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Calibration of an Accelerometer Using GPS Measurements

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A COURSE PROJECT REPORT

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ABSTRACT

To make up for the situation where the GPS signals may not be available under certain circumstances, it is necessary to add another sensor like accelerometer so as to get a better estimate of the vehicle location information. Nevertheless, any constant deviation or bias in the accelerometer can be integrated over time to generate inaccurate speed and position estimation. Therefore, the calibration of accelerometer should to applied to get enhanced estimation.

In this course project, discrete time Kalman filter algorithm is applied combined with accelerometer data as well as the GPS signal. Vehicle is equipped with accelerometer to measure acceleration at 200 Hz frequency and a GPS to measure the frequency position and speed at 5 Hertz. The Dynamic model based on Kalman filter is derived between differences true model and accelerometer output using the Euler integral to get velocity and position so as to measure the model from the difference outputs between the GPS and accelerometer outputs. Then, a Kalman filter algorithm recursively estimates vehicle velocity and position as well as accelerometer bias. By applying this, the error can be minimized.

In conclusion, the results show that in a time period of 30 seconds, the Kalman filter gives an excellent estimation in spite of the bias in the accelerometer. A Monte Carlo simulation for 5000 times realizations is carried out to comfirm that the Kalman filter's properties are satisfied by showing that the orthogonality, error conditional variance, and residuals are rather small, which is nearly 0 all the time. All in all, Kalman filter syndicates information from accelerometer and GPS successfully with the implementation of a minimum variance estimator.

Keywords: Kalman Filter, Minimum Variance Estimator, Accelerometer Calibration, GPS

Content

1 INTRODUCTION

In the real time, it is possible that the GPS signals will not be available, for instance, the vehicle is going through a submarine tunnel or the signal is interfered. In order to make up for the situation where the GPS signals may not be available under certain circumstances, it is necessary to add another sensor like accelerometer so as to get a better estimate of the vehicle location information. Nevertheless, any constant deviation or bias in the accelerometer can be integrated over time to generate inaccurate speed and position estimation. Therefore, the calibration of accelerometer should to applied to get enhanced estimation.

In this project, estimating the actual location information and Speed and acceleration of one-dimensional vehicles in inertial systems is the goal. Accelerometer is being used to measure the vehicle's trajectory despite of being affect by a constant unknown offset as well as a Gaussian white noise. When using acceleration information in order to obtain velocity or position, this deviation will occur over time and accumulate. In the meanwhile, GPS noise is also corrupted by Gaussian white noise. The shortcomings of GPS, compared to accelerometers, the data is rather noisy and statistically less reliable. Due to each sensor being imperfect, it can be used for combining data from both sensors in order to overcome these limitations and get a good estimate by Kalman filter.

Kalman filter algorithm is wildly implemented to optimally blend the measurements. Now that goal is calibrating the accelerometer sensor, the difference between the true value and accelerometer model ought to be based so as to drive the dynamic model of the vehicle, which is Kalman filter's framework. Another affecting thing is that the frequency is not the same for these two sensor despite synchronized. Therefore, only these signals are both available can the Kalman filter work. To check the performance, the error should be emphasized. Monte Carlo simulation for huge number of realization can be a quite reliable test to confirm the implemented Kalman filter algorithm with the test of its property. Based on all, the optimal method of processing the information is achieved with a minimum variance estimator.

2 THEORY AND ALGORITHM

2.1 True Model of Motion

We can have the vehicle acceleration have this model so as to be use to simulation is:

$$a(t) = a\sin(2\pi\omega t), m/s^2$$

Where $a = 10m/s^2, \omega = 0.1rad/s$

The position and velocity can be obtained by integration over time for the equation before. Thus, we can get:

$$v(t) = v(0) + \frac{a}{2\pi\omega} - \frac{a}{2\pi\omega}\cos(2\pi\omega t)$$
$$p(t) = p(0) + (v(0) + \frac{a}{2\pi\omega})t - \frac{a}{4\pi^2\omega^2}\sin(2\pi\omega t)$$

Where the initial values of velocity and position are:

$$v(0) \sim N\left(\overline{v}_0,\ M^v{}_0\right), x(0) \sim N\left(\overline{p}_0,\ M^p{}_0\right)$$

They go with values of the mean and variance as given above. In this case, the initial conditions are random numbers under Gaussian distribution.

Based on the equations above, it can be obtained that the actual true acceleration, velocity and position of the vehicle in 30 seconds, which shows under the plots in Figure 1 respectively.

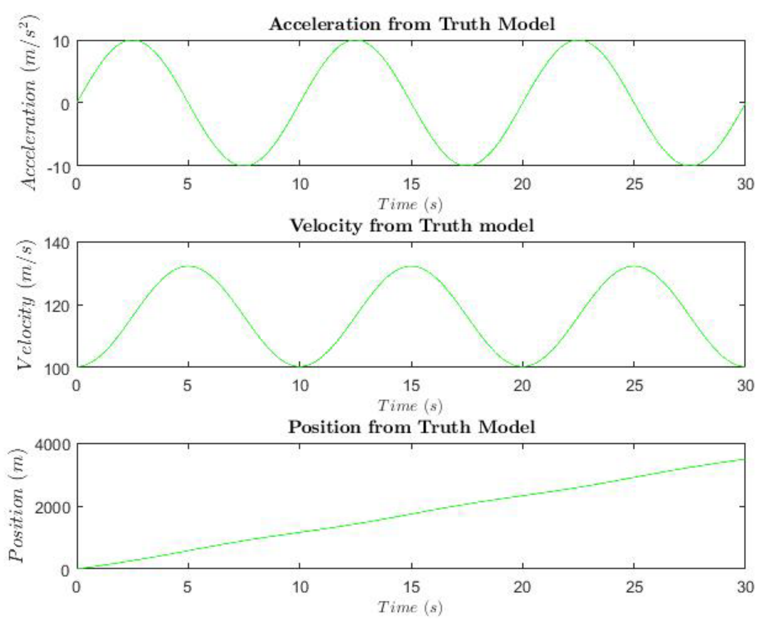


Figure 1: Vehicle's Accelerometer, Velocity, and Position with True Values

2.2 Model of Accelerometer

As is known to all, the acceleration is measured by an accelerometer with a sample rate of 200 Hz at sample time t_j. The acceleration model is:

$$a_c(t_j) = a(t_j) + b_a + w(t_j)$$

It can be discerned that it is modeled with additive white Gaussian noise:

$$w \sim N(0,\ W)$$

Where w goes with zero mean and variance $W = 0.0004\ (m/s^2)^2$. On the other hand, The accelerometer has a bias b_a with a priori statistic $b_a \sim N(0,\ 0.01(m/s^2)^2)$.

Based on the sampling rate 200Hz, it can be known that $t_{j+1} - t_j = \Delta t = 0.005\ s$

Using this formula, the velocity and position of the vehicle can be obtained by the Euler integration of accelerometer's mode shown below.

$$v_c(t_j+1) = v_c(t_j) + a_c(t_j)\Delta t$$
$$p_c(t_j+1) = p_c(t_j) + v_c(t_j)\Delta t + a_c(t_j)\frac{\Delta t^2}{2}$$

The initial condition for these two function are: $v_c(0) = \overline{v}_0 = 100,\ p_c(0) = \overline{p}_0 = 0.$

We remove the constant bias in the GPS measurement and Kalman filter due to the fact that it is integrated over time for both velocity and position calculations, which contributes to a large growth of error accumulation over time. Figure 2 shows the error between the accelerometer measurements and the truth model.

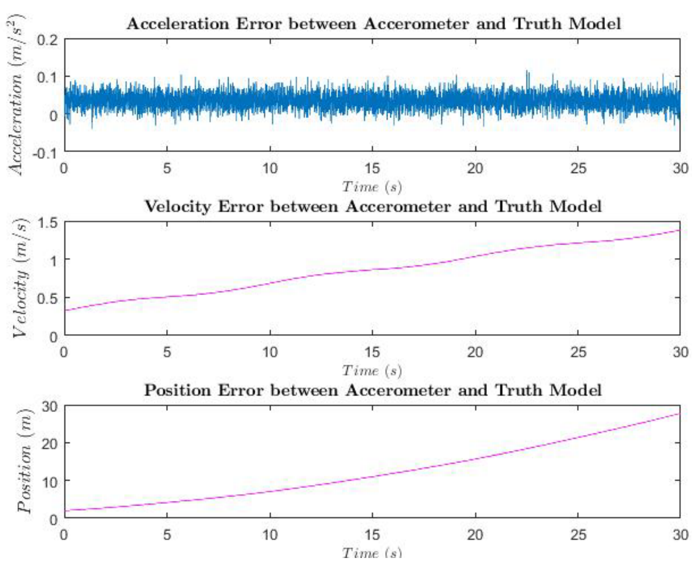


Figure 2: Error Between the Accelerometer Measurements and the Truth Model

2.3 GPS Measurements and Model

A GPS receiver is used to measure the position and velocity in an inertial space. The synchronized sample time together with accelerometer is 5 Hz whenever it is available, which is rather small and slow compared with accelerometer. It can be combined that the GPS measurements as well as accelerometer data, then an optimal better estimation of the vehicle's accelerometer, velocity, and position can be get along with iteration of calculations.

It is rather important to be synchronized otherwise it will mess up, since the actual data measurements of the GPS is not acquired directly, which is assumed as corrupted values of position and velocity from true model of motion. They are named $p(t_i)$ and $v(t_i)$ respectively.

As shown blow, the GPS model:

$$z_p(t_j) = p(t_j) + \eta_p(t_j)$$
$$z_v(t_j) = v(t_j) + \eta_v(t_j)$$

The actual position and actual velocity can be gained in the time interval of

$$t_{j+1} - t_j = 40\Delta t = 0.02\ s$$

Their *a priori* statistics are:

$$x_0 \sim N(0, 100m^2)$$
$$v_0 \sim N(100m, 1(m/s)^2)$$

And Gaussian white noise available in the GPS measurement of the position and velocity are assumed to be independent of each other:

$$\begin{bmatrix} \eta_{1i} \\ \eta_{2i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1m^2 & 0 \\ 0 & 0.0016m/s^2 \end{bmatrix} \right)$$

As a description, Figure 3 shows the differences between the GPS's measurements of position and velocity and those values obtained by accelerometer's outputs. In the meanwhile, Figure 4 shows the white noises in the GPS measurements from the error between GPS measurement and true values.

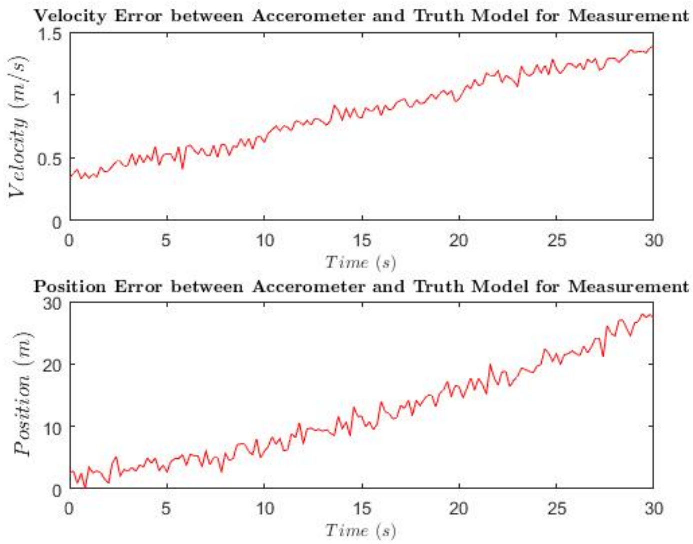


Figure 3: Error of GPS's Output and Accelerometer's Output

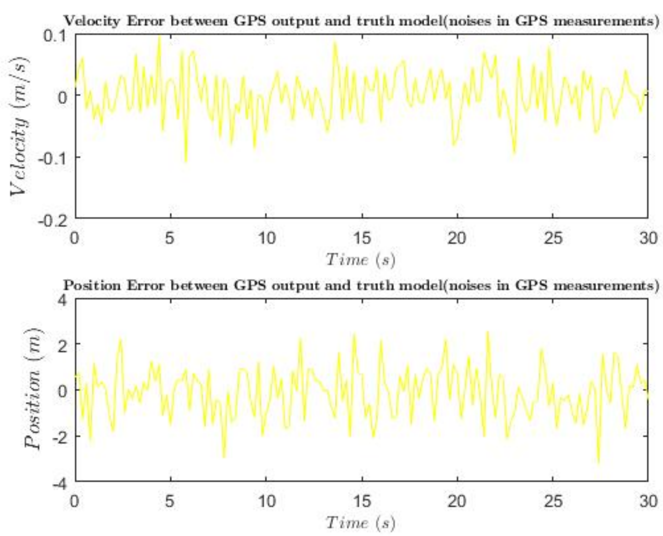


Figure 4: Error of GPS's Output and True Values (White Noise of GPS)

2.4 Dynamic Model of Combined Measurement Systems

It is needed to drive the system model with equations with state variables to apply Kalman filter. One rather important thing to mention is the constant bias from the accelerometer model. Therefore, we are going to make the system model for the Kalman filter being independent of the actual acceleration a(t) and instead dependent on b_a.

Under previous analysis, the accelerometer data can be adapted to determine the dynamic model. The bias should be one of the state variable instead so that we can gain the interest in estimating the bias as well as getting the accelerometer output from the dynamic model.

We make the assumption that the actual or true acceleration is integrated by the same Euler integration formula as the accelerometer so

$$v_E\ (t_j+1)=v_E(t_j)+a(t_j)\Delta t$$
$$p_E\ (t_j+1)=p_E(t_j)+v_E(t_j)\Delta t+a(t_j)\frac{\Delta t^2}{2}$$

The initial conditions are:

$$p_0=p_E(0)\sim N(0,100m^2)$$
$$v_0=v_E(0)\sim N(100m,1(m/s)^2)$$

So as to get the model of Kalman filter, we subtract the accelerometer Euler integrated equations from the true and real Euler integrated equations. According to this, the actual acceleration is eliminated but the bias ought to be left.

It is noted that it is assumed that the true position along with velocity can be attained by the same Euler integration because when subtracting accelerometer equation permitted us to eliminate the real acceleration and bias is left over.

$$\begin{bmatrix}\delta p_E\\ \delta p_E\\ b_a\end{bmatrix}_{j+1}=\begin{bmatrix}1 & \Delta t & 0.5\Delta t^2\\ 0 & 1 & -\Delta t\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\delta p_E\\ \delta p_E\\ b_a\end{bmatrix}_j-\begin{bmatrix}0.5\Delta t^2\\ \Delta t\\ 0\end{bmatrix}w_j$$

Where we can have:

$$\delta p_E(t_0)=\delta p_E(t_0)-\delta p_c(t_0)\sim N(0,100m^2)$$
$$\delta v_E(t_0)=\delta v_E(t_0)-\delta v_c(t_0)\sim N(0,1\ (m/s^2)^2)$$
$$E[w(t_j)]=0$$
$$E[w(t_j)w(t_i)^T\ ]\ =\ 0.0004\delta_{i,j}$$
$$b_a(0)\sim N(0,0.01\ (m/s)^2)$$

2.5 Measurement Equations

The measurements are corrupted values of $p(t_i)$ and $v(t_i)$ as

$$z_p(t_j)\ =p(t_j)+\eta_p(t_j)$$
$$z_v(t_j)=v(t_j)+\eta_v(t_j)$$

The actual position and actual velocity can be gained in the time interval of

$$t_{j+1}\ -\ t_j\ =\ 40\Delta t\ =\ 0.02\ s$$

Knowing the statistics of the noises, a convenient method to put the measurement equations so as to construct the Kalman filter. Subtracting computed accelerometer position and velocity from the GPS measurements as:

$$\delta z^p(t_j) = \delta p(t_j) + \eta^p(t_j)$$
$$\delta z^v(t_j) = \delta v(t_j) + \eta^v(t_j)$$

Where

$$\delta p(t_j) = p(t_j) - p_c(t_j)$$
$$\delta v(t_j) = v(t_j) - v_c(t_j)$$

Figure 5 compares the differences between truth model and accelerometer's output against the difference between the GPS output and accelerometer's output in the same plot.

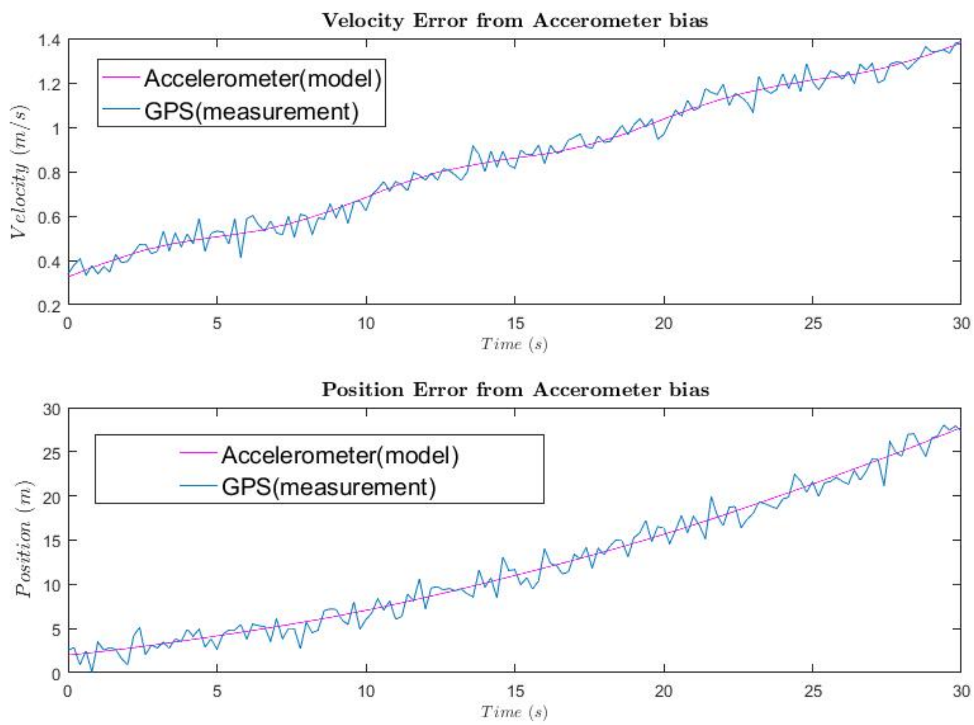


Figure 5: The Differences Between Truth Model and Accelerometer's Output Against the Difference Between The GPS Output and Accelerometer's Output

2.6 Kalmen Filter Implementation

We assume the approximation relationship:

$$\delta v_E(t_j) \approx \delta v(t_j)$$
$$\delta p_E(t_j) \approx \delta p(t_j)$$

The Kalman filter can be constructed along with the measurement model and dynamic model of position and velocity error along with the bias.

Approximate posteriori conditional mean $\delta\hat{x}(t_j)$ can be defined as below:

$$\delta\hat{x}(t_j) = \begin{bmatrix} \delta\hat{p} \\ \delta\hat{v} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} \hat{p} - p_c \\ \hat{v} - v_c \\ \hat{b} \end{bmatrix}$$

The state estimate is deemed to be the estimate of the actual position and velocity that is what the measurement will use.

Due to the fact that the accelerometer and GPS work in separate sampling time, the way propagating and updating states ought to be applied using control statement. In other words, GPS provides data every 0.2 seconds while accelerometer provides every 0.005 seconds so if the data from GPS is not available, we propagate mean and variance:

$$x(t_{j+1}) = \Phi x(t_j)$$
$$M(t_{j+1}) = \Phi M(t_j)\Phi^T + \Gamma W \Gamma^T$$

In the else condition, data from GPS is available along with the accelerometer data as stated earlier, the following equations will be arranged to propagate or update the states and covariance matrices:

$$P(t_j) = (M(t_j)^{-1} + H^T V^{-1} H)^{-1}$$
$$K(t_j) = P(t_j) H^T V^{-1}$$
$$\hat{x}(t_j) = \overline{x}(t_j) + K(t_j)(z(t_j) - Hx(t_j))$$
$$x(t_{j+1}) = \Phi x(t_j)$$
$$M(t_{j+1}) = \Phi P(t_j)\Phi^T + \Gamma W \Gamma^T$$

Where

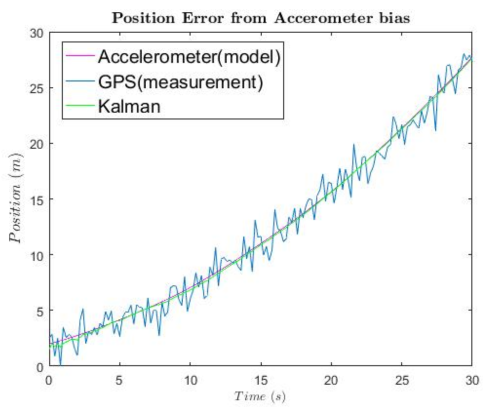
$$\Phi = \begin{bmatrix} 1 & \Delta t & 0.5\Delta t^2 \\ 0 & 1 & -\Delta t \\ 0 & 0 & 1 \end{bmatrix}, M_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

$$W = 0.0016$$

$$V = \begin{bmatrix} 1 & 0 \\ 0 & 0.0016 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 0.5\Delta t^2 \\ \Delta t \\ 0 \end{bmatrix}, \Delta t = 0.005s$$

In Figure 6, the error the estimated state parameters of the Kalman filter is compared with the other errors mentioned before. In the meanwhile, Figure 7 gives a sight of the actual bias and estimated bias trend.



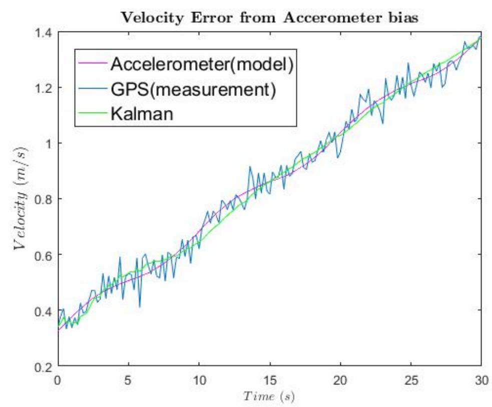


Figure 6 Error of Estimated State Parameters Compare

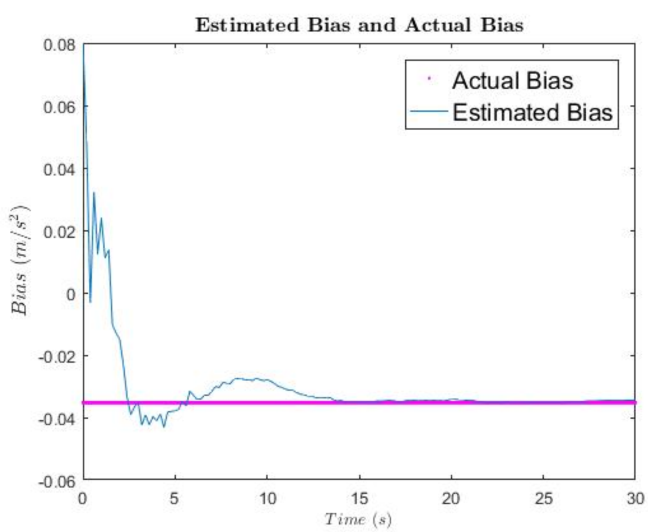


Figure 7 Bias Estimation and Actual Bias

3 RESULTS AND PERFORMANCE ANALYSIS

3.1 State Estimation

Firstly, based on following equation:

$$\delta x(t_j) = \begin{bmatrix} \delta p(t_j) \\ \delta v(t_j) \\ b(t_j) \end{bmatrix} = \begin{bmatrix} p(t_j) - p_c(t_j) \\ v(t_j) - v_c(t_j) \\ b(t_j) \end{bmatrix}$$

The actual *a priori* estimation error is defined as

$$e(t_j) = \delta x(t_j) - \delta \overline{x}(t_j) = \begin{bmatrix} p(t_j) - p_c(t_j) \\ v(t_j) - v_c(t_j) \\ b_a(t_j) \end{bmatrix} - \begin{bmatrix} \overline{p}(t_j) - p_c(t_j) \\ \overline{v}(t_j) - v_c(t_j) \\ \overline{b}_a(t_j) \end{bmatrix} = \begin{bmatrix} p(t_j) - \overline{p}(t_j) \\ v(t_j) - \overline{v}(t_j) \\ b_a(t_j) - \overline{b}_a(t_j) \end{bmatrix}$$

The recursive discrete time equations can be solved through business software Matlab. The estimated state parameters of the Kalman filter can be calculated and analyzed using the interface of Matlab. It can be shown that the estimates of errors of position, velocity, and accelerometer bias as well as the filter error variance for one realization in the figures below.

Noted that the error vector for each discrete time of *posteriori* is defined as

$$e(t_j) = \begin{bmatrix} e_p(t_j) \\ e_v(t_j) \\ e_{b_a}(t_j) \end{bmatrix} = \begin{bmatrix} p(t_j) - p_c(t_j) - \delta \hat{p}(t_j) \\ v(t_j) - v_c(t_j) - \delta \hat{v}(t_j) \\ b_a(t_j) - \hat{b}_a(t_j) \end{bmatrix} = \begin{bmatrix} p(t_j) - \hat{p}(t_j) \\ v(t_j) - \hat{v}(t_j) \\ b_a(t_j) - \hat{b}_a(t_j) \end{bmatrix}$$

Where $\overline{(\cdot)}(t_j)$ denotes the a priori or estimation error propagated in Kalman filter. However, at the measurement time $t_i = t_j$, $e(t_i) = \delta x(t_i) - \delta \overline{x}(t_i^-)$ and posteriori estimation error $e(t_i) = \delta x(t_i) - \delta \hat{x}(t_i^+)$ where t_i^- means before the measurement and t_i^+ means after the measurement update.

Figure 8 gives the graphs of the error of position and velocity between filtered and modeled for accelerometer along with the $\pm\sigma$ bounds. Figure 9 is about the bias and the $\pm\sigma$ bounds are also given.

At last, we can plot the trend of K gains, which suggests that this method is adaptive changing how much the filter relies on information from either of dynamics or measurements. Figure 10 shows this.

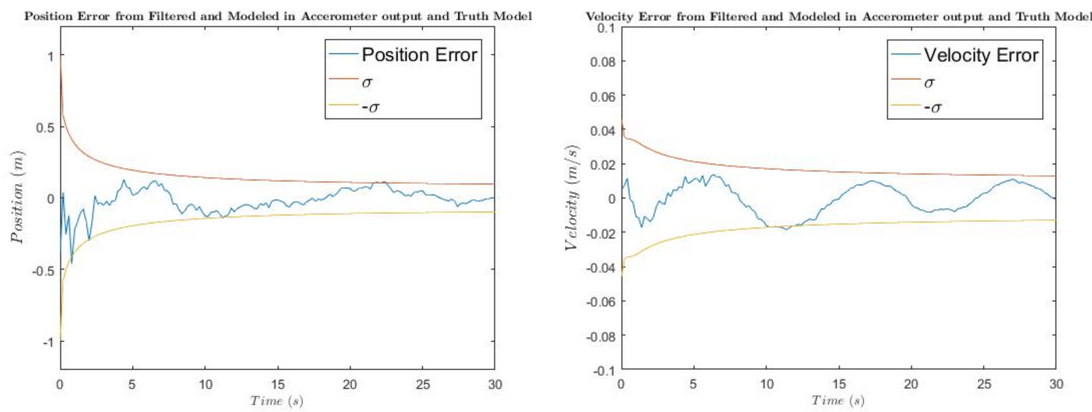


Figure 8: The Error of Position and Velocity between Filtered and Modeled for Accelerometer along with The $\pm\sigma$ Bounds.

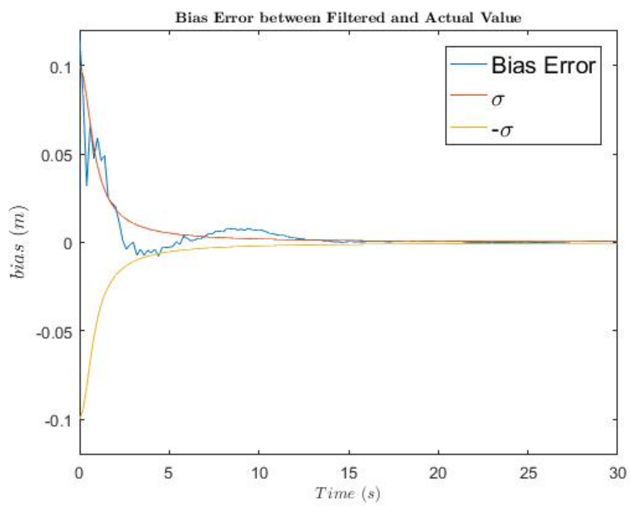


Figure 9: Accelerometer Bias Error

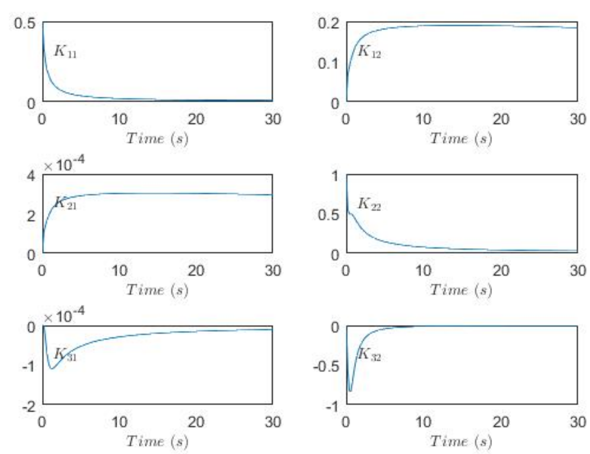


Figure 10: Kalman Gains Changing over Time

3.2 Monte Carlo Simulation

A Monte Carlo simulation is to be constructed to find the ensemble averages over a set of realizations. Let $e^l\left(t_i\right)$ represent the actual error for realization one. This vector can be described as:

$$e^l(t_j)=\begin{bmatrix} e^l{}_p(t_j) \\ e^l{}_v(t_j) \\ e^l{}_{b_a}(t_j) \end{bmatrix}=\begin{bmatrix} p^l(t_j)-p^l{}_c(t_j)-\delta\hat{p}^l(t_j) \\ v^l(t_j)-v^l{}_c(t_j)-\delta\hat{v}^l(t_j) \\ b^l{}_a(t_j)-\hat{b}^l_a(t_j) \end{bmatrix}=\begin{bmatrix} p^l(t_j)-\hat{p}^l(t_j) \\ v^l(t_j)-\hat{v}^l(t_j) \\ b^l{}_a(t_j)-\hat{b}^l_a(t_j) \end{bmatrix}$$

To obtain each realization, an initial condition for $p\left(0\right)$, $v\left(0\right)$, ba are generated from a Gaussian noise generator. The state estimate is determined from measurements where the measurement noise for $\eta^p\left(t_i\right),\eta^v\left(t_i\right)$ are generated at each measurement time t_i from a Gaussian noise generator. The process noise $w\left(t_j\right)$ in the accelerometer model is generated at each propagation time t_j .

We first show that The ensemble average of $e^l(t_j)$ which produces the actual mean and is obtained directly from a Monte Carlo simulation is close to zero.

If having the number of realizations huge enough, then we can produce a good approximate statistic of model and performance so that it should be close to zero for all $t_i \in \left[0,\ 30\right]$. The mathematical formulation to produce the actual mean is

$$e^{ave}(t_i)=\frac{1}{N_{ave}}\sum_{l=1}^{N_{ave}}e^l(t_i)\approx 0,\forall t_i\in\ \left[0,\ 30\right]$$

The results of the Monte Carlo simulation for the expected value of the error are shown in Figure 11. We choose

$$N_{ave}=5000$$

Ensemble average producing the actual error variance P^{ave}

$$P^{ave}=\frac{1}{N_{ave}-1}\sum_{l=1}^{N_{ave}}[e^l(t_i)-e^{ave}(t_i)][e^l(t_i)-e^{ave}(t_i)]^T$$

Where $N_{ave}-1$is used for unbiased variance from small sample theory. The matrix

$P^{ave}\left(t_i\right)$ should be close to $P\left(t_i\right)$ which is a posteriori error covariance matrix in the Kalman filter algorithm for a single run. This is an important check to verify that the modeling is approximately correct and the Kalman filter has been programmed and implemented correctly.

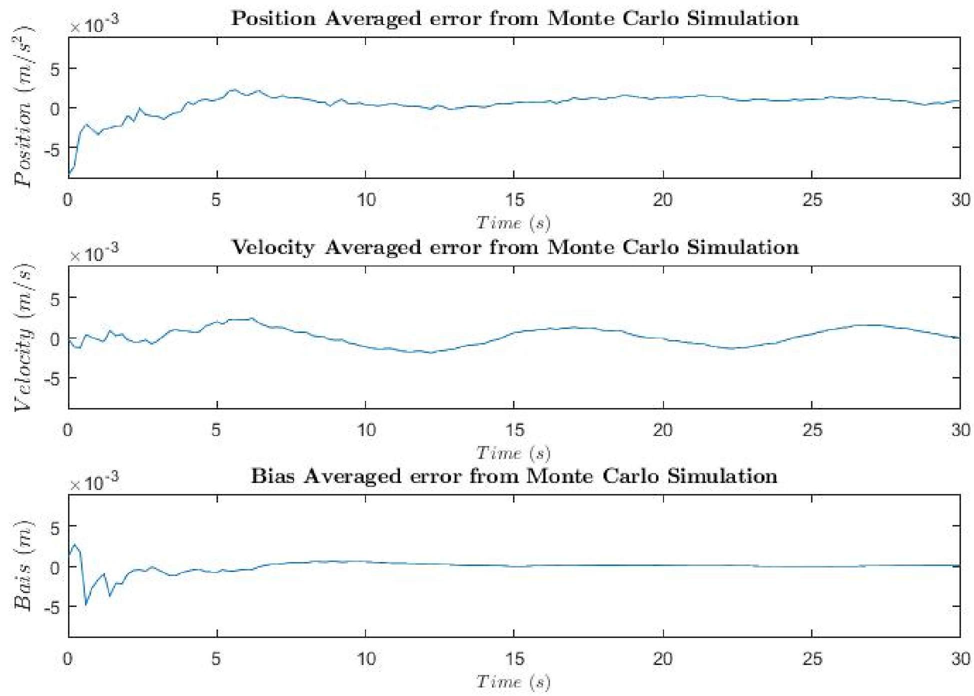


Figure 10: Average Error Among Estimated and Actual Position, Velocity, and Bias Estimation for 5000 Realizations.

This can be expressed as

$$P^{ave}(t_i) - P(t_i) \approx 0, \forall t_i \in [0,\ 30], P^{ave}(t_i) - P(t_i) \in R^{3\times 3}$$

Which shows in Figure 11.

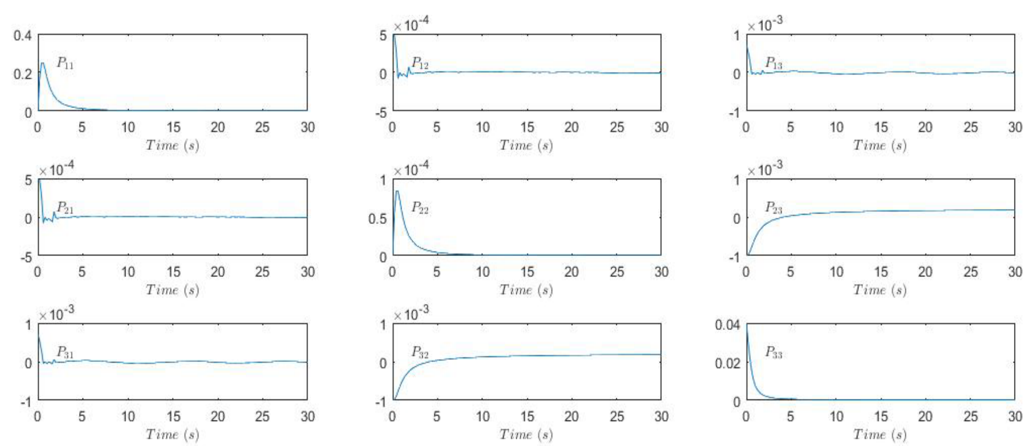


Figure 11: Variance Zero Check

Likewise, the orthogonality of the error in estimates with the estimate is checked by average

$$P^{ave} = \frac{1}{N_{ave}} \sum_{l=1}^{N_{ave}} [e^l(t_i) - e^{ave}(t_i)]\hat{x}(t_i)^T \approx 0, \forall t_i \in [0,\ 30]$$

One thing to be mentioned is that, the $\hat{x}(t_i)$ is independent on the realization times. Total nine figures at each time step can be generated due to the same dimension $R^{3\times 3}$ as before. Figure 12 shows this process and it is about zero all the time.

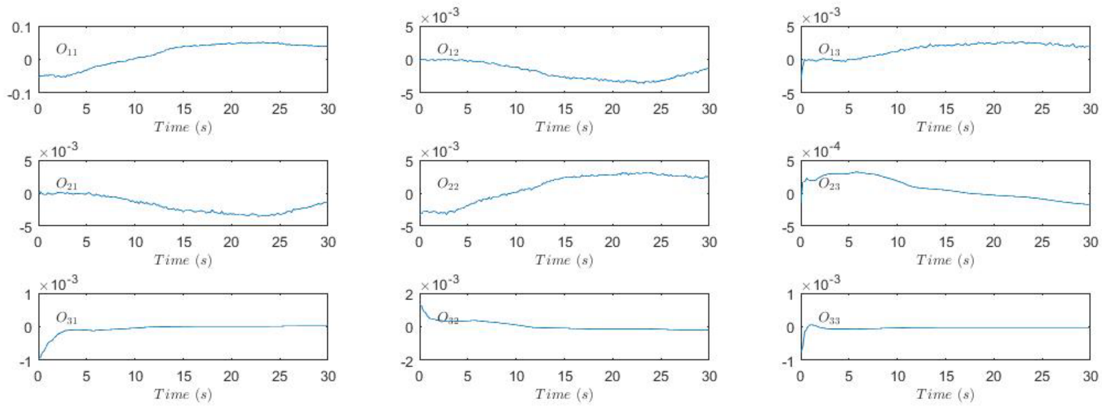


Figure 12: Orthogonality Check

Finally, the independence of the residuals can to be checked. The residual for a realization can be

$$r^l(t_i) = \begin{bmatrix} \delta z^{pl}(t_i) - \delta \overline{z}^{pl}(t_i) \\ \delta z^{vl}(t_i) - \delta \overline{z}^{vl}(t_i) \end{bmatrix}$$

$$\frac{1}{N_{ave}} \sum_{l=1}^{N_{ave}} r^l(t_i) r^l(t_m)^T \approx 0, \forall t_m < t_i$$

$r^l(t_i)$ is the residual at time at time t_i.The time chosen are following

$$\frac{1}{N_{ave}} \sum_{l=1}^{N_{ave}} r^l(20) r^l(30)^T = \begin{bmatrix} 0.0486 & -0.0023 \\ 0.0002 & 0.0002 \end{bmatrix} \approx 0$$

A conclusion can be drawn here about the performance analysis is that we expect very small results for this checking under the large times of trials, which suggests that correct and nice implementation of the Kalman filter.

4 Discussion and Conclusion

To sum up, in this course report, the system is deemed as a stochastic system and the difference between the integration of the accelerometer output and the GPS as a new measurement with an associated stochastic discrete time system that is nearly independent of the acceleration profile. So the minimum variance estimator is achieved. In the meanwhile, the results show that in a time period of 30 seconds, the Kalman filter gives an excellent estimation in spite of the bias in the accelerometer. A Monte Carlo simulation for 5000 times realizations is carried out to confirm that the Kalman filter's properties are satisfied by showing that the orthogonality, error conditional variance, and residuals are rather small, which is nearly 0 all the time. All in all, Kalman filter syndicates information from accelerometer and GPS successfully with the implementation of a minimum variance estimator.

To be concrete, focusing on true model and Kalman filter estimations, it can be vividly seen that the filtered and estimated position and velocity are integrated from measurements and model with dynamics, particularly, less noisy than the measurements. Moreover, the posteriori error values from the covariance matrix give the upper-bound and lower-bound. The errors lying in the bounds well present a good stochastic performance as general despite some values are out in this process which can be revisited in Figure 8.

Digging into the process of Kalman filter. In Figure 10, the Kalman gain will converge to one constant value over time, suggesting that the algorithm is working to adjust the errors. Looking deeply, it can be found that the K_{12} the only one that is not going to zero, which, actually is affected by the GPS's velocity measurement. Because the bias in the accelerometer, it is less accurate in the state dynamic model. Therefore, the system will depend on more on measurement. However, as the process goes along with time long enough, the filter itself can get a good estimation of states, particularly, the bias, the filter then selects to start relying more on the dynamic model, which is now more reliable than the measurements. It is clear that the noise variance in the Kalman process is less than the position measurement noise variance. Moreover, it is comparable to the noise variance in the velocity measurement. So the model dynamic is better with less noise, the Kalman will tend to be unchanged but the data for K_{12} is not zero for the reason that it is still valuable. As a result, measurements will be still a good source of information. By the way, integrating the accelerometer output twice will tend to make the output be easier to be affect by error in dynamics.

Monte Carlo simulation is trivial trial in this project. With all many trials and realizations, the Kalman filter yields a good error elimination with the performance of error being almost zero all the time. What's more, the error conditional variance checks, the orthogonality checks as well as the residual checks should all approximately equal zero, according to the property of Kalman filter.

Reference

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