Collatz Conjecture Run-time Issue

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1 Collatz Conjecture

The Collatz Conjecture, also known as the $3x + 1$ mapping or the $3n + 1$ problem was a conjecture proposed by Lothar Collatz. It states that given any integer n, if n is even, multiply it by $1/2$ and if it is odd, multiply it by 3 and add 1. If you were to recursively do this an infinite amount of times, it would eventually reach the number 1, no matter the starting number.

One computer scientist named Marijn Heule is attempting to solve this conjecture. Heule is a computer scientist at Carnegie Mellon University and uses a computerized proof technique called SAT for solving famous math problems such as The Pythagorean triples problem in 2016, Schur number 5 in 2017, and Keller's conjecture in dimension seven. Heule now has his eyes on the biggest math problem, the Collatz conjecture. While other computer scientists doubt SAT can help solve the Conjecture, Scott Anronson of the University of Texas belives that Heule might be able to solve it [3].

2 Biography of Lothar Collatz

Lothar Collatz was a German mathematician who worked in numerical analysis. He studied at the University of Greifswald in 1928. He was awarded his doctorate in 1935 for his dissertation Das Differenzenverfahren mit höherer Approximation für lineare Differentialgleichunge.

Dr. Collatz was born on July 6, 1910. He entered the University of Greifswald in 1928,

moving to Munich, Göttingen, and finally Berlin where he studied under Alfred Klose. In 1943, Collatz was appointed to a professorship at the Technical University of Hanover. He would hold this position for nine years till he moved to the University of Hamburg where he founded the Institute of Applied Mathematics [2].

In Collatz's early years, he published important books. In 1945, he wroteEigenwertprobleme und ihre numerische Behandlung which lead to boundary value problems for ordinary and partial differential equations. In 1937 he asked the famous Collatz Conjecture explained in Section 1 [2].

Collatz received many honours for his contributions, including the German Academy of Scientists Leopoldina. He would later die in Varna, Bulgaria, while attending a mathematics conference [2].

3 Mathematical Analysis of the Collatz Conjecture [4]

3.1 Recursive Piecewise function

The Collatz Conjecture is a Recursive Piecewise function that follows the following rules [4]:

$$a_n = \begin{cases} \frac{1}{2}a_{n-1} & for a_{n-1}\ even \\ 3a_{n-1}+1 & for a_{n-1}\ odd \end{cases}$$

Using this function, the Collatz Conjecture claims any number will reach 1. For example I will use the number 13 [4].

$$
\begin{aligned}
a_1 &= \text{ 13 is odd, therefore } 3(13)+1 = 40 \\
a_2 &= \text{ 40 is even, therefore } \frac{1}{2}(40) = 20 \\
a_3 &= \text{ 20 is even, therefore } \frac{1}{2}(20) = 10 \\
a_4 &= \text{ 10 is even, therefore } \frac{1}{2}(10) = 5 \\
a_5 &= \text{ 5 is odd, therefore } 3(5)+1 = 16 \\
a_6 &= \text{ 16 is even, therefore } \frac{1}{2}(16) = 8 \\
a_7 &= \text{ 8 is even, therefore } \frac{1}{2}(8) = 4 \\
a_8 &= \text{ 4 is even, therefore } \frac{1}{2}(4) = 2 \\
a_9 &= \text{ 2 is even, therefore } \frac{1}{2}(2) = 1
\end{aligned}
$$

3.2 Unusual patterns and/or unusual entropy

When it comes to the relationship between steps and the starting number, there is no distinguishable pattern between the two. Below is a table that shows starting numbers 1 through 10 as well as the corresponding steps needed to achieve reaching number 1.

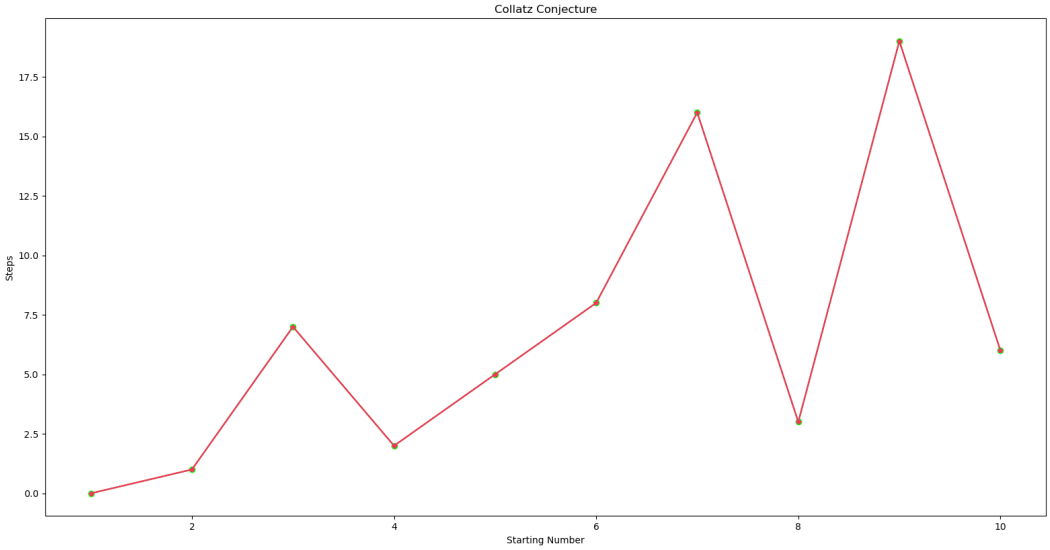

Figure 1

Starting Number	Steps
1	0
2	1
3	7
4	2
5	5
6	8
7	16
8	3
9	19
10	6

As you can see, with this small sample size it shows a chaotic pattern takes place. A graphic representation is shown below.

With a program I've developed, I have come to the conclusion that there is no set related to the starting number and the number of steps it takes to reach 1. I have determined that the integer with the highest number of steps going to the number 100,000 is 77,031 with the

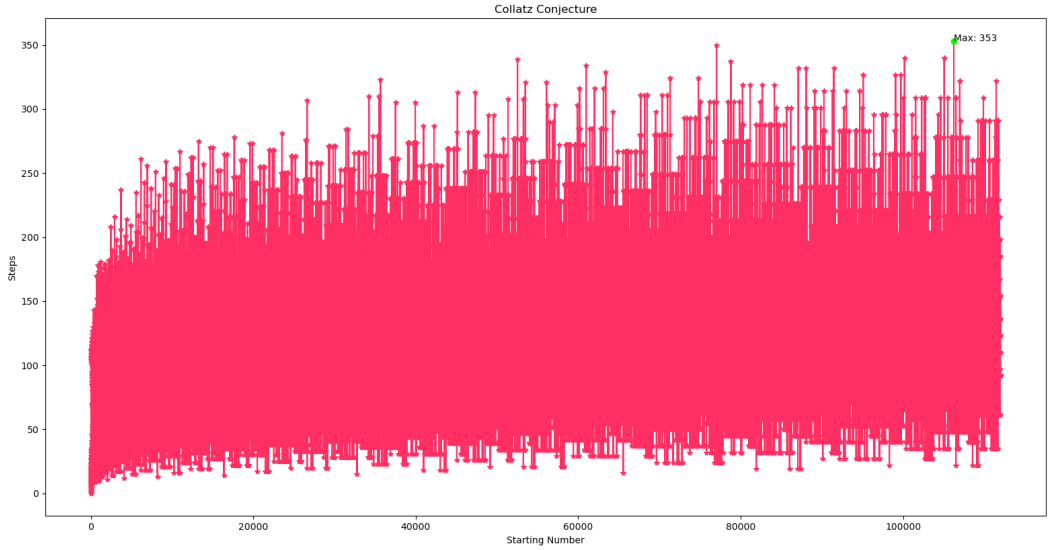

Figure 2

steps being 350 steps. The odd part is, the next number, 77,032, only has 76 steps making the relation between starting number and steps redudnted. Below in figure 2, you can see the starting number and corresponding step. The Green Dot indicated the number with the highest steps.

3.3 Big O and Run time of recursive Algorithm [1]

Knowing that there is no relation between starting number and the number of steps required to complete the task, the run time of the recursive algorithm is dependent on n, being the number of steps taken, making *Big O* of this problem $O(n)$. The problem with this is that n is not the starting value but instead the number of steps. There is no known relation between the starting value and the number of steps so the Big O of this algorithm is n where the starting value is dependent of a unknown function. This means the run time is unknown and can only be known if a relation can or is found. Below is the code that the $O(n)$ was derived from [1].

```
public class CollazStepCounter {
```

```
        public static void main(String[] args) {
        System.out.println("Running");
        int n = 0;
        int max = 0;
        int maxN = 0;
        for (int i = 1; i <= 1000000; i++) {
            n = i;
            int count = 0;
            while (n != 1) {
                if (n % 2 == 0) {
                    n = n / 2;
                } else {
                    n = 3 * n + 1;
                }
                count++;
            }
            if (count > max) {
                max = count;
                maxN = i;
            }
        }
        System.out.println("The number " + maxN +
        "has the longest chain with " + max + " steps.");
    }
}
```

4 Final Remarks

While the Collatz Conjecture is a famous and well known problem in mathematics's, there has not been many famous attempts at solving it. The main reason for this is that the

Collatz Conjecture does not hold any key knowledge if it were to be solved. Why would you spend your whole life trying to prove something that doesn't reward you? While this is true, it still remains as on of the most well known conjectures in mathematics. It is so well known, it had a appearance in Marvel's movie "Black Panther: Wakanda Forever"

References

[1] Big-o notation (article) — algorithms. https://www.khanacademy.org/computing/computer-science/algorithms/asymptotic-notation/a/big-o-notation.

[2] Lothar collatz - biography. https://mathshistory.st-andrews.ac.uk/Biographies/Collatz/.

[3] Kevin Hartnett and substantive Quanta Magazine moderates comments to facilitate an informed. Computer scientists attempt to corner the collatz conjecture, Jun 2021.

[4] Eric W. Weisstein. Collatz Problem. https://mathworld.wolfram.com/.