


Stat 441/505 Winter 2019
Instructor: Majid Nabipoor
Assignment 1

There are totally 10 equally weighted questions. Include all necessary R codes and output in your answers.

Q1.

Consider simple linear regression with one covariate:

 $Y = \beta_0 + \beta_1 X + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2),$

show that $\hat{\beta}_0, \hat{\beta}_1$ are unbiased estimators, and calculate their variance.

Q2.

Suppose we have a data set with five predictors, $X_1 = \text{GPA}$, $X_2 = \text{IQ}$, $X_3 = \text{Gender}$ (1 for Female and 0 for Male), $X_4 = \text{Interaction between GPA and IQ}$, and $X_5 = \text{Interaction between GPA and Gender}$. The response is starting salary after graduation (in thousands of dollars). Suppose we use least squares to fit the model, and get $\hat{\beta}_0 = 50$, $\hat{\beta}_1 = 20$, $\hat{\beta}_2 = 0.07$, $\hat{\beta}_3 = 35$, $\hat{\beta}_4 = 0.01$, $\hat{\beta}_5 = -10$.

- (a) Which answer is correct, and why?
- i. For a fixed value of IQ and GPA, males earn more on average than females.
 - ii. For a fixed value of IQ and GPA, females earn more on average than males.
 - iii. For a fixed value of IQ and GPA, males earn more on average than females provided that the GPA is high enough.
 - iv. For a fixed value of IQ and GPA, females earn more on average than males provided that the GPA is high enough.
- (b) Predict the salary of a female with IQ of 110 and a GPA of 4.0.
- (c) True or false: Since the coefficient for the GPA/IQ interaction term is very small, there is very little evidence of an interaction effect. Justify your answer.

Q3.

This question involves the use of simple linear regression on the `Auto` data set.

- (a) Use the `lm()` function to perform a simple linear regression with `mpg` as the response and `horsepower` as the predictor. Use the `summary()` function to print the results. Comment on the output. For example:
- Is there a relationship between the predictor and the response?
 - How strong is the relationship between the predictor and the response? P, R^2
 - Is the relationship between the predictor and the response positive or negative?
 - What is the predicted `mpg` associated with a `horsepower` of 98? What are the associated 95 % confidence and prediction intervals?
- (b) Plot the response and the predictor. Use the `abline()` function to display the least squares regression line.
- (c) Use the `plot()` function to produce diagnostic plots of the least squares regression fit. Comment on any problems you see with the fit.

Q4.

This question should be answered using the `Carseats` data set.

- (a) Fit a multiple regression model to predict `Sales` using `Price`, `Urban`, and `US`.
- (b) Provide an interpretation of each coefficient in the model. Be careful—some of the variables in the model are qualitative!
- (c) Write out the model in equation form, being careful to handle the qualitative variables properly.
- (d) For which of the predictors can you reject the null hypothesis $H_0 : \beta_j = 0$?

- (e) On the basis of your response to the previous question, fit a smaller model that only uses the predictors for which there is evidence of association with the outcome.
- (f) How well do the models in (a) and (e) fit the data?
- (g) Using the model from (e), obtain 95 % confidence intervals for the coefficient(s).
- (h) Is there evidence of outliers or high leverage observations in the model from (e)?

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Q5.

It is well-known that ridge regression tends to give similar coefficient values to correlated variables, whereas the lasso may give quite different coefficient values to correlated variables. We will now explore this property in a very simple setting.

Suppose that $n = 2$, $p = 2$, $x_{11} = x_{12}$, $x_{21} = x_{22}$. Furthermore, suppose that $y_1 + y_2 = 0$ and $x_{11} + x_{21} = 0$ and $x_{12} + x_{22} = 0$, so that the estimate for the intercept in a least squares, ridge regression, or lasso model is zero: $\hat{\beta}_0 = 0$.

- (a) Write out the ridge regression optimization problem in this setting.
- (b) Argue that in this setting, the ridge coefficient estimates satisfy $\hat{\beta}_1 = \hat{\beta}_2$.
- (c) Write out the lasso optimization problem in this setting.
- (d) Argue that in this setting, the lasso coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$ are not unique—in other words, there are many possible solutions to the optimization problem in (c). Describe these solutions.

Q6.

In this exercise, we will generate simulated data, and will then use this data to perform best subset selection.

- (a) Use the `rnorm()` function to generate a predictor X of length $n = 100$, as well as a noise vector ϵ of length $n = 100$.
- (b) Generate a response vector Y of length $n = 100$ according to the model

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \epsilon,$$

where $\beta_0, \beta_1, \beta_2$, and β_3 are constants of your choice.

- (c) Use the `regsubsets()` function to perform best subset selection in order to choose the best model containing the predictors X, X^2, \dots, X^{10} . What is the best model obtained according to C_p , BIC, and adjusted R^2 ? Show some plots to provide evidence for your answer, and report the coefficients of the best model obtained. Note you will need to use the `data.frame()` function to create a single data set containing both X and Y .
- (d) Repeat (c), using forward stepwise selection and also using backwards stepwise selection. How does your answer compare to the results in (c)?
- (e) Now fit a lasso model to the simulated data, again using X, X^2, \dots, X^{10} as predictors. Use cross-validation to select the optimal value of λ . Create plots of the cross-validation error as a function of λ . Report the resulting coefficient estimates, and discuss the results obtained.
- (f) Now generate a response vector Y according to the model

$$Y = \beta_0 + \beta_7 X^7 + \epsilon,$$

and perform best subset selection and the lasso. Discuss the results obtained.

Q7.

In this exercise, we will predict the number of applications received using the other variables in the `College` data set.

- (a) Split the data set into a training set and a test set.

- (b) Fit a linear model using least squares on the training set, and report the test error obtained.
- (c) Fit a ridge regression model on the training set, with λ chosen by cross-validation. Report the test error obtained.
- (d) Fit a lasso model on the training set, with λ chosen by cross-validation. Report the test error obtained, along with the number of non-zero coefficient estimates.
- (e) Comment on the results obtained. How accurately can we predict the number of college applications received? Is there much difference among the test errors resulting from these three approaches?

Q8.

Suppose we collect data for a group of students in a statistics class with variables X_1 = hours studied, X_2 = undergrad GPA, and Y = receive an A. We fit a logistic regression and produce estimated coefficient, $\hat{\beta}_0 = -6$, $\hat{\beta}_1 = 0.05$, $\hat{\beta}_2 = 1$.

- (a) Estimate the probability that a student who studies for 40 h and has an undergrad GPA of 3.5 gets an A in the class.
- (b) How many hours would the student in part (a) need to study to have a 50 % chance of getting an A in the class?

Q9.

This question should be answered using the **Weekly** data set, which is part of the **ISLR** package. This data is similar in nature to the **Smarket** data from this chapter's lab, except that it contains 1,089 weekly returns for 21 years, from the beginning of 1990 to the end of 2010.

- (a) Produce some numerical and graphical summaries of the **Weekly** data. Do there appear to be any patterns?
- (b) Use the full data set to perform a logistic regression with **Direction** as the response and the five lag variables plus **Volume** as predictors. Use the summary function to print the results. Do any of the predictors appear to be statistically significant? If so, which ones?
- (c) Compute the confusion matrix and overall fraction of correct predictions. Explain what the confusion matrix is telling you about the types of mistakes made by logistic regression.

- (d) Now fit the logistic regression model using a training data period from 1990 to 2008, with `Lag2` as the only predictor. Compute the confusion matrix and the overall fraction of correct predictions for the held out data (that is, the data from 2009 and 2010).
- (e) Repeat (d) using LDA.
- (f) Repeat (d) using QDA.
- (g) Which of these methods appears to provide the best results on this data?
- ✓ (h) Experiment with different combinations of predictors, including possible transformations and interactions, for each of the methods. Report the variables, method, and associated confusion matrix that appears to provide the best results on the held out data.

Q10.

In this problem, you will develop a model to predict whether a given car gets high or low gas mileage based on the `Auto` data set.

- (a) Create a binary variable, `mpg01`, that contains a 1 if `mpg` contains a value above its median, and a 0 if `mpg` contains a value below its median. You can compute the median using the `median()` function. Note you may find it helpful to use the `data.frame()` function to create a single data set containing both `mpg01` and the other `Auto` variables.
- (b) Explore the data graphically in order to investigate the association between `mpg01` and the other features. Which of the other features seem most likely to be useful in predicting `mpg01`? Scatterplots and boxplots may be useful tools to answer this question. Describe your findings.
- ✓ (c) Split the data into a training set and a test set.
- (d) Perform LDA on the training data in order to predict `mpg01` using the variables that seemed most associated with `mpg01` in (b). What is the test error of the model obtained?
- (e) Perform QDA on the training data in order to predict `mpg01` using the variables that seemed most associated with `mpg01` in (b). What is the test error of the model obtained?
- (f) Perform logistic regression on the training data in order to predict `mpg01` using the variables that seemed most associated with `mpg01` in (b). What is the test error of the model obtained?