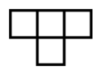
Assignment 4
due Feb 17, 2014

Exercise 1 (10 points)**.**
Prove or disprove that every nonnegative integer can be written as the sum of three (not necessarily distinct) square numbers $(0, 1, 4, 9, 16, 25, \ldots)$.

Exercise 2 (40 points)**.**
Prove or disprove that an 8×8 chessboard from which all 4 corner cells are removed can be covered by 15 so-called T-tetrominos, which are pieces of the following form:



As in the lecture, the pieces can be rotated.

Exercise 3 (50 points)**.**
We use $\mathscr{P}(A)$ to denote the powerset of A. Prove or disprove the following propositions:

1. $\{\emptyset\} \subseteq \{\{\emptyset\}, \emptyset, \{\{\emptyset\}, \emptyset\}\}$
2. $|\{\{\emptyset\}, \emptyset, \{\{\emptyset\}, \emptyset\}\} - \{\{\emptyset, \{\emptyset\}\}, \{\{\emptyset\}\}\}| = 2.$
3. $\mathscr{P}(\{\emptyset\}) - \{\emptyset\} = \{\emptyset\}$
4. $\{\emptyset\} \times \emptyset = \emptyset$
5. For every pair of sets (A, B) we have $\mathscr{P}(A \times B) = \mathscr{P}(A) \times \mathscr{P}(B)$