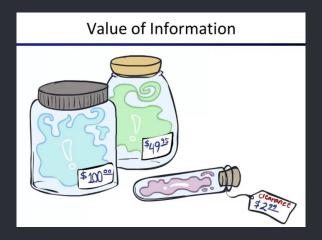


# **Decision Trees 2**

**Lecture 21** 

**STA 371G** 

# What Is It Worth to Know More About an Uncertain Event?





Value of Information

- Value of Information
- Bevo: The Movie Example

- Value of Information
- Bevo: The Movie Example
- Expected Value of Perfect Information

- Value of Information
- Bevo: The Movie Example
- Expected Value of Perfect Information
- Expected Value of Imperfect Information

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- How much is information worth, and if it costs a given amount, should you purchase it?
- The expected value of perfect information, or EVPI, is the most you would be willing to pay for perfect information.

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- This leads to an investigation of the value of information

# Example: Marketing Strategy for Bevo: The Movie

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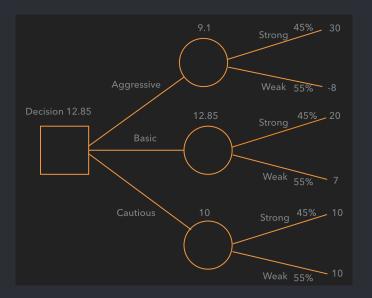
- (A) Aggressive: Large expenditures on television and print advertising.
- (B) Basic: More modest marketing campaign.
- (C) Cautious: Minimal marketing campaign.

# Payoffs for Bevo: The Movie

The net payoffs depend on the market reaction to the film.

	Market Reaction	
Decisions	Strong	Weak
Aggressive	30	-8
Basic	20	7
Cautious	10	10
Probability	0.45	0.55

#### Decision Tree for Bevo: The Movie



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EVPI = (EV with perfect information) - (EV with no information)

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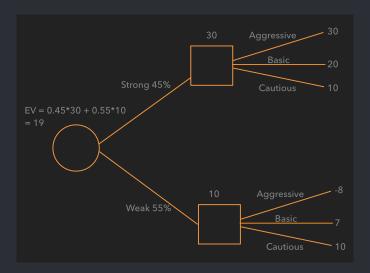
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- EVPI = 19 12.85 = 6.15

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- Step 2: Rearrange the tree the reflect the receipt of the information and calculate the new EV
- Step 3: Compare the EV's with and without the information





# What about imperfect information?

Suppose that Myra the movie critic has a good record of picking winners.

- For movies where the audience reaction was strong, Myra has historically predicted that 70% of them would be strong.
- For movies where the audience reaction was weak, Myra has historically predicted that 80% of them would be weak.

Remember that the probability of a strong reaction is 45% and of a weak reaction is 55%.



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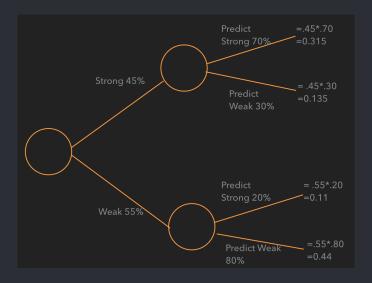
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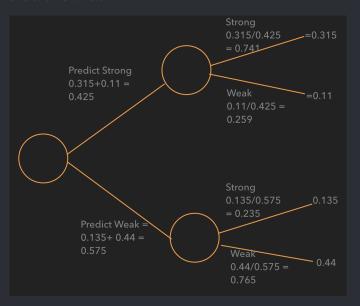
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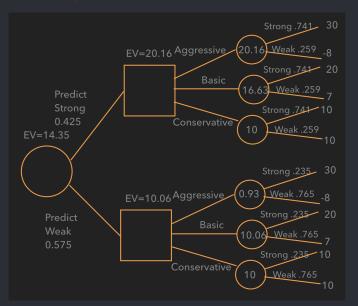
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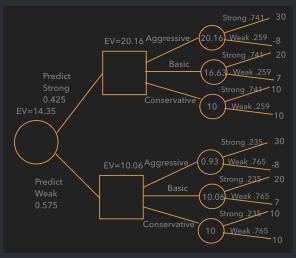


# Tree with imperfect information



### Myra's information is worth paying for

It changes the decision and adds 14.35 - 12.85 = 1.5 in value. (Compare this to the 6.15 the clairvoyant's prediction was worth.)



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- Sometimes there is more than one correct way to draw a decision tree for a decision