DISCREET STOCHASTIC CONTROL NOTES

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1. Introduction

1.1. **Setup.** Let $\mathcal{S}, \mathcal{A}, \mathcal{W}$ be finite sets, $(W_t)_{t=0}^{\infty}$ a sequence of $\mathcal{W}$-valued random variables and for each $t \in \mathbb{N}_0$, $f_t : \mathcal{S} \times \mathcal{A} \times \mathcal{W} \to \mathcal{S}$. For each $s \in \mathcal{S}$, let $\mathcal{A}_s \subset \mathcal{A}$. For each $t \in \mathbb{N}_0$, let $\pi_t : \mathcal{S} \to \mathcal{A}$ such that for each $s \in \mathcal{S}$, $\pi_t(s) \in \mathcal{A}_s$. Define $\pi = (\pi_t)_{t=0}^{\infty}$. For each $t \in \mathbb{N}_0$, let $v_t : \mathcal{S} \times \mathcal{A} \times \mathcal{W} \to \mathbb{R}$.

Let $s_0 \in \mathcal{S}$. We define the dynamical system $(S_t)_{t=0}^{\infty}$ of $\mathcal{S}$-valued random variables by:

$$S_t = \begin{cases} s_0 & t = 0 \\ f(S_{t-1}, \pi_{t-1}(S_{t-1}), W_{t-1}) & t > 0 \end{cases}$$

This system is said to be a **stochastic control system** with states $\mathcal{S}$, actions $\mathcal{A}$, disturbances $(W_t)_{t=0}^{\infty}$, transitions $(f_t)_{t=0}^{\infty}$, policy π and values $(v_t)_{t=0}^{\infty}$.

2. Finite Horizon