The common explanation for proving that there are more numbers in between 0 and 1, than numbers between 0 and Infinity, goes like this…

Let's create a table with all the numbers between 0 and Infinity on one side,

and assign each a random number between 0 and 1.

0	0.876896576876987698768754135679876987263495870928345972 09…
1	0.123546234589234957823458927345098273450987234095872093 48…
2	0.563897208974098750984279084709854089549387904908234523 45…
…	…
Infinity	0.298143570913452345432435666453524563474567478567814357 2…8

Now each number 0 - Infinity has a number from 0 - 1.

But wait, we're missing a number. We can create a new number by going through each decimal place and adding one to it. If we get a nine we'll just wrap it around to a zero.

0	0.**8**76896576876987698768754135679876987263495870928345972 09…
1	0.1**2**354623458923495782345892734509827345098723409587209348…
2	0.563**3**897208974098750984279084709854089549387904908234523 45…
…	…
Infinity	0.298143570913452345432435666453524563474567478567814357 2…**8**
?	0.(8+1)(2+1)(3+1)…(8+1)

So now we have another number in between 0 and 1, but we already used all our numbers from 0 - Infinity. This proves that there are more numbers in between 0 and 1 than 1 and Infinity, right?

Well I'd like to prove it wrong. Let's switch things up a little bit. Let's create a table with all the numbers between 0 and 1 on one side, and assign it a number between 0 - Infinity. However to visualize the places we aren't using we'll place X's.

0.8767…	…XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX0
0.1235…	…XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX1
0.5638…	…XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX2
…	…
0.298…8	Infinity

Now each number 0 - 1 has a number from 0 - Infinity.

But wait, we're missing a number. We can create a new number by going through each place value and adding one to it. If we get an X we'll put it at 1. If we get a nine we'll just wrap it around to a zero.

0.8767…	…XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX**0**
0.1235…	…XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX**X**1
0.5638…	…XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX**X**X2
…	…
	…(X+1)(X+1)(0+1)

Now we have another number between 0 - Infinity, but we already used up all our numbers between 0 - 1.

So we just proved that there are more numbers between 0 - Infinity than 0 - 1. But before we just proved the opposite. So what is it?