



THE UNIVERSITY OF TEXAS AT AUSTIN  
McCOMBS SCHOOL OF BUSINESS

# Decision Trees 2

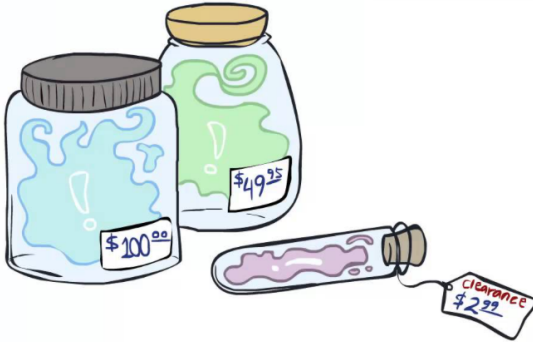
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Lecture 21

STA 371G

# What Is It Worth to Know More About an Uncertain Event?

## Value of Information



## Key topics for today

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- Value of Information
- Bevo: The Movie Example

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- Expected Value of Perfect Information

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- Expected Value of Imperfect Information

## Value of information

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## Value of information

- Sometimes information can lead to better decisions.
- How much is information worth, and if it costs a given amount, should you purchase it?
- The expected value of perfect information, or EVPI, is the most you would be willing to pay for perfect information.

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- Information usually comes at a price. You want to know whether the information is worth its price
- This leads to an investigation of the value of information

## Example: Marketing Strategy for *Bevo: The Movie*

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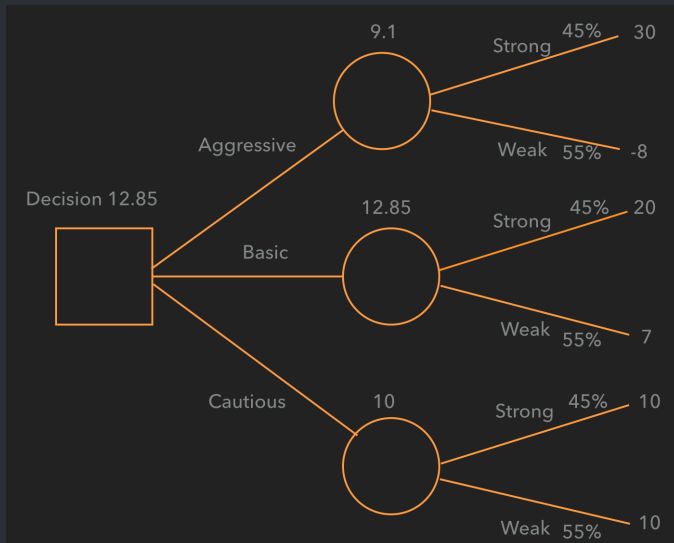
- (A) Aggressive: Large expenditures on television and print advertising.
- (B) Basic: More modest marketing campaign.
- (C) Cautious: Minimal marketing campaign.

## Payoffs for *Bevo: The Movie*

The net payoffs depend on the market reaction to the film.

Decisions	Market Reaction	
	Strong	Weak
Aggressive	30	-8
Basic	20	7
Cautious	10	10
Probability	0.45	0.55

## Decision Tree for *Bevo: The Movie*



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$EVPI = (EV \text{ with perfect information}) - (EV \text{ with no information})$

## Finding EVPI with a payoff table

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- $EVPI = 19 - 12.85 = 6.15$

## Finding EVPI with a decision tree

- Step 1: Set up tree without perfect information and calculate EV by rolling back

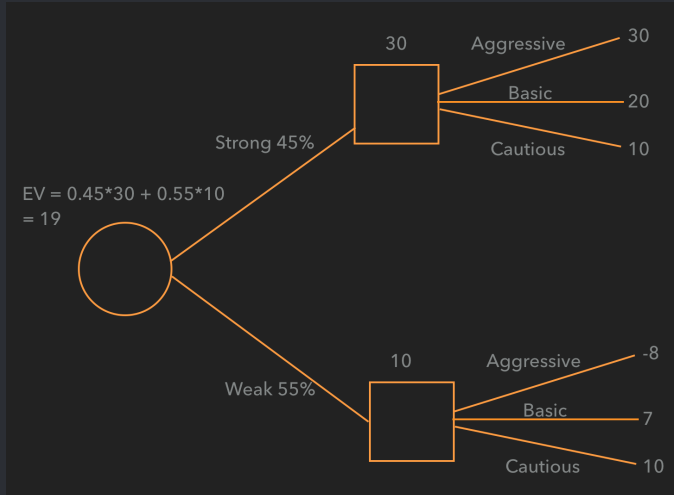
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- Step 2: Rearrange the tree to reflect the receipt of the information and calculate the new EV
- Step 3: Compare the EV's with and without the information

# Finding EVPI with a decision tree



## What about imperfect information?

Suppose that Myra the movie critic has a good record of picking winners.

- For movies where the audience reaction was strong, Myra has historically predicted that 70% of them would be strong.
- For movies where the audience reaction was weak, Myra has historically predicted that 80% of them would be weak.

Remember that the probability of a strong reaction is 45% and of a weak reaction is 55%.



Suppose  $S$  and  $W$  means the audience reaction was strong or weak, respectively, and  $PS$  and  $PW$  means that Myra's prediction was strong or weak, respectively. Let's translate what we know:



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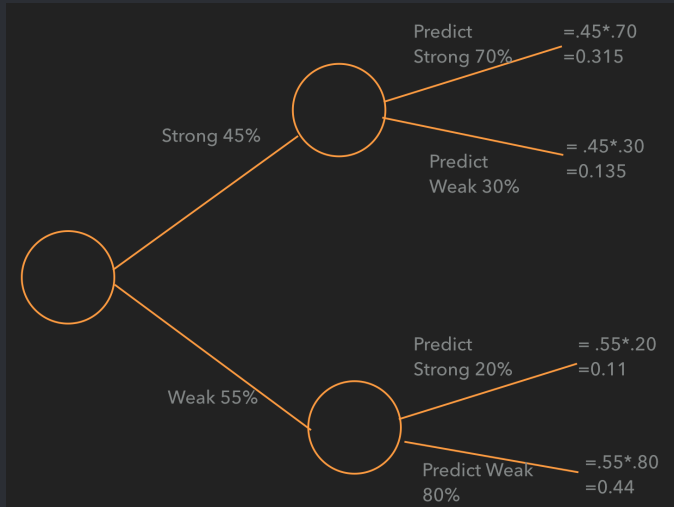
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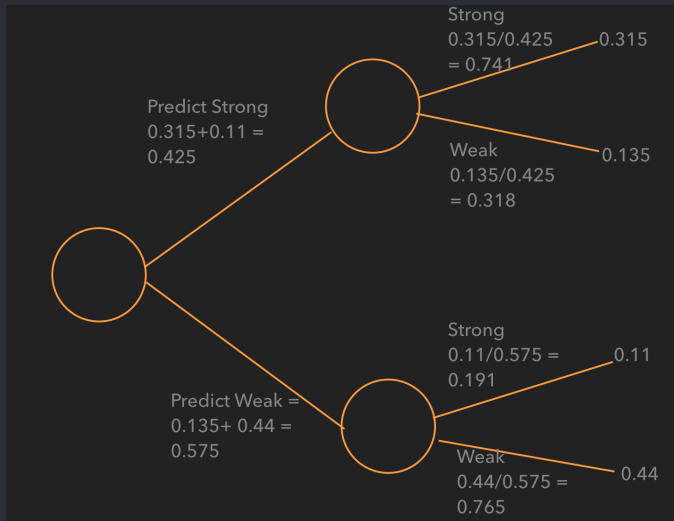
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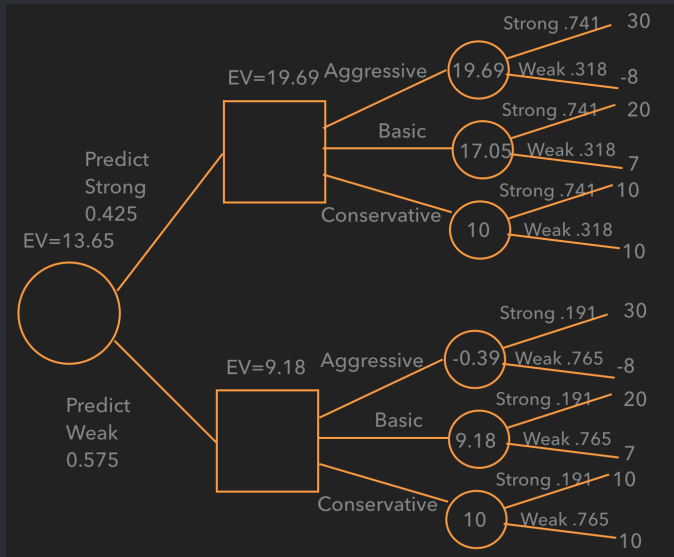
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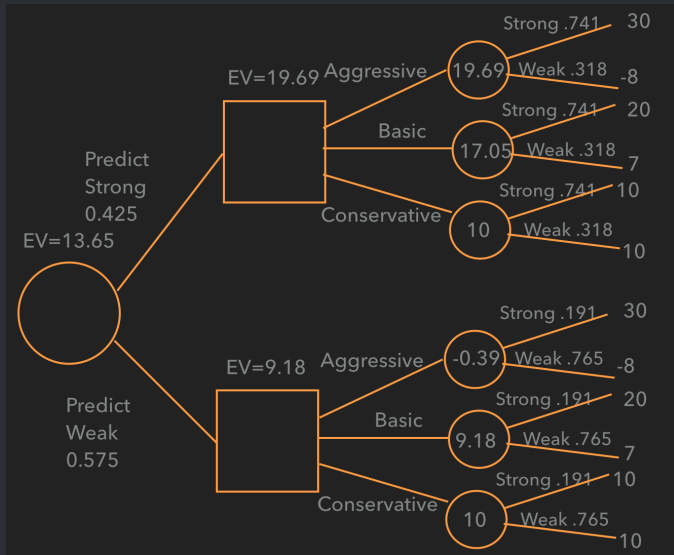


## Tree with imperfect information



## Myra's information is worth paying for

It changes the decision and adds  $13.65 - 12.85 = 0.80$  in value





## Things to remember about the value of information

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- Sometimes there is more than one correct way to draw a decision tree for a decision