Assignment 1
due Jan 27, 2014

- Learn and understand what we covered in class.
- To check if you understood the material, do some exercises from the textbook that have solutions provided in the back of the book.
- Come to the office hours to ask questions. Also ask your fellow students for help and discuss the material.

On this assignment sheet we are only interested in nonnegative functions, so we use the following simplified version of the big-O notation. We define that $x^2 + 16 \in O(x^2)$ if there exists a pair or nonnegative real numbers (C, k) such that

- for all real numbers $x \geq k$ we have that $x^2 + 16 \leq C \cdot x^2$.

Such a pair is called a *witness pair to* $x^2 + 16 \in O(x^2)$. On this assignment sheet, witness pairs are always restricted to have nonnegative C and nonnegative k.

Exercise 1 (10 points)**.**
Prove that $x^2 + 16 \in O(x^2)$.

Exercise 2 (20 points)**.**
Prove that there exist two different witness pairs to $x^2 + 16 \in O(x^2)$.

Exercise 3 (35 points)**.**
Two pairs (C, k) and (D, ℓ) of nonnegative real numbers are called *incomparable*, if one of the following two conditions holds:

- $C > D$ and $k < \ell$
  or
- $C < D$ and $k > \ell$.

Prove that there exist two incomparable witness pairs to $x^2 + 16 \in O(x^2)$.

Exercise 4 (35 points)**.**
Draw a graphical representation of the set of all witness pairs (C, k) to the relationship $x^2+16 \in O(x^2)$. One axis should be the C-axis and one axis should be the k-axis. Explain your approach.