Assignment 2
due Feb 3, 2014

Exercise 1 (50 points)**.**
Recall that a set S of logical operators is called a *functionally complete set* if every compound proposition is logically equivalent to a compound proposition that

- involves only logical operators from S, propositional variables, and parantheses
- and uses none of the constants `true` or `false`.

Consider the boolean operator NOR that has the following truth table.

p	q	p NOR q
T	T	F
T	F	F
F	T	F
F	F	T

Prove that NOR is functionally complete on its own, i.e., the set S that contains only the single logical operator NOR is a functionally complete set.

The following exercises in section 1.3 of the textbook can guide you: 42, 43, 45, 48, 49, 50.

Exercise 2 (25 points)**.**
The situation is as in the previous exercise, but we use $\vee$ ("OR") instead of NOR. Prove that $\vee$ is not functionally complete on its own.

Exercise 3 (25 points)**.**
The compound proposition

$$\neg((p \vee \neg s) \wedge (q \vee \neg r \vee \neg s) \wedge \neg(t \wedge s)) \wedge (p \vee \neg p)$$

can be thought of as a list of 33 characters from the following table.

$\neg$	$\vee$	$\wedge$	p	q	r	s	t	(	)

Write down a logically equivalent compound proposition using at most 14 characters of this table and prove the equivalence.

Operator precedence is important for this exercise. Note that by operator precedence we have $p \vee q \wedge r \equiv p \vee (q \wedge r)$, where the left hand side has 5 characters and the right hand side has 7.