THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Diagnostics & Transformations 1

Lecture 12

STA 371G

Predicting the fuel economy (miles per gallon) for different car models of the 70s.

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“LINE” assumptions:

- Linearity
- Independent errors
- Normally distributed errors
- Equal Variance (Homoscedasticity)

Predicting MPG from Horsepower

```r
model <- lm(MPG ~ HP, data=auto_mpg)
summary(model)
```

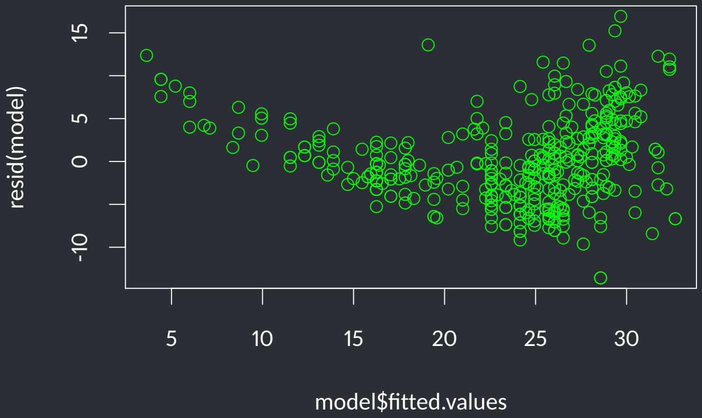
```
Call:
lm(formula = MPG ~ HP, data = auto_mpg)

Residuals:
     Min       1Q   Median       3Q      Max
-13.5710  -3.2592  -0.3435   2.7630  16.9240

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 39.935861   0.717499   55.66   <2e-16 ***
HP          -0.157845   0.006446  -24.49   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.906 on 390 degrees of freedom
Multiple R-squared:  0.6059,Adjusted R-squared:  0.6049
F-statistic: 599.7 on 1 and 390 DF,  p-value: < 2.2e-16
```

```r
plot(model$fitted.values, resid(model), col='green')
```



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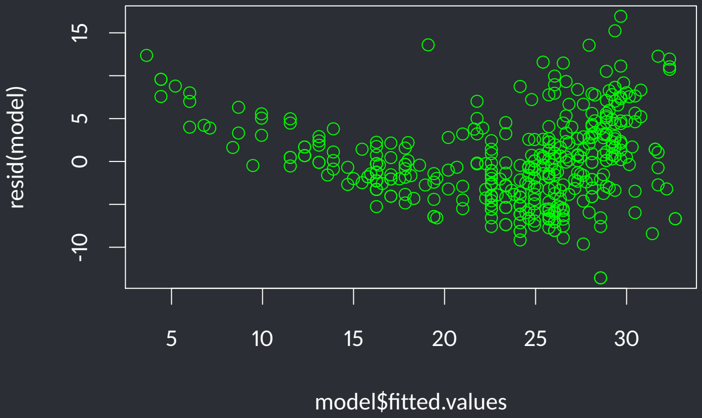
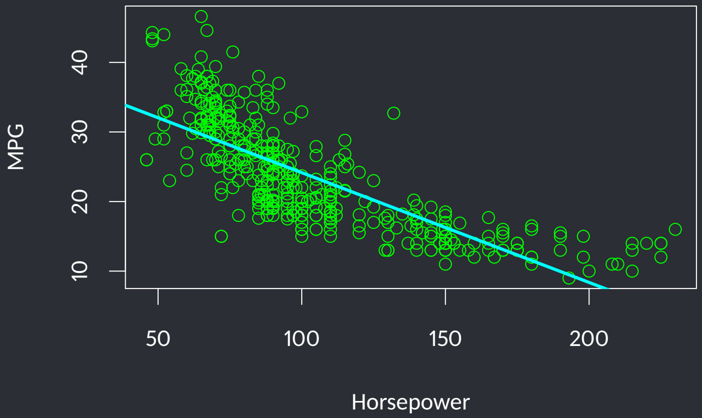


The trend in the residuals implies linearity issues.
The “funnel” implies equal variance issues.

Addressing the linearity issue

The relation between MPG and horsepower does not seem to be linear.

```r
plot(auto_mpg$HP, auto_mpg$MPG, col='green',
     xlab='Horsepower', ylab='MPG')
abline(model, col='cyan', lwd=3)
```



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If we could horizontally shift the data on the far right towards the left, the plot would look more linear.

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So let's predict the MPG of a car not from the horsepower, but from a "transformation" of the horsepower.

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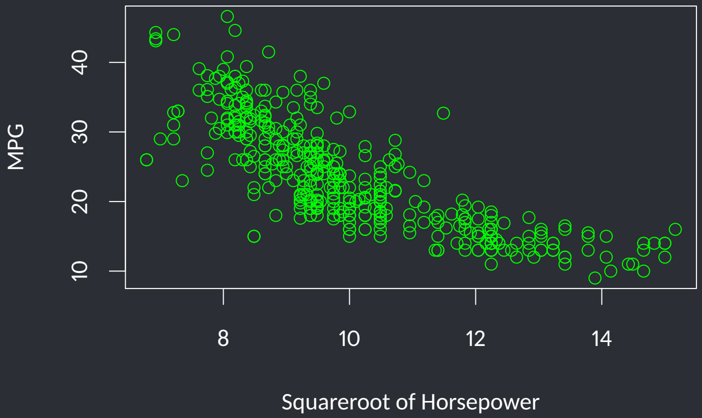
So let's predict the MPG of a car not from the horsepower, but from a "transformation" of the horsepower.

For example, the relation between MPG and HP is not linear, but the one between MPG and $\sqrt{\text{HP}}$ could be!

Addressing the linearity issue

```
auto_mpg$HP_sqrt <- sqrt(auto_mpg$HP)
plot(auto_mpg$HP_sqrt, auto_mpg$MPG, col='green',
     xlab='Squareroot of Horsepower', ylab='MPG')
```



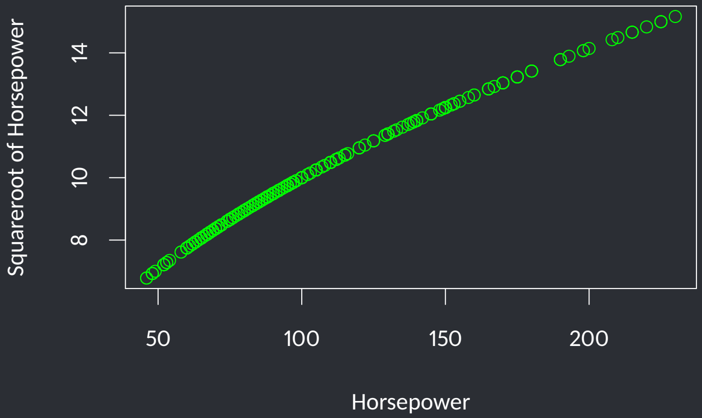
Addressing the linearity issue

It indeed seems a bit better. Notice the change in the range of the horizontal axis.

Addressing the linearity issue

It indeed seems a bit better. Notice the change in the range of the horizontal axis. It has changed from [49,225] to [7,15]. The shift is larger for the data on the far right.

```
plot(auto_mpg$HP, auto_mpg$HP_sqrt, col='green',
    xlab='Horsepower', ylab='Squareroot of Horsepower')
```

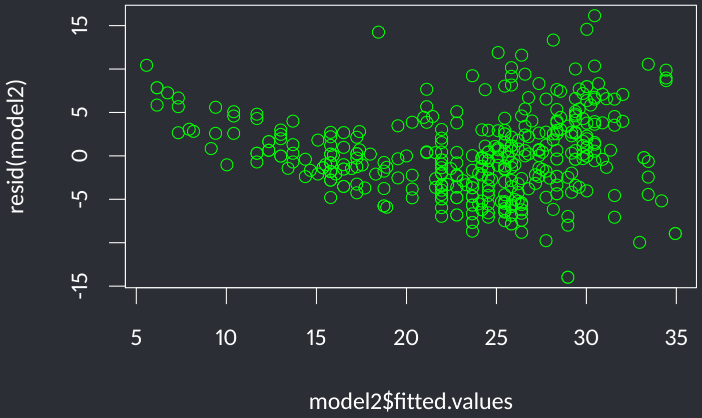


Addressing the linearity issue

```
model2 <- lm(MPG ~ HP_sqrt, data=auto_mpg)
summary(model2)
```

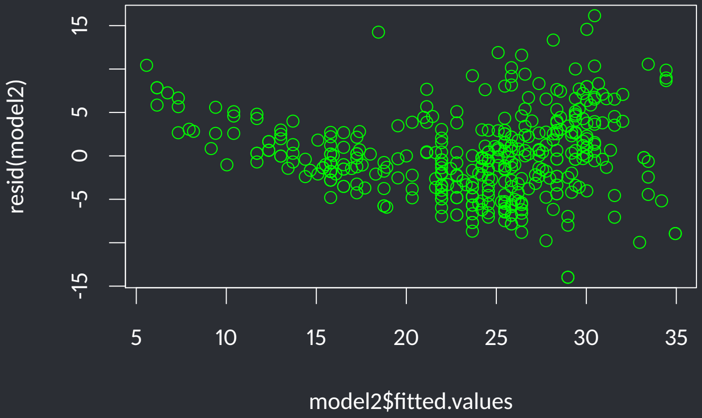
```

Call:
lm(formula = MPG ~ HP_sqrt, data = auto_mpg)

Residuals:
     Min       1Q   Median       3Q      Max 
-13.9768  -3.2239  -0.2252   2.6881  16.1411 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   58.705      1.349   43.52   <2e-16 ***
HP_sqrt       -3.503      0.132  -26.54   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.665 on 390 degrees of freedom
Multiple R-squared:  0.6437,Adjusted R-squared:  0.6428 
F-statistic: 704.6 on 1 and 390 DF,  p-value: < 2.2e-16
```

```r
plot(model2$fitted.values, resid(model2), col='green')
```



The trend flattened a bit.

```r
plot(model2$fitted.values, resid(model2), col='green')
```



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Can we do better? Let's try some other transformation.

Logarithmic transformation

One of the most common transformations is the logarithmic transformation with base e (natural logarithm).

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$$e = 2.7182818284\ldots$$

$$e^2 = 7.389$$

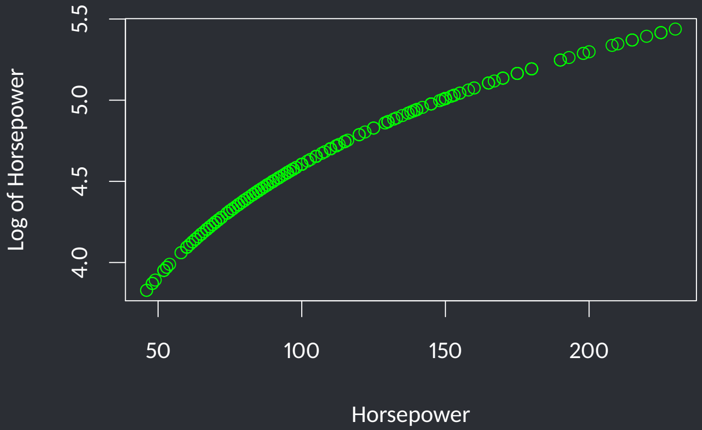
$$\log(e^2) = 2$$

In general:

$$y = e^x \qquad \longleftrightarrow \qquad \log y = x.$$

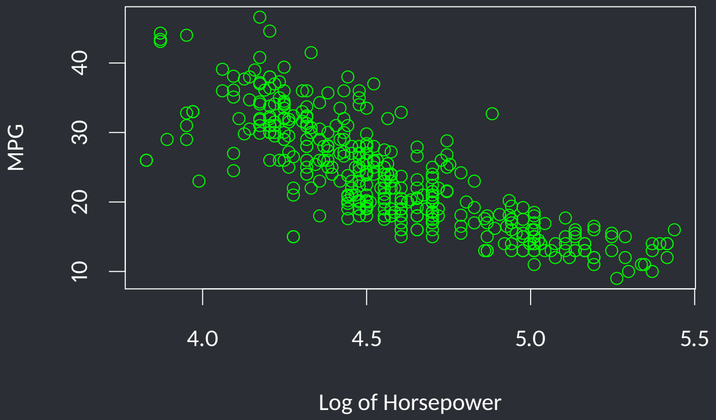
Logarithmic transformation

```
auto_mpg$HP_ln <- log(auto_mpg$HP)
plot(auto_mpg$HP, auto_mpg$HP_ln, col='green',
    xlab='Horsepower', ylab='Log of Horsepower')
```



Logarithmic transformation

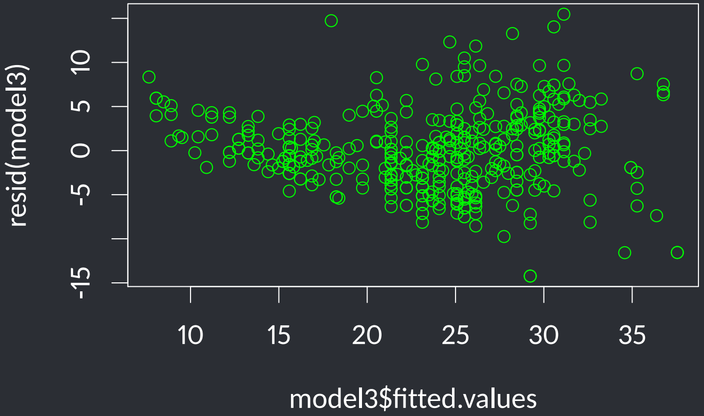
```
plot(auto_mpg$HP_ln, auto_mpg$MPG, col='green',
     xlab='Log of Horsepower', ylab='MPG')
```



```r
model3 <- lm(MPG ~ HP_ln, data=auto_mpg)
summary(model3)
```

```

Call:
lm(formula = MPG ~ HP_ln, data = auto_mpg)

Residuals:
     Min       1Q   Median       3Q      Max 
-14.2299  -2.7818  -0.2322   2.6661  15.4695 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 108.6997     3.0496   35.64   <2e-16 ***
HP_ln       -18.5822     0.6629  -28.03   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.501 on 390 degrees of freedom
Multiple R-squared:  0.6683,Adjusted R-squared:  0.6675 
F-statistic: 785.9 on 1 and 390 DF,  p-value: < 2.2e-16
```

```r
plot(model3$fitted.values, resid(model3), col='green')
```



The trend flattened even more.

Transforming a Predictor

It is equivalent to "cutting the distribution of *X* into vertical slices and changing the spacing of the slices."

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It does not affect the vertical locations of the data (MPG did not change!).

When the nonlinearity is the biggest issue in the model, transforming the predictor is a good start.

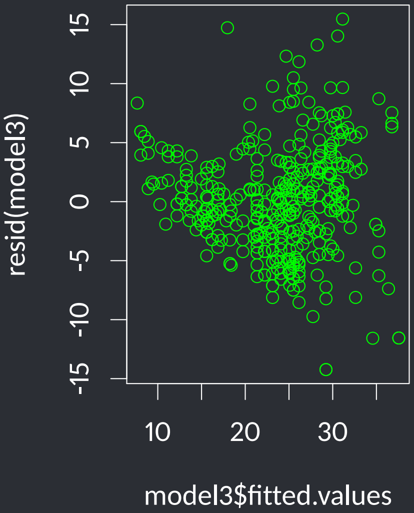
Finding the right transformation is a bit of art, field knowledge and trial and error.

Transforming a Predictor

x	$\sqrt{x}$	$\log x$
1	1	0
10	3.16	2.3
100	10	4.61
1000	31.62	6.91
10000	100	9.21
100000	316.23	11.51

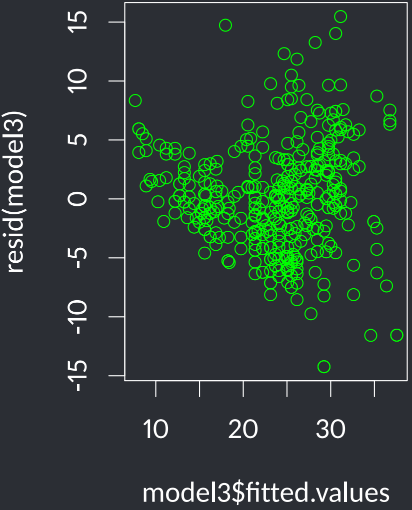
Addressing the equal variance issue

- The (unexplained) variance in the response is higher in some regions.



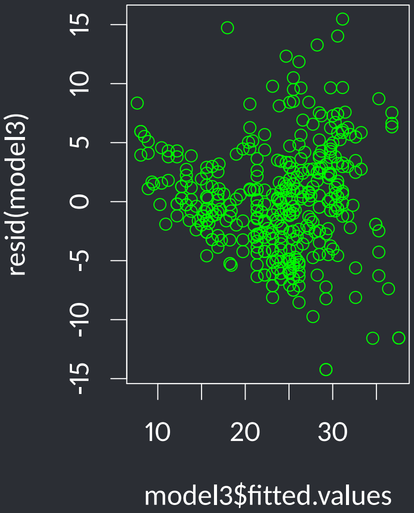
Addressing the equal variance issue

- The (unexplained) variance in the response is higher in some regions.
- Remember how log-transformation shrinks larger numbers more than it shrinks smaller numbers.



Addressing the equal variance issue

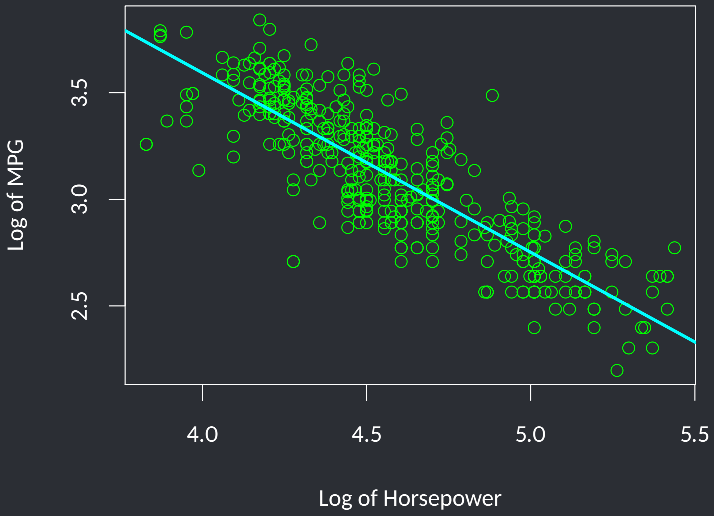
- The (unexplained) variance in the response is higher in some regions.
- Remember how log-transformation shrinks larger numbers more than it shrinks smaller numbers.
- Log-transformation of the response often helps with fixing heteroscedasticity (and non-normality)!



```r
auto_mpg$MPG_ln <- log(auto_mpg$MPG)
model4 <- lm(MPG_ln ~ HP_ln, data=auto_mpg)
summary(model4)
```

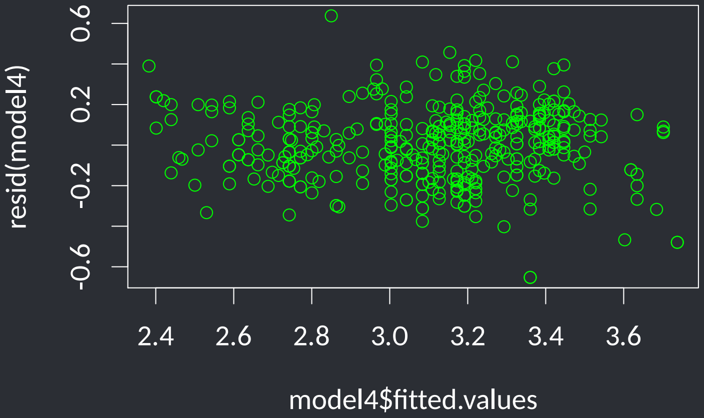
```

Call:
lm(formula = MPG_ln ~ HP_ln, data = auto_mpg)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.6523 -0.1218  0.0079  0.1163  0.6373 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   6.9606     0.1215    57.3   <2e-16 ***
HP_ln        -0.8418     0.0264   -31.9   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.18 on 390 degrees of freedom
Multiple R-squared:  0.723,Adjusted R-squared:  0.722 
F-statistic: 1.02e+03 on 1 and 390 DF,  p-value: <2e-16
```

```r
plot(auto_mpg$HP_ln, auto_mpg$MPG_ln, col='green',
     xlab='Log of Horsepower', ylab='Log of MPG')
abline(model4, col='cyan', lwd=3)
```



Beautiful!

```
plot(model4$fitted.values, resid(model4), col='green')
```



Look how awesome our models are getting

Predictor (X)	Response (Y)	R^2	Residual SE
HP	MPG	0.61	4.91
$\sqrt{\text{HP}}$	MPG	0.64	4.66
$\log \text{HP}$	MPG	0.67	4.5
$\log \text{HP}$	$\log \text{MPG}$	0.72	0.18

Interpretation of β values

The model before the transformations was:

$$\widehat{\text{MPG}} = 39.94 - 0.16 \cdot \text{HP}$$

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The interpretation of -0.16 is “for each unit of increase in the horsepower, the MPG estimate reduces by 0.16”.

After transforming the predictor or the response, this interpretation does not hold!

Interpretation of β values

When the square root of HP is used:

$$\widehat{\text{MPG}} = 58.71 - 3.5 \cdot \sqrt{\text{HP}}$$

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When the square root of HP is used:

$$\widehat{\text{MPG}} = 58.71 - 3.5 \cdot \sqrt{\text{HP}}$$

The interpretation of -3.5 is "for each unit of increase in the square root of the horsepower, the MPG estimate reduces by 3.5".

Interpretation of β values

Similarly, in the following model:

$$\widehat{\text{MPG}} = 108.7 - 18.58 \cdot \log \text{HP}$$

Interpretation of β values

Similarly, in the following model:

$$\widehat{\text{MPG}} = 108.7 - 18.58 \cdot \log \text{HP}$$

The interpretation of -18.58 is "for each unit of increase in the natural logarithm of the horsepower, the MPG estimate reduces by 18.58".

Transformation strategy

- If the model has two or three of the equal variance, normality and linearity issues, try transforming Y.

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- There is no rule for which transformations will work in all cases; trial and error may be required.

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- After transforming the response, if the nonlinearity is not fixed, try transforming the predictor(s) as well.
- There is no rule for which transformations will work in all cases; trial and error may be required.
- Remember, the interpretations of the coefficients will change after you transform one or more variables!