Inference for $\boldsymbol{\beta}$

Simple Linear Regression

$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

- Non-Simultaneous
    - $\hat{\beta}_0 \pm t\left(1 - \frac{\alpha}{2}, n-2\right) \times SE(\hat{\beta}_0)$ where $SE(\hat{\beta}_0) = \sqrt{s^2\left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)}$
    - $\hat{\beta}_1 \pm t\left(1 - \frac{\alpha}{2}, n-2\right) \times SE(\hat{\beta}_1)$ where $SE(\hat{\beta}_1) = \sqrt{\frac{s^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$
- Simultaneous
    - Bonferroni
        - $\hat{\beta}_j \pm t\left(1 - \frac{\alpha}{2k}, n-2\right) \times SE(\hat{\beta}_j)$ where k is the number of parameters for which we are constructing intervals (can be less than p).
    - Working-Hotelling
        - $\hat{\beta}_j \pm \sqrt{2F(1-\alpha, 2, n-2)} \times SE(\hat{\beta}_j)$

Multiple Linear Regression

$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_{p-1} x_{i(p-1)}$ where p is the number of parameters including the intercept.

- Non-Simultaneous
    - $\hat{\beta}_j \pm t\left(1 - \frac{\alpha}{2}, n-p\right) \times SE(\hat{\beta}_j)$ where $SE(\hat{\beta}_j) = \sqrt{s^2 (X^T X)^{-1}_{jj}}$ where $(X^T X)^{-1}_{jj}$ is the j^{th} diagonal of $(X^T X)^{-1}$.
- Simultaneous
    - Bonferroni
        - $\hat{\beta}_j \pm t\left(1 - \frac{\alpha}{2k}, n-p\right) \times SE(\hat{\beta}_j)$ where k is the number of parameters for which we are constructing intervals (can be less than p).
    - Working-Hotelling
        - $\hat{\beta}_j \pm \sqrt{pF(1-\alpha, p, n-p)} \times SE(\hat{\beta}_j)$

Inference for Y

Simple Linear Regression

$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

- Non-Simultaneous
  - Mean Response at one x_h
    - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2}, n-2\right) \times \sqrt{s^2\left(\frac{1}{n} + \frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)}$
  - Single y_h at one x_h
    - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2}, n-2\right) \times \sqrt{s^2\left(1 + \frac{1}{n} + \frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)}$
  - Multiple (m) y_h at one x_h
    - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2}, n-2\right) \times \sqrt{s^2\left(\frac{1}{m} + \frac{1}{n} + \frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)}$
- Simultaneous
  - Mean Response at multiple (g) x_h
    - Bonferroni
      - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2g}, n-2\right) \times \sqrt{s^2\left(\frac{1}{n} + \frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)}$
    - Working-Hotelling
      - $\hat{y}_h \pm \sqrt{2F(1-\alpha, 2, n-p)} \times \sqrt{s^2\left(\frac{1}{n} + \frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)}$
  - Single y_h at multiple (g) x_h
    - Bonferroni
      - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2g}, n-2\right) \times \sqrt{s^2\left(1 + \frac{1}{n} + \frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)}$
    - Scheffe
      - $\hat{y}_h \pm \sqrt{gF(1-\alpha, g, n-p)} \times \sqrt{s^2\left(1 + \frac{1}{n} + \frac{(x_h-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)}$

Multiple Linear Regression

$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_{p-1} x_{i(p-1)}$

- Non-Simultaneous
  - Mean Response at one $\boldsymbol{x}_h$
    - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2}, n-p\right) \times \sqrt{s^2 \boldsymbol{x}_h^T (X^T X)^{-1} \boldsymbol{x}_h}$
  - Single y_h at one x_h

        - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2}, n-p\right) \times \sqrt{s^2(1+\boldsymbol{x}_h^T(X^TX)^{-1}\,\boldsymbol{x}_h)}$
    - Multiple (m) y_h at one x_h
        - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2}, n-p\right) \times \sqrt{s^2\left(\frac{1}{m}+\boldsymbol{x}_h^T(X^TX)^{-1}\,\boldsymbol{x}_h\right)}$
- Simultaneous
    - Mean Response at multiple (g) x_h
        - Bonferroni
            - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2g}, n-p\right) \times \sqrt{s^2\boldsymbol{x}_h^T(X^TX)^{-1}\,\boldsymbol{x}_h}$
        - Working-Hotelling
            - $\hat{y}_h \pm \sqrt{pF(1-\alpha, p, n-p)} \times \sqrt{s^2\boldsymbol{x}_h^T(X^TX)^{-1}\,\boldsymbol{x}_h}$
    - Single y_h at multiple (g) x_h
        - Bonferroni
            - $\hat{y}_h \pm t\left(1-\frac{\alpha}{2g}, n-p\right) \times \sqrt{s^2(1+\boldsymbol{x}_h^T(X^TX)^{-1}\,\boldsymbol{x}_h)}$
        - Scheffe
            - $\hat{y}_h \pm \sqrt{gF(1-\alpha, g, n-p)} \times \sqrt{s^2(1+\boldsymbol{x}_h^T(X^TX)^{-1}\,\boldsymbol{x}_h)}$