def plot_sound_level_as_func_of_stream_vel(
    freq=2500,
    angle_of_attack=10,
    chord_length=0.1,
```

```
suc_side_disp_thick=0.01,
         ax=None,
         label=None
     ):
         if ax is None:
             fig, ax = plt.subplots(dpi=100)
         # The velocities on which we want to evaluate the model
         vel = np.linspace(X[:, 3].min(), X[:, 3].max(), 100)[:, None]
         # Make the input for the model
         freqs = freq * np.ones(vel.shape)
         angles = angle_of_attack * np.ones(vel.shape)
         chords = chord_length * np.ones(vel.shape)
         sucs = suc_side_disp_thick * np.ones(vel.shape)
         # Put all these into a single array
         XX = np.hstack([freqs, angles, chords, vel, sucs])
         ax.plot(vel, best_model(XX), label=label)
         ax.set_xlabel('Velocity (m/s)')
         ax.set_ylabel('Scaled sound pressure level (decibels)')
[]: fig, ax = plt.subplots(dpi=100)
     for aofa in [0, 5, 10]:
         plot_sound_level_as_func_of_stream_vel(
             angle_of_attack=aofa, ax=ax, label="Angle of attack={0:1.2f}".
      →format(aofa)
         )
```

plt.legend(loc="best");



1.4 Problem 2 - Classification with DNNs

Dr. Ali Lenjani kindly provided this homework problem. It is based on our joint work on this paper: Hierarchical convolutional neural networks information fusion for activity source detection in smart buildings. The data come from the Human Activity Benchmark published by Dr. Juan M. Caicedo.

So the problem is as follows. You want to put sensors on a building so that it can figure out what is going on inside it. This has applications in industrial facilities (e.g., detecting if there was an accident), public infrastructure, hospitals (e.g., did a patient fall off a bed), etc. Typically, the problem is addressed using cameras. Instead of cameras, we will investigate the ability of acceleration sensors to tell us what is going on.

Four acceleration sensors have been placed in different locations in the benchmark building to record the floor vibration signals of other objects falling from several heights. A total of seven cases cases were considered:

- bag-high: 450 g bag containing plastic pieces is dropped roughly from 2.10 m
- bag-low: 450 g bag containing plastic pieces is dropped roughly from 1.45 m
- ball-high: 560 g basketball is dropped roughly from 2.10 m
- ball-low: 560 g basketball is dropped roughly from 1.45 m
- j-jump: person 1.60 m tall, 55 kg jumps approximately 12 cm high
- d-jump: person 1.77 m tall, 80 kg jumps approximately 12 cm high

• w-jump: person 1.85 m tall, 85 kg jumps approximately 12 cm high

Each of these seven cases was repeated 115 times at five different building locations. The original data are here, but I have repackaged them for you in a more convenient format. Let's download them:

```
[]: # !curl -O 'https://dl.dropboxusercontent.com/s/n8dczk7t8bxOpxi/
→human_activity_data.npz'
```

Here is how to load the data:

```
[]: data = np.load("human_activity_data.npz")
```

This is a Python dictionary that contains the following entries:

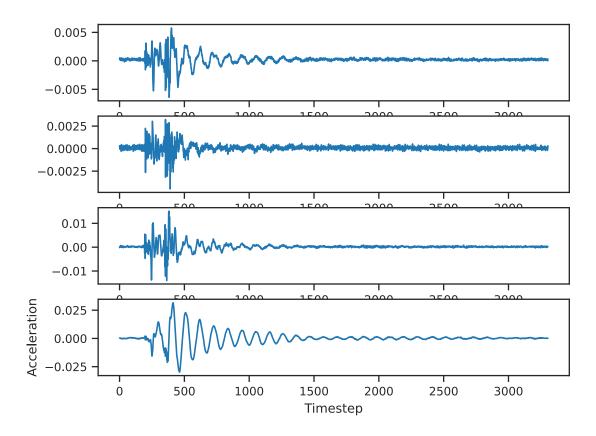
```
[]: for key in data.keys():
    print(key, ":", data[key].shape)

features : (4025, 4, 3305)
```

labels_1 : (4025,) labels_2 : (4025,) loc_ids : (4025,)

Let's go over these one by one. First, the **features**. These are the accelertion sensor measurements. Here is how you visualize them:

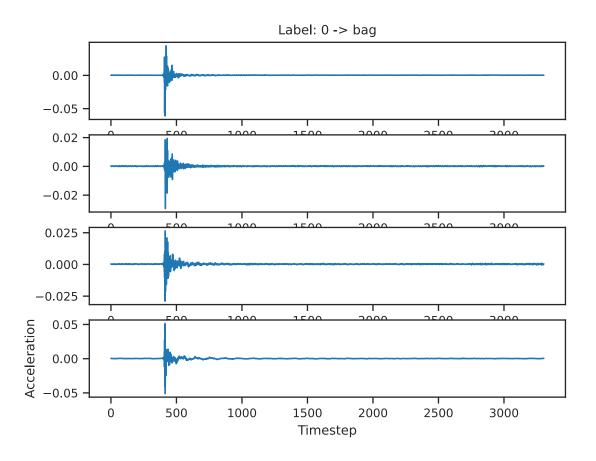
```
[]: fig, ax = plt.subplots(4, 1, dpi=100)
# Loop over sensors
for j in range(4):
        ax[j].plot(data["features"][0, j])
ax[-1].set_xlabel("Timestep")
ax[-1].set_ylabel("Acceleration");
```

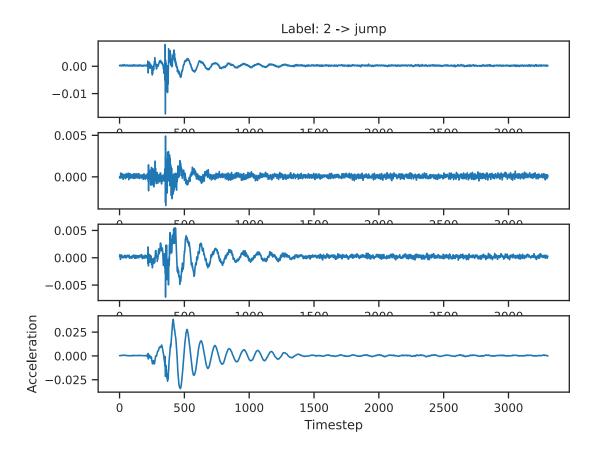


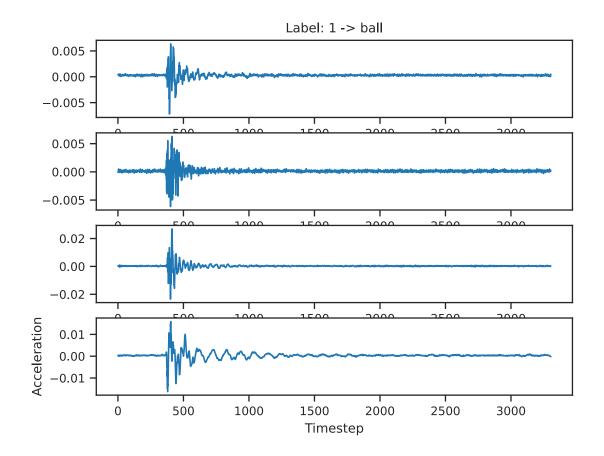
The second key, labels_1, is a bunch of integers ranging from 0 to 2 indicating whether the entry corresponds to a "bag," a "ball" or a "jump." For your reference, the correspondence is:

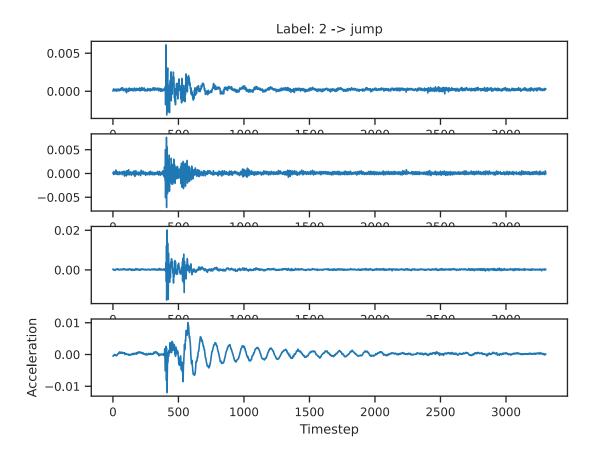
```
[]: LABELS_1_TO_TEXT = {0: "bag", 1: "ball", 2: "jump"}
```

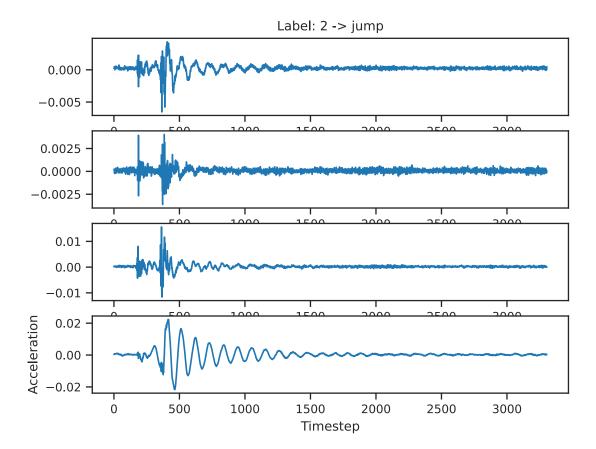
And here are a few examples:











The array labels_2 includes integers from 0 to 6 indicating the detailed label of the experiment. The correspondence between integers and text labels is:

Finally, the field loc_ids takes values from 0 to 4 indicating five distinct locations in the building. Before moving forward with the questions, let's extract the data in a more covenient form:

```
[]: # The features
X = data["features"]
# The labels_1
y1 = data["labels_1"]
```

```
# The labels_2
y2 = data["labels_2"]
# The locations
y3 = data["loc_ids"]
```

1.4.1 Part A - Train a CNN to predict the high-level type of observation (bag, ball, or jump)

Fill in the blanks in the code blocks below to train a classification neural network that will take you from the four acceleration sensor data to the high-level type of each observation. You can keep the network structure fixed, but you can experiment with the learning rate, the number of epochs, or anything else. Just keep in mind that for this particular dataset, it is possible to hit an accuracy of almost 100%.

Answer:

The first thing that we need to do is pick a neural network structure. Let's use 1D convolutional layers at the very beginning. These are the same as the 2D (image) convolutional layers but in 1D. The reason I am proposing this is that the convolutional layers are invariant to small translations of the acceleration signal (just like the labels are). Here is what I propose:

```
[]: import torch
     import torch.nn as nn
     import torch.nn.functional as F
     class Net(nn.Module):
         def __init__(self, num_labels=3):
             super(Net, self).__init__()
             # A convolutional layer:
             # 3 = input channels (sensors),
             # 6 = output channels (features),
             #5 = kernel size
             self.conv1 = nn.Conv1d(4, 8, 10)
             # A 2 x 2 max pooling layer - we are going to use it two times
             self.pool = nn.MaxPool1d(5)
             # Another convolutional layer
             self.conv2 = nn.Conv1d(8, 16, 5)
             # Some linear layers
             self.fc1 = nn.Linear(16 * 131, 200)
             self.fc2 = nn.Linear(200, 50)
             self.fc3 = nn.Linear(50, num_labels)
         def forward(self, x):
             # This function implements your network output
             # Convolutional layer, followed by relu, followed by max pooling
             x = self.pool(F.relu(self.conv1(x)))
             # Same thing
```

```
x = self.pool(F.relu(self.conv2(x)))
# Flatting the output of the convolutional layers
x = x.view(-1, 16 * 131)
# Go through the first dense linear layer followed by relu
x = F.relu(self.fc1(x))
# Through the second dense layer
x = F.relu(self.fc2(x))
# Finish up with a linear transformation
x = self.fc3(x)
return x
```

```
[]: # You can make the network like this:
net = Net(3)
```

Now, you need to pick the right loss function for classification tasks:

```
[ ]: cnn_loss_func = nn.CrossEntropyLoss()
```

Just like before, let's organize our training code in a convenient function that allows us to play with the parameters of training. Fill in the missing code.

```
[]: from sklearn.model_selection import train_test_split
     def train_cnn(X, y, net, n_batch, epochs, lr, test_size=0.33):
         A function that trains a regression neural network using stochastic gradient
         descent and returns the trained network. The loss function being minimized \Box
      \hookrightarrow is
          `loss_func`.
         Parameters:
                          The observed features
         X
                         The observed targets
                       The network you want to fit
         net
                    - The batch size you want to use for stochastic optimization
         n\_batch
                     - How many times do you want to pass over the training \Box
         epochs
      \hookrightarrow dataset.
                          The learning rate for the stochastic optimization algorithm.
                          What percentage of the data should be used for testing\sqcup
         test_size -
      \hookrightarrow (validation).
         11 11 11
         # Split the data
         X_train, X_test, y_train, y_test = train_test_split(X, y,_
      →test_size=test_size)
         # Turn all the numpy arrays to torch tensors
         X_train = torch.Tensor(X_train)
         X_test = torch.Tensor(X_test)
```

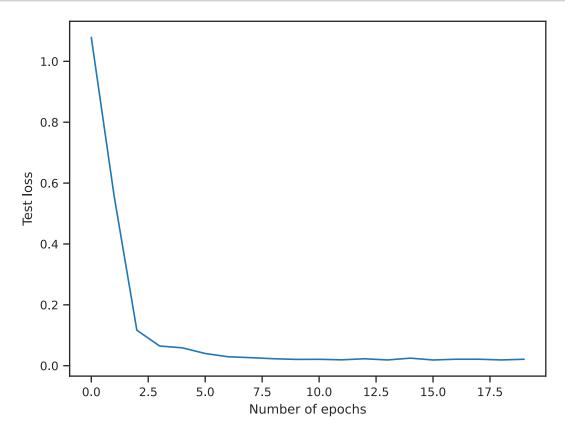
```
y_train = torch.LongTensor(y_train)
  y_test = torch.LongTensor(y_test)
  # This is pytorch magick to enable shuffling of the
  # training data every time we go through them
  train_dataset = torch.utils.data.TensorDataset(X_train, y_train)
  train_data_loader = torch.utils.data.DataLoader(train_dataset,
                                                    batch_size=n_batch,
                                                    shuffle=True)
  # Create an Adam optimizing object for the neural network `net`
  # with learning rate `lr`
  optimizer = torch.optim.Adam(params=net.parameters(), lr=lr)
  # This is a place to keep track of the test loss
  test_loss = []
   # This is a place to keep track of the accuracy on each epoch
  accuracy = []
  # Iterate the optimizer.
   # Remember, each time we go through the entire dataset we complete an_{\sqcup}
→ `epoch`
  # I have wrapped the range around tqdm to give you a nice progress bar
  # to look at
  for e in range(epochs):
       # This loop goes over all the shuffled training data
       # That's why the DataLoader class of PyTorch is convenient
      for X_batch, y_batch in train_data_loader:
           # Perform a single optimization step with loss function
           # cnn_loss_func(y_batch, y_pred, reg_weight)
           # Hint 1: You have defined cnn_loss_func() already
           # Hint 2: Consult the hands-on activities for an example
           # your code here
           optimizer.zero_grad()
          y_pred = net(X_batch)
           loss = cnn_loss_func(y_pred, y_batch) # + reg_weight *_
\hookrightarrow l2\_reg\_loss(params)
           loss.backward()
           optimizer.step()
       # Evaluate the test loss and append it on the list `test_loss`
       y_pred_test = net(X_test)
```

Now experiment with the epochs, the learning rate, and the batch size until this works.

```
Epoch 1: accuracy = 0.42664, test loss = 1.0785764455795288
Epoch 2: accuracy = 0.72837, test loss = 0.5590016841888428
Epoch 3: accuracy = 0.97818, test loss = 0.11661700904369354
Epoch 4: accuracy = 0.98871, test loss = 0.06478121131658554
Epoch 5: accuracy = 0.98721, test loss = 0.0587134025990963
Epoch 6: accuracy = 0.99097, test loss = 0.04019084572792053
Epoch 7: accuracy = 0.99473, test loss = 0.029450371861457825
Epoch 8: accuracy = 0.99323, test loss = 0.026471437886357307
Epoch 9: accuracy = 0.99549, test loss = 0.02297958731651306
Epoch 10: accuracy = 0.99699, test loss = 0.020777974277734756
Epoch 11: accuracy = 0.99549, test loss = 0.021033180877566338
Epoch 12: accuracy = 0.99850, test loss = 0.019437361508607864
Epoch 13: accuracy = 0.99549, test loss = 0.02288350835442543
Epoch 14: accuracy = 0.99850, test loss = 0.018923873081803322
Epoch 15: accuracy = 0.99549, test loss = 0.024927642196416855
Epoch 16: accuracy = 0.99774, test loss = 0.018860435113310814
Epoch 17: accuracy = 0.99699, test loss = 0.021309170871973038
Epoch 18: accuracy = 0.99699, test loss = 0.02136903442442417
Epoch 19: accuracy = 0.99774, test loss = 0.019002089276909828
Epoch 20: accuracy = 0.99699, test loss = 0.021310392767190933
```

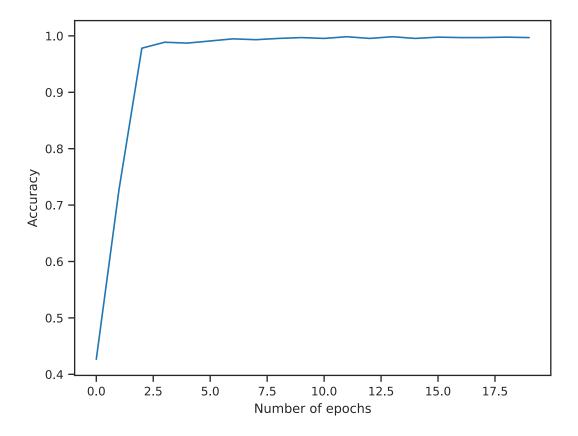
Plot the evolution of the test loss as a function of epochs.

```
[]: fig, ax = plt.subplots(dpi=100)
    ax.plot(test_loss)
    ax.set_xlabel("Number of epochs")
    ax.set_ylabel("Test loss");
```



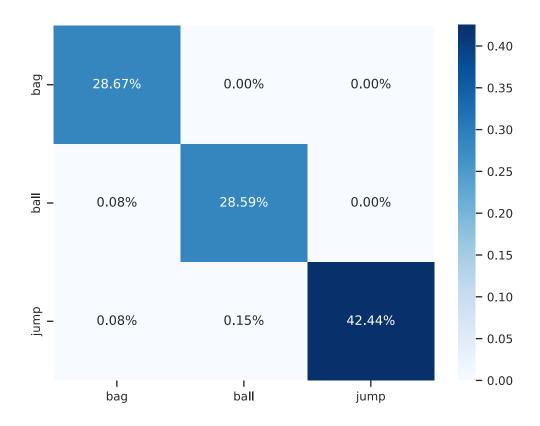
Plot the evolution of the accuracy as a function of epochs.

```
[]: fig, ax = plt.subplots(dpi=100)
    ax.plot(accuracy)
    ax.set_xlabel("Number of epochs")
    ax.set_ylabel("Accuracy");
```



Plot the confusion matrix.

```
[]: from sklearn.metrics import confusion_matrix
     # Predict on the test data
     y_pred_test = trained_model(X_test)
     \# Remember that the prediction is probabilistic Epoch
     # We need to simply pick the label with the highest probability:
     _, y_pred_labels = torch.max(y_pred_test, 1)
     # Here is the confusion matrix:
     cf_matrix = confusion_matrix(y_test, y_pred_labels)
[]: sns.heatmap(
         cf_matrix / np.sum(cf_matrix),
         annot=True,
         fmt=".2%",
         cmap="Blues",
         xticklabels=LABELS_1_TO_TEXT.values(),
         yticklabels=LABELS_1_TO_TEXT.values(),
     );
```



1.4.2 Part B - Train a CNN to predict the the low-level type of observation (bag-high, bag-low, etc.)

Repeat what you did above for y2.

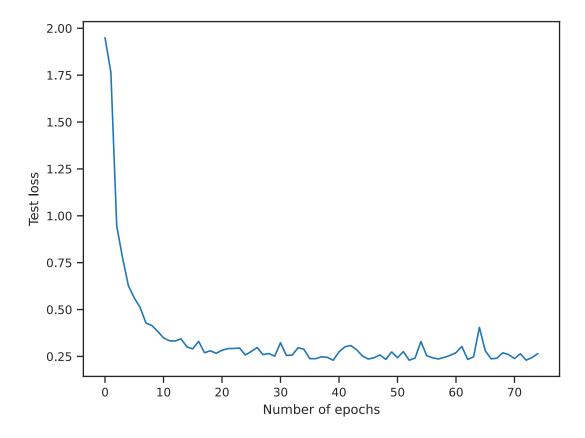
```
# )
# epochs = 50
\# lr = 0.0008
\# n_batch = 30
# trained_model, test_loss, accuracy, X_train, y_train, X_test, y_test =_

→ train_cnn(
      X, y2, trained_model, n_batch, epochs, lr
Epoch 1: accuracy = 0.13093, test loss = 1.9496824741363525
Epoch 2: accuracy = 0.22423, test loss = 1.7648767232894897
Epoch 3: accuracy = 0.53047, test loss = 0.9458544850349426
Epoch 4: accuracy = 0.59142, test loss = 0.7757077813148499
Epoch 5: accuracy = 0.65839, test loss = 0.6264389753341675
Epoch 6: accuracy = 0.70429, test loss = 0.5621477961540222
Epoch 7: accuracy = 0.76373, test loss = 0.5113839507102966
Epoch 8: accuracy = 0.82092, test loss = 0.4279845356941223
Epoch 9: accuracy = 0.81941, test loss = 0.4150756597518921
Epoch 10: accuracy = 0.81941, test loss = 0.3838397264480591
Epoch 11: accuracy = 0.85403, test loss = 0.34898486733436584
Epoch 12: accuracy = 0.85854, test loss = 0.33369866013526917
Epoch 13: accuracy = 0.84725, test loss = 0.3323861360549927
Epoch 14: accuracy = 0.85327, test loss = 0.34370261430740356
Epoch 15: accuracy = 0.87133, test loss = 0.301111102104187
Epoch 16: accuracy = 0.88036, test loss = 0.29033809900283813
Epoch 17: accuracy = 0.86757, test loss = 0.33066239953041077
Epoch 18: accuracy = 0.88713, test loss = 0.26970377564430237
Epoch 19: accuracy = 0.88337, test loss = 0.2800876200199127
Epoch 20: accuracy = 0.88939, test loss = 0.2667470872402191
Epoch 21: accuracy = 0.87810, test loss = 0.28279614448547363
Epoch 22: accuracy = 0.87735, test loss = 0.291701078414917
Epoch 23: accuracy = 0.88036, test loss = 0.29264646768569946
Epoch 24: accuracy = 0.88412, test loss = 0.2946324646472931
Epoch 25: accuracy = 0.89014, test loss = 0.25764045119285583
Epoch 26: accuracy = 0.88187, test loss = 0.27653223276138306
Epoch 27: accuracy = 0.88488, test loss = 0.297753244638443
Epoch 28: accuracy = 0.89090, test loss = 0.2596733272075653
Epoch 29: accuracy = 0.88864, test loss = 0.2656620442867279
Epoch 30: accuracy = 0.88939, test loss = 0.25115013122558594
Epoch 31: accuracy = 0.86080, test loss = 0.3230888843536377
Epoch 32: accuracy = 0.89165, test loss = 0.25565099716186523
Epoch 33: accuracy = 0.89391, test loss = 0.25713035464286804
Epoch 34: accuracy = 0.87961, test loss = 0.296455442905426
Epoch 35: accuracy = 0.88412, test loss = 0.2880666255950928
Epoch 36: accuracy = 0.90068, test loss = 0.23832035064697266
Epoch 37: accuracy = 0.90068, test loss = 0.23792560398578644
```

X, y2, trained_model, n_batch, epochs, lr

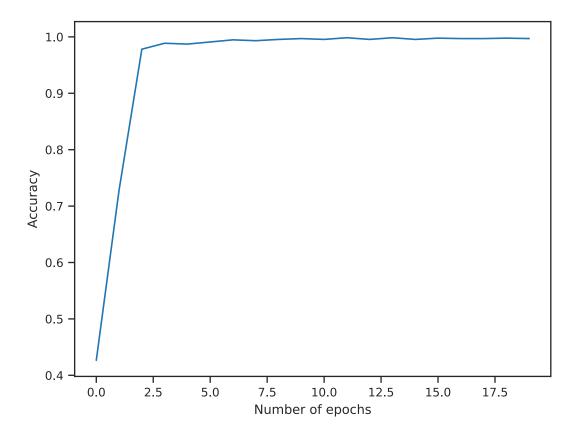
```
Epoch 39: accuracy = 0.89992, test loss = 0.2452692985534668
    Epoch 40: accuracy = 0.90594, test loss = 0.22946150600910187
    Epoch 41: accuracy = 0.89391, test loss = 0.27420902252197266
    Epoch 42: accuracy = 0.88111, test loss = 0.30122965574264526
    Epoch 43: accuracy = 0.88412, test loss = 0.3083687722682953
    Epoch 44: accuracy = 0.89315, test loss = 0.2860420048236847
    Epoch 45: accuracy = 0.90745, test loss = 0.25271111726760864
    Epoch 46: accuracy = 0.90293, test loss = 0.2362031787633896
    Epoch 47: accuracy = 0.90293, test loss = 0.2430894821882248
    Epoch 48: accuracy = 0.90594, test loss = 0.25807514786720276
    Epoch 49: accuracy = 0.90369, test loss = 0.23403804004192352
    Epoch 50: accuracy = 0.89391, test loss = 0.2741195559501648
    Epoch 51: accuracy = 0.90444, test loss = 0.243416428565979
    Epoch 52: accuracy = 0.90143, test loss = 0.2754834294319153
    Epoch 53: accuracy = 0.91046, test loss = 0.22989299893379211
    Epoch 54: accuracy = 0.90971, test loss = 0.2410309761762619
    Epoch 55: accuracy = 0.88187, test loss = 0.3293669521808624
    Epoch 56: accuracy = 0.90519, test loss = 0.25362786650657654
    Epoch 57: accuracy = 0.91196, test loss = 0.24274834990501404
    Epoch 58: accuracy = 0.91046, test loss = 0.2364710420370102
    Epoch 59: accuracy = 0.91196, test loss = 0.24519456923007965
    Epoch 60: accuracy = 0.90519, test loss = 0.2555801570415497
    Epoch 61: accuracy = 0.91046, test loss = 0.26999884843826294
    Epoch 62: accuracy = 0.89014, test loss = 0.3031378388404846
    Epoch 63: accuracy = 0.91573, test loss = 0.23372623324394226
    Epoch 64: accuracy = 0.90519, test loss = 0.2478390634059906
    Epoch 65: accuracy = 0.88187, test loss = 0.40484586358070374
    Epoch 66: accuracy = 0.90895, test loss = 0.28060588240623474
    Epoch 67: accuracy = 0.91874, test loss = 0.23709408938884735
    Epoch 68: accuracy = 0.91874, test loss = 0.2406998574733734
    Epoch 69: accuracy = 0.91046, test loss = 0.2693856954574585
    Epoch 70: accuracy = 0.90895, test loss = 0.25943586230278015
    Epoch 71: accuracy = 0.91573, test loss = 0.23877692222595215
    Epoch 72: accuracy = 0.90820, test loss = 0.2642674446105957
    Epoch 73: accuracy = 0.92099, test loss = 0.23007147014141083
    Epoch 74: accuracy = 0.91798, test loss = 0.24462997913360596
    Epoch 75: accuracy = 0.91347, test loss = 0.2649286091327667
[]: fig, ax = plt.subplots(dpi=100)
     ax.plot(test loss)
     ax.set xlabel("Number of epochs")
     ax.set_ylabel("Test loss");
```

Epoch 38: accuracy = 0.89842, test loss = 0.24776367843151093



Plot the evolution of the accuracy as a function of epochs.

```
[]: fig, ax = plt.subplots(dpi=100)
    ax.plot(accuracy)
    ax.set_xlabel("Number of epochs")
    ax.set_ylabel("Accuracy");
```



Plot the confusion matrix.

```
[]: from sklearn.metrics import confusion_matrix
     # Predict on the test data
     y_pred_test = trained_model(X_test)
     \# Remember that the prediction is probabilistic Epoch
     # We need to simply pick the label with the highest probability:
     _, y_pred_labels = torch.max(y_pred_test, 1)
     # Here is the confusion matrix:
     cf_matrix = confusion_matrix(y_test, y_pred_labels)
[]: sns.heatmap(
         cf_matrix / np.sum(cf_matrix),
         annot=True,
         fmt=".2%",
         cmap="Blues",
         xticklabels=LABELS_2_TO_TEXT.values(),
         yticklabels=LABELS_2_TO_TEXT.values(),
     );
```

