Carson McLean
CSC 2515
HW 1
2018-09-26

1 (a)

$Z = (X-Y)^2 = X^2 - 2XY + Y^2$

$\mathbb{E}(Z) = \mathbb{E}(X^2 - 2XY + Y^2)$

$= \mathbb{E}(X^2) - \mathbb{E}(2XY) + \mathbb{E}(Y^2)$

$= \mathbb{E}(X^2) - 2(\mathbb{E}(X)\mathbb{E}(Y)) + \mathbb{E}Y^2$ 2 is constant, X, Y independent

$= \int_0^1 x^2 \frac{1}{1-0} - (2 \cdot \int_0^1 x \frac{1}{1-0} \cdot \int_0^1 y \frac{1}{1-0}) + \int_0^1 y^2 \frac{1}{1-0}$

$= \frac{1}{3} - (2 \cdot \frac{1}{2} \cdot \frac{1}{2}) + \frac{1}{3} = \frac{2}{3} - \frac{1}{2} = \boxed{\frac{1}{6}}$

$\mathbb{E}(X) = \int_a^b x f_X(x)\,dx$ continuous random variables

$\mathbb{E}(g(X)) = \int_a^b g(x) f_X(x)\,dx$

Uniform PDF $= \frac{1}{b-a} = f(x)$

$Var(Z) = \mathbb{E}(Z^2) - (\mathbb{E}(Z))^2 = \mathbb{E}(x^4 - 4x^3Y + 6X^2Y^2 - 4XY^3 + Y^4) - (\frac{1}{6})^2$

$= \left[\int_0^1 x^4 \frac{1}{1-0} - (4 \cdot \int_0^1 x^3 \frac{1}{1-0} \cdot \int_0^1 y \frac{1}{1-0}) + (6 \cdot \int_0^1 x^2 \frac{1}{1-0} \cdot \int_0^1 y^2 \frac{1}{1-0}) - (4 \cdot \int_0^1 x \frac{1}{1-0} \int_0^1 y^3 \frac{1}{1-0}) + \int_0^1 y^4 \frac{1}{1-0}\right] - \frac{1}{36}$

$= \frac{1}{5} - (4 \cdot \frac{1}{4} \cdot \frac{1}{2}) + (6 \cdot \frac{1}{3} \cdot \frac{1}{3}) - (4 \cdot \frac{1}{2} \cdot \frac{1}{4}) + \frac{1}{5}] - \frac{1}{36}$

$= \frac{1}{5} - \frac{1}{2} + \frac{2}{3} - \frac{1}{2} + \frac{1}{5} - \frac{1}{36} = \frac{1}{15} - \frac{1}{36} = \boxed{\frac{7}{180}}$

1 b

$$= \sum_{i=1}^{d} Z_i =$$

$R = Z_1 + \dots + Z_d = dZ$ where Z is a random variable as seen in 1 a $Z_i = (X_i - Y_i)^2$

$\mathbb{E}(R) = \mathbb{E}(dZ) = d\,\mathbb{E}(Z)$ by removing the constant d From 1 a, $\mathbb{E}(Z) = 1/6$

$= \boxed{d/6}$

$Var(R) = Var(Z_1 + \dots + Z_d) = Var\left(\sum_{i=1}^{d} Z_i\right)$

By Bienaymé Formula

$Var\left(\sum_{i=1}^{d} Z_i\right) = \sum_{i=1}^{d} Var(Z_i) = d\,Var(Z_i)$ From 1 a, $Var(Z) = 7/180$

$= \boxed{\dfrac{d7}{180}}$