Presentation

Carson James

May 13, 2022

Definition

We define
$\Lambda_+^{n\times r} = \{\Sigma \in \mathbb{R}^{n\times r} : \Sigma \text{ is diagonal and positive semi-definite}\}$ and
$O_n = \{U \in \mathbb{R}^{n\times n} : U \text{ is orthogonal}\}$.

Model

1. Fix $M \in \mathbb{R}^{n \times n_M}$columns of M are orthogonal and set $P_M = M(M^TM)^{-1}M^T$?
2. Choose $\Sigma_Z \in \Lambda_+^{n_M \times r}$, $\Sigma_X \in \Lambda_+^{n \times p}$, $U_Z \in O_r$ and $U_X \in O_p$.
3. Set $V_Z^T = \Sigma_Z U_Z$ and $V_X^T = \Sigma_X U_X$.
4. Set $J_Z = MV_Z^T$ and $J_X = MV_X^T$.
5. Choose $I_Z \in \mathbb{R}^{n \times r}$, $I_X \in \mathbb{R}^{n \times p}$ such that $\mathcal{C}(I_Z) \cup \mathcal{C}(I_X) \subset \mathcal{C}(I - P_M)$.
6. Choose $E_X \in \mathbb{R}_{n \times p}$ with $(E_X)_{i,j} \sim N(0, \sigma^2)$
7. Set $Z = J_Z + I_Z$ and $X = J_X + I_X + E_X$

Then $\mathcal{C}(M) \perp \mathcal{C}(I_Z), \mathcal{C}(I_X)$.

Model

1. Fix $M \in \mathbb{R}^{n \times n_M}$columns of M are orthogonal and set $P_M = M(M^T M)^{-1} M^T$?
2. Choose $\Sigma_Z \in \Lambda_+^{n_M \times r}$, $\Sigma_X \in \Lambda_+^{n \times p}$, $U_Z \in O_r$ and $U_X \in O_p$.
3. Set $V_Z^T = \Sigma_Z U_Z$ and $V_X^T = \Sigma_X U_X$.
4. Set $J_Z = M V_Z^T$ and $J_X = M V_X^T$.
5. Choose $I_Z \in \mathbb{R}^{n \times r}$, $I_X \in \mathbb{R}^{n \times p}$ such that $\mathcal{C}(I_Z) \cup \mathcal{C}(I_X) \subset \mathcal{C}(I - P_M)$.
6. Choose $E_X \in \mathbb{R}_{n \times p}$ with $(E_X)_{i,j} \sim N(0, \sigma^2)$
7. Set $Z = J_Z + I_Z$ and $X = J_X + I_X + E_X$

Then $\mathcal{C}(M) \perp \mathcal{C}(I_Z), \mathcal{C}(I_X)$.

Model

We consider a modification of the planted partition model which is a submodel of the stochastic block model with n nodes and r blocks (for now $r = 2$).

1. Choose $U \in \mathbb{R}^{n\times 2}$ such that for each $i \in \{1, \dots, n\}$,

   $$U_{i,j} = \begin{cases} 1 & \text{node } i \text{ is in block } j \\ 0 & \text{else} \end{cases}$$

   as in the stochastic block model. We choose

   $$U = \begin{pmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{pmatrix}$$

2. Choose $W \in \mathbb{R}^{n\times 1}$, with $W = (1, -1, \dots, 1, -1)^T$ and

Model

We consider another modification of the planted partition model which is a submodel of the stochastic block model with n nodes and r blocks (for now $r = 2$).

1. Choose $U \in \mathbb{R}^{n \times 2}$ such that for each $i \in \{1, \dots, n\}$,

$$U_{i,j} = \begin{cases} 1 & \text{node } i \text{ is in block } j \\ 0 & \text{else} \end{cases}$$

   as in the stochastic block model. We choose

$$U = \begin{pmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{pmatrix}$$

2. Choose $W \in \mathbb{R}^{n \times 1}$, with $W = (1, -1, \dots, 1, -1)^T$ and