INTRODUCTION TO FOURIER ANALYSIS

CARSON JAMES

Contents

1. The Fourier Transform on $\mathbb{R}^n$

1.1. Schwartz Space.

Definition 1.1.1. Let $f \in C^\infty(\mathbb{R}^n)$ and $\alpha, \beta \in \mathbb{N}_0^n$. We define

$$\|f\|_{\alpha,\beta} = \sup_{x \in \mathbb{R}^n} |(1 + x^\beta)\partial^\alpha f(x)|$$

We define Schwartz space, denoted $\mathcal{S}$, by

$$\mathcal{S} = \{f \in C^\infty(\mathbb{R}^n) : \text{ for each } \alpha,\, \beta \in \mathbb{N}_0^n,\, \|f\|_{\alpha,\beta} < \infty\}$$

1.2. The Convolution.

1.3. The Fourier Transform on $L^1(\mathbb{R}^n)$.

Definition 1.3.1. Let $f \in L^1(\mathbb{R}^n)$. We define the **Fourier transform of** f, denoted $\hat{f} : \mathbb{R}^n \to \mathbb{C}$ by

$$\hat{f}(\xi) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbb{R}^n} f(x) e^{-i}$$