



THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Time Series: Smoothing & Moving Averages

Lecture 19

STA 371G

Trend & Seasonality in Time Series



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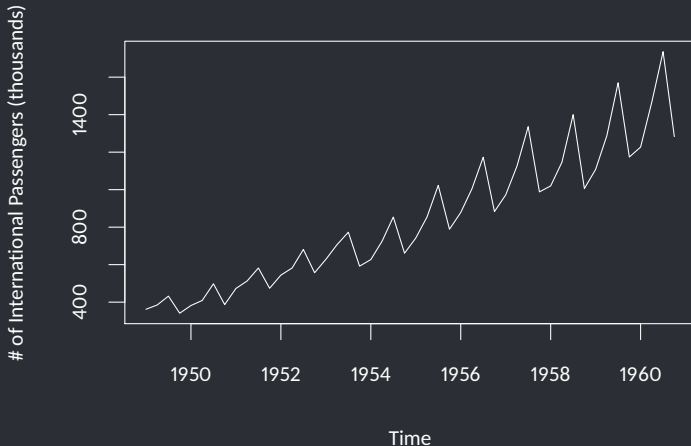
Trend & Seasonality in Time Series



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- Two main factors that make data non-stationary: trends and seasonality.
- Sales, economic, activity, etc. data often show strong seasonality.

Airline Passengers 1949-1960

```
# Convert data into time series, starting from 1st quarter of 1949.  
air <- ts(air_passengers$number, start=c(1949,1), frequency=4)  
# Frequency: 4 data points per year (this is quarterly data)  
plot(air, ylab="# of International Passengers (thousands)")
```



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- In this case, a season is a year.
- The frequency, the number of observations in a season, is 4.
- The data behaves similarly in the same quarter of different years: every year, the number of passengers peaks in Q3 and dips in Q4.

Exploring the data

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- The trend in the data makes it difficult to see the impact of seasonality in the data: is Q2 higher than the Q1 because it is a more active traveling period of the year or is it just that more people are using air travel as time goes by?



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- Overall, there is an upward trend in the number of airline passengers from 1949-1960.
- The seasonality in the data makes it difficult to see how the trend changes over years: is the number is increasing at the same rate every year, or does the rate vary in different years?
- The trend in the data makes it difficult to see the impact of seasonality in the data: is Q2 higher than the Q1 because it is a more active traveling period of the year or is it just that more people are using air travel as time goes by?
- Smoothing the data helps identify the trend and seasonal effects.



Smoothing the data: one-sided moving average

A (one-sided) moving average of span 4 (MA) is a simple average of all observations over the previous 4 time periods, and can be used to forecast the next time period.

y_t : Number of passengers traveled at time t

One-sided moving average at time t :

$$\hat{y}_{t+1} = \frac{1}{4}y_t + \frac{1}{4}y_{t-1} + \frac{1}{4}y_{t-2} + \frac{1}{4}y_3$$

Each quarter has equal weight in the moving average.

Smoothing the data: one-sided moving average

	Y49Q1	Y49Q2	Y49Q3	Y49Q4	Y50Q1	Y50Q2	...
t	1	2	3	4	5	6	...
y_t	362	385	432	341	382	409	...
\hat{y}_t	NA	NA	NA	NA	380	385	...

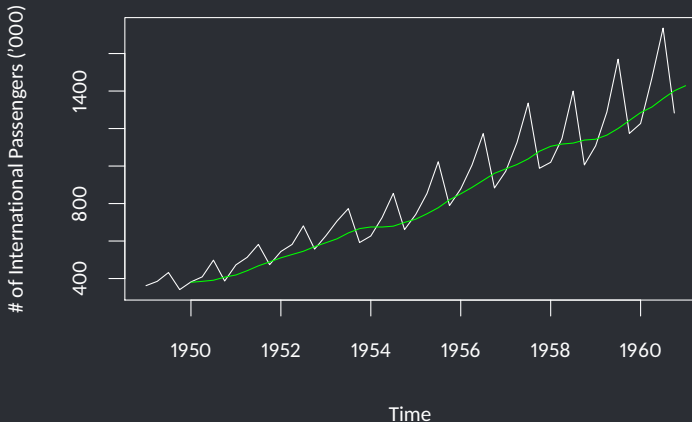
Smoothing the data: one-sided moving average

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y_t	362	385	432	341	382	409	...
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$$\begin{aligned}\hat{y}_5 &= \frac{1}{4}y_1 + \frac{1}{4}y_2 + \frac{1}{4}y_3 + \frac{1}{4}y_4 \\ &= \frac{1}{4}362 + \frac{1}{4}385 + \frac{1}{4}432 + \frac{1}{4}341 \\ &= 380\end{aligned}$$

Smoothing the data: one-sided moving average

```
# One sided moving average  
air_ma_one <- (lag(air, -1) + lag(air, -2) + lag(air, -3) + lag(air, -4))/4  
plot(air, ylab="# of International Passengers ('000)")  
lines(air_ma_one, col='green')
```



Smoothing the data

Smoothed data better shows the slow-downs in the trend around 1954 and 1959.

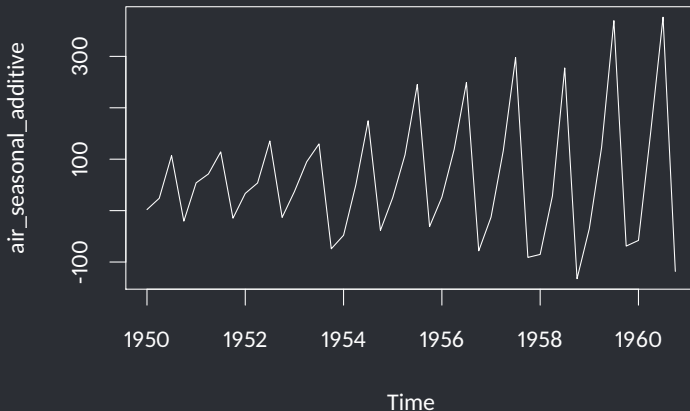
Let's also see the effect of the seasonality by eliminating the trend in the data. This is called **detrending**.

Smoothing the data

When trend and seasonality are additive:

$$\text{Data} = \text{Trend} + \text{Seasonality} + \text{Randomness}$$

```
air_seasonal_additive <- air - air_ma_one  
plot(air_seasonal_additive)
```

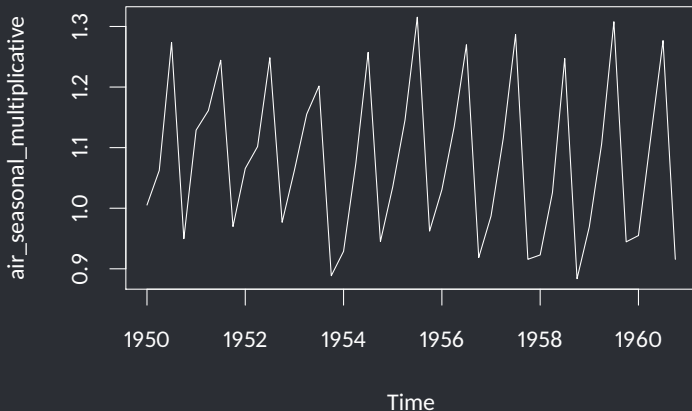


Smoothing the data

When trend and seasonality are multiplicative:

$$\text{Data} = \text{Trend} \times \text{Seasonality} \times \text{Randomness}$$

```
air_seasonal_multiplicative <- air / air_ma_one  
plot(air_seasonal_multiplicative)
```



Forecasting using Simple Exponential Smoothing

So far, we have smoothed the data to better observe the trend and the seasonality, and to make predictions about future values.

The problem with moving averages is that it ignores previous quarters beyond the last 4. We could extend the span to a larger number—but then we weight less recent data as much as more recent data!

Simple exponential smoothing (SES) is a way to deal with these issues.

Simple Exponential Smoothing

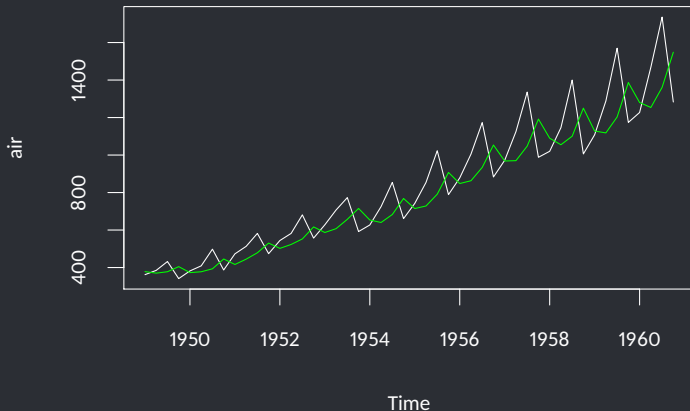
We'll use *every* previous data point to forecast the next one, but using a weight that decreases exponentially:

$$\begin{aligned}\hat{y}_{t+1} &= \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2\hat{y}_{t-2} + \cdots \\ &= \sum_{k=0}^{t-1} \alpha(1-\alpha)^k y_{t-k}.\end{aligned}$$

\hat{y}_{t+1} carries a portion of all past observations; the more recent the observation is, the more weight it has (since $(1-\alpha)^k \rightarrow 0$ as k increases).

Simple Exponential Smoothing

```
library(forecast)
air_ses_05 <- ses(air, alpha=0.5)
plot(air)
lines(air_ses_05$fitted, col='green')
```

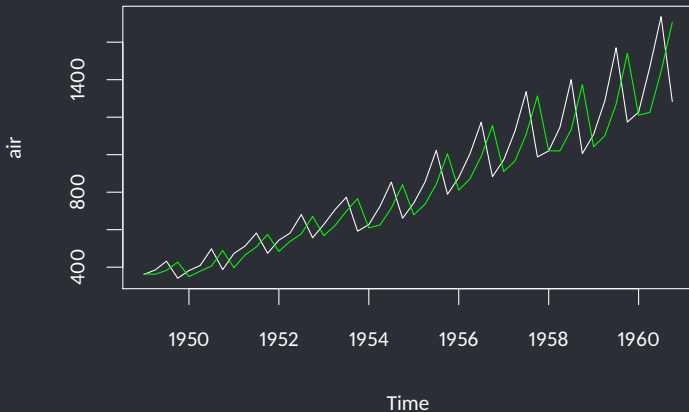


Simple Exponential Smoothing

- α determines how aggressively the forecast will react to the observed error.
- A larger α means a more aggressive adjustment (i.e. closer forecast to the observed value), but can also cause “overfitting.”

Simple Exponential Smoothing

```
air_ses_09 <- ses(air, alpha = 0.9)
plot(air)
lines(air_ses_09$fitted, col='green')
```



Simple Exponential Smoothing

- Regardless of the size of the α parameter, there seems to be always a delay in the forecast.

Simple Exponential Smoothing

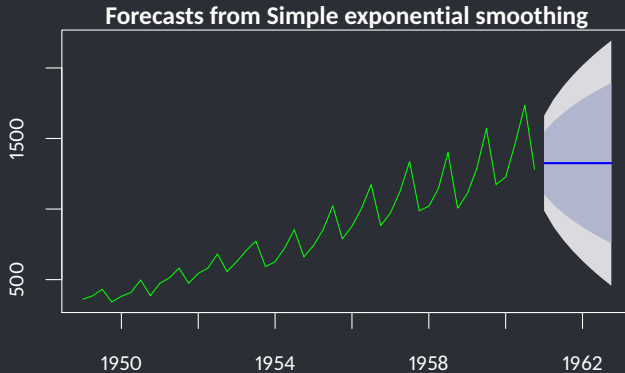
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Simple Exponential Smoothing

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- This is because the SES method is for stationary data, i.e., when there is no trend or seasonality.
- When trend and seasonality exist, SES (like MA) predicts the same value for all future observations after a certain point.

Simple Exponential Smoothing

```
air_ses_09 <- ses(air, alpha = 0.9, h=8)  
plot(air_ses_09, col='green')
```



Simple Exponential Smoothing

air_ses_09

	Point	Forecast	Lo 80	Hi 80	Lo 95	Hi 95
1961	Q1	1325.377	1105.3915	1545.363	988.9380	1661.817
1961	Q2	1325.377	1029.4167	1621.338	872.7445	1778.010
1961	Q3	1325.377	969.2991	1681.456	780.8027	1869.952
1961	Q4	1325.377	917.9579	1732.797	702.2830	1948.472
1962	Q1	1325.377	872.3988	1778.356	632.6064	2018.148
1962	Q2	1325.377	831.0206	1819.734	569.3240	2081.431
1962	Q3	1325.377	792.8480	1857.907	510.9440	2139.811
1962	Q4	1325.377	757.2343	1893.520	456.4776	2194.277

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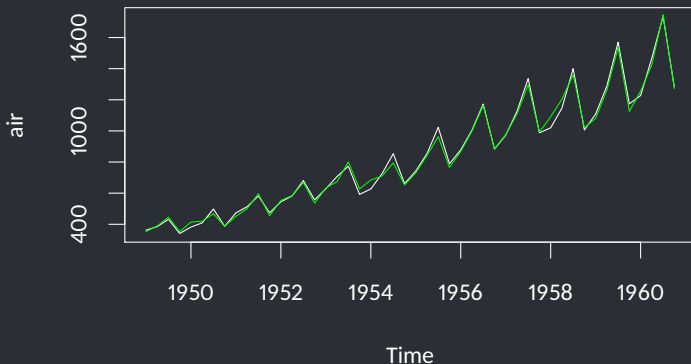
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- Holt's Method estimates the trend, and smooths it (with smoothing parameter β).
- Winter's Method estimates the seasonal components, and smooths them (with smoothing parameter γ).

What if there is a trend and/or seasonality?

We can use `holt` when there is a trend, or `hw` when there is both a trend and seasonality. You can specify α , β , and/or γ , or omit them and let R find the optimal values that minimize prediction error:

```
air_hw <- hw(air, seasonal='multiplicative')  
plot(air)  
lines(air_hw$fitted, col='green')
```



```
plot(air_hw, col='white')
```

Forecasts from Holt-Winters' multiplicative method

