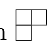
Discrete Mathematics MATH 302 - 501 & 502, Spring 2014

Assignment 5
due Feb 24, 2014

Exercise 1 (40 points)**.**
Prove that for all nonempty finite sets A there exists a bijection between the set E_A of subsets of A having even cardinality and the set O_A of subsets of A having odd cardinality.

Exercise 2 (40 points)**.**
As in our checkerboard exercises we consider tilings of shapes, but this time the shape does not necessarily arise from removing cells from a checkerboard. So let S be any shape that is made of cells, formally let $S \subseteq \mathbb{N} \times \mathbb{N}$ be a finite subset. If we scale S by a factor of 2, we get a larger shape which we call $2S$. Formally we define

$$\begin{aligned} 2S &:= \{(2x, 2y) \mid (x, y) \in S\} \\ &\cup \{(2x + 1, 2y) \mid (x, y) \in S\} \\ &\cup \{(2x, 2y + 1) \mid (x, y) \in S\} \\ &\cup \{(2x + 1, 2y + 1) \mid (x, y) \in S\}. \end{aligned}$$

Let $c(S)$ denote the number of tilings of S with pieces of the form  (as always, pieces can be rotated). Prove that for all shapes S we have $c(S) \leq c(2S)$.

Exercise 3 (20 points)**.**
The same situation as in the previous exercise. Prove that there exists a shape S such that $c(S) \neq c(2S)$.