Decision Trees 2
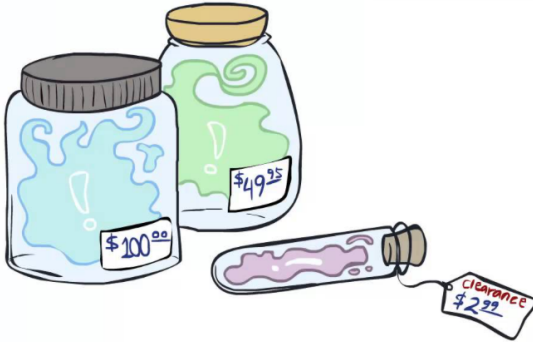
Lecture 21

STA 371G

What Is It Worth to Know More About an Uncertain Event?

Key topics for today

- Value of Information

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- Bevo: The Movie Example

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- Value of Information
- Bevo: The Movie Example
- Expected Value of Perfect Information

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- Value of Information
- Bevo: The Movie Example
- Expected Value of Perfect Information
- Expected Value of Imperfect Information

Value of information

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- How much is information worth, and if it costs a given amount, should you purchase it?
- The expected value of perfect information, or EVPI, is the most you would be willing to pay for perfect information.

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- This leads to an investigation of the value of information

Example: Marketing Strategy for *Bevo: The Movie*

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- (A) Aggressive: Large expenditures on television and print advertising.
- (B) Basic: More modest marketing campaign.
- (C) Cautious: Minimal marketing campaign.

Payoffs for *Bevo: The Movie*

The net payoffs depend on the market reaction to the film.

	Market Reaction	
Decisions	Strong	Weak
Aggressive	30	-8
Basic	20	7
Cautious	10	10
Probability	0.45	0.55

Decision Tree for *Bevo: The Movie*
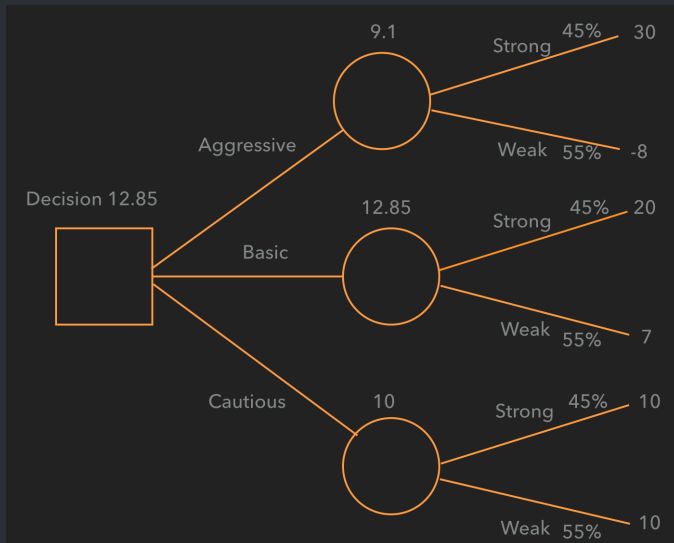
Expected Value of Perfect Information (EVPI)

How valuable would it be to know what was going to happen?

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EVPI is: EV with perfect information - EV with no information

Finding EVPI with a payoff table

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- EVPI = 19 − 12.85 = 6.15

Finding EVPI with a decision tree

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- Step 3: Compare the EV's with and without the information

Finding EVPI with a decision tree
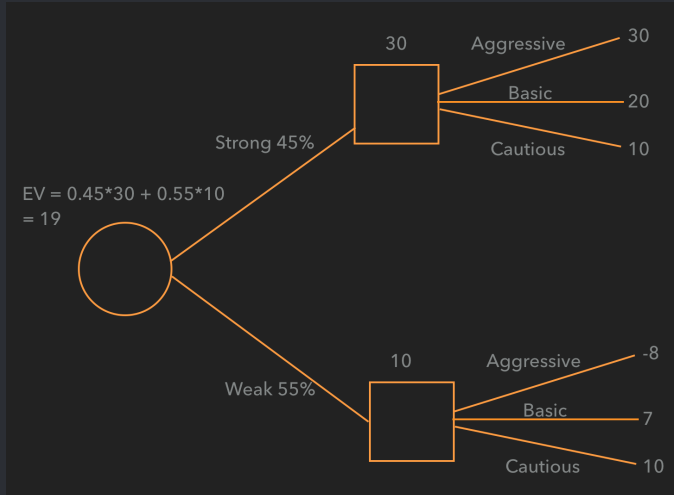
What about imperfect information?

Suppose that Myra the movie critic has a good record of picking winners.

- For movies where the audience reaction was strong, Myra has historically predicted that 70% of them would be strong.
- For movies where the audience reaction was weak, Myra has historically predicted that 80% of them would be weak.

Remember that the probability of a strong reaction is 45% and of a weak reaction is 55%.

Suppose *S* and *W* means the audience reaction was strong or weak, respectively, and *PS* and *PW* means that Myra's prediction was strong or weak, respectively. Let's translate what we know:

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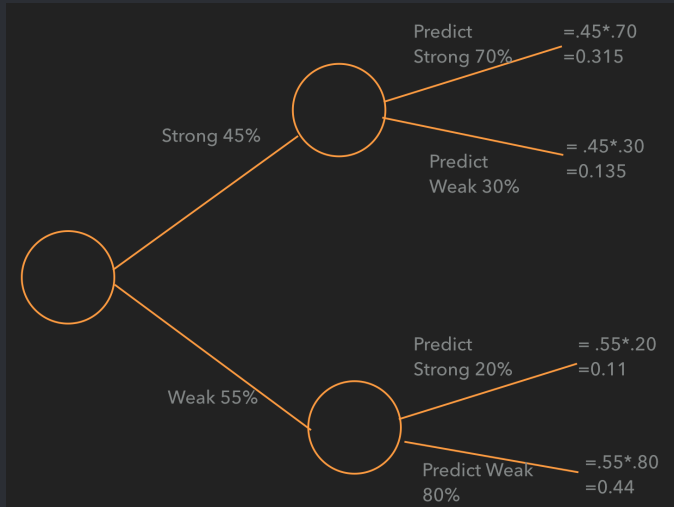 $P(PS|S) = .7, \qquad P(PW|S) = .3$

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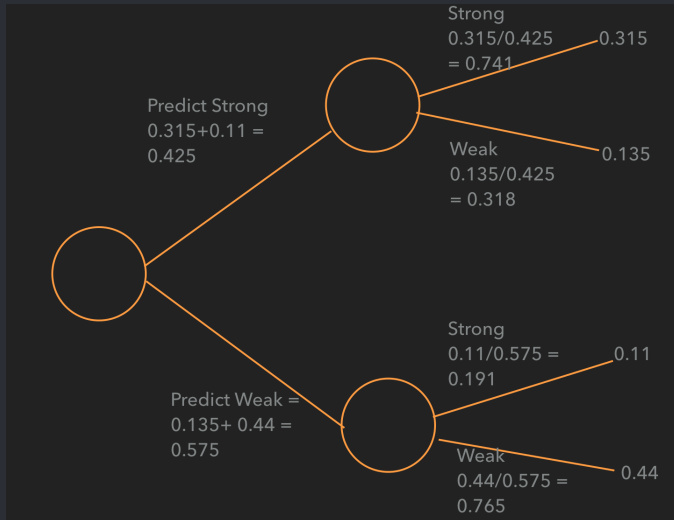 $P(PW|W) = .8, \qquad P(PS|W) = .2$

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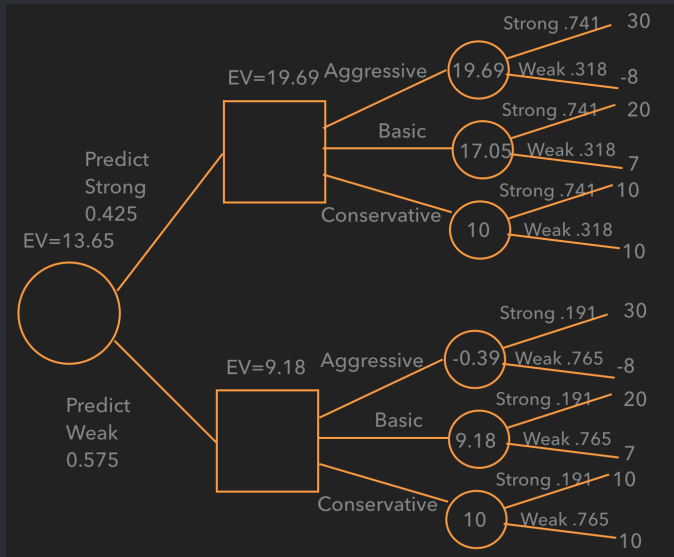 $P(S) = .45, \qquad P(W) = .55$

Reverse the tree!

Strong 45%

Predict Strong 70% =.45*.70 =0.315

Predict Weak 30% = .45*.30 =0.135

Weak 55%

Predict Strong 20% = .55*.20 =0.11

Predict Weak 80% =.55*.80 =0.44

Reverse the tree!

Predict Strong
0.315+0.11 =
0.425

Strong
0.315/0.425
= 0.741

0.315

Weak
0.135/0.425
= 0.318

0.135

Predict Weak =
0.135+ 0.44 =
0.575

Strong
0.11/0.575 =
0.191

0.11

Weak
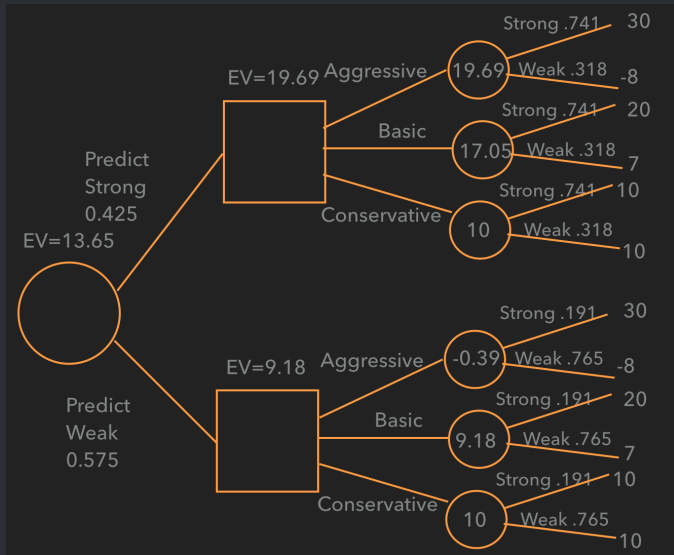0.44/0.575 =
0.765

0.44

Tree with imperfect information

Myra's information is worth paying for

It changes the decision and adds 13.65 - 12.85 = 0.80 in value

Things to remember about the value of information

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- Perfect information is more valuable that any imperfect information
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- Information has non-zero value if and only if it changes some decision
- Sometimes there is more than one correct way to draw a decision tree for a decision