Catalyst's Approach to Teaching Mathematics

Preface

We offer self-standing math learning programs aimed at helping students pass the standardized tests they need to graduate from high school and get into college. To most effectively implement these programs, we've created our own programs, materials and technologies specifically designed for one-on-one tutoring. In this paper, we describe the structure and underlying educational philosophy of these programs and materials. In Appendix A, we outline the motivation for this effort.

In this position paper, our purpose is to be clear about our motivation, our understanding of the issues at hand, and the decisions we have made regarding math education pedagogy. This way, as Catalyst matures and as we work with more collaborators, we can explicitly reinforce the strong points and systemically fix the problems.

In the ensuing discussion, we make frequent reference to current issues in math education, such as the ideas and debates surrounding the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* and the associated mathematics education reform movement. Further, we make frequent reference to several major education theories in American education, most notably direct instruction versus constructivist or discovery based learning.

All high-level terminology, background information, and history are left to the appendices. Further, these appendices provide additional insight into our own educational philosophy.

In Appendix B we provide some history on and definitions of "direct instruction" as compared to "constructivist" instructional approaches. A focal debate is "should a teacher provide explicit instruction before a student attempts a problem, or should a student construct her or his own strategy to a new problem with the tutor facilitating that construction?"

QUICK LINKS

Executive Summary

Program Terminology

Educational Philosophy

Notes and Future

APPENDICES

Appendix A: Motivation

Appendix B: Major Theories In Education

Appendix C: Mathematics Education Reform

Appendix D: Multiple Modes of Learning

Appendix E: Sample "Printable" Materials

Appendix C introduces the NCTM standards, highlights the major points of great importance to Catalyst, and details the debate that has surrounded the introduction of reform texts both at a national and local level. In addition, we contextualize the *Standards* within the larger debate surrounding direct instruction versus constructivism and talk about the recently release NCTM Curriculum Focal Points, an important document that already is guiding how we edit and add content to our database.

Lastly, Appendix D outlines two major educational theories we reference concerning multiple modes of learning: Bloom's Taxonomy and Gardner's Theory of Multiple Intelligences. Appendix E contains sample Catalyst materials that we reference throughout this paper.

Thank you for taking the time to understand our philosophy and taking interest in our development,

The Catalyst Board of Directors

Executive Summary

At Catalyst we offer more than homework help: we offer self-standing learning programs where we use our own teaching and assessment materials specifically designed for one-on-one tutoring. Our primary focus is helping student to pass the standardized tests necessary for graduation from high school and admission into college.

To reach these goals, often we have to go back and work on conceptual gaps in a student's knowledge base. To most effectively do this, we've created our own programs, materials and technologies specifically designed for one-on-one tutoring. In this paper, we describe the structure and underlying educational philosophy of these programs and materials. In Appendix A, we outline the motivation for this effort.

CATALYST MATERIALS

We store our materials in a searchable, online database found at http://www.catalystlc.org. Tutors use this website to keep themselves organized (especially when working with many students) and to get all the materials needed for the current tutoring session. They draw new materials from a library of "printables" – which are PDF documents they print out and cover one-on-one with a student using paper and pencil.

Our library is organized into "content standards", "strands" and "lessons". Content Standards are based on the NCTM Content Standards, such as the "Number and Operation" content standard. A strand is a large concept such as "Fractions" that exists within a content standard. Lessons are more specific sub concepts as "Exploring Equivalent Fraction Form" and contain specific learning objectives and all necessary materials (including such items as manipulatables). A lesson may implement one strand but usually implement several strands (and, in turn, also implements several content standards).

Currently, library content development is guided by 1) the National Council of Teachers of Mathematics K-8 *Curriculum Focal Points* and 2) the mathematic Graduation Requirement Examinations (GREs) set by the Minnesota Department of Education. Originally, our materials were developed to help students pass the Minnesota Basic Skills Test (MBST) math exam; we will be editing our database for necessary changes needed to help students pass the 8th and 11th grade Minnesota Comprehension Assessment II exams which replaced the MBST in 2006. Major content that will be available in our 2007 database includes:

- Understanding number notation, place value, number magnitude, and number lines.
- Basic arithmetic (+, -, x, ÷) using whole numbers, fractions, decimals and percentages.
- Understanding and working with prime numbers and factoring.
- Understanding exponential notation and computing with scientific notation.
- Rounding, estimation, "mental math", and calculator computation.
- Understanding and calculating with positive and negative numbers.
- Understanding, measuring, and calculating length, area, and volume.
- Understanding, naming, and converting between common units of measurement for length, area, and volume.
- Names and properties of two and three dimensional shapes.
- Understanding and measuring angles.
- Understanding, reading and creating plots, charts, tables, and graphs.
- Introductory statistics such as mean, median, surveys, and basic probability.

All material is covered computationally and contextually. With these programs in place, we will begin building on this work to extend the library with content (e.g. algebra) needed for the 11th grade MCA-II and ACT/SAT tests.

EDUCATIONAL PHILOSOPHY

A substantial debate on how best to teach mathematics emerged when the National Council of Teachers of Mathematics (NCTM) released its first *Principles and Standards* document in 1989 and launched the mathematics education reform movement. Generally, reformers cast this debate as the teaching of fundamental mathematical concepts in real-world applications versus the more traditional "skill-and-drill" approach that centers on isolated computational/algorithmic fluency and route memorization of mathematical facts. The reform debate, however, is also a continuation of a larger (and older) debate surrounding constructivism or "discovery learning" versus explicitly teacher-led direct instruction.

At Catalyst, we place strong emphasis on the NCTM process standards. We strongly believe that conceptual foundations are more important than disconnected route memorization and isolated repetitive algorithmic practice. Students need to be able to talk about mathematics; to be able reason with and compare different mathematical approaches; and to be able to use multiple representations to express mathematical concepts in order solve problems.

Foundational conceptual knowledge is lasting and it is generative; that is, it makes learning increasingly easier and easier. However, it was not the intent of the *Principles and Standards* nor is it our intent to avoid the building of computational and algorithmic fluency. But, following the NCTM guidelines, we stress practice and fluency *after* strong conceptual foundations are in place.

While we avoid a "skill-and-drill" approach, we encase our lessons with a formal structure that explicitly ensures our tutors follow all necessary teaching steps so that our students do not get lost. At the beginning of each lesson, we outline learning objectives, provide tutors with an "into" that explicitly directs a student's attention to the new concept being learned and review previously learned concepts. At the end, we use a "closure" section where students actively reflect on what was covered.

For the presentation of new content, we sometimes use constructivist based methods and sometimes use more explicit direct instruction methods. Whenever reasonable, we introduce new information through relatively small open-ended problems that a student attempts to solve before direct tutor demonstration. Throughout, we provide tutors with prompting strategies that help students to solve the problem by making connections to knowledge they've built in the past.

Certain concepts – especially new definitions or notation and most formal algorithms – we explicitly and directly cover with a student. A tutor then demonstrates how to use the new knowledge and then leads the student through guided practice to reinforce the newly acquired concept or skill.

In general, our use of constructivism is far more restrained and explicitly structured than what is found in many recent reform curriculums. There are two major reasons. Like all curriculum developers, we have our own interpretation on how to best implement the NCTM process *Standards*. But more importantly, we are a supplemental education organization and are designing our material to meet a very different set of constraints than found in the traditional classroom.

Nevertheless, we believe there is a major and important difference between having a student first tackle a new problem head-on as opposed to the more traditional process of first using direct telling and demonstration followed by practice. The former is more engaging, interactive, and increases the student's responsibility in the learning process. But most importantly, it actively forces a student to draw on and make the necessary connections with past knowledge.

After new information has been presented, we reinforce this knowledge through extended practice. While we do have many sections that stress fluency (e.g.. "Compute It") we have many more that approach the subject matter from a variety of viewpoints (our "Say It", "Cut It", "Draw It", "Graph It", "Prove It", "Judge It", and "Move It" sections to name a few). Drawing on the NCTM process Standards, Bloom's Taxonomy of Educational Objectives, and Gardner's Theory of Multiple Intelligences, our goal is to deepen and extend our student's conceptual understanding. At every chance, we tie today's lesson concept with other concepts a student had already learned through our "Connections" sections.

Throughout, we provide both "checks for understanding" and formal assessment. Our programs are student paced: a tutor does not move on to new concepts until a student has sufficiently mastered already presented material. This is simply a continuation of the fundamental premise of Catalyst: every student will excel if given the access and time to master all necessary prerequisite knowledge.

Program Terminology

Our materials and curriculum are organized very differently than traditional textbooks. Most teaching materials are designed for a specific grade-level with the assumption that the teacher will present the material sequentially over the school year. Since most curriculums are also designed to be implemented over several years, concepts are slowly introduced and parts of a concept are found throughout a multiyear curriculum.

At Catalyst, we face a different set of circumstances than those found in a traditional classroom. Therefore, we do not divide our material by grade level. Rather, we've divided all of our programs and materials by concept.

Our online library of materials is divided into **content standards, strands, lessons, and printables** – each one conceptually smaller and less abstract than the last.

These concepts and the overall structure and logic of our materials library are best illustrated through example. Please note, however, that we are a startup organization and, in these early years, we are heavily editing and reorganizing our materials. Please contact us for a complete and up to date description of our library content.

CONTENT STANDARDS AND STRANDS

There are five content standards in our library. They are derived from the *NCTM Principles and Standards* (2000). They are:

- Number and Operation.
- Geometry.
- Measurement
- Data Analysis and Probability
- and Algebra

Key Terminology

Session. The actual 1 to 2 hour tutoring session.

Content Standards. Content Standards are the largest conceptually groups in our library and are derived from the *NCTM Principles and Standards* (2000).

Strands. Strands are large concepts within a content standard such as "Fractions, Decimals, and Percents".

Lessons. Lessons centers on a relatively small concept. For example, we have a lesson called "Exploring Equivalent Fraction Form". It has students explore and reason with the concept of Equivalent Fraction Form (e.g. 1/3 = 2/6) using manipulatables and pictures.

Lessons are categorized by strand. While some lessons contain only a single strand, most implement several strands since we are developing an integrated curriculum.

Each lesson contains learning objectives that are both objective and subjective in nature.

Printables. Each lesson is composed of printables – PDF documents our tutors print out and work on directly with a student using paper and pencil. Printables go by different names such as handout, worksheet, or quiz.

For example, handouts are used to introduce the concept and explore the underlying conceptual frameworks. We use handouts to ensure that tutors follow a solid presentation strategy when introducing new information.

Each content standard is further divided into strands. For example, let's take the *Number and Operation* and the *Data Analysis and Probability* content standards.

The Number and Operation content standards centers on rational number representation, computation, and estimation. Consequently, the strands within the Number and Operation content standard are:

- Magnitude and Place Value
- Addition and Subtraction
- Multiplication and Division
- Factoring and Prime Numbers
- Mental Math

- Rounding and Estimation
- Using a Calculator
- Fractions, Ratios, Decimals and Percents
- Positive and Negative Numbers
- Exponents and Roots
- and Scientific Notation

The Data Analysis and Probability content standards centers on representation and statistics. It is concerned with the multiple ways mathematical ideas can be expressed, the understanding and analysis of data, and the mathematics of chance. Currently, within our Data Analysis and Probability content standard are:

- 3D Objects, Cutouts, and Manipulatables
- 2D Pictures and Drawings
- Number Lines
- XY Plots
- Frequency Plots
- Tables
- Charts and Graphs
- Basic Probability
- Mean and Median
- Surveys and Sample Populations

LESSONS

Lessons are divided into relatively small *concepts* such as "Place Value -- Ones, Tens, Hundreds, Thousands" and "Fractions on a Number Line". Each lesson contains a set of learning objectives that can be either objective or subjective in nature. For example, for the "Fractions on a Number Line" lesson we have the learning objective "After this lesson, you'll be able to use a number line to plot and order fractions and mixed numbers."

We strive to make our objectives as objective as possible. However, we also use subjective objectives such as "After this lesson, you'll be able to verbally compare and contrast whole numbers and fractions" as found in our "Fractions – Pieces of a Whole" lesson.

Lessons are organized by strand. Due to the integrated nature of our curriculum, most lessons are categorized by several strands. For example, the "Fractions- Pieces of a Whole" lesson is under both the *Number and Operations* "Fractions" strand, the *Data Analysis and Probability* "Three Dimensional Objects, Cutouts, and Manipulatables" strand, and the *Data Analysis and Probability* "Two Dimensional Pictures and Drawing" strand.

Our entire math library is arranged in roughly a linear order. Firstly, this is to ensure that any given lesson does not require concepts found in later lessons. But also this allows us to integrate the content of our library.

For example, after the lessons "Exploring Equivalent Fraction Form" and "Exploring Fraction Magnitude" (where students use drawings, cutouts, fraction strips, and number lines to explore the concept of fraction magnitude) there is the lesson "Using a 12 Inch Ruler". Here, students get practice using an English ruler to examine fractions as well as work on concepts from the *Measurement* content standard strands.

Each lesson contains a series of documents called printables which are PDF documents that a tutor prints on a laserjet printer and then covers with a student using paper and pencil. Unlike traditional textbooks -- which are designed to be just one of the tools a classroom teacher uses to teach -- each printable is self-standing.

That is, each lesson contains all the materials, manipulatables, check points, prompting strategies, notes, and other necessary lesson elements. This is to ensure a tutor has them in hand without having to manage several different sets of materials. Further, they explicitly guide a tutor to cover necessary steps of material exposition – such as "intos" and "closure" – which an experienced teacher always encases around traditional curriculums.

TYPES OF PRINTABLES

There are several types of printables in our library: handouts, worksheets, quizzes, practice sheets, cutouts, and scaffolds.

Handouts are the first printables within a lesson; the majority of our lessons contain this starting handout. They are used to introduce new material to a student. Here, we lay out the learning objectives we'll use to determine whether a student has, in fact, understood the presented content. Then, we provide an "into" and review. New information is then introduced. Then, practice and multiple ways of understanding the introduced concepts are explored.

Worksheets are used to further cement the conceptual ideas introduced in the handout. They continue to introduce more and more ways to understand and explore the lesson concept; that is, they stress the multiple modes of understanding necessary for deep conceptual understanding. When appropriate, and after foundational concepts are in place, we provide ample practice to build student fluency.

Quizzes tie back to the learning objectives first laid out in the handouts. In fact, it is the quizzes that truly define the learning objectives for a given lesson. They provide the assessment tools a tutor needs to determine whether to move on, to work more on the given lesson, or decide that the current lesson is too advanced and to work on more fundamental concepts first.

PRINTABLE SECTIONS

Our printables are broken into sections that are given labels such as "Connections", "Investigate", "How To", "Definitions", "Scaffold", "Cut It", "Draw It", "Compute It", "Judge It", "Prove It", "Say It", "Create It", "Teach It", and "Closure".

"Connections" are found on the first page of every handout. Here, we explicitly direct a student attention to the fact that a new subject is going to be covered. This section contains an "into" (or hook) and review of necessary information needed to complete the current lesson.

"Investigate", "How To", and "Definition" sections are used to introduce new information. "Investigate" introduces new information through a carefully controlled open-ended problem. During "How To" sections a tutor explicitly shows a student how to perform an algorithm. "Definition" sections cover new terminology or notation that a tutor explicitly (directly) covers with a student.

"Scaffold" sections are usually self-standing documents that, much like they sound, are temporary support structures such as diagrams or "cheat-sheets" that a tutor lets a student use when solving problems. This is especially important for English Language Learner (ELL) students who benefit from scaffolds that list English terminology of mathematical concepts.

"It" sections (such as "Judge It", "Prove It", "Watch It", "Think It" and "Compute It") provide multiple modes of building conceptual understanding. Each section has a specific intent. For example, in "Cut It" students use concrete tactile "cutouts" to represent and solve a problem. In "Compute It" sections we provide basic arithmetic and equation manipulation practice. In "Say It" sections we talk and write about the mathematical concept at hand. In "Think It" we work on "mental math" and work on computing (whether exact or estimated) without calculators *or* paper and pencil.

"Cut It" sections contain **cutouts** that are objects such as fractions strips, pizzas, money, blocks, foldable shapes, and other 2D drawings that students can cut out and play with. While we also use 3D objects, we favor cutouts whenever possible so we have all necessary materials at hand.

Our use of labeled sections serves several purposes: each section is heavily tied with different teaching styles and modes of learning that we're explicitly enforcing; they help us intentionally design our materials; and we train our tutors on the educational intent of each section so that they approach a given section in an appropriate frame of mind.

Educational Philosophy

At base, we're working to implement the goals and processes outlined in the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics (2000)*.

But like all groups that have created materials based on the NCTM Standards, we have our own interpretation on how to best implement them. Further, being a supplementary learning organization as opposed to a K-12 classroom, we have many unique circumstances that influence our implementation. So while we use the Standards as our foundation, there are many ideas taken from direct instruction and other educational theories that also influence our systems and materials.

BIG IDEAS: THE NCTM STANDARDS

Drawing from the *Principles and Standards*, we strongly focus on first instilling the foundational "Big Ideas" or foundational conceptual ideas by getting our students to openly talk about, reason with, and compare mathematical concepts. And while we do very much stress computational fluency, we first make sure that conceptual foundations are in place; we don't substitute conceptual understanding for mindless calculation. To quote the NCTM:

Principles and Standards recognize that computational proficiency alone is not enough. In today's world students" basic arithmetic skills must include the ability to choose what numbers to use and what operation is appropriate for carrying out the computation, deciding if the results make sense, and then making a decision about what to do next. Reasoning, problem solving, making connections, communicating, and using representations all come into play. Having both computational skills and conceptual understanding will enable students to solve problems that they encounter in their daily lives.

-- *Principles and Standards FAQ*

We stress the importance of "Big Ideas" and conceptual understanding in several ways. To start, we get students exploring a concept -- often using more concrete methods -- *before* jumping right in and showing them the numerically fastest, most general procedure or algorithm. That is, we don't show our students the key before they've had adequate time to see the lock they're trying to open.

For example, in Appendix E are a few of the beginning lessons from our *Fractions* strand. Here, in the first lesson *Fractions – Pieces of a Whole* we first underline the necessary terminology and notation used to talk about and denote fractions in the English language.

After introducing and providing some initial practice on notation, we let students explore the concept using 3D pizza manipulatables (check our "Cut it" section and provided pizza manipulatables in Handout #1 of this lesson). We then proceed to a "Say It" section where we get them to compare whole numbers and fractions and to provide verbal responses to the question: "In your own words, why do we need fractions and how are they different than whole numbers?"

Looking through Appendix E, you'll notice that we do not introduce the use of division to reduce a fraction to lowest form for many sessions. First, we get students playing with objects like fraction strips, drawing pictures, making connections between the whole number line and a fractional number line, and having students change a fraction's form using pictures and objects. Throughout, we stress verbal descriptions. Then, we introduce the more abstract concept of reducing a fraction directly using division.

And even this we do not present in a vacuum; rather, we immediately stress that lowering a fraction's form using division is the opposite of "raising" a fraction's form using multiplications – a skill student's use when finding common denominators.

This illustrates a trend: whenever possible we present ideas in their totality so student's can right away begin to classify and contextualize their knowledge. We believe it's easier to understand a concept if you can see the "big picture" first.

Here we should note that part of our mission is to create rich materials that allow us to go back and work on fundamental problems with a student's knowledge base. It would be easier for us to show them right away how to reduce a fraction directly in order to compare fractions for equality. Even if they could perform this isolated task shortly after demonstration, they still might have major conceptual misunderstandings regarding fractions that will, in some way or another, show up later on (for example, when using fractions in algebraic equations).

To quote the 2002 NCTM Yearbook *Making Sense of Fractions, Ratios, and Proportions*:

Before they learn anything in our classrooms, students engage in activities in the everyday world where they generate ideas about fractions, ratios, and proportionality . . . Mathematically successful students manage to connect these two *these two bodies of knowledge; students who never really understand do not. [page 3].*

Many of our students, for one reason or another, missed building these foundations. We work to rebuild them.

Take another example from Appendix E. A skill many students have conceptual problems with is the multiplications and division of fractions. This concept is not only important to understanding decimals but also an underpinning to algebraic manipulation.

To introduce this concept, in *Handout #1: Multiplying Whole Numbers and Fractions* we have students physically work with a bag of cookies -- "let's say you have 8 cookies, how many cookies is in ¾ of bag". We do this well before we get to the faster, more formal algorithm of cross-cancellation, followed by multiplication, followed by reducing to lowest form.

Again, we want students to get a physical feel for the concept before we move onto more abstract symbol manipulation. In fact, when we do introduce the formal algorithm we explicitly tell them:

REMEMBER! It's more important that you understand what's going on and can draw a picture to represent the problem than remember these steps.

You might forget the steps, but if you understand what's going on you'll always get to the right solution, even if it takes a little longer.

MASTERY LEARNING

A major advantage of one-on-one tutoring is the ability to assign and pace material introduction based on a student's current level of understanding. Drawing on the work of direct instructionists in general and Bloom's "Mastery Learning" in particular (outlined in Appendix B) we carefully structure our teaching and assessment materials so we can meet a student where there are at academically. Further, we don't move on to new concepts until more foundational concepts are fully understood.

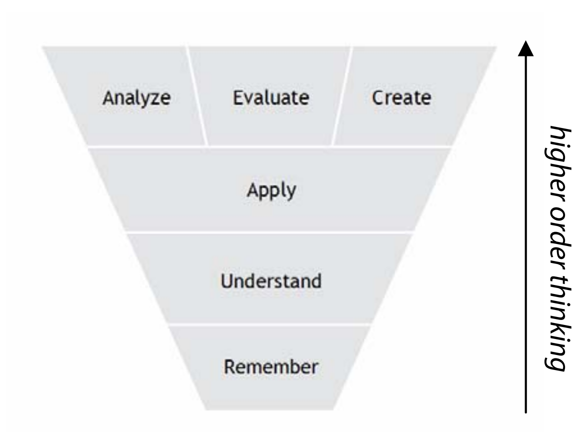
First, our materials are divided into relatively small learning units. This is a balancing act. While we stress 'big ideas' there's obviously a limit to how big of a picture a student can see at one time. If you cover too much material at once, you're at risk of introducing new concepts before more fundamental concepts have been fully understood. In fact, the whole point of Catalyst is that what's most often preventing a student from reaching their full academic potential is that they don't have the prerequisite knowledge.

Each lesson contains a set of learning objectives which are first presented at the beginning of the lesson handout. The assessment quizzes are the realizations of those objectives. After a lesson has been introduced and when a tutor feels a student understands the concept and it has been sufficiently explored, we then give a student a quiz covering this material at the beginning of each session.

If they complete a series of two to three quizzes satisfactorily, the tutor 'closes' that lesson. Then, a tutor can add a new topic to a student's program. If a student is struggling, a tutor makes a judgment call: continue to teach the concept or decide that the lesson is too advanced and a more foundational lesson must be taught first.

A few points of are worthy of highlight:

- Learning objectives and quizzes are not always absolutely demonstrable "objective" objectives and questions. Using Bloom's Taxonomy of Education Objectives as a guide, we explicitly



Bloom's Taxonomy of Educational Objectives

create learning goals which stretch students beyond simple recall and computation.

Clearly, if a student can't complete certain mathematical operation there's a problem – and we catch these problems through the course of a lesson. But some problems which require students to reason, communicate, create, and judge are often much harder to grade as right or wrong. We don't want a maniacal desire for easy to grade tests to preclude us from including richer mathematical content.

We do however try to make our objectives as precise as possible. Just because making precise objectives is difficult doesn't mean we don't strive to do so.

- It's impossible to completely linearize the presentation of mathematics – there are so many overlapping and complementary ideas. But again, there are some concepts that clearly rest on other concepts and we make sure that these foundational concepts are in place before moving on.

As can be seen, we call on our tutors to make many subjective judgments as they decide how to pace the introduction of new material. While this is much harder than setting hard "objective "objectives and using questions that always have a right or wrong answer, we believe the benefits of this approach outweigh the risks. In our opinion, due to the relatively small concept covered in each lesson, we feel that our tutors will be able to make sound judgments as to when to continue practice, when to move on, and when to back up.

LESSON STRUCTURE

In addition to objectives, there are other elements normally associated with direct instruction and Hunter's seven step lesson plan that we formally include in our printables to ensure a tutor is executing an effective presentation (for more information on the Seven Step Lesson Plan, please see Appendix B).

To start, at the beginning of each lesson we include an "Into" section with a review of concepts needed for the current lesson. This explicitly directs a student's attention to the fact that they are beginning a new concept and allows them time to name and remember other ideas and skills they've used in the past. Together with the learning objectives provided at the beginning of the handouts, the first page of the handout contains the full anticipatory set.

At the end of each handout, we include a "Closure" section where a student is asked to reflect on what was learned. Here, we ask tutors to engage in interactive communication with the student and to ask them what they found hard or easy.

In between the "Connections" and the "Closure" of the handout, we first present new information and then lead students through guided/independent practice. Normally, formal lesson plans of this structure are associated with direct teacher input (telling) and demonstration (modeling) followed by guided practice. We, however, present new information in two major ways.

INVESTIGATE: THE USE OF CONSTRUCTIVISM AND PROBLEM BASED LEARNING

Whenever possible, we introduce new information through an "Investigate" section. Here, we provide a student space to construct their own solutions to a problem rather than having them simply mimic the tutor's preferred process or algorithm.

We start by letting a student solve a problem in any way they can think of using whatever tools and knowledge their comfortable using. Then we provide tutors with prompting strategies they can use to help students explore their own knowledge base to come up with a solution to the problem (also, we often later in a lesson include "Judge It" sections where we have students compare and contrast their preferred approach versus a more general, faster algorithm).

For example, take page 2 of the Handout in the lesson "Exploring Equivalent Fraction Form". This is the first time within this strand that we introduce the concept that a fraction can look different but be equal in magnitude.

Here, we're exploring that 3/4 = 6/8. To introduce this concept, **we do not** start by simply telling the student that two fractions can look different but be equal in magnitude, then draw some pictures to show that this is true and further prove it by using division to reduce 6/8 to 3/4.

We have them jump right in by correcting an imaginary answer to a test. Using tutor notes, we tip up our tutors to the intent of the section and provide prompting strategies to solve the problem and make the necessary connections. We know they can use manipulatables and draw pictures to represent fractions, so the most obvious prompting strategy: "if your student is frozen, have your student represent each fraction using pizza cutouts or by drawing pictures".

At Catalyst, we feel this is a more powerful approach. First, it allows a tutor to see the exact stumbling points. It can be quite amazing what misconceptions (or you could say misconnections) a student brings to a problem; a teaching opportunity can be lost if a student is first shown to solve the problem the way the tutor would solve it.

But most important, it greatly increases student responsibility. That is, we don't want students to feel that they don't have to think for themselves because the tutor will always just show them how to do it. We're creating structures that not only teach content but also build up our student's intellectual independence and critical thinking skills. Throughout this process, we support our students through scaffolding and prompting and help redirect unproductive lines of thinking.

DEFINITIONS AND HOW TO: THE USE OF EXPLICIT INSTRUCTION

There are many times, however, where introducing new material through the use of an open ended problem is not appropriate – especially when learning terminology, notation, conventions, and formal algorithms. Here, we directly introduce terminology and notation through our "Definition" sections and mathematical algorithms and procedures through our "How To" sections. For example:

- **Definition**. In Handout #1 of the lesson "Fractions – Pieces of a Whole" we explicitly cover the terminology and notation used when working with fractions. It's quite obvious that students aren't going to discover the word "numerator" by themselves.

- **How To**. In Handout #1 of the lesson "Calculating Equivalent Fraction Form" we have a tutor directly show a student how change a fraction's form using multiplication and division. This is after they've had ample time to explore this concept using other methods. After this exploration, we want our students to see how to perform this algorithm precisely.

PRACTICE / MULTIPLE APPROACHES

After new information has been introduced (whether constructed or taught explicitly), we provide a space so a student can practice a newly introduced concept. We believe that it is critical that new information be introduced in small chunks and that students are given a chance to use this information before moving onto new concepts. If not, you are again at risk of moving on before a student has had a chance to master necessary prerequisite knowledge.

Sometimes we simply have a student directly practice an algorithm or procedure that was just shown to them. For example, on page 2 of *Handout #1: Fractions – Pieces of a Whole* we introduce the concept of a fraction and show a student how to "break up" a shape and talk about the meaning of and label the numerator and the denominator. Then, on the next page, have the student practice this concept six times. Throughout our lessons, we provide students with space to practice computation and build fluency (usually using our "Compute It" or "Solve It" sections).

But we also explore the new concept from a variety of angles and work to understand the new information in a variety of way that stress multiple modes of understanding (e.g. spatial, linguistic, logical, etc.). This is heavily tied with the need to use multiple representations to tackle concepts and problems.

Students need to have a concrete, hands-on understanding of what is being talked about. They need to be able to use available tools they learned to use like rulers, number lines, fraction strips, and pictures to represent and solve problems. They need to be able draw pictures and use common conventions such as number lines and graph paper. And they need to be able to write and speak about mathematical concepts in English. And, of course, they need to be able to use numbers and mathematical symbols.

And, most importantly, they need to see how these different approaches and representations are connected. We strive to get students seeing how all these ideas are related by having them compare and contrast different approaches, to talk about what's good and what's bad about a given strategy.

For example, in "Calculating Equivalent Fraction Form" on page 4 we include a "Judge It" section where we have students show that 1/2 = 2/4, 8/10 = 4/5, and 1/3 = 250/750 by using either pictures or division. We then ask them to say which method they like better and why.

This is a more difficult approach than just showing students a single algorithm or procedure and then reinforcing it through repeated practice. But, we believe the fostering of multiple representations and algorithms provides the deepest and most lasting knowledge. Again from the NCTM yearbook *Making Sense of Fractions, Ratios, and Proportions*:

We must help students make sense of expressions like "3/5" and "x/3 = 8/12" in ways that (1) connect to their ideas and (2) address honestly the mathematics of rational numbers. But to do so, we need to listen for their ideas, which are often quite different from what we are teaching and are sometimes only partially correct [page 4].

Students with the strongest understandings can use many different approaches. To help students connect their constructed knowledge with what we hope to teach them, we should listen for these different approaches and strategies, support them when we hear and understand them, and help all students build a rich repertoire of fraction strategies [page 10].

MULTIPLE MODES OF LEARNING

We believe much of the recent NCTM call for deeper conceptual understanding, the fostering and analysis of multiple approaches, and the use of multiple representations can be viewed as effort to teach children through multiple modes of learning (please see Appendix C for more information).

In total, the NCTM publications; Bloom's Taxonomy of Educational Objectives; and Gardner's Theory of Multiple Intelligences all stress a similar theme: students learn best when given a chance to explore information from multiple vantage points and make connections between these different ways of understanding a problem.

AN INTEGRATED CIRRUCILLUM

As a final note, another way we strive to implement multiple modes of learning is through the use of an integrated curriculum. For example, when working with fractions and decimals, we naturally inline lessons covering measurement, money, and basic probability to help reinforce the recently learned arithmetic.

First, an integrated curriculum is a more efficient way to teach a student important and necessary mathematics. Or using the common metaphor you can "kill two birds with one stone".

But most important, as stressed by the NCTM, an integrated curriculum leads to the deepest level understanding as students are forced to make connections between various areas of mathematics.

Notes and Future

Working towards implementing the NCTM *Standards* and creating integrated materials that get students thinking about mathematical concepts from a variety of vantage points requires a substantial effort; it would be a lot easier to make and grade a large number of "practice" worksheets that simply drill students on basic arithmetic and algebraic manipulation.

While we do require such practice, we want richer materials that lead students to explore concepts they're struggling with in new and deeper ways. We want to get students playing with objects, drawing pictures, and explicitly talking about mathematics *as well as* working with numbers and symbols. Without these materials, we believe most tutors will rush over concepts *they've* already mastered and forget the multitude of experiences they gathered that helped them reach this level of understanding.

Of course, as we've noted earlier, this is a balancing act: we're constantly balancing "concept building" with "fluency practice". Striking this balance runs throughout our material development process. For example, we've noted that we both strive to introduce the "Big Ideas" in mathematics while breaking lessons into small concepts to ensure that students have the prerequisite knowledge needed to succeed. Also, while we stress multiple approaches and multiple representations, we also are careful not to overwhelm students by presenting problems that are too difficult before they've had time to work on simpler problems that reinforce newly introduced terms and concepts.

We do now and always will adjust this balance. Catalyst is a startup organization and we are continually editing our existing material based on field feedback. Further, we are adding new materials that reflect our growing understanding. And lastly, right now our materials our more focused on basic numerical algorithms. We know now that, in the future, we want to create more activity based materials that better contextualize newly learned concepts and we are working to create such materials.

We know this editing will happen and we are designing it into the system: a major technology that Catalyst has developed is material development technology that makes editing and organizing materials extremely easy. And, there are no textbooks to reprint!

But as an organization we owe it to our financial sponsors, our tutors, our community, and our students to be explicit about the system we are designing. That is what this document is all about. This way, as we grow, we can systemically fix the problems and reinforce the strong points.

The fruit of this process is a technology that will lead tutors to be as effective as possible. Like direct instruction systems, the goal is to identify the specific procedures that lead to measurable results and enforce them through program and material structure. That doesn't mean it has to be rigid – you can just as easily reinforce communication and reasoning as rote memorization. But it does mean that it has to be intentional and explicit and that our tutors will follow it. This way, all of students and our tutors using our materials anywhere in the world will benefit from our work and our growing experience on how best to tutor students in a one-on-one environment.

Appendix A: Motivation

The Need for Catalyst's Self-Standing Learning Programs

At Catalyst our mission is to create self-standing learning programs and materials that exist outside of any assigned work given to our students at school. This is a substantially greater effort than simply offering homework help (which we also offer). But we feel it's a very worthy effort for the following reasons:

Foundational Repair. Many of our students are far behind their grade level at school since they have been pushed through a system based on their age and not on their academic capability. For example, working on an algebra problem concerning graphing linear equations is often not productive because your student does not have a strong conceptual understanding of fractions, decimals, or negative numbers. In the long run, it's faster to go back and work on these foundational skills so our students can get the most out of school.

Standardized Tests. A major goal of Catalyst is to help students best perform on standardized tests – both graduation requirement examinations (GREs) as set by the Minnesota Department of Education and college entrance examinations such as the ACT or SAT. A major part of our mission is to help students with these exams so they graduate, get into college, and -- even more importantly -- feel empowered in a society that places such high emphasis on these examinations.

Continuity of Instruction. Homework help is usually an extremely ad hoc process as tutors are constantly struggling to understand what a student has been taught in school. Then by the next session, the students has progressed several lessons and the tutor is forced to jump ahead knowing that previously material was not adequately addressed. By making our own self-standing programs, tutors can feel a sense of empowerment as they manage the material and knowledge they are teaching. Further, they can make sure that what they teach is – even if takes time – is learned. In fact, as we grow and begin to seek more volunteer tutors, we believe that providing this continuity of instruction will make the tutoring process a much more fulfilling and rewarding experience and, therinefore, lead to higher level of volunteer dedication.

Summer Instruction. A major way that Catalyst will serve the community and the schools of our students will be summer instruction. During these times, the students will not have assigned homework that a tutor can use from which to teach.

Non-Traditional Students. The mission of Catalyst is to work with all students – even those who are not currently enrolled in a class. While all of our students currently are attending a regular K-12 school, in the future we envision that many of our students may not be currently enrolled in a mathematics class. For example, they may be studying for a Graduation Equivalency Degree (GED) or enrolled in some other type of non-mathematics class (such as a nursing class) but struggling due to their deficiency in mathematics, reading or writing.

Equal Access. For-profit learning centers offer such self-standing learning programs that serve families who can afford high one-on-one tutoring rates. A very central part of our mission is to ensure that the majority of our students who cannot possibly afford for-profit learning centers have access to the same academic advantages afforded to their more affluent counterparts.

Market Need. We belief that no organization is offering the comprehensive tutoring programs that we are offering. The majority of self-standing tutoring programs are offered by for-profit companies which, in our opinion, are heavy on marketing and low on substance. We feel that most for-profit programs offer very surface level instruction (i.e. simple arithmetic instruction) and do not work on developing the strong conceptual foundations necessary for continued academic performance.

The Motivation for our Materials

Self-standing learning programs require self-standing curriculums – which includes more than just materials but also a plan for pacing material presentation and a plan for assessment.

At Catalyst, we have decided to make our own materials – called printables – which seamlessly integrate into our database which tracks a student's program information. Before making this decision, we have carefully thought about using existing materials and examined several texts for applicability to our programs. We have decided that, despite the major effort it entails, it makes more sense to compose materials specifically designed for our system and programs for the following reasons:

Existing Materials Designed for Classroom. Almost exclusively, the sets of comprehensive materials available for purchase are designed for classrooms use and therefore set up to be "paced" in a classroom environment.

While true of both traditional (direct instruction) and NCTM reform based (constructivist) textbooks, it is especially true of the more modern approaches that often call for many day (or even multiweek) explorations that require extensive group work. Clearly, the existing materials are designed for a teacher who can expect solid daily attendance throughout a school year.

Also, these texts are heavily "grade" oriented and each year-long text contains material specific to that grade. Single subjects, therefore, span multiple textbooks and they are introduced in one year and continually presented in increasingly complex form each increasing year.

At Catalyst, being a supplementary education organization as opposed to a school, we do not work with students by grade level but by "content" level. For example, we'll work with a student who, being older may be in a more advanced mathematics class like algebra but is struggling due to missing foundational work on fractions.

Our materials are organized by concept and not by age level. When working with the student described above, we simply get our "fractions" materials and do not have to manage three or four textbooks that contain all the necessary information.

NCTM Standards. The scant materials that are available – such as those found on EdHelper – consist mainly of worksheets that contain series of arithmetic problems or single algebra equations to be reduced or solve.

Our materials are designed to implement both the content and process NCTM *Principles and Standards*. Our materials don't just focus on computation and symbolic manipulation; we work towards the higher-order cognitive skills of communication, evaluation, reasoning, and proof. That is not to say we don't provide practice in these other areas (we very much do), but not before strong conceptual understandings are in place.

A "Full" Lesson. Being designed for classroom use, available materials are generally just one-part of an experienced teacher's class. Usually, the pacing of the material (syllabus) and an individual lesson plan that teaches what is in the book are usually not, in practice, provided within the materials set.

At Catalyst, our materials contain all the elements a tutor should follow to ensure a successful lesson. This includes such elements as objectives, standards, into and review (anticipatory set), and closure.

In addition, along the way, we also provide tutor with scaffolds or "prompting" strategies so tutors have access to tried and true methods to get their students to think – an especially important point when working with NCTM standards inspired problems.

Students Access. With very few exceptions, the bulk of available materials are owned by for-profit corporations (even those developed by universities are non-profits are still, at the publishing level, owned by companies). Therefore, we would have to pay for all textbooks and ancillary materials such as workbooks. Having surveyed this cost, we felt this would mean our materials would have to stay with Catalyst and not the student.

Our materials are designed to be printed on black and white three-hole punched paper. This way, students take home material we worked on to review when completing homework assignments.

Database Integration. Our materials are stored in a portable document format (PDF). This way, a tutor can use our online database to access and manage documents.

Our tutors work with many different students covering many different subjects at once. One of the consequences of one-on-one tutoring is greatly increased book keeping since every student has his or her own learning program and is studying different materials. By integrating our curriculum with our online database, we greatly reduce the amount of time tutors spend bookkeeping and increase their time spent face-to-face with a student teaching.

Appendix B: Major Theories in Education

Direct Instruction, Constructivism and Cognitive Strategies

The landscape of educational and cognitive development theories is a complex one, to say the least. Over the years, different terminology has been used to describe the same concept. Further, certain ideas are grouped together even though they are conceptually not equivalent.

It is with this disclaimer that we will define the major concepts that influence our approach. Further, we outline what we believe are the most pressing aspects of current debates in educational and pedagogy design that affect the design of our learning programs. We hasten to oversimplify, as the lines are not as clear cut as any discussion of them will make them sound.

In our opinion, the most pressing issue for Catalyst in contemporary education theory -- especially within mathematics education theory -- is the debate between explicit, direct instruction versus problem-based, constructivist learning[1]. This debate is well-known in the education community. In reading, it centered on phonics based approaches versus whole reading. In mathematics, it has been the nexus of the debate that arose when the National Council of Teachers of Mathematics (NCTM) released their first *Standards* document in 1989 and started the mathematics education reform movement.

A most pressing issue in this debate to Catalyst is the use of teacher input and demonstration (often referred to as *modeling* in educational theory). That is, should the tutor show the student how they would solve the problem (just show the student their preferred algorithm) or should a student try and assimilate his or her knowledge to arrive at the answer with the tutor acting more as a guide and facilitator along the way? But it also affects other aspects as well, including material organization and assessment.

[1] Often, each of these "camps" goes by different names. Sometimes direct instruction is referred to as expert/novice instructions and constructivist instruction is referred to as inquiry, discovery, or exploratory based learning (in spirit, constructivist learning is also extremely closely aligned with Montessori based approaches).

DIRECT (EXPLICIT) INSTRUCTION

Due to its relatively long history, direct instruction has multiple interpretations. Further, being a highly political issue, people at times have wanted to be associated with or to be distanced from the term depending on the educational circumstances of the day.

While trying not to over simplify, we believe that there are three increasing formal definitions of direct instruction that serve as useful concepts to further this discussion.

For starters, the defining feature of direct instruction is the use of *direct teacher input and modeling* – that is, the process where a teacher (the expert) first explains the concept to the student (the novice) and then demonstrates the process that the teacher uses to solve the problem. Then, the student attempts to solve the problem in the same way that the teacher showed them with continually less assistance (modeling) by the teacher.

Generally, direct instruction is often what most people think of when they think of teaching; that is, it seems to remind them of how they were taught when they went to school or is generally the strategy they take when explaining a new concept to somebody else. But more importantly to Catalyst, it's often the mode of teaching used by tutors who working in an ad hoc manner without specific materials and training (e.g. homework help).

However, there is a more theoretically formal stain of direct instruction that often is tied (both in praise and criticism) to work of behaviorists who set out to make the learning process a well-controlled and exact science.

The defining structure found in most direct instruction systems is well illustrated by 1) the "Mastery Learning" approach as devised by Benjamin Bloom and the 2) the "Seven Step Lesson Plan" as devised by Madeline Hunter.

First, let's look at "Mastery Learning" as defined by Bloom:

1. Major objectives representing the purposes of the course or unit define mastery of the subject.

Definitions of "direct instruction"

1. Any teaching method where a teacher explicitly gives information/instruction or demonstrates (models) a process first, followed by student practice. At times, this is often referred to as "traditional" instruction.

2. A more formal method of dividing curriculum into logical, finite sections and creating a structured plan for both explicitly teaching and explicitly assessing each section as outlined by Bloom and Hunter.

3. A highly controlled and carefully paced method of teaching where each lesson plan is formally scripted and teachers are given formal instructions on how to teach/assess every section of a lesson. Typically this is referred to as Direct Instruction (emphasis on capitalization) and associated with curriculums such as DISTAR (**D**irect **I**nstruction **S**ystem for **T**eaching **A**rithmetic and **R**eading), Connecting Math Concepts, Word Problems Made Easy, and the work of Siegfried Engelmann.

2. The substance is divided into relatively small learning units, each with their own objectives and assessment.

3. Learning materials and instructional strategies are identified; teaching, modeling, practice, formative evaluation, retouching, and reinforcement, etc., and summative evaluation are included.

4. Each unit is preceded by brief diagnostic tests.

5. The results of diagnostic tests are used to provide supplementary instruction to help student(s) overcome problems.

Note: Time to learn must be adjusted to fit aptitude. NO STUDENT IS TO PROCEED TO NEW MATERIAL UNTIL BASIC PREREQUISITE MATERIAL IS MASTERED.

-- Notes from Benjamin Bloom Lecture at ACSA, April, 1987.

In a similar vein, Madeline Hunter formally defined what elements should be in any given lesson plan. Generally, this is referred to the Hunter Seven Step lesson plan and consists of:

1. Objectives (what will the student learn).

2. *Standards (statement of what methods/questions will be used to assess mastery).*

3. *Anticipatory set and review (the into or hook that grabs a student's attention and review of necessary prior knowledge).*

4. *Teaching: input (telling) and modeling (showing) of the process, concept, algorithm, etc.*

5. *Guided practice / check for understanding.*

6. *Closure (review of what was taught; having students explain back to teacher what was taught).*

7. *Independent practice (reinforcement).*

-- For more information see Joyce and Weil, Models of Teaching: Mastery Learning and Direct Instruction, 3rd Edition

This, of course, is followed up by assessment of retention (which was outlined in the Standards).

The major characteristics to note are:

- The use of objectives and standards which, generally, are defined in a behavioral manner. For example, a common direct instruction objective might read as "the student will demonstrate mastery of the concept of adding single digit numbers by solving 9 out of 10 addition problems with no errors for three consecutive days".

- The clear emphasis on modeling and demonstration of the concept by the teacher first followed by emphasis on practice until "mastery" is reached.

- "Mastery Learning" predisposes that material can be broken into a logical series of "steps" that can be arranged in a linear order so if a student is having problems with a certain lesson, one can always "back-up" and reteach lower-order skills first to ensure mastery.

- But of greatest importance, in our opinion, is somewhat subtle in its consequences: the emphasis on modeling and then repeated practice assumes that you have enough problems to demonstrate and model. At times, that leads one to create lessons that cover fairly well-defined skills and concepts – like reducing a fraction to lowest form – because then you can easily create the many problems necessary for modeling, guided practice, independent practice and assessment. For example, in mathematics this often translates to "first show a student how do it, then just change the numbers a little and let the student try".

The influence of Bloom and Hunter are great: the seven step lesson plan became ubiquitous in late twentieth century American schools as the way to organize a successful material presentation. Yet, the degree (critics would say the "severity") to which direct instruction was implemented varied from school to school, from textbook to textbook.

Taken to its logical extreme, a strand of instruction dubbed Direct Instruction (emphasis on the capitalization) emerged in the late sixties (its birth sometimes traced to section of Project Follow Through – the most funded government educational experiment in American history). Led by Siegfried "Ziggy" Endlemann, Direct Instruction is characterized by highly structured, critics might say rigid, learning environments where teachers are given scripts that guide them step by step through a lesson.

To illustrate the distinction, take the following passage on teaching fractions from *Designing Effective Mathematics Instruction: A Direct Instruction Approach*, a book that teaches how to create Direct Instruction math curriculums:

During the first days, all examples should include circles divided into parts. After several weeks, other shapes (e.g. squares, rectangles, triangles) can be included in exercises . . . Examples that yield 1 as a denominator should not be included when fractions are initially presented, but can be introduced about a week after initial instruction. Thereafter, about 1 in every 10 diagrams should be an example with 1 as a denominator. These examples are important, since they present a conceptual basis for exercises in which students convert a whole number to a fraction by division (e.g. 8 = 8/1).

-- From Designing Effective Mathematics Instruction: A Direct Instruction Approach. 4th Edition. Page 252.

While it is this level of rigidness/scriptedness that forms a common target of critics, it's important to point out that behind the methods of Bloom, Hunter, and Direct Instruction is a simple yet powerful concept: *every* student can learn if given access to the prerequisite knowledge and it is made sure that they do, in fact, learn that knowledge that was presented.

In this view, the reason so many students fall behind is because – for whatever reasons – they miss the background knowledge they need to understand the lesson being presented today. And, in a spiral effect, they continue to slip further and further behind. To quote Bloom, "There is a difference between "80% of students will master the material" and "each student will master at least 80% of the material"." In its raw simplicity and when removed from its implementation, there are few that could argue with this fundamental premise.

CONSTRUCTIVISM

Like direct instruction, constructivism resists simple definition and categorization; especially in light of how some interpret this method to mean that each teacher/class should create their own methods of teaching and curriculums (sometimes also called "authentic curriculums").

Further, there are many strands and variations often going by names such as inquiry, discovery, exploratory, minimally-led, problem-based learning, or implicit instruction.

There are, however, three concepts worthy of highlight which are generally common to all constructivist approaches. They are the ideas that:

1. People do not passively receive new knowledge but actively construct their own knowledge.

2. People do not learn by accumulating a series of small, discrete pieces of information (i.e. "steps"). Rather, it is when they have a "light-bulb" moment and see the entire picture or system as a whole that the most useful and lasting learning takes place.

3. People learn best by doing – that is, by applying their knowledge in real-world, concrete, hands-on situations as opposed to simply remembering detached, abstract theories and ideas.

Major Constructivism Concepts

Piaget: The Construction of New Knowledge. Jean Piaget posited that a student does not passively *receive* new knowledge but must *construct* new knowledge by making connections with prior knowledge and then *personally transforming* (synthesizing) new knowledge. Further, a student only performs this process when they bump into a new problem they cannot solve with their current knowledge. That is, they are out of *equilibrium* and they construct new knowledge *in order* to get back to a steady-state of understanding.

Gestalt: "Light Bulb" Moments and "Big Ideas". The German Gestalt theorists argued that learning happens when people see "the whole" or "the big picture" as opposed to the behaviorist idea that people learn by completing a series of small, discrete -- and, they argue, isolated -- steps. This is often associated with the "Aha" moments in learning or the "light bulb" moments. That is, those times the whole picture (or whole system) is understood. Also very related is the notion of "Big Concepts" or "Big Ideas" being the most important – and hence they should be taught *first*, before the details of implementation.

Dewey: Concrete, Hands-On Learning. John Dewey – one of America's first philosophers and educational theorists -- is commonly associated with the idea of project based or activity based learning. Dewey was a strong believer that students didn't learn as much by passively sitting in a classroom listening to a teacher give instructions (the knowledge gained in these environments he called "static, cold-storage knowledge"). Rather, he advocated that children learn by doing, by "bumping" into the real-world problems and "struggling" with them until they find a solution. It is this struggle that leads to the highest form of learning, understanding, and retention -- or, in his own words, "information severed from thoughtful action is dead, a mind-crushing load".

For more information see Phillips and Soltis, *Perspectives on Learning, 4th Edition*

Theoretically, the major difference between explicit, direct instruction and implicit, constructivist based instruction again centers on teacher input and demonstration (modeling).

In constructivist based approaches, the teacher does not stand at the front of the class and give the students information, show them how they complete the problem, and then guide them through practice. Rather, students jump into a problem "head-first" and tackle it without being shown how to solve it.

Generally, constructivists urge that the teacher act as a "facilitator" rather than as the traditional "fountain of knowledge". In this mode, they should minimally prompt students to explore their own knowledge base to find a solution to the problem.

Consequences of this approach are:

- The acceptance and fostering of several methods or algorithms to solve a problem rather than simply "showing" a student the "best" or "fastest" method or algorithm. For example, even when working basic fraction problems there are usually several ways to solve a given problem.

- That methods and strategies for solving a problem should be "caught not taught". That is, a method for solving a problem that a student realizes on his or her own is far more useful and lasting than a strategy simply handed down by a teacher.

- A higher level of emphasis on having students justify their methods and explaining how they solved a problem.

In practice, constructivist approaches have transcended how to teach any one given idea or algorithm. Due to it's foundation in favoring "Big Ideas" and activity based learning, constructivist classrooms and curriculums are often characterized by:

- Students jumping head first into a problem *without being told what's going to be taught*. Then, they work to understand what's being asked, think of what skills they need to solve it, and then begin to work towards a solution.

- Longer, more involved problems that may take days or, in the extreme, even weeks to complete (for example, the Integrated Math Project Curriculum).

- Group work, collaboration, and communication over individual seat work.

- Problems that incorporate many skills at once as opposed to working on singular ideas in isolation. This is true of both individual lessons and entire K-12 curriculums; constructivists are often critical of the "layer-cake" style of traditional or direct instruction approaches that more systemically divide and present content (i.e. arithmetic, pre-algebra, algebra, geometry, statistics, etc.)

When using a constructivist approach, there is often a subtle difficulty that arises that is quite opposite to the problem that arises when using direct instruction. Constructivist problems, due to their more complicated and interrelated nature, avoid duplication. So if a student is struggling and your only recourse is that you must show the student how to solve a problem, it can be difficult to recreate a "similar" problem where they can then try it on their own without assistance. Further, this puts greater strain on any practice and assessment material development because, by definition, the problems resist this type of characterization. Put more simply, what if a student doesn't "catch" the concept the first time around, what do you do then?

COGNITIVE STRATEGIES: BARAK ROSHENSHINE

An often hurled criticism of direct instruction is that it breaks under the weight of its own rigidness and structure. Perhaps, critics argue, for something as simple as adding single digit numbers or learning how to focus a microscope one can break the task into a series of small, easily followed steps that be mastered. But quickly, students need to learn "skills" that are not so easy to define and systemize. An obvious example is forming a persuasive argument. But one can argue that such examples abound even in "basic math" curriculums (such as the processes one uses to solve a ratio/proportion problem).

This is the major idea found in the work of Barak Roshenshine. For such problem, he posits, teachers need to develop and foster their students cognitive strategies. In his own words:

Cognitive strategies are heuristics. A cognitive strategy is not a direct procedure; it is not an algorithm to be precisely followed. Rather, a cognitive strategy is a heuristic or guide that serves to support or facilitate the learner as she or he develops internal procedures that enable them to perform the higher level operations. Teaching students to generate questions about their reading is an example of a cognitive strategy. Generating questions does not directly lead, in a step-by-step manner, to comprehension. Rather, in the process of generating questions, students need to search the text and combine information, and these processes serve to help students comprehend what they read.

-- J.W. Lloyd, E.J. Kameanui, and D. Chard (Eds.) Issues in educating students with disabilities. Mahwah, N.J.: Lawrence Erlbaum (1997). Chapter 10. Pages 197-221.

An important concept from Roshenshine's work is that of the *scaffold*. A scaffold, much like it sounds, is a temporary support structure a teacher places around a student when they are learning that, gradually, is reduced over time as they gain confidence and independence.

Major types of scaffolding include:

1. Procedural prompts or facilitators (that is, careful hints that don't give the answer away).

2. Connections to past knowledge and talking about how the problem is similar or different to a more familiar problem.

3. Diagrams, figures, or cue cards that a student can look at when solving a problem.

4. Thinking aloud with a student as choices are being made.

5. Anticipate and discuss potential difficulties and regulate the difficulty of the material. This can include creating a simpler example (perhaps on the fly) first, before a student jumps into the more difficult problem.

As can be seen, Roshenshine stresses the importance of "well-connected" knowledge structures as vital to higher understanding and retention.

So is Roshenshine, with his method of Cognitive Strategies and his importance of "well-connected" learning structures, more of a direct instructionist or more of a constructivist in his approach?

In a certain light, cognitive strategies seem to be tools that a constructivist teacher could use when acting as a "facilitator" during problem exploration. Further, there is heavy emphasis on connections to past knowledge and seeing any piece of information as part of a "web" of concepts (a focus on the "whole" as opposed to the "piece"). And certainly, there is acceptance that many skills and ideas are too complicated to be broken down into a systematic approach as characterized by Direct Instruction.

However, Roshenshine and texts incorporating cognitive strategy development generally are more similar to the direct instruction of Bloom and Hunter. There is an emphasis 1) on *explicit teacher-led* development of strategies; 2) of presenting material in small, manageable steps; 3) and of moving from guided practice with prompting to independent practice to name just a few similarities.

In fact, in his 1997 paper "The Case for Explicit, Teacher-led, Cognitive Strategy Instruction" he writes of constructivist approaches that call for minimally led instruction:

This focus on discovery learning is reminiscent of William Heard Kilpatrick's project method of 1918, of the discovery learning of the 1950"s and of the open classrooms of the 1970"s. This shows how ardently we practice recycling in education. But although I place glass bottles and newspaper on the curb each Monday, I am reluctant to discard our tremendous accomplishments in cognitive strategy instruction. Results still count, and cognitive strategy research has produced results and a technology for future research and application.

To those who would discard teacher-led cognitive strategy instruction for discovery learning, I have a simple quote from a recent movie, modified slightly to fit education:

"Show me the data!" "Show -- me -- the data!"

-- Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL. March 24-28, 1997.

Roshenshine and other proponents of cognitive strategy development believe the strategies *should be taught* and not left to the risky chance that a student will catch them on their own.

Appendix C: Mathematics Education Reform

The NCTM Standards Based Reform and "The Math Wars"

In 1989, the National Council of Teachers of Mathematics (NCTM) released the first edition of the *Principles and Standards for School Mathematics*. With it, they also released a substantial and at times heated public debate concerning how to fix America's perceived lagging performance in mathematics. The NCTM *Principles and Standards* – which were recently updated and released in 2000 – call for a major revision to how mathematics is taught. NCTM and its proponents labeled their efforts the "math reform movement".

But due to the ferocity of the response of many critics many, including those within the media, dubbed it the "The Math Wars". "Other than the war in Iraq, I don't think there's anything more controversial to bring up than math," says Barry Graff, a top Alpine School administrator in the Alpine School District in Utah. "The debate will drive us eventually to be in the right place[2]."

Usually, this debate is characterized as a fight over long and involved real-world problems using calculators versus "back-to-basics" computation skills. But the debate of importance to Catalyst is that, in practice, the NCTM Standards Based reform based materials and classrooms place heavy emphasis on constructivist methods and a strong de-emphasis on explicit and direct instruction.

This debate has been made even more complicated by the September 2006 release of the NCTM *Curriculum Focal Points*; we address this document in more detail in the next section. Some have characterized the Focal Points as a reversal of early stances; others argue it is a further clarification of the original Standards released to prevent misinterpretation. We agree with the latter.

The Focal Points address content and not the core issue on how best to teach mathematics. We feel the core of the process Standards -- which focus on conceptual understanding over mindless application of formulas and algorithms -- remain very much at the core of the NCTM reform movement.

The *Principles and Standards* outline five content standards (what should be taught) and five process standards (how the content should be taught) and outline each set of standards within four K-12 grade bands: K-2, 3-5, 6-8, and 9-12. The five content standards cover: *Number and Operations* (includes arithmetic); *Algebra*; *Geometry*; *Measurement*; and *Data analysis and Probability*.

The five process standards outline that, within each content standards, students should be developing ability in: *Problem Solving*; *Reasoning and Proof*; *Connections* (understanding how mathematical ideas are related); *Communication* (being able to explain and justify their work); and *Representation* (expressing mathematical ideas in a variety of formats such as graphically and symbolically). For more information, visit www.nctm.org/standards/.

A major argument put forth by the NCTM is that, traditionally, far too much emphasis is placed on teaching isolated computational concepts and algorithms (such as long division, reducing fractions to lowest form, or finding the roots of a polynomial) and having students practice these algorithms until they can complete them independently, from memory, or "in their sleep" (this method of instruction is sometimes critically referred to as "skill and drill"). Or, in their own words:

Many adults are quick to admit that they are unable to remember much of the mathematics they learned in school. In their schooling, mathematics was presented primarily as a collection of facts and skills to be memorized. The fact that a student was able to provide correct answers shortly after studying a topic was generally taken as evidence of

[2] New Report Urges Return to Basics In Teaching Math, Wall Street Journal, September 12, 2006; Page A1

understanding. Students" ability to provide correct answers is not always an indicator of a high level of conceptual understanding (Standards 2000 Discussion Draft, p. 33).

At Catalyst, we are strongly influenced by the NCTM *Principles and Standards*, particularily, the process standards – which emphasize how one should teach. In our own words, we want to talk about two ideas contained in the *Principles and Standards* process standards which have greatly influenced our approach:

The Emphasis on "Big Ideas" and Conceptual Understanding. The NCTM calls for students to understand *first* the foundational concepts which are being explored. Further, conceptual understanding (e.g. "what is it, exactly, that I'm doing and why") is more important than computational or algorithmic speed or fluency. Not that both aren't important; it's just that conceptual understanding is *more* important and, further, conceptual understanding should always come *before* fluency practice.

Just to note, this is quite different to the approach of typical direct instruction approaches, which tend to progress in the reverse: start with tiny little pieces, practice all those pieces first and, through mastery of these small pieces, deeper conceptual understanding can then take place.

In our opinion, the core to this idea is simple: foundational conceptually knowledge is lasting and it is generative; that is, it makes learning increasingly easier and easier. The practice of route memorization may be the faster way to get through tomorrow's test. But for every year, for every subject and for every test, you have to keeping cramming more formulas and procedures into short term memory. You don't have the foundational knowledge to say "oh, this isn't new information. It's just like the concepts I learned before, only a little different".

The Emphasis on Making Connections and Multiple Modes of Learning. In order to build conceptual knowledge, you need to make connections with other knowledge you have learned in the past; to form a map in you head of what this new concept "is" in relation to other concepts you have already mastered.

A student that understands that, at core, Algebra is generalized arithmetic; that understands that variable notation is simply a convention to represent the idea "take any number, any number in the world"; and that can draw back on his or her connections to arithmetic and see Algebra as an extension of arithmetic will do far better in the long run than a student who simply learns the formal mechanisms for plotting linear equations.

Of course, in order to best be in a position to "see" how things are connected, you need to tackle problems in multiple ways and from multiple vantage points. Route calculation is not enough: you need to be talking about math and reasoning about ideas; you need to understand multiple ways to represent mathematically ideas; and you need to be making connections about how these ways to talk about mathematics are similar and how they are different.

To note, this process is achieved in a variety of ways. Sometimes it's well designed problems which tease out these connections; other times it's the tutor explicitly talking about the connections between, say, arithmetic and Algebra and explicit problems asking student to write about the connections. Also, this process doesn't happen all at once – of course a student can't understand that Algebra is generalized arithmetic until he or she has had some time and practice to play with Algebraic notation and formulas. Rather, it's a continual process throughout the study.

What's important, we believe, is the NCTM is stressing that, traditionally, no emphasis was placed on conceptual understanding. The few students who made these connections excelled; the rest gave up on a subject that, at core, made no sense to them.

NSF FUNDED, NCTM STANDARDS BASED CURRICULUMS

The *Standards*, in and of themselves, are not a curriculum – an intentional objective. And while the *Standards* are littered with examples, an exact curriculum that details materials, pacing, and assessment is a matter of interpretation.

In practice, throughout the 1990s – and with heavy backing from the National Science Foundation – several curriculums appeared that were "NCTM Standards based". For more information visit http://www.nsf.gov/pubs/2002/nsf02084/.

Without worry of sounding controversial the:

- Other publications by the NCTM such as the Ihttp://illuminations.nctm.org/ website and the many NCTM yearbooks
- And very much the NSF backed, Standards-Based curriculums

are highly constructivist, discovery learning based approaches to math education. (The degree to which the *Principles and Standards* themselves enforce constructivism is a matter of interpretation; however, they too are often highly constructivist in spirit).

In fact, at root, the math reform movement can be seen as the larger debate between direct instruction ("skill and drill") versus constructivist based learning. In fact, a very similar debate exists in reading in the controversy surrounding phonics based reading (direct instruction) versus whole reading (constructivist).

And while the Standards do not, explicitly, recommend exact teaching styles, in practice, the NCTM publications and Standards based curriculums clearly deemphasize the (direct instruction) methods of: breaking a subject into small, discrete pieces; behavioral objections and the use of direct teacher input and modeling; and the use guided and independent practice of the explicitly presented concept.

For example, take a problem from the Illuminations website aimed at teaching children the concept of number magnitude (shown at right). Clearly, pulling a single example will color the picture we're creating; for a complete accurate picture, please browse one of the listed reform-based curriculums website or texts.

NSF Funded, Standards-Based Curriculums

Elementary School

- Everyday Mathematics
  http://everydaymath.uchicago.edu/
- Investigations in Number, Data and Space
  http://investigations.terc.edu
- Math Trailblazers
  http://www.mathtrailblazers.com/

Middle School

- Connected Mathematics
  http://connectedmath.msu.edu/
- Middle School Math Through Application (MMAP)
  http://mmap.wested.org

High School

- Contemporary Mathematics in Context
  http://www.wmich.edu/cpmp/
- Interactive Mathematics Program
  http://www.mathimp.org/
- MATH Connections
  http://www.mathconnections.org/
- SIMMS Integrated Mathematics
  http://www.montana.edu/~wwwsimms

For more information visit:
http://www.nsf.gov/pubs/2002/nsf02084/chap1_4.htm
or *www.edc.org/mcc*.

But our goal is not to distort in a pro or anti reform based manner. Rather, we're trying to give an accurate depiction of the reform movement and how it contrasts to direct instruction. And this problem is, in our opinion, very representative of the overall spirit and approach of Standards based curriculums (for another example, read Chapter 1 of the NSF *Foundations* document).

Even a cursory comparison between:

- This problem and other NSF funded NCTM Standards based texts and
- Direct instruction texts such as DISTAR and those advocated by *Designing Effective Mathematics Instruction: A Direct Instruction Approach*

points to the vastness in difference of approach.

LOCAL AND NATIONAL CRITICISM

Such drastic change was not meet without its critics. In California – after adopting reform texts -- there was large protest by mathematicians (such as Jim Milgram of Stanford University) and parents against reform math texts (such as Everyday Mathematics). In response, California has generally dropped reform texts from their curriculum standards.

At the national level in 1999, the U.S. Department of Education released a report strongly recommending reform texts over traditional ones. In response Milgram and more than 200 mathematicians and scientists, including four Nobel laureates, took out a full-page ad in the *Washington Post* asking the federal agency to withdraw their endorsement.

Of equal interest has been the rise of "grass-root" protest groups, usually composed of concerned parents and mathematicians such as:

- http://www.wheresthemath.com/,
- http://www.nychold.com/

Sample NCTM Based Learning Activity (Grade 5-8)

Learning Objectives -- students will:

- *Develop intuition about number relationships*
- *Estimate computational results*
- *Develop skills in using appropriate technology*

Begin the investigation by telling the following story:

Just as you decide to go to bed one night, the phone rings and a friend offers you a chance to be a millionaire. He tells you he won $2 million in a contest. The money was sent to him in two suitcases, each containing $1 million in one-dollar bills. He will give you one suitcase of money if your mom or dad will drive him to the airport to pick it up. Could your friend be telling you the truth? Can he make you a millionaire?

Involve students in formulating and exploring questions to investigate the truth of this claim. For example:

Can $1,000,000 in one-dollar bills fit in a standard-sized suitcase? If not, what is the smallest denomination of bills you could use to fit the money in a suitcase?

Could you lift the suitcase if it contained $1,000,000 in one-dollar bills? Estimate its weight.

Calculators should be available to facilitate and expedite the computation for analysis.

Note: The dimensions of a one-dollar bill are approximately 6 inches by 2.5 inches. Twenty one-dollar bills weigh approximately 0.7 ounces.

You may wish for students to locate these facts about dollar bills on their own, using internet or other appropriate resources. The students will also need to determine the dimensions of a "standard" suitcase.

-- http://illuminations.nctm.org/LessonDetail.aspx?id=L252

"The Math Wars" in Minnesota

Several districts in Metro area have been adopting reform based mathematics– including Minneapolis Public Schools, Eden Prairie Public Schools, Prior Lake Public Schools among others (some districts have allowed parents and students to choose between "traditional" versus "reform" based classrooms.

And while so many experience educators and districts have chosen the reform math is a tacit endorsement of the effectiveness of such programs, these decisions have been meet by strong criticism by both parents and teachers alike. While creating a complete picture is beyond the scope of this document, here's a few selected Star Tribune article that illustrate what the debate looks like at the local level:

- On March 2, 2003 the Star Tribune published "New math divides districts: Some parents, teachers want to return to basics" that interviewed both parents and teachers unhappy with the reform texts. The article outlines how many parents – perplexed and highly suspicious of very unfamiliar textbooks and homework problems – have been highly active in getting their kids out of reform classrooms:

  When parents don't understand the math their kids bring home, they lose confidence in it, said Minneapolis math coordinator Anne Bartel. ``We get parents who are engineers and see [integrated math] and say, `Get my kid out of this'", she said. "We also get parents who didn't do well in math saying, `Get my kid back into something I at least recognize.'"

  But some parents and teachers believe that these programs, while more difficult to implement, need to be fostered:

  Stillwater parent Ann Malwitz says integrated math has done wonders for her second- and fourth-grade sons, who, she said, are getting the basics - and a whole lot more.

  ``Boy, what I see it doing for their confidence. . . . And it makes sense to them and they see how it's important in their lives,"" she said.

- In 2003, some 1200 parents organized as the Parents for Better Math presented Eden Prairie's school district with a petition asking for a switch to a "proven" math curriculum with "a traditional learning approach." The parents claim the reform math programs lead to lower test scores.

  -- "Eden Prairie school board hears parents" views on reform math curriculum". Star Tribune. January 14th, 2003.

- In an April 26, 2006 article entitled "Schools demystifying math", the Star Tribune reported on the efforts in Burnsville to use $448,000 from the U.S. Department of Education to better train teachers to most successfully implement the new reformed-based programs. And while a survey of teachers in the district highlighted that many teachers have strong concerns, many also believe that, once the bumps are ironed out, the end results are better and more lasting:

  "This program is showing that kids are more capable [with math] than what we once thought," [6th grade teacher Jennifier] Walls said. "Kids are going to do better in math if we stick with it."

- Recently, Minnesota Public Radio's Midmorning aired *Back to Basics* covering the implications of the recently released NCTM Curriculum Focal Points.

- In Eden Prairie a similar group of 1200 parents formed under the banner of "Parents for Better Math" urged the district to drop reform texts.

- And -- the most visible of the bunch – Mathematically Correct.

The homepage of Mathematically Correct is a good summary of the many of these groups core argument against the reform movement:

The advocates of the new, fuzzy math have practiced their rhetoric well. They speak of higher-order thinking, conceptual understanding and solving problems, but they neglect the systematic mastery of the fundamental building blocks necessary for success in any of these areas. Their focus is on things like calculators, blocks, guesswork, and group activities and they shun things like algorithms and repeated practice. The new programs are shy on fundamentals and they also lack the mathematical depth and rigor that promotes greater achievement.

In our opinion, a majority of the criticism has not centered on the actual goals of the NCTM but the implementation. For example: A cornerstone of direct instruction is the use of clearly written, demonstrable objectives and standards so teachers and parents know when a student is struggling or not. Critics of reform programs argue that constructivist approaches have "mushy" objectives (i.e. "students will develop intuition about number relationships") that are far too subjective to provide meaningful guidance. They argue that the reform programs are not clear on what needs to be taught and how one determines whether or not learning actually took place. Further, again due to the complex, interrelated nature of the problems, there's no path for students to practice learned concepts until they have, using direct instruction terminology, reached "mastery".

Another common criticism focuses on the insistence of having students devise their own strategies and the fostering of multiple methods/algorithms to solve a problem overwhelms and confuses students. In the end, they argue, students don't even know what they are being taught. Rather, critics argues, students should be taught how to perform individual math skills and algorithms (and apply them in simple story problems) *first* before jumping into more complicated real-world, open ended problems and exercises where they are asked to evaluate, compare and contrast competing methods. In the end, some critics argue, the NCTM reform texts are just too difficult.

To leave this highly complex debate that is being played out at both the national and local levels, we wish to highlight a question very important to us. Is the NCTM reform movement's critique of traditional, direct instruction "skill and drill" approaches a:

- Fundamental criticism of direct instruction versus constructivism in general or is it a
- Or a criticism that, in practice, direct instruction approaches stop at such a shallow level of learning and understanding?

Using Bloom's Taxonomy of Educational Objectives as a guide, can you not in a direct instruction approach work "higher" order levels of thinking by explicitly leading students to communicate, evaluate, and analyze mathematical ideas? Is there no way to bring explicitly teacher led direct instruction beyond "skill and drill"?

The NCTM Curriculum Focal Points

In September of 2006, the NCTM released their Curriculum Focal Points. At times, this has been characterized as a full reversal of the philosophy outlined in the original NCTM *Principles and Standards*. For example, see *New Report Urges Return to Basics In Teaching Math*, Wall Street Journal, September 12, 2006; Page A1.

A story from EdWeek posted on the NCTM website presents a more balanced view using more neutral terminology:

More than 15 years after its publication of influential national standards in mathematics, a leading professional organization has unveiled new, more focused guidelines that describe the crucial skills and content students should master in that subject in elementary and middle school.

The National Council of Teachers of Mathematics today released "Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics," a document that supporters hope will encourage the polyglot factions of state and local school officials, textbook publishers, and teachers to set clearer, more common goals for math learning.

While the report is being published by the NCTM, it was reviewed by numerous math experts from across the country, some of whom have strongly disagreed with the organization's past positions on essential skills. The new NCTM document reflects an attempt to overcome those conflicts and focus on a number of crucial, agreed-upon concepts.

"I would hope that this has a large impact, because I believe it gets it right," said R. James Milgram, a Stanford University mathematics professor and a critic of the NCTM's previously issued national standards. He was one of 14 individuals who provided an outside, formal review of the document. "I would like to hope that this represents a new era of cooperation," he added. "I hope that what this represents is an end to the math wars."

The NCTM's publication of voluntary national standards in 1989 served as a guidebook for many states' drafting of academic standards in math—or expectations for what students should know in that subject. Yet the content of those state standards, which typically serve as blueprints for state tests, varies enormously from state to state. Some of the documents are packed with nearly 100 expectations per grade level, NCTM officials say. Supporters of the Focal Points report can help state and local officials pare down those goals.

"States and school districts are looking for guidance," NCTM President Francis M. "Skip" Fennell said in an interview. The message behind the Focal Points is that "this is the blueprint," he said. "We start here."

The Focal Points are not meant to replace the previous NCTM standards, Mr. Fennell said, but rather serve as a next step offering more focused help in the earlier grades. The new document, in fact, includes links showing where the previous standards are covered in the Focal Points.

http://www.nctm.org/news/ext_articles/2006_0912_edweek.htm

Many in the NCTM have claimed that the *Focal Points* build on the *Principles and Standards* and do not replace them. Further, the Focal Points are content related and not process (i.e. how to teach) standards. Therefore, the same emphasis on 'Big Ideas' and foundational concepts over route memorization; on communication, reasoning and problem solving; on connections between mathematical ideas and the use of multiple mathematical representations still apply. But they are at least a formal admittance that something was getting lost in the interpretation of the *Principles and Standards* and that further clarity was needed.

Judging the affect this document will have on the debate over math education is difficult to determine, especially since so much of the controversy surrounding the *Principles and Standard* centered more on the actual NSF funded curriculums (which are interpretations of the *Principles and Standards)* than the actual *Principles and Standards* themselves. Based on the Focal Points, there most likely will be a returned emphasis on basic computation and memorization of basic facts. But more importantly, the focal points will probably lead to curriculums which cover fewer core concepts but do so in much greater depth.

For Catalyst, this document is a great help. We began the development of our library with the material necessary to pass the Minnesota Basic Skill Test (MBST) mathematics test so much of our curriculum was already inline with the *Focal Points*. However, as we revise and add new material to our library, this document serves as an excellent guide to focus our attention. We hope that in the near future the NCTM elaborates on how to use the process standards (especially if they are changes) to implement the *Focal Points*.

Appendix D: Multiple Modes of Learning

Bloom's Taxonomy and Gardner's Multiple Intelligences

Benjamin's Bloom Taxonomy of Educational Objectives and Howard Gardner's Theory of Multiple Intelligences are used heavily in the development of our programs and materials. While being very different theories, at core is a common thesis important to Catalyst: students learn best when you excite more than a single mode of learning and present information to a student in a variety of ways.

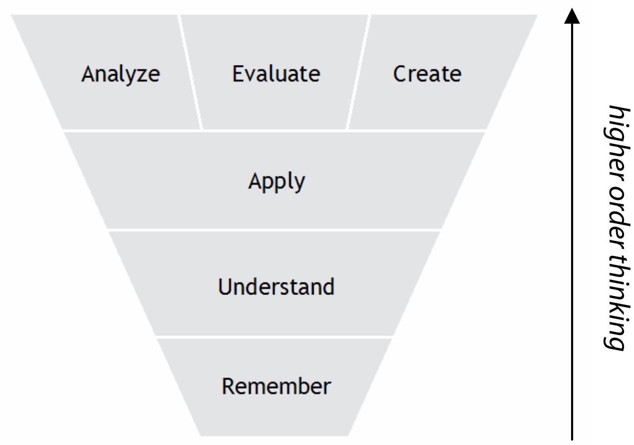
Bloom's Taxonomy states that you must progress students from beyond their starting point of simple recall to a "higher order" of thinking where they are judging, discussing, and creating ideas from the initially taught content. Without this process, information is easily forgotten and cannot be applied to new situations.

Gardner's Theory of Multiple intelligences has a slightly different bent: it states that some students are stronger at approaching certain problems (i.e. abstract versus physical) in a specific way and therefore educators must try to engage and accommodate all learners. Further, all students need practice approaching problems in a variety of ways so their "weaker" intelligences are not neglected.

BLOOM'S TAXONOMY

At first glance, Bloom's taxonomy might appear as quite constructivist in approach as it calls for students to do much more than simply solve simple problems over and over again.

In reality, however, we believe this concept can be applied in either a direct instruction or a constructivist based learning environment. In fact, Bloom, who also defined the "Mastery Learning" approach as outlined in Appendix A, created this taxonomy as a guide for creating direct instruction objectives and standards.



Further, Bloom felt that the higher orders of application, analysis, evaluation, and synthesis should follow after the fundamentals of remembrance (recall) and understanding (an unfortunately ambiguous label) took place.

At times, this taxonomy is presented in a linear manner, with each of the six categories representing higher level of thinking than the last. We, however, prefer to represent this taxonomy as in the diagram shown with analysis, evaluation, and synthesis (creation) as equal order.

As presented in [Wikipedia](), the six division of the taxonomy are

1. Knowledge (Remember)

> Exhibit memory of previously-learned materials by recalling facts, terms, basic concepts and answers
>
> - Knowledge of specifics - terminology, specific facts
> - Knowledge of ways and means of dealing with specifics - conventions, trends and sequences, classifications and categories, criteria, methodology

- Knowledge of the universals and abstractions in a field - principles and generalizations, theories and structures

2. Comprehension (Understand)

Demonstrative understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas

- Translation
- Interpretation
- Extrapolation

3. Application

Using new knowledge. Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.

4. Analysis

Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations

- Analysis of elements
- Analysis of relationships
- Analysis of organizational principles

5. Synthesis (Create)

Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions

- Production of a unique communication
- Production of a plan, or proposed set of operations
- Derivation of a set of abstract relations

6. Evaluation

Present and defend opinions by making judgments about information, validity of ideas or quality of work based on a set of criteria

- Judgments in terms of internal evidence
- Judgments in terms of external criteria

Gardner's Theory of Multiple Intelligences

THOUGHT

- **Verbal-linguistic.** To do with words, spoken or written. People who specialize in this area are generally good at writing, oration and (to a lesser extent) learning from lectures. They also tend to have broad vocabularies and learn languages very easily.

- **Logical-mathematical.** To do with numbers, with logic and abstractions. Those who favor this intelligence generally excel in mathematics and computer programming, and are often jacks of all trades by virtue of logic. Careers might include those involving science and computer programming. A common criticism of this intelligence is that some people feel that logical ability in general is more strongly associated with verbal than with mathematical intelligence; for example, the old Analytic section of the GRE correlated more strongly with the Verbal section than the Mathematical. One possibility is that formal, symbolic logic, and strict logic games are under the command of mathematical intelligence, while skills at fallacy hunting, argument construction, etc. are under the command of verbal intelligence.

SENSATE

- **Visual-spatial.** To do with visual perception and spatial judgment. People in this group are generally possessed of high hand-eye coordination, can interpret art well and can tessellate objects (as in loading a truck) easily. Such people might work as artists, artisans and engineers. One of the most common criticisms of the whole frame work of the theory of multiple intelligences is the extremely high degree of correlation between visual and mathematical intelligence. There are several responses to this line of criticism, the most common being that though they may share several different factors they can be distinguished and have been demonstrated to vary by enormous quantities in some cases.

Gardner's 8 Intelligences

Thought

- Verbal-linguistic.
- Logical-mathematical.

Sensate

- Visual-spatial.
- Body-kinesthetic.
- Auditory-musical.

Communicational

- Interpersonal.
- Intrapersonal.

Innate / Instinctual

- Naturalistic.

From http://en.wikipedia.org/wiki/Multiple_intelligence

- **Body-kinesthetic.** To do with muscular coordination, movement and doing. In this category, people generally are more adept at sports and dance, and work better when moving. In addition, they learn better by doing things and interacting with them physically. Most dancers, gymnasts and athletes are in this category.

- **Auditory-musical.** To do with hearing. Those good with this tend to be better singers and have better pitch, in addition to liking music more. Music also helps people in this category work better, and those here will also learn better from lectures.

  Aural capabilities have physiological and psychological similarities to other gifts associated with the processing of any input by the brain/mind. Those with "perfect pitch" have the ability to identify and differentiate notes to an exact degree, without a reference pitch. Also, most have the ability to play one or more musical instruments with exceptional ease and style, or to compose music of exceptional quality (such as Wolfgang Amadeus Mozart). Many other traits are indicative of a musical/auditory genius.

COMMUNICATIONAL

- **Interpersonal communication.** To do with interaction with others. People categorized here are usually extroverts, and good with people. They can be charismatic and convincing and diplomatic. They tend to learn better when people are involved, e.g., in discussions. People in these fields often become politicians or educators.

- **Intrapersonal communication.** To do with oneself. People categorized here are most often introverts and have very complex philosophies. These people often end up in religion or psychology, and like to be alone. One of the major areas of attack on the theory of multiple intelligences, it is alleged that a concept like intrapersonal intelligence is vague and immeasurable, and hence not a proper study for psychology. Others question whether intrapersonal intelligence can really be considered intelligence, and claim that it instead should be considered more a personality trait, and a set of desires. (Information headhunting, someone else put it into a well-written connection with the above: Intrapersonal intelligence is first and foremost the ability to objectively examine and judge oneself, including one's own weaknesses and strengths, motivations and desires; perhaps often with the purpose of improving one's understanding of the general human experience. It is in basic terms, a sense of insight into one's nature).

INNATE / INSTINCTUAL

- **Naturalistic.** A late addition to Gardner's theory is that of a "naturalist" intelligence. Naturalist intelligence enables human beings to recognize, categorize and draw upon certain features of the environment. It combines a description of the core ability with a characterization of the role that many cultures value. From an interview with Howard Gardner by Ronnie Durie in Mindshift Connection, a publication of Zephyr Press:

  "The core of the naturalist intelligence is the human ability to recognize plants, animals, and other parts of the natural environment, like clouds or rocks. All of us can do this; some kids (experts on dinosaurs) and many adults (hunters, botanists, anatomists) excel at this pursuit.

  "While the ability doubtless evolved to deal with natural kinds of elements, I believe that it has been hijacked to deal with the world of man-made objects. We are good at distinguishing among cars, sneakers, and jewelry, for example, because our ancestors needed to be able to recognize carnivorous animals, poisonous snakes, and flavorful mushrooms."

Appendix E: Sample Catalyst Printables

Attached are several printables covering fractions. We start by introducing the concept and slowly work up to fraction arithmetic. We have included only select printables from each lesson. Printables were chosen to best illustrate the philosophy of our approach.

Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

Into: Pieces of a Whole

LESSON OBJECTIVE

When you complete this lesson, you will be able:

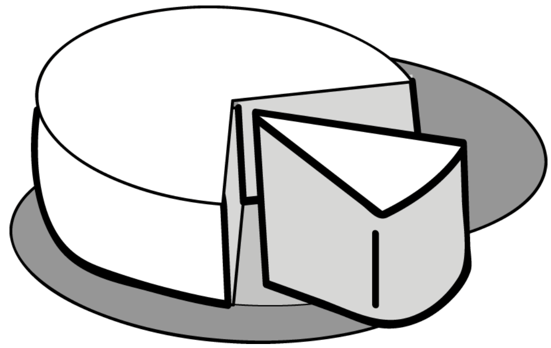
- To verbally compare and contrast the concept of whole numbers and fractions.

- To define numerator and denominator.

- To say and write out fractions in English.

- To represent fractions using manipulatables and simple drawings.

Talk About . . .



• **Whole Numbers.** For example: *We already know about **whole numbers**, right? They are the regular numbers we use to **count things** like 2 pizzas, 3 birds, 5 bedrooms, 101 dollars, 1,203 people or 150,023 stars. They are used to describe **whole things**.*

• **Fractions versus Whole Numbers.** For example: *Why do we need fractions? To describe things that **are not whole** like a **part of pizza** or **part of a room**. But fractions are numbers just like whole numbers, they are used to describe how much or how little you have of something.*

In your own words, and with your tutor's help, define:

Whole:

Piece:

TUTOR PROMPT

Don't worry if your students can't solve the second 'apple' question exactly.

Use it as a motivator and just very much highlight that whole numbers cannot be used to solve the problem.

Answer the question: 'Tom, Ping, and Rocio share 6 apples. How many apples did each person get?'

Now take the question: 'Tom, Ping, and Rocio share 1 apple. How many apples did each person get?'

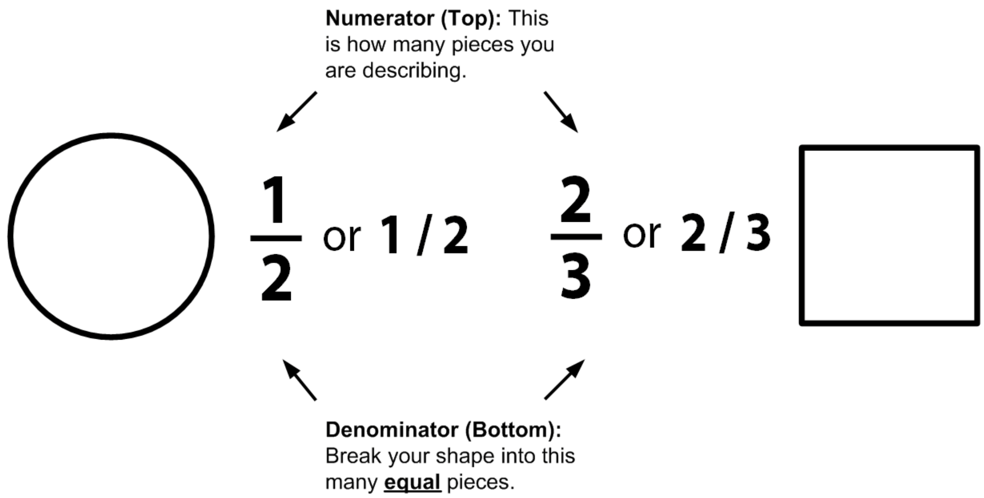
Can you answer the question above using whole numbers? Why or why not?

Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

Definitions: Numerator -- Big Word for 'Top'

A fraction let's you break a **whole** into **pieces**. Finish drawing in this picture with your tutor for the two fractions given:



DEFINITION: FRACTION

A mathematical **convention** that allows you to describe objects or concepts **that do not have to be whole**.

It is written as:

$$\frac{\textbf{described pieces}}{\textbf{total number of pieces}}$$

DEFINITION: DENOMINATOR

The **bottom number** of a fractions. It represents how many **equal** size pieces you broke the whole into. It represents how many **pieces you have total**.

DEFINITION: NUMERATOR

The **top number** of a fraction. It represents how many pieces you are **describing** with your fraction.

Answer true or false. If false, tell your tutor why correct it.

When working with fractions, the sizes of the pieces can be different:	**true** | **false**
The numerator in a fraction represents how many pieces you broke your whole into:	**true** | **false**
The denominator in a fraction represents how many pieces you are describing (are talking about):	**true** | **false**
There is only **one way** to write a fraction:	**true** | **false**
The numerator is on the top:	**true** | **false**
The denominator is on the top:	**true** | **false**

TUTOR PROMPT

Before answering the questions, be sure to point out that a fraction is just a conventional notation; that is, just the way we write something just as the letter 'a' or a '+' sign is just the way we write something.

Be sure to stress that there are several way to write the same fraction, from the fraction bar being completely horizontal to a much more slanted bar.

Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

Draw It: Breaking Up Shapes

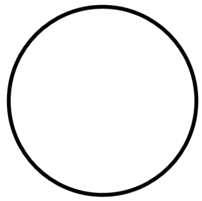
REMEMBER!

The top number in a fraction is called the **numerator**.

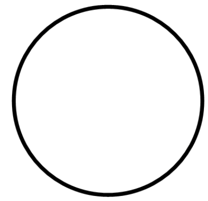
The bottom number in a fraction is called the **denominator**.

Q1 Use the circles to represent the fraction. Add the labels 'numerator' and 'denomi and write out the fraction **in English**.

$$\frac{3}{4}$$

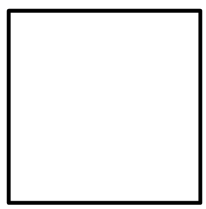


$$\frac{2}{3}$$

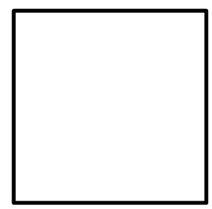


Q2 Use the squares to represent the fraction. Add the labels 'numerator' and 'denominator' and write out the fraction **in English**.

$$\frac{3}{8}$$

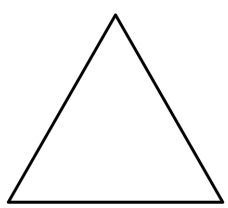


$$\frac{5}{6}$$

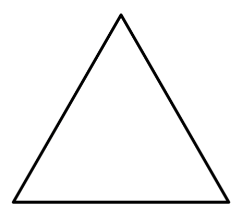


Q3 Use the triangles to represent the fraction. Add the labels 'numerator' and 'denominator' and write out the fraction **in English**.

$$\frac{1}{2}$$



$$\frac{1}{3}$$



Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

Cut It

DIRECTIONS

Get the pizza cutouts. Use the **pans** to represent the whole.

Then, cut up the **pizzas** to solve the following problems.

TUTOR PROMPT

Stress 'how many pieces' versus 'how much of pizza'. That is, you ate '3' pieces but you ate '3/4' of a pizza, etc.

Also, continue to make up examples past the presented ones in this exercise.

Also, continue to reinforce the different ways to write fractions.

Get your pizza cutouts. Use them to express 1/2 as a fraction:

Use them to express $\frac{2}{2}$ as a fraction.

Use them to express 0/2 as a fraction:

You get a pizza and cut it into 4 equal pieces. You then eat 3 pieces. Show **how much of the pizza you ate** using the cutouts. Then, write the fraction numerically and in English:

Show **how many pieces** you ate. Write that numerically.

Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

Say It

You and a group of your friends order five pizzas. How many pizzas did you order?

Did you need **whole numbers** or **fractions** to solve this problem?

Susan, Musha, and Satish order a pizza. If they share **equally**, how much **of a pizza** do they each get?

Did you need **whole numbers** or **fractions** to solve this problem?

? In your own words, why do we need fractions and how are they different than whole numbers?

? What does the word 'fracture' mean. How do you think that relates to the word **fraction**?

Closure

In this lesson, we learned:

- The difference between **whole numbers** and **fractions**.

- That fractions can be used to denote **non-whole** amounts like **three-fifths** of a whole pizza.

? In your own words, what does it mean for something to be 'whole':

? Why do we need fractions:

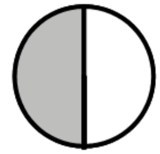
Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

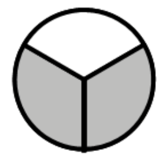
Scaffold: Fractions in English

NOTE TO TUTOR

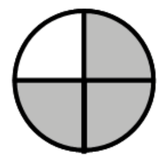
Point out the use of hyphen.

half $\longleftrightarrow \dfrac{\#}{2}$ | $\dfrac{1}{2}$ = 

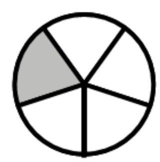
one-half

third $\longleftrightarrow \dfrac{\#}{3}$ | $\dfrac{2}{3}$ = 

two-thirds

fourth $\longleftrightarrow \dfrac{\#}{4}$ | $\dfrac{3}{4}$ = 

three-fourths

fifth $\longleftrightarrow \dfrac{\#}{5}$ | $\dfrac{1}{5}$ = 

one-fifth

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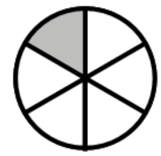
Handout #1

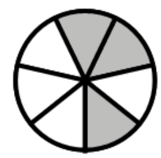
FRAC-PW: Fractions -- Pieces of a Whole

Scaffold: Fractions in English

sixth $\leftrightarrow \dfrac{\#}{6}$ | $\dfrac{1}{6} =$ 

one-sixth

---

seventh $\leftrightarrow \dfrac{\#}{7}$ | $\dfrac{3}{7} =$ 

three-sevenths

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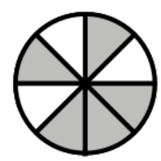
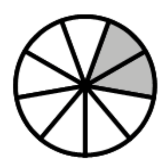
eighth $\leftrightarrow \dfrac{\#}{8}$ | $\dfrac{5}{8} =$ 

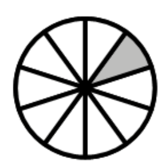
five-eighths

---

ninth $\leftrightarrow \dfrac{\#}{9}$ | $\dfrac{2}{9} =$ 

two-ninths

---

tenth $\leftrightarrow \dfrac{\#}{10}$ | $\dfrac{1}{10} =$ 

one-tenth

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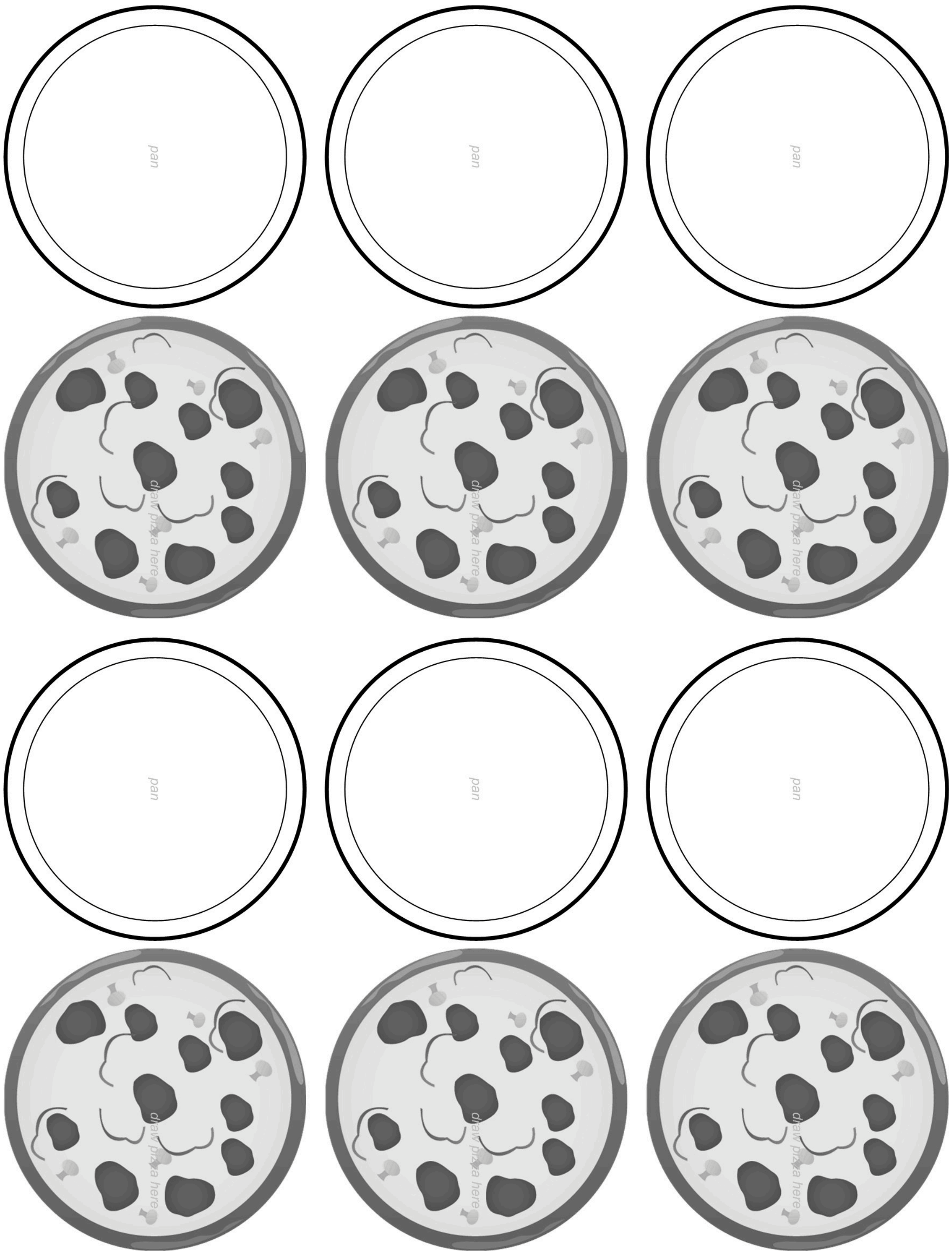
Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

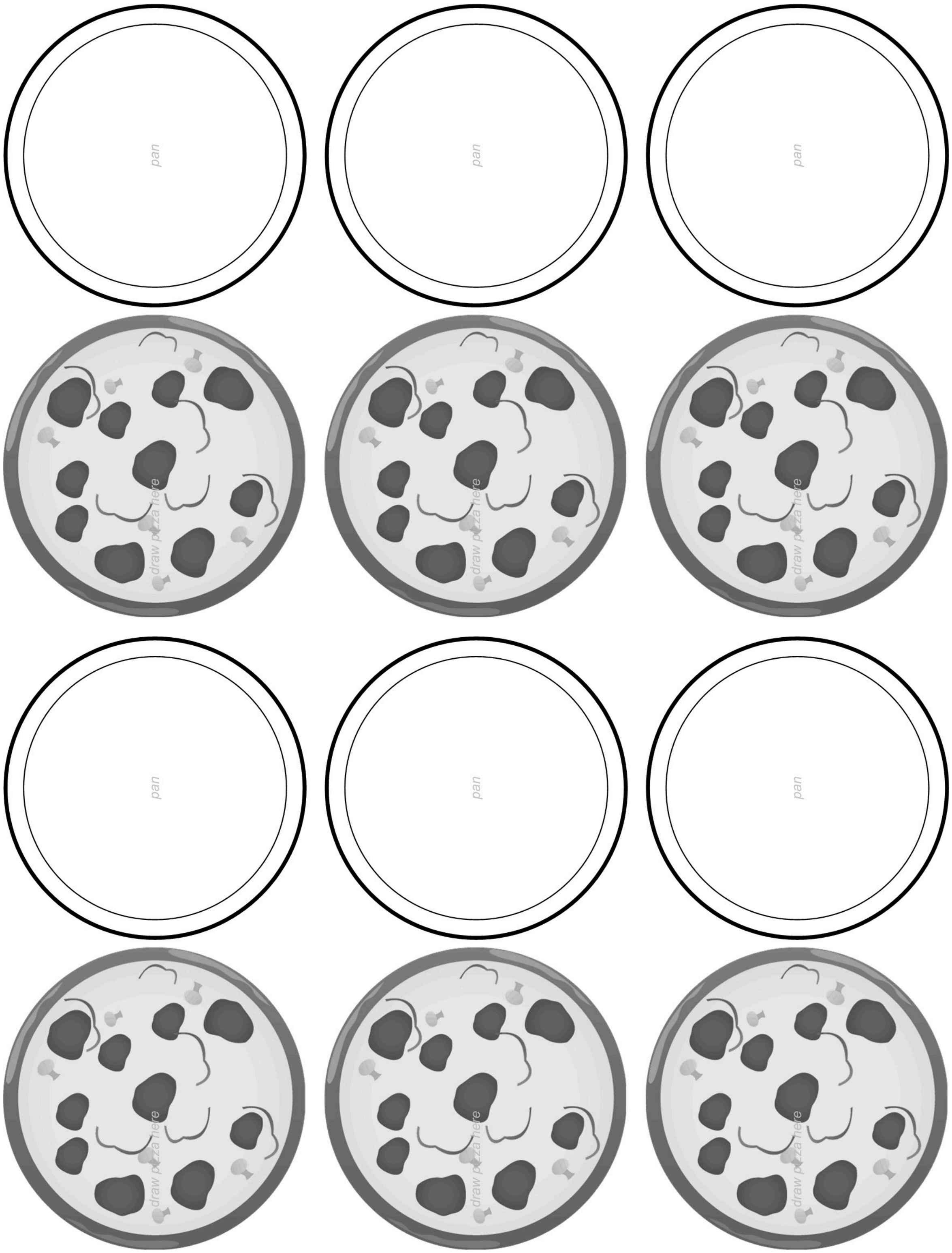
-- cutouts --



Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

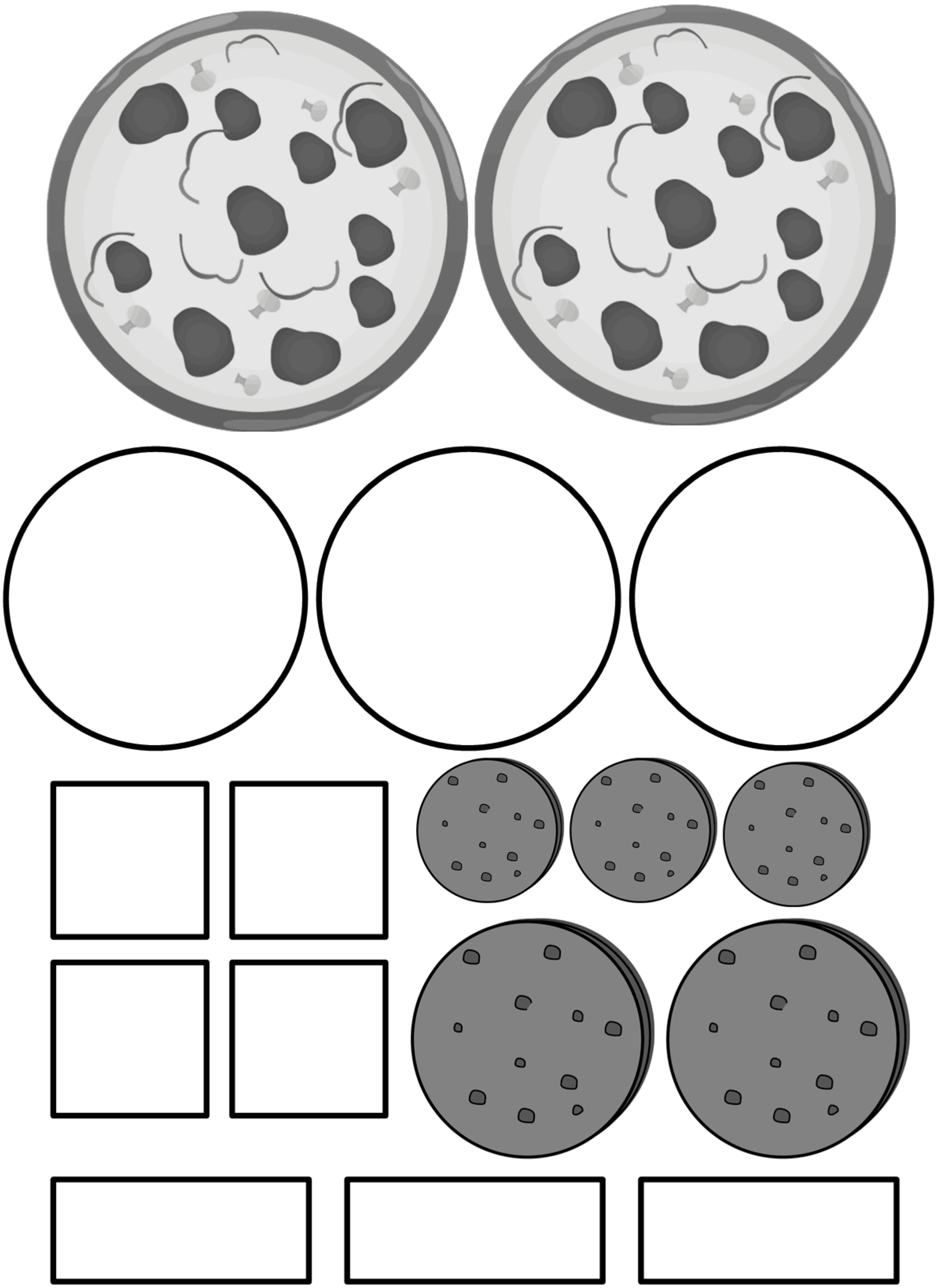
-- cutouts --



Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

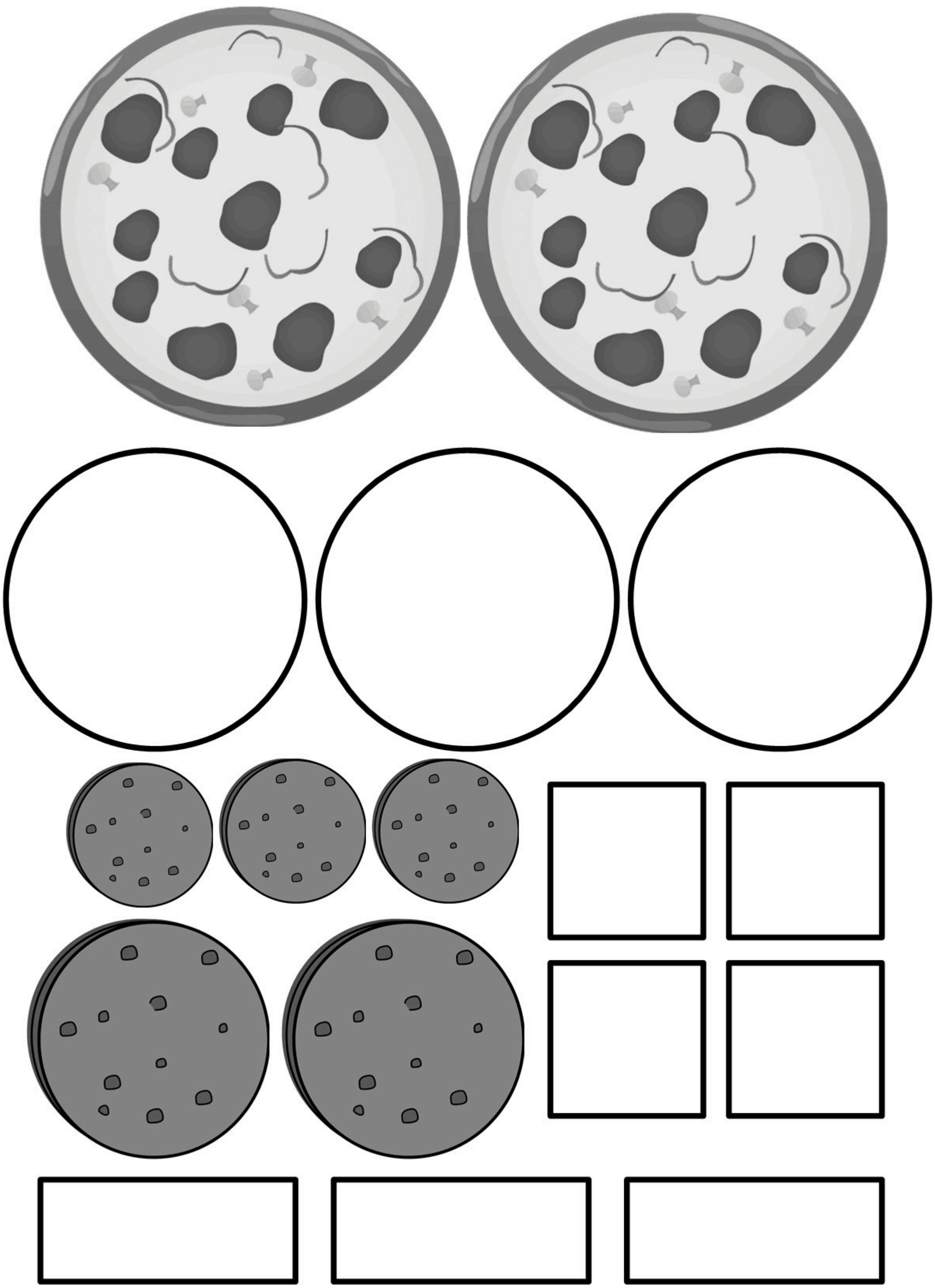
-- cutouts --



Handout #1

FRAC-PW: Fractions -- Pieces of a Whole

-- cutouts --



Worksheet #1

FRAC-PW: Fractions -- Pieces of a Whole

Draw It: Whole into Pieces

Remember, a **whole** is a concept like

- 'a whole cookie'
- 'one whole pizza'
- 'one whole apple'
- 'a whole cake'

With your tutor's help, think of some other 'whole things' and write them here:

1

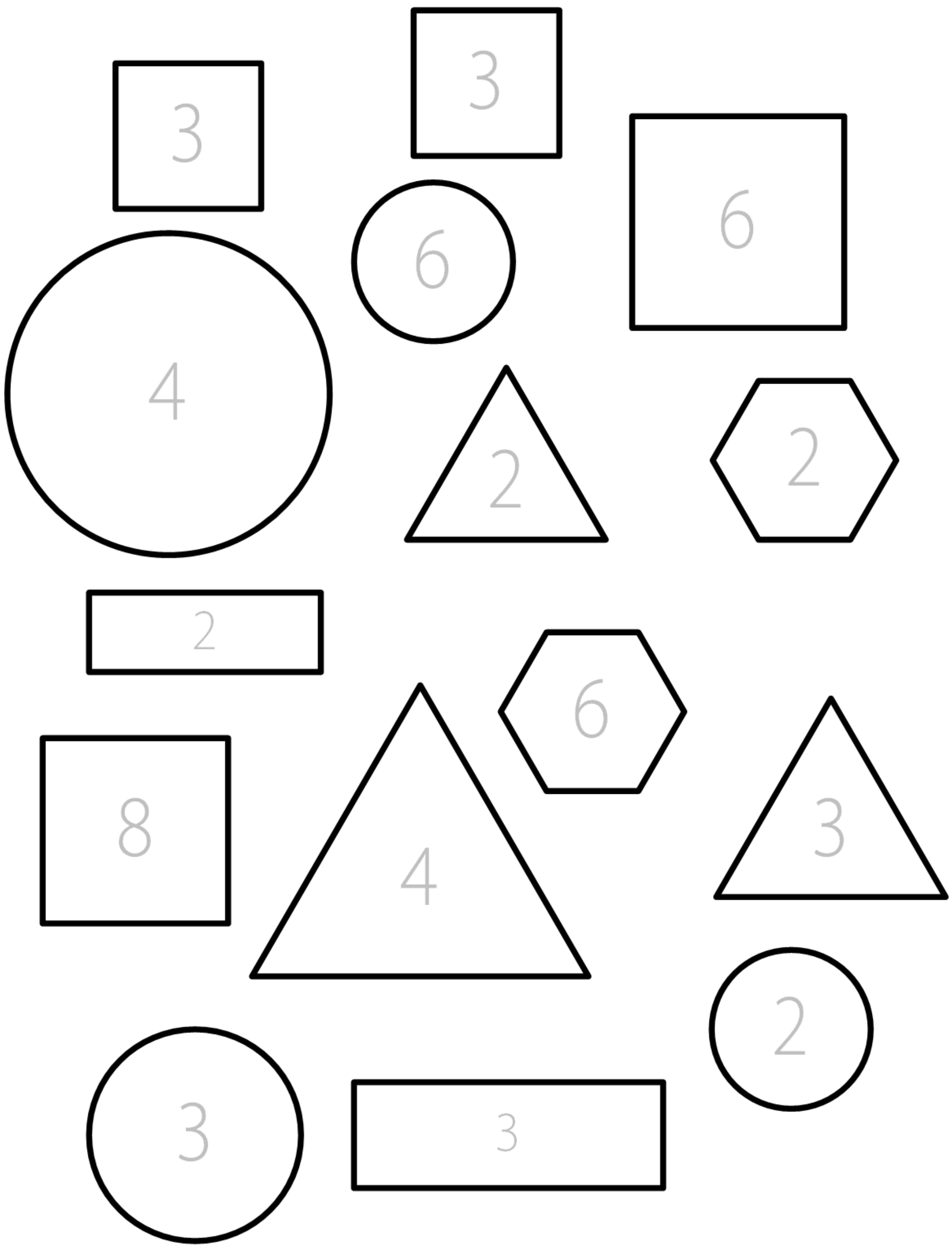
2

3

Often, to represent a **whole** people use **pictures** like pictures of **circles, squares, rectangles, or triangles**.

To your right are pictures of **whole** objects. Break them into the number **pieces** written on them.

Do the pieces have to be equal or not?



Worksheet #1

FRAC-PW: Fractions -- Pieces of a Whole

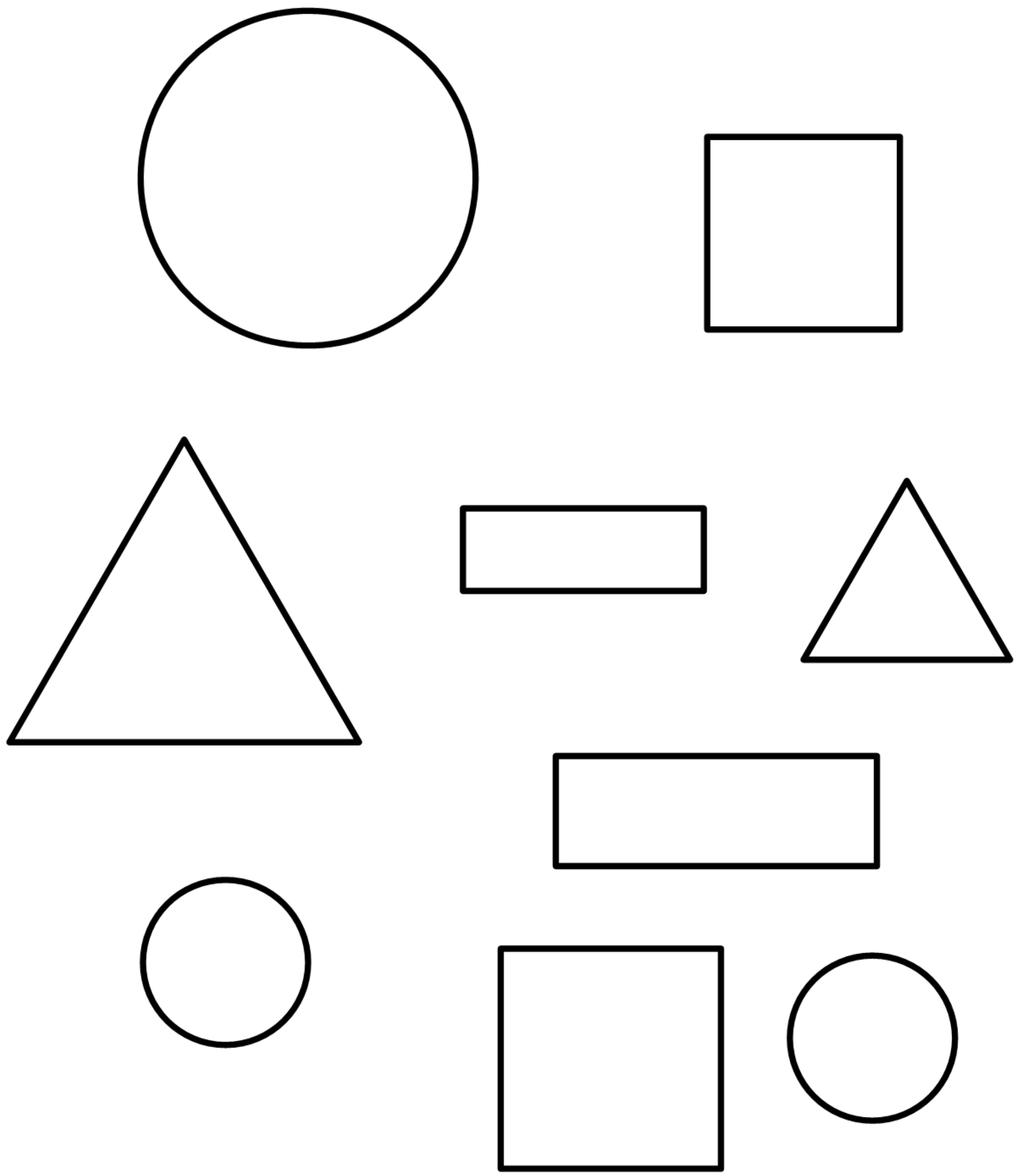
Create It: Make a Fraction

In this exercise, you make up the problems. For each shape, make up a fraction and use the shape to represent the fraction. Be sure to **write down** your fraction somewhere near the shape.

NOTE TO TUTOR

Prompt your student to think of fractions with a numerator of 0 and a denominator of 1.

Also, continue to say fractions aloud in English.



? Think up a shape **AND** think up a fraction. Draw the shape and use it to represent your fraction.

Worksheet #2

FRAC-PW: Fractions -- Pieces of a Whole

Cut It: How Many Pieces vs. How Much of Pizza

DIRECTIONS

Use the pizza and pizza pan cutouts (included) to answer the problems.

Cut the pizza, but leave the pans whole to represent the 'whole' and place the cut up pizzas on the pans.

After using your cutouts, also draw a picture to represent the situation.

NOTE TO TUTOR

Remember to stress 'how many pieces' versus 'how much of a pizza'.

Let's say you make a pizza and you cut it into four pieces. You then eat three pieces.

Q1 How many **pieces of pizza** did you eat?

Q2 How **much of the pizza** did you eat? Answer both on paper and show with cutouts:

Q3 In your own words, explain why these two questions are different:

Worksheet #2

FRAC-PW: Fractions -- Pieces of a Whole

Cut It: Pieces of Pizza

DIRECTIONS

Use the pizza and pan cutouts to answer the questions to the right.

Remember, after using your cutouts, also draw a picture to represent the situation **and** write out the fraction numerical and in English.

WARNING!

If you do not have enough information, talk to your tutor about what you are missing.

NOTE TO TUTOR

Use extra pizzas to keep making up examples and use the cutouts to explore all the ideas presented in this lesson. Especially point out '1 whole pizza' and 'pieces of a whole pizza'.

Also, continue to point out the numerator and denominator.

NOTE TO TUTOR

Talk alot about the fraction 1/1. Reinforce the idea of 'if you don't cut up your whole how many pieces do you have -- you have one big piece'.

Also, talk alot about fractions with 0 in the numerator. Reinforce the idea of 'if you don't have any pieces you have 0 pieces'.

Q4 Judy and Samson ordered three pizzas. Show how many pizzas they ordered.

Q5 Shukri makes a pizza. He then cuts the pizza into 4 pieces and eats 1 piece. Show how much pizza **he ate** as a fraction.

Q6 Susan orders a pizza and eats 3 pieces. Express how much she has left as a fraction.

Q7 Rocio orders a pizza and **does not cut the pizza**. Express how much pizza she has left as a fraction.

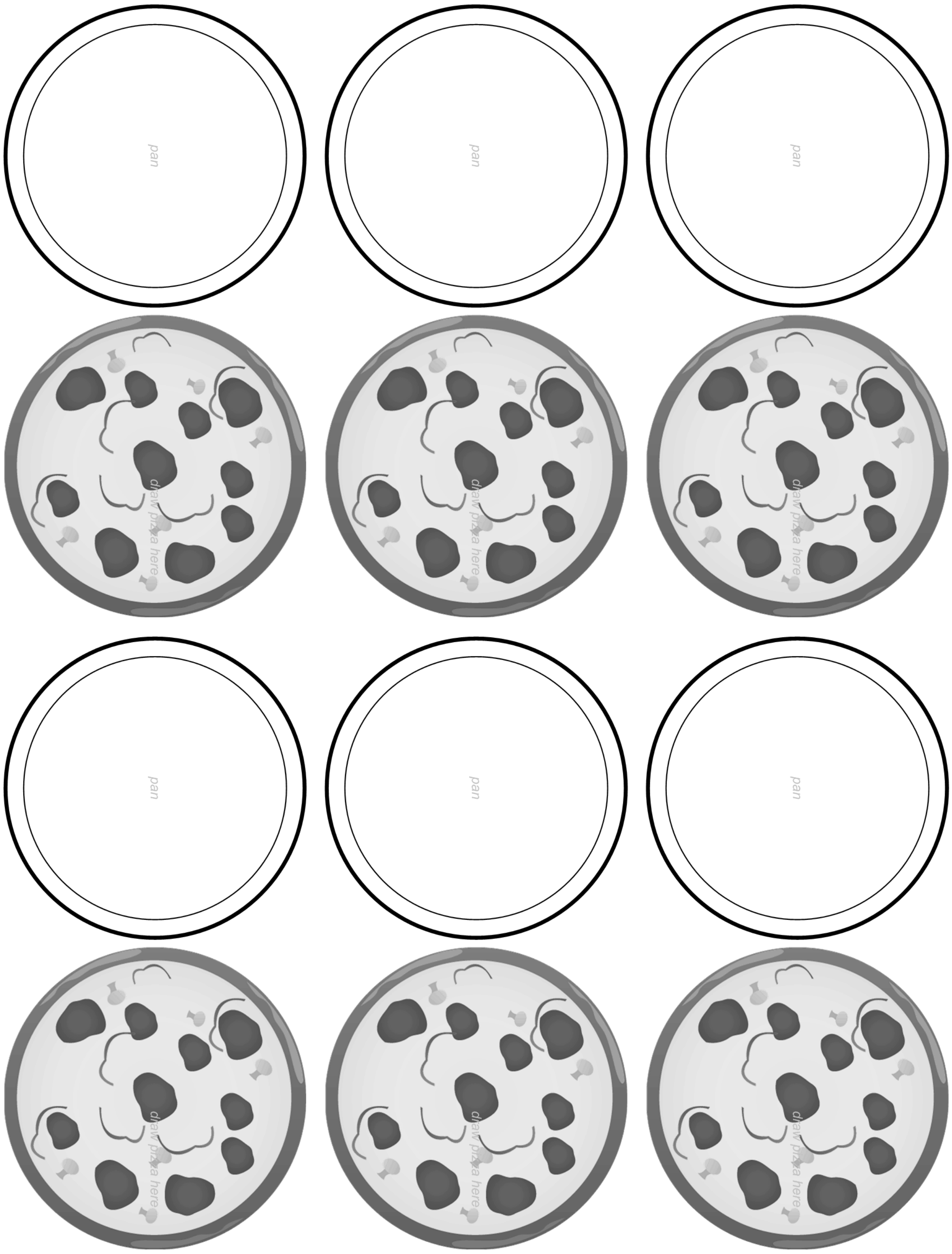
Q8 Jacque orders a pizza and cuts the pizza into four pieces. He does not eat any pizza. Express how much pizza he has left and how much pizza **he ate** as a fraction.

Q9 Mohammed orders a pizza and cuts the pizza into five pieces. How much pizza did he eat expressed as a fraction.

Worksheet #2

FRAC-PW: Fractions -- Pieces of a Whole

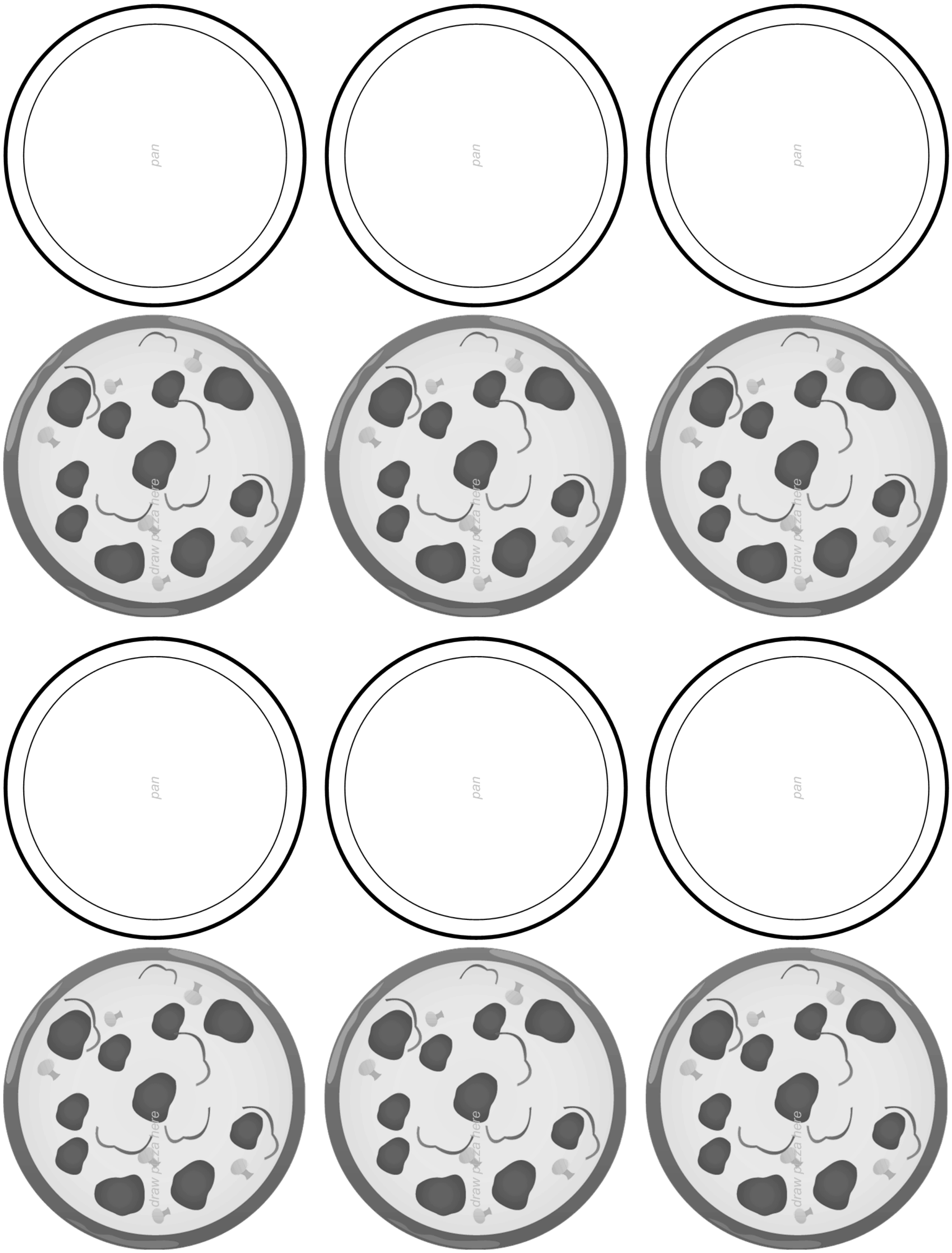
-- cutouts --



Worksheet #2

FRAC-PW: Fractions -- Pieces of a Whole

-- cutouts --



Worksheet #3

FRAC-PW: Fractions -- Pieces of a Whole

Questions

Q1 Use the squares to represent the fraction. Add the labels 'numerator' and 'denominator' and write out the fraction **in English**.

TUTOR PROMPT

"How many pieces are you cutting your whole into?"

"How many pieces are you describing (do you have)?"

$$\frac{1}{1}$$

$$\frac{7}{10}$$

Q2 Use the circles to represent the fraction. Add the labels 'numerator' and 'denominator' and write out the fraction **in English**.

$$\frac{3}{5}$$

$$\frac{1}{3}$$

Q3 Jax makes a pie and cuts it into 5 equal pieces. He then eats 1 piece and his friend Tomas eats 2 pieces, and his friend Jamal isn't hungry so he doesn't eat any pieces.

Draw a picture and express how much of the pie each person ate as a fraction.

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Worksheet #3

FRAC-PW: Fractions -- Pieces of a Whole

Questions

Q4 Give an example of a **whole number**:

Give an example of a **fraction**:

In your own words, describe the difference between whole numbers and fractions:

Q5 Make up a fraction and represent it using a circle.

Define 'denominator' and answer 'what does it tell you about a fraction?'. Use the fraction from above in your explanation (use it as a diagram):

Define 'numerator' and answer 'what does it tell you about a fraction?'. Use the fraction from above in your explanation (use it as a diagram):

Worksheet #4

FRAC-PW: Fractions -- Pieces of a Whole

Correct It: How Many Pieces or How Much of a Pizza?

On a test, Musha ran into the following question:

You make a pizza which you cut into six equal pieces. You then eat 5 pieces. How many pieces of pizza did you eat?

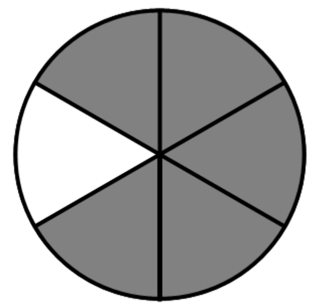
Here is her answer:

Step 1: We'll I have a pizza which I need to cut into 6 equal pieces.

Step 2: So I'm going to use a circle to be my pizza — this is my 'whole pizza'. Then I'm going to 'cut it' into six equal pieces.

Step 3: Then, I ate five pieces so I'm going to shade in 5 of them.

Step 4: So my answer is $\frac{5}{6}$.



NOTE TO TUTOR

Get them to think about 'how much of the pizza did Musha eat' versus 'how many pieces of pizza did Musha eat'.

Come up with other examples and explore them with the student.

Check **each one of Musha's steps**. Explain why it is either right or wrong:

Step 1:

Step 2:

Step 3:

Step 4:

Can you **change the question** so Musha's answer is correct?

Worksheet #4

FRAC-PW: Fractions -- Pieces of a Whole

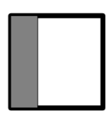
Questions

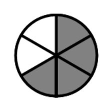
Q1 Draw a circle -- use it to represent the fraction $\frac{5}{6}$.

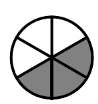
Q2 Ali orders a pizza and cuts it into four pieces. He then eats one. Draw a picture showing how much pizza he has left:

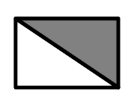
Q3 Susan and Musha get a cookie. Susan breaks the cookie into 4 pieces. Susan then eats 1 piece and Musha does not eat any. Using fractions express 1) how much of the cookie Susan ate 2) how much of the cookie Musha ate and 3) how much of the cookie is left.

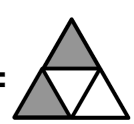
Q4 Sunny drew some pictures to represent fractions below. Are they right or wrong answers? In your own words, explain why or why not. If they are wrong, correct them.

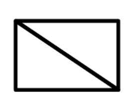
$\frac{1}{2}$ = 

$\frac{4}{5}$ = 

$\frac{1}{6}$ = 

$\frac{1}{2}$ = 

$\frac{3}{4}$ = 

$\frac{0}{2}$ = 

Worksheet #5

FRAC-PW: Fractions -- Pieces of a Whole

Cut It: Make Your Own Fraction Strips

Take out your cutout page. The big strip is labeled 1. This is your 'whole'.

NOTE TO TUTOR

Be sure to reenforce that the 'whole' is one. This is just what we decided to call a whole. The 'fractions' or 'pieces' are in reference to this 'whole'.

Fill in the fractions for the rest of the spaces. Then, cut them into 'strips' but not into individual 'pieces' (each strip should be as long as the '1' strip).

Q1 Using your fraction strips make the depict the following fractions:

$$\frac{0}{2} \qquad \frac{1}{2} \qquad \frac{2}{2}$$

NOTE TO TUTOR

Guide them on a couple of example first. For example, fill out the 1/3 strip. Then, try and let them fill out as much of the chart as they can. Remember to say out loud each fraction in English.

And before you cut up the strips, have them study the strips first -- to look at the patterns formed by the blocks. Openly ask questions about the structure of the strips.

Q2 Using your fraction strips make the depict the following fractions:

$$\frac{2}{5} \qquad \frac{1}{7} \qquad \frac{8}{8}$$

NOTE TO TUTOR

Continue to make up problems and explore many different fractions.

Q3 Circle which is bigger: $\frac{1}{10}$ or $\frac{1}{7}$

NOTE TO TUTOR

Remember to save these strips; they are used later many times.

Q4 Circle which is bigger: $\frac{1}{9}$ or $\frac{1}{2}$

Worksheet #5

FRAC-PW: Fractions -- Pieces of a Whole

Cut It: Make Your Own Fraction Strips

Q5 Using your fraction strips make the depict the following fractions:

$$\frac{5}{5} \qquad \frac{3}{5} \qquad \frac{1}{5}$$

Q6 Using your fraction strips make the depict the following fractions:

$$\frac{2}{4} \qquad \frac{5}{10} \qquad \frac{1}{2}$$

Q7 Circle which is bigger: $\frac{1}{3}$ or $\frac{1}{4}$

Q8 Circle which is bigger: $\frac{1}{5}$ or $\frac{1}{6}$

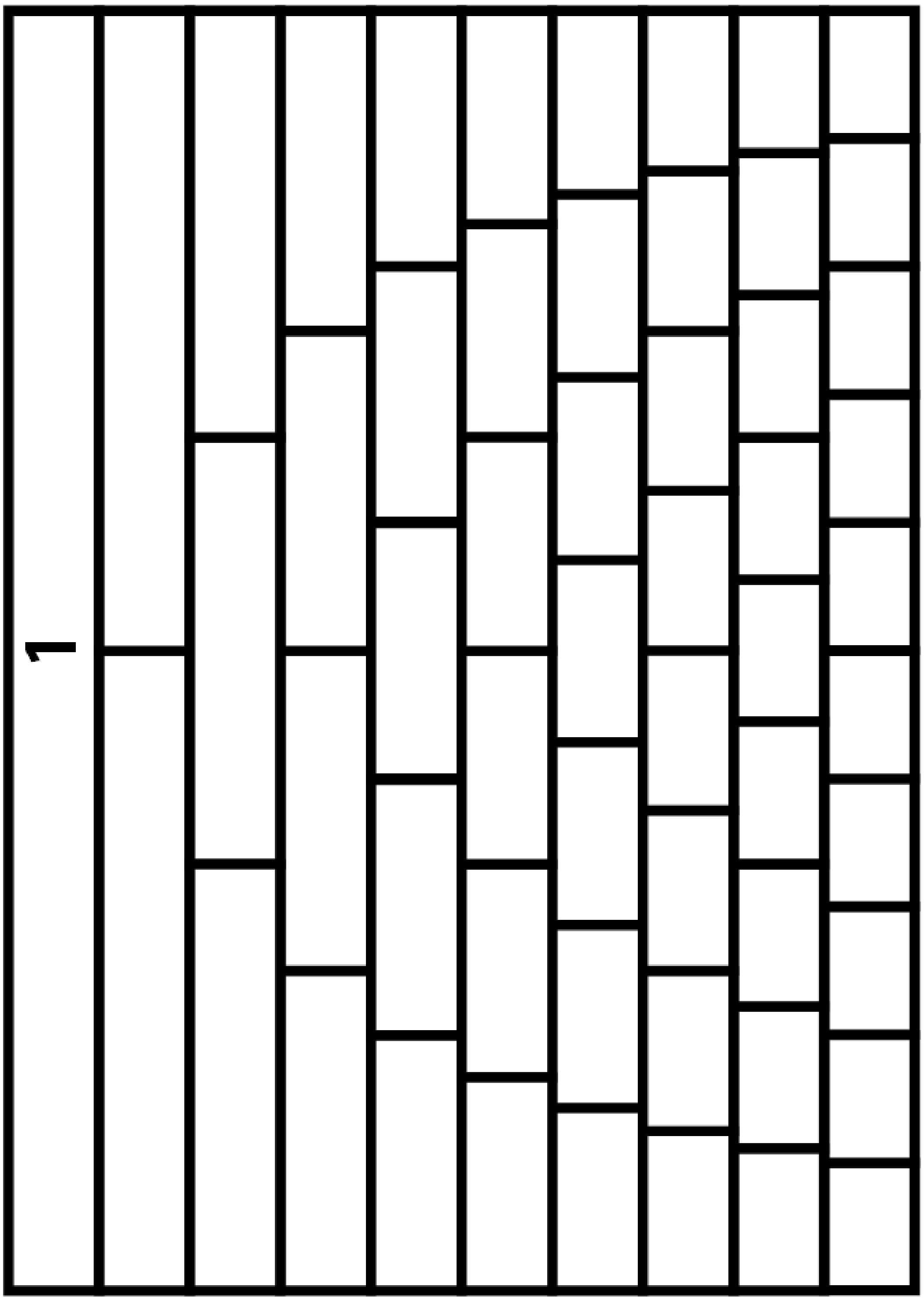
Q9 Circle which is bigger: $\frac{1}{5}$ or $\frac{1}{7}$

Q10 Circle which is bigger: $\frac{1}{9}$ or $\frac{1}{10}$

Worksheet #5

FRAC-PW: Fractions -- Pieces of a Whole

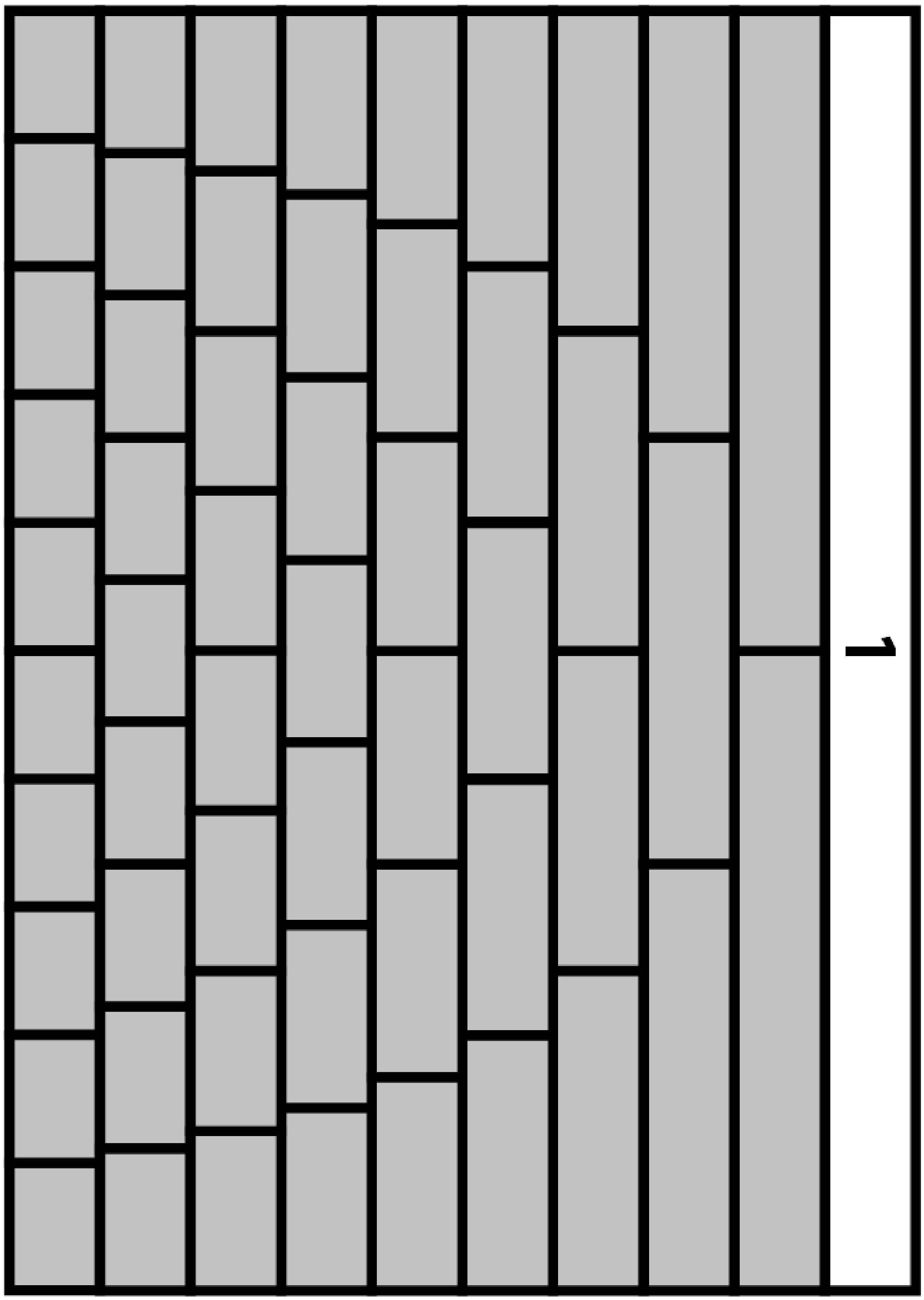
-- cutouts --



Worksheet #5

FRAC-PW: Fractions -- Pieces of a Whole

-- cutouts --



Handout #1

FRAC-MN: Mixed Numbers

LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To represent mixed numbers using and from manipulatables, pictures, and descriptions.

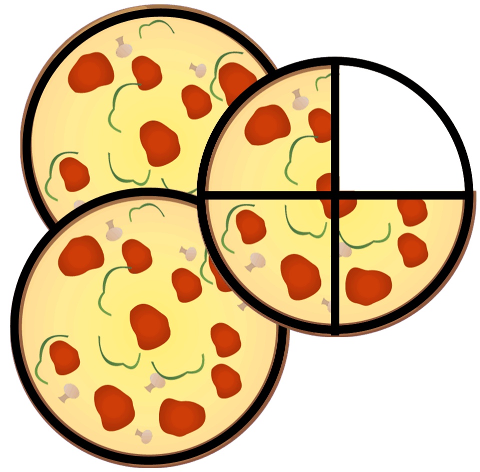
- To be able to say and write mixed numbers in English.

Into: Mixed Numbers -- Fractions and Whole Numbers Together At Last

Talk About . . .

- **Whole Numbers versus Fractions.** For example: *Whole numbers describe* ***complete or whole*** *things like 3 cars and 5 people. Fractions can be used to describe things that have been* ***broken into pieces*** *like* ***three-fourths*** *of a cake.*

- **Can Whole Numbers and Fractions be Together?.** For example: *But can we have a number that has both* ***whole numbers and fractions*** *in it?*



Today, we're going to learn more about subtraction.	**true** | **false**
You can use fractions to describe something that is not whole, like a cake that has been cut up into pieces.	**true** | **false**
You can use whole numbers to describe something that is not whole, like a pizza that has been cut up into pieces.	**true** | **false**
All anyone ever needs are whole numbers. There is never a use for fractions.	**true** | **false**

REVIEW

Draw a picture to represent 5/6. Also, write out this fraction in English.

Handout #1

FRAC-MN: Mixed Numbers

Investigate

DIRECTIONS

Get the pizza cutouts. Use the **pans** to represent the whole.

Then, use the **pizzas** to solve the following problems. **Sometimes you'll need to cut up the pizzas and sometimes you'll need to leave them whole.**

Always say and write out the numbers in English and point out the numerator and the denominators of every fraction.

TUTOR PROMPT

Here, we're informally introducing mixed numbers -- that is numbers that have both a whole number and a fraction.

Stress the fact that fractions do not have to be by themselves; that you can have a whole and fractional amount at the same time.

TUTOR PROMPT

Continue to make up similar examples using the more pizzas other the shapes (cookies, circles, etc). Generally, it's good first to use the shapes to represent whole numbers first before cutting. For example, 'Okay, show me 2 cookies'. Alright, now show me 1 1/2 cookies.

If you need to, cut some more pizza cutouts.

Get your pizza cutouts. Can you use them to express **two and one-fourth pizzas**.

Use them to express $\mathbf{1\frac{1}{2}}$ as a fraction.

You and your friends get three pizzas. You cut the first pizza into six equal pieces. You then eat 5 of those pieces. Show **how much pizza you have left** using the cutouts:

[?] On the question above, show with the cutouts **how much** pizza you and your friends ate:

[?] On the question above, **how many pieces** of pizza did you and your friends eat?

Handout #1

FRAC-MN: Mixed Numbers

Draw It

Draw any shape or picture you want to describe the fraction **three-fourths**. Also, write out the fraction in numerically.

DEFINITION: MIXED NUMBER

A number that has both a whole number and a fraction like the number **two and five-sixths**. This is written numerically as:

2 5/6 or $2\frac{5}{6}$

Use any shape you want to describe the fraction **zero-halfs**. Also, write out the fraction in numerically.

TUTOR PROMPT

Continue to make up examples (get scratch paper if you need it).

Use any shape you want to describe the number **three and three-fourths**. Also, write out the number numerically.

Handout #1

FRAC-MN: Mixed Numbers

Say It

In your own words, explain why the following is **true or false**:

*You **can not** have a number that has both a whole number and a fraction in it.*

In your own words, what's a mixed number?

Create It

TUTOR PROMPT

Have your student make up a couple of examples (get scratch paper if you need it).

Make up a mixed number -- that is, a number that has both a whole number and a fraction.

Then, draw a picture to represent your mixed number -- use any shape you want, like a circle, squares, pizzas, cookies, or any other object you want to draw.

Closure

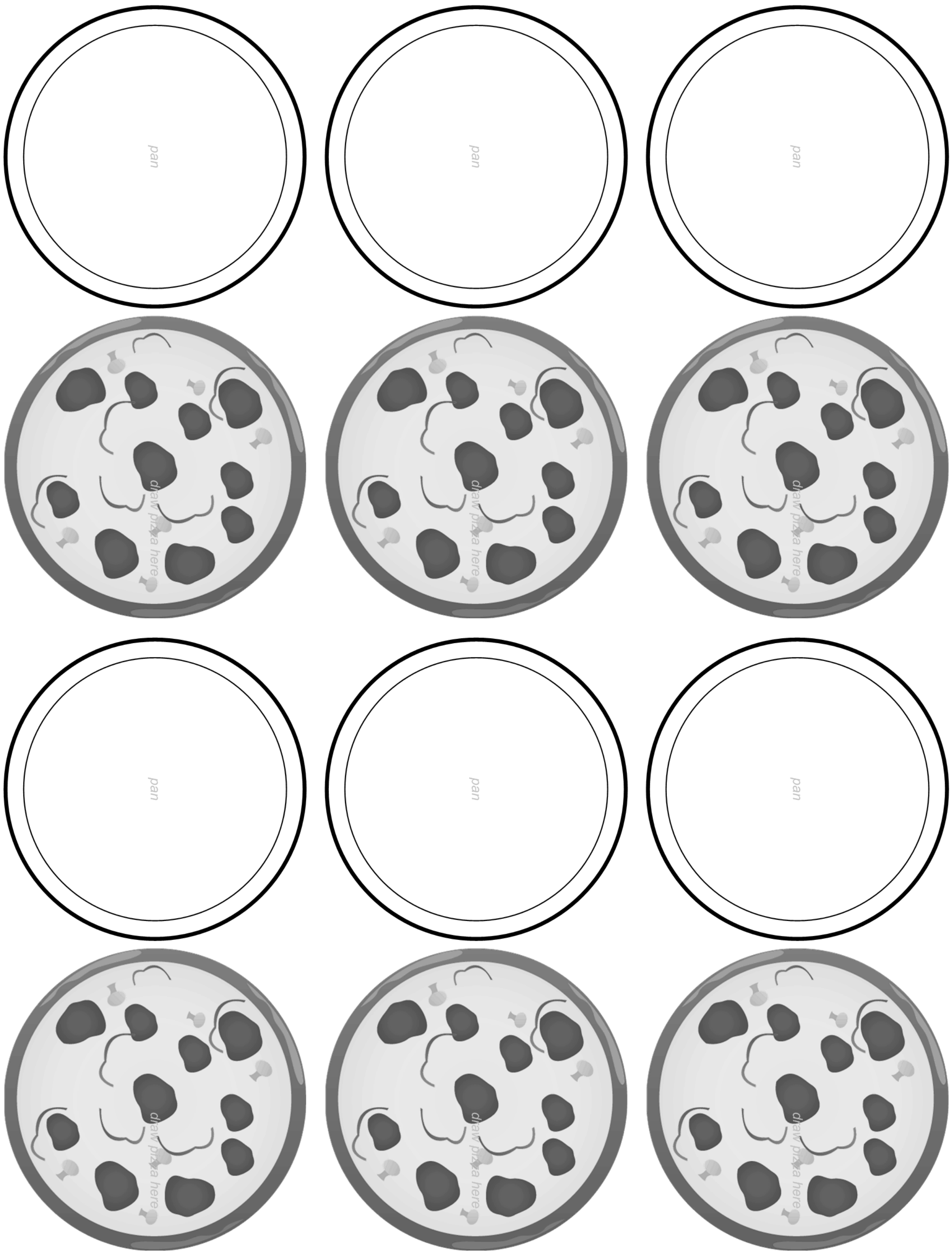
What did we learn about today?

Do **mixed numbers** make sense or are they confusing?

Handout #1

FRAC-MN: Mixed Numbers

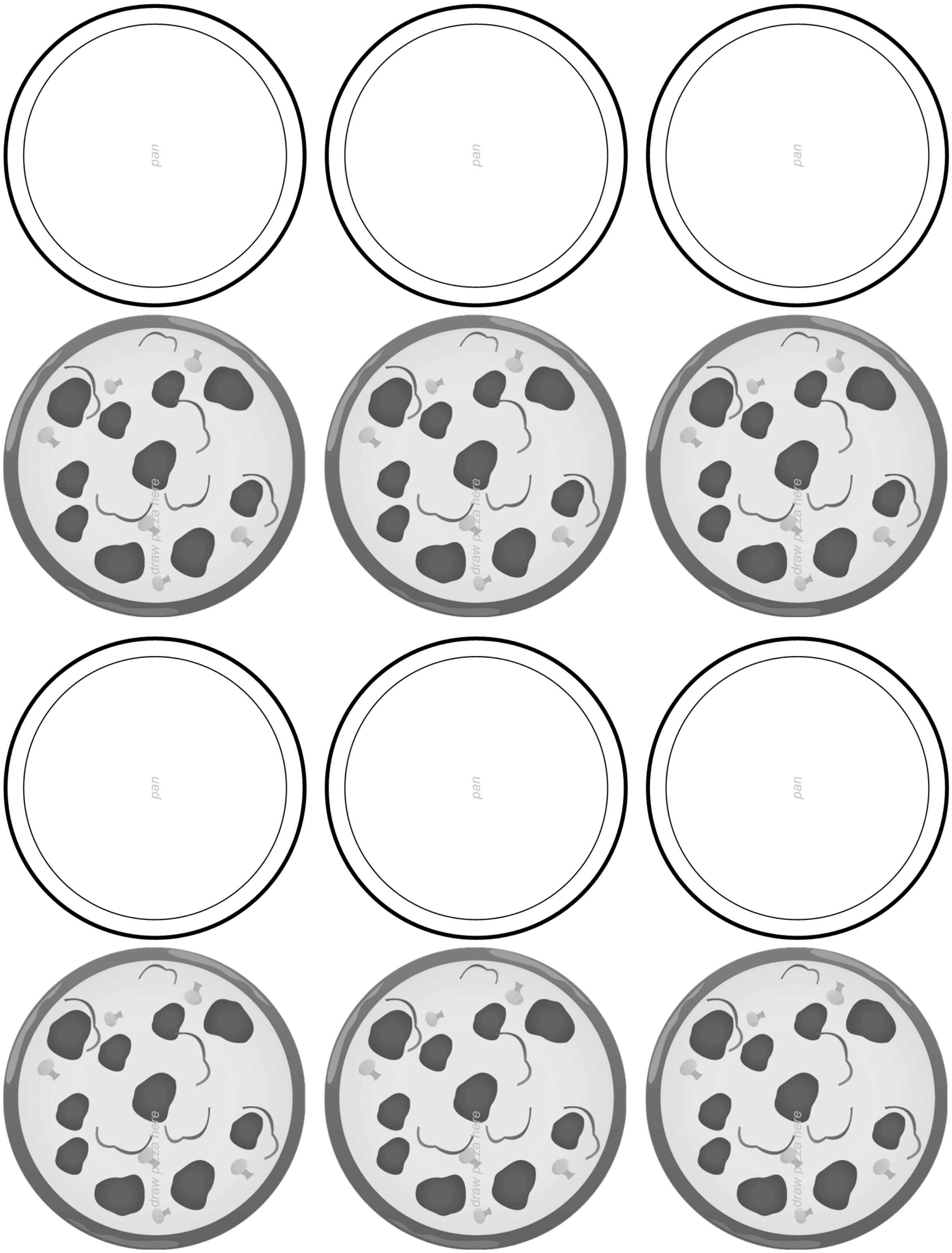
-- cutouts --



Handout #1

FRAC-MN: Mixed Numbers

-- cutouts --



Handout #1

FRAC-NL: Fractions on a Number Line

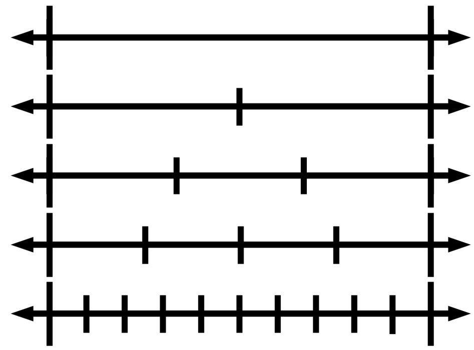
Into: Fractions as Number Lines

LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To use a number line to plot and order fractions and mixed numbers.

Talk About . . .



- **Whole Numbers Lines.** For example: *We know we can use a* ***number line*** *to plot and order* ***whole numbers****.*
- **Fractions On a Number Line.** For example: *But can we plot* ***fractions*** *on a number line? Remember, fractions are* ***just numbers*** *just like* ***whole numbers*** *are just numbers.*
- **Breaking a Unit Number Line.** For example: *But even more importantly, just like we can break up circles, squares, cookies and pizzas into equal parts we can* ***break up a unit number line*** *into equal parts.*

? Fractions can be plotted on a number line just like whole numbers: **true** | **false**

We can break up a unit number line into **equal parts** just like we can break up a circle into **equal parts**: **true** | **false**

REVIEW

? Draw a number line. Label it from 0 to 10 and use it to plot the numbers **0, 3, 5, 8, and 10**.

? Make up a fraction and represent it with a picture. Point out the **denominator** and the **numerator**.

What does the **denominator of your fraction** represent?

Handout #1

FRAC-NL: Fractions on a Number Line

Investigate: Breaking the Number Line

Break the number line below into **10 divisions** and then label the 'tick marks' with the missing **whole numbers** 1 through 9. Then **draw 1, 4, 6** circles.

DEFINITION: TICK MARKS

The divisions on a number line.

NOTE TO TUTOR

Make sure you point out the endpoints as 0 and 10 on the top number line and ***0 and 1*** *on the bottom number line. Point out that the scales are different.*

NOTE TO TUTOR

After the 'inner' fractions are labeled, push your student to label the endpoints (i.e. 0/3 and 3/3 of a pizza).

Now **break up the number line below into three pieces** and label it with **the correct fractions** and draw circles to represent 1/3 and 2/3.

NOTE TO TUTOR

If you feel it's necessary, grab some pizza cutouts and put them on the number line to further show the connection.

Handout #1

FRAC-NL: Fractions on a Number Line

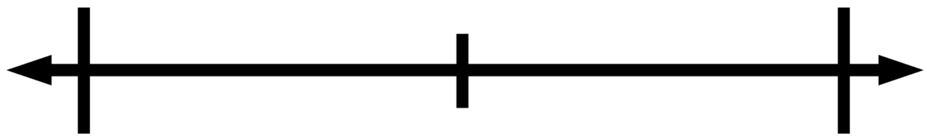
Plot It

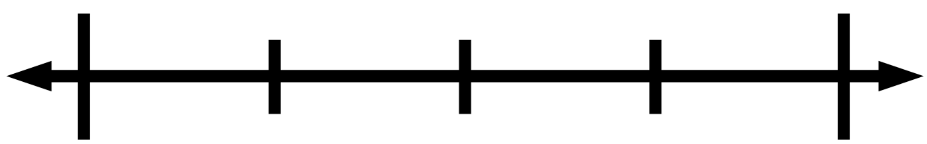
On the number lines below, label the left endpoint as '0' and the right endpoint as '1'. Then, fill in the 'tick marks' with the correct fractions. Finally, draw a picture to represent each fraction on the number line:

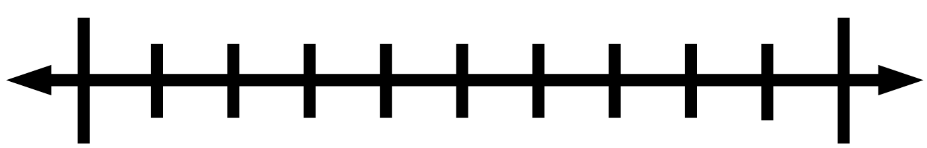
NOTE TO TUTOR

Get out your student's fraction strips (from a previous lesson) and have them compare the strips to the number line.

Ask them if they see any similarities between the strips and the number line.







Handout #1

FRAC-NL: Fractions on a Number Line

Investigate

Use the number line below to plot the following numbers:

$$\frac{3}{4},\ 2,\ 2\frac{1}{2},\ 1,\ 1\frac{5}{6},\ 0$$

TUTOR PROMPT

After having seen only a 'unit' number line, your student may struggle to figure out how to 'break up' the line from, say, 2 to 3 into halves in order to plot 2 1/2.

If your student is getting confused, have them explain their confusion. Minimally prompt them to make the connection on their own.

? Draw a picture over each of your plotted numbers to represent each number -- use any shape you want (like circles, squares, pizzas, cookies, or anything else you can think of).

Handout #1

FRAC-NL: Fractions on a Number Line

Create It: Make Your Own Line

Draw a unit number line -- that is, draw a number line and **label the endpoints as 0 and 1**.

Then, break it **fifths**. Label the tick-marks with the correct fractions and draw pictures to represent each fraction:

Now, draw a new number line below and use it to plot the following numbers. Look at the numbers **first** -- you need to decide how big of a number line to draw!

$$\frac{1}{2},\quad 5,\quad 4\frac{1}{5},\quad 2\frac{2}{3},\quad 3,\quad 3\frac{3}{4}$$

Handout #1

FRAC-NL: Fractions on a Number Line

Prove It

Prove each of the following as **true or false**. Justify your answers (make up an example to show why it is true or false):

You can use a number line to plot whole numbers. **true | false**

You can use a number line to plot fractions. **true | false**

You can use a number line to plot mixed numbers. **true | false**

Closure

What did you learn in this lesson?

Can you break up a number line just like you can break up a square or a pizza?

Worksheet #1

FRAC-NL: Fractions on a Number Line

Investigate

DIRECTIONS

Label the fractions on the number lines to the right.

Before you get started, grab the **scaffold** at the back and fill it out. Use it to plot the fractions.

Let's start with an easier problem:

Use the number line below to plot the following fractions:

$$\frac{3}{4},\quad \frac{1}{4},\quad \frac{2}{4},\quad \frac{4}{4},\quad \frac{0}{4},\quad 0$$

Now, let's make it a little bit harder. Try on your own first. If you are having problems, tell your tutor what you are finding confusing.

Use the number line below to plot the following numbers:

$$\frac{0}{2},\quad \frac{3}{4},\quad \frac{1}{3},\quad \frac{1}{2},\quad \frac{0}{4},\quad \frac{2}{3},\quad 0$$

Worksheet #1

FRAC-NL: Fractions on a Number Line

Investigate

Now let's take an even *harder* problem.

Now, let's make it a little bit harder. Try on your own first. If you are having problems, tell your tutor what you are finding confusing.

Use the number line below to plot the following numbers:

$$\frac{2}{3},\ \frac{1}{4},\ \frac{5}{6},\ \frac{1}{2},\ \frac{1}{5},\ \frac{1}{6},\ 1$$

Worksheet #1

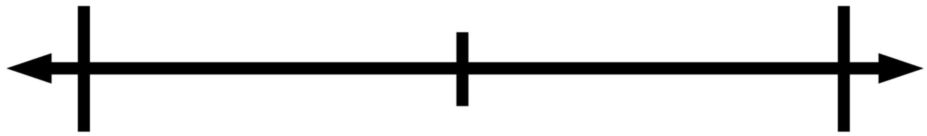
FRAC-NL: Fractions on a Number Line

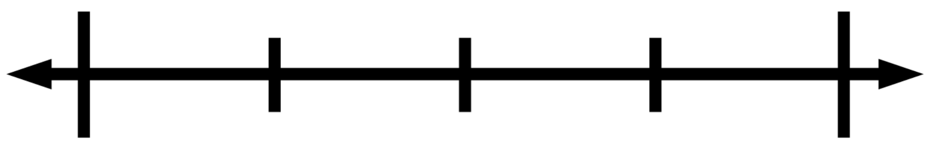
Scaffold

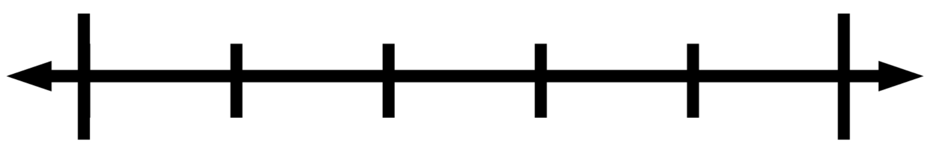
DIRECTIONS

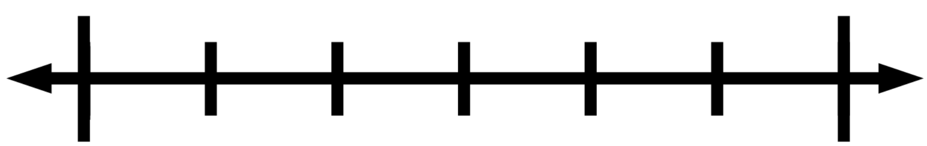
Fill in the endpoints as **0 and 1**. Then, label all the fractions.

You can use these number lines to help you solve the problems in this worksheet.









Worksheet #2

FRAC-NL: Fractions on a Number Line

Investigate

Now let's take an even *harder* problem.

Use the number line below to plot the following numbers. Don't worry about being exact -- if you need to just **estimate where the number goes**.

$$\frac{3}{4},\quad \frac{1}{3},\quad 2\frac{2}{3},\quad 2,\quad 1\frac{3}{4},\quad 2\frac{2}{4},\quad 1\frac{1}{2},\quad 2\frac{1}{4},\quad 0$$

Worksheet #2

FRAC-NL: Fractions on a Number Line

Create It: Make Your Own Line

Draw a unit number line -- that is, draw a number line and **label the endpoints as 0 and 1**.

Then, break it **fifths**. Label the tick-marks with the correct fractions and draw pictures to represent each fraction:

Now, draw a new number line below and use it to plot the following numbers. Look at the numbers **first** -- you need to decide how big of a number line to draw!

$$\mathbf{\frac{1}{3},\ \ 7,\ \ 4\frac{3}{4},\ \ 2\frac{2}{3},\ \ 3,\ \ 2\frac{1}{4}}$$

Worksheet #3

FRAC-NL: Fractions on a Number Line

Judge It: Shapes Versus Number Lines

Use the shape below to represent the fraction $\frac{1}{5}$.

NOTE TO TUTOR

Prompt your student to notice that using shapes, we can only represent one fraction at a time whereas a number line let's us see many fractions at once.

Use the shape below to represent the fraction $\frac{4}{5}$.

Now, break up the number line below into 5 pieces and label the endpoints as 0 and 1 and label the 'tick-marks':

Q1 In your own words, is breaking up a shape like a rectangle similar or different to breaking up a number line:

Q2 In your own words, what is different about using a number line to represent fractions than using pictures of shapes:

Worksheet #3

FRAC-NL: Fractions on a Number Line

Plot It

DIRECTIONS

On the number lines to the right, label the endpoints as **0 and 1**. Then, **break up your number lines** to solve the following problems.

NOTE TO TUTOR

Get out your students fraction strips and make comparisons between the strips and the number lines.

Break up the number line below into **thirds**. Label it and draw a picture above each 'tick-mark' to represent the correct fraction:

Break up the number line below into **fifths**. Label it and draw a picture above each 'tick-mark' to represent the correct fraction:

Break up the number line below into **fourths**. Label it and draw a picture above each 'tick-mark' to represent the correct fraction:

Handout #1

FRAC-EFF: Exploring Equivalent Fraction Form

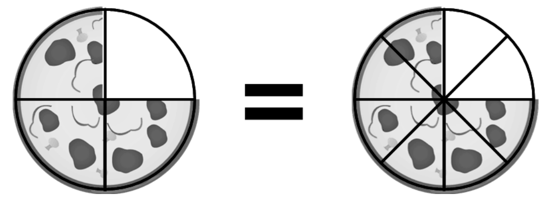
Into: Fraction Shape Shifters

LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To determine whether two fractions are equal using pictures, fraction strips, number lines, or other manipulatables.

Talk About . . .



- **Can Two Whole Numbers Look Different But Be Equal.** For example: *Can you think of two whole numbers that look different but mean the same number of things?*
- **Can Two Fractions Look Different But Be Equal.** For example: *Look at the pizzas. What fractions are they? Are they equal or different? Take your fraction strips, for example, look at 1/2 and 2/4. Are they the same size or are they different?*

Two whole numbers that look different can be equal. **true | false**

Two different fractions that look different can be equal. **true | false**

REVIEW

In your own words, define 'denominator'. Make up a fraction to use as an example in your description:

NOTE TO TUTOR

After student completion, be sure to point out 'how much of the cake' versus 'how many pieces' -- use this to reinforce the concept 'how much of the whole' and 'how many pieces from the whole' and, in general, 'whole' and 'pieces' once again before moving on.

Satish takes a cake and cuts it into eight pieces. He then eats three pieces. How much cake does Satish have left?

Write out the answer as a number and in English. Also, draw a picture to represent you answer

Handout #1

FRAC-EFF: Exploring Equivalent Fraction Form

Investigate: How Big Is 1 Piece?

Solomon ran into the following question on a recent test:

*You get a **large** pizza and cut it into 4 pieces -- you then eat three pieces.*

*Your friend also orders a **large** pizza and cuts her pizza into 8 pieces -- she then eats six pieces.*

Who ate more pizza -- you or your friend?

Here's is Solomon's answer:

Wow -- this is really easy.

Step 1: I ate 3 pieces and my friend ate 6 pieces.

Step 2: The pizzas are equal size.

Step 3: So my friend ate more pizza, because she ate 6 pieces and I only ate 3 pieces.

Is Solomon right or wrong?

First, use pictures to represent the situation. Then, use the pictures to **say whether Solomon is right or wrong:**

NOTE TO TUTOR

The main observation to be made is that fractions are not equal even if the numerators are equal.

At this point, allow the student to make observations about the fractions (which is bigger, etc.) without formally showing how to prove this is true.

NOTE TO TUTOR

If you have the time, also use pizza cutouts to represent the situation.

Handout #1

FRAC-EFF: Exploring Equivalent Fraction Form

Investigate: Different but Equal

To the right are a bunch of fractions -- each one looks different **but some** are equal in **size or magnitude**.

Put all fractions that are **equal in size or magnitude** in the same boxes.

Each time you use a fraction, put an X through it so you know you used it!

You can use **fraction strips** or pictures to complete this activity.

NOTE TO TUTOR

In the beginning, allow students to explore ways of finding equal fractions without directly showing them how you would do it. Then, show them how to do a couple using fractions strips and pictures.

NOTE TO TUTOR

To get started, tell students just to pick a fraction, then place it in a box. Then pick another, if it's not equal to any of the other fractions put it in a new box.

$\frac{2}{4}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{2}{8}$ $\frac{2}{5}$ $\frac{0}{4}$ $\frac{0}{8}$ $\frac{2}{3}$ $\frac{0}{1}$ $\frac{4}{8}$ $\frac{4}{10}$ $\frac{0}{10}$

$\frac{0}{2}$ $\frac{4}{4}$ 0 1 $\frac{1}{3}$ $\frac{0}{9}$ $\frac{2}{6}$ $\frac{0}{5}$ $\frac{1}{4}$

$\frac{3}{3}$ $\frac{3}{4}$ $\frac{0}{1}$ $\frac{6}{8}$ $\frac{3}{9}$ $\frac{6}{6}$ $\frac{5}{10}$ $\frac{4}{6}$

$\frac{10}{10}$ $\frac{0}{3}$ $\frac{1}{1}$ $\frac{5}{5}$ $\frac{6}{9}$ $\frac{9}{9}$ $\frac{8}{8}$ $\frac{0}{6}$

? What did you notice about fractions that **have the same numerator and denominator**:

What did you notice about fractions that **have the 0 for a numerator**:

Handout #1

FRAC-EFF: Exploring Equivalent Fraction Form

Prove It

Prove the following statements either **true or false**. Justify all of your answers with an example.

Whole numbers that look different can be equal. **true | false**

DEFINITION: FRACTION FORM

Two fractions can have different **form**, or look different, but be equal in **size or magnitude**.

For example:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

Fractions that look different can be equal. **true | false**

If the numerator and denominator of a fraction are the same, the fraction is equal to 1. **true | false**

Closure

What did we learn about fractions today?

Define 'equivalent fraction form' in your own words:

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Worksheet #1

FRAC-EFF: Exploring Equivalent Fraction Form

Questions

To the right are a bunch of fractions -- each one looks different **but some** are equal in **size or magnitude**.

Put all fractions that are **equal in size or magnitude** in the same boxes.

Each time you use a fraction, put an X through it so you know you used it!

You can use **fraction strips** or pictures to complete this activity.

$\frac{1}{2}$ $\frac{2}{2}$ $\frac{2}{4}$ $\frac{2}{8}$ $\frac{2}{5}$ $\frac{0}{4}$ $\frac{0}{8}$ $\frac{2}{3}$ $\frac{0}{1}$ $\frac{4}{8}$ $\frac{4}{10}$ $\frac{0}{10}$

$\frac{0}{2}$ $\frac{4}{4}$ 0 1 $\frac{1}{3}$ $\frac{0}{9}$ $\frac{2}{6}$ $\frac{0}{5}$

$\frac{1}{4}$ $\frac{3}{3}$ $\frac{3}{4}$ $\frac{0}{1}$ $\frac{6}{8}$ $\frac{3}{9}$ $\frac{6}{6}$ $\frac{5}{10}$ $\frac{4}{6}$

$\frac{10}{10}$ $\frac{0}{3}$ $\frac{1}{1}$ $\frac{5}{5}$ $\frac{6}{9}$ $\frac{9}{9}$ $\frac{8}{8}$ $\frac{0}{6}$

NOTE TO TUTOR

In this worksheet, keep pushing your student to understand a other methods. But by this time, they having probably developed a way to do this quickly.

If they can just 'see' the answer, allow them to do this but prompt them to explain how they are 'seeing' the answer.

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Worksheet #1

FRAC-EFF: Exploring Equivalent Fraction Form

Questions

To the right are a bunch of fractions -- each one looks different **but some** are equal in **size or magnitude**.

Put all fractions that are **equal in size or magnitude** in the same boxes.

Each time you use a fraction, put an X through it so you know you used it!

You can use **fraction strips** or pictures to complete this activity.

$\frac{0}{7}$ $\frac{3}{5}$ $\frac{6}{10}$ $\frac{2}{2}$ $\frac{2}{8}$

$\frac{2}{5}$ $\frac{0}{4}$ $\frac{0}{8}$ $\frac{4}{5}$ $\frac{0}{1}$ $\frac{9}{15}$ $\frac{4}{10}$ $\frac{0}{10}$

$\frac{0}{2}$ $\frac{4}{4}$ 0 1 $\frac{1}{3}$ $\frac{0}{9}$ $\frac{2}{6}$ $\frac{0}{5}$

$\frac{1}{4}$ $\frac{3}{3}$ $\frac{3}{4}$ $\frac{0}{1}$ $\frac{6}{8}$ $\frac{3}{9}$ $\frac{6}{6}$ $\frac{12}{20}$ $\frac{12}{15}$

$\frac{10}{10}$ $\frac{0}{3}$ $\frac{1}{1}$ $\frac{5}{5}$ $\frac{8}{10}$ $\frac{9}{9}$ $\frac{8}{8}$ $\frac{0}{6}$ $\frac{7}{7}$

Worksheet #2

FRAC-EFF: Exploring Equivalent Fraction Form

Investigate: Using a Number Line

DIRECTIONS

First, grab the scaffold at the back of this document. On this scaffold, label the **endpoints** on the number lines as **0 and 1** and fill in the correct fractions on the number line.

To the right are some fractions you need to plot on the provided number line. Use the scaffold to help you plot the numbers.

NOTE TO TUTOR

What's being introduced is the fact that a number line can be used to find and represent equivalent fractions.

Prompt your student to make this connection.

Use the number line below to plot the following fractions.

$$\frac{3}{4},\ \frac{2}{2},\ \frac{1}{3},\ \frac{4}{4},\ \frac{4}{6},\ \frac{2}{3},\ \frac{3}{6},\ \frac{2}{4},\ \frac{1}{2},\ \frac{5}{6},\ \frac{1}{4}$$

Now, use your work to **put all fractions that are equal in the same box**:

Worksheet #2

FRAC-EFF: Exploring Equivalent Fraction Form

Judge It: Three Ways to Compare Fractions

Use fraction strips to show that two-thirds is equal to four-sixths:

Use pictures to show that **one-half** is equal to **two-fourths**:

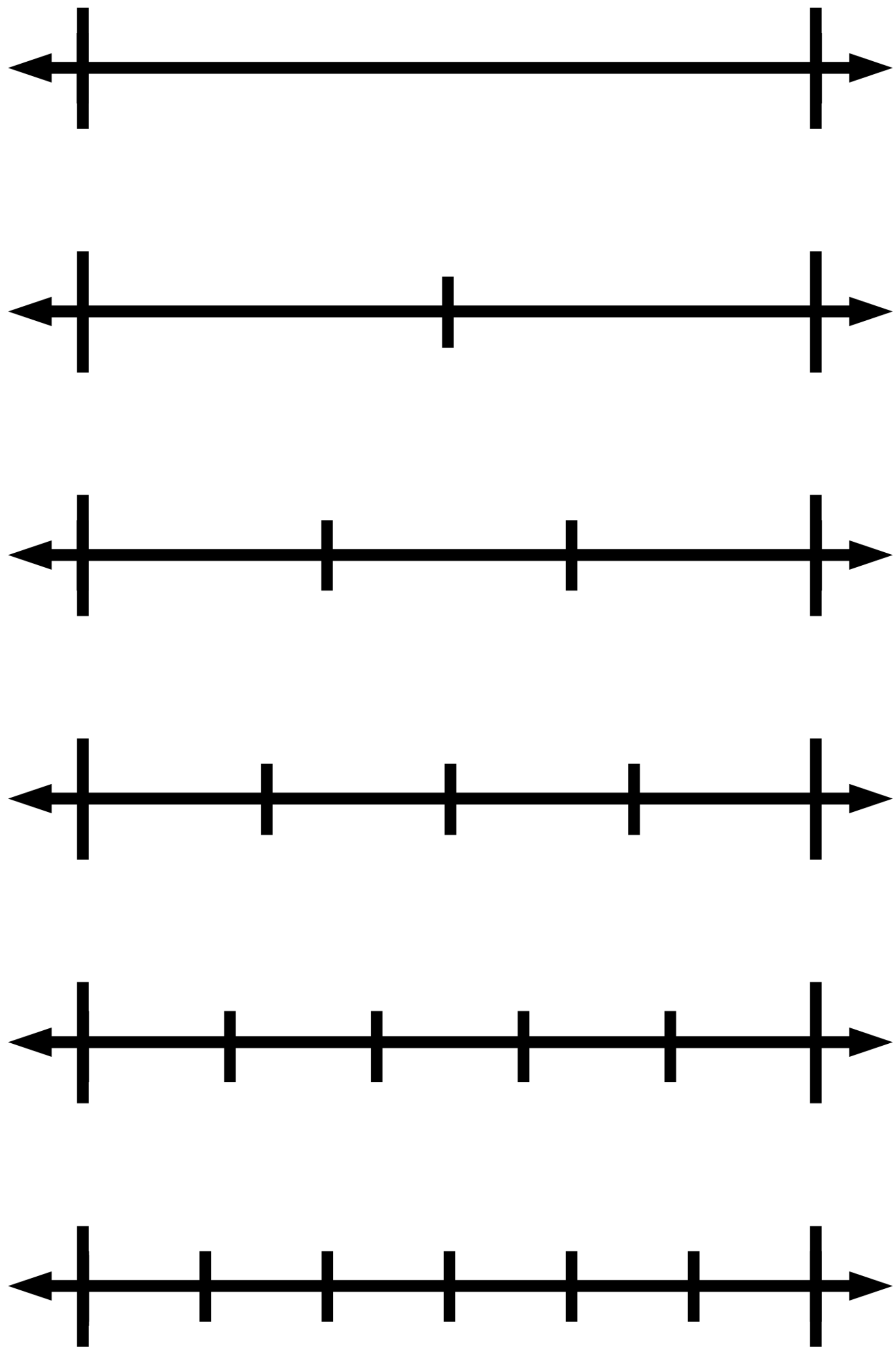
Use the number line below to show that **four-sixths** is equal to **two-thirds**:

Which method do you like best?

Worksheet #2

FRAC-EFF: Exploring Equivalent Fraction Form

Scaffold



Handout #1

FRAC-EFM: Exploring Fraction Magnitude

Into: How Big is a Fraction?

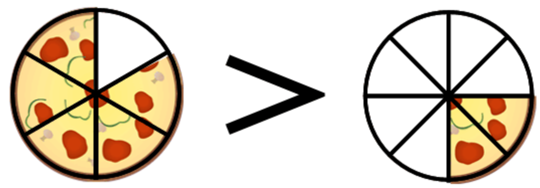
LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To be able to order fractions using pictures, fraction strips, and number lines.

Talk About . . .

• **Fractions That Look Different Can Be Equal.** For example: *And we already saw that some fractions can* ***look different or have different form*** *but be equal in size.*



• **Can A Fraction Be Bigger or Smaller Than Another Fraction?.** For example: *Can you* ***order fractions*** *just like you can order whole numbers? How can we tell if* ***one fraction*** *is bigger or smaller than another?*

If two fractions look different, they can not be equal. **true | false**

Two fractions that look different **can be** equal. **true | false**

REVIEW

Use **= , < , or >** to compare the **size or magnitude** of the following numbers:

1 _____ 2

9 _____ 5

100 _____ 200

52 _____ 52

999 _____ 1,000

91 _____ 19

? Represent **three and two-thirds** numerically. Then, draw a picture to represent this mixed number:

Handout #1

FRAC-EFM: Exploring Fraction Magnitude

Investigate: Which Fraction is Bigger?

DIRECTIONS

Use pictures or fraction strips to solve the problems to the right.

Use **= , < , or >** to compare the **size or magnitude** of the following numbers:

$\frac{1}{2}$ _____ $\frac{3}{4}$

$\frac{1}{3}$ _____ $\frac{2}{6}$

NOTE TO TUTOR

Be sure to use both pictures and fraction strips. If your student is preferring one method over another, try showing them with the other method.

$\frac{1}{3}$ _____ $\frac{4}{5}$

$\frac{1}{3}$ _____ $\frac{1}{4}$

$\frac{4}{4}$ _____ $\frac{3}{3}$

$\frac{0}{10}$ _____ $\frac{1}{2}$

Handout #1

FRAC-EFM: Exploring Fraction Magnitude

Correct It: Comparing Pieces of Pizza

DIRECTIONS

Correct Solomon's test answer.

Draw pictures to help you see whether or not he answered the problem correctly. If you want, get some pizza cutouts to also represent the situation.

Salomon ran into the following question on a recent test:

*You get a **large** pizza and cut it into 4 pieces -- you then eat three pieces.*

*Your friend also orders a **large** pizza and cuts her pizza into 6 pieces -- she then eats three pieces.*

Who ate more pizza -- you or your friend?

Here's is Salomon's answer:

Wow -- this is really easy.

Step 1: I ate three pieces and my friend ate 3 pieces.

Step 2: The pizzas are equal size.

Step 3: So we ate the same amount of pizza, because she ate 3 pieces and I ate 3 pieces.

? Is Salomon right or wrong? Depict the situation using pizza cutouts or pictures (you choose the method!).

Then, correct **each step** and explain why he is **right or wrong**:

Handout #1

FRAC-EFM: Exploring Fraction Magnitude

Questions

DIRECTIONS

Use picture, fraction strips, a number line, or any method you want to compare the fractions below.

Use **= , < , or >** to compare the **size or magnitude** of the following numbers:

$\frac{1}{2}$ ______ $\frac{3}{4}$

$\frac{1}{3}$ ______ $\frac{2}{6}$

$\frac{1}{3}$ ______ $\frac{2}{5}$

$\frac{1}{2}$ ______ $\frac{2}{2}$

$\frac{1}{8}$ ______ $\frac{1}{3}$

$\frac{5}{5}$ ______ $\frac{2}{3}$

$\frac{1}{7}$ ______ $\frac{1}{3}$

$\frac{3}{5}$ ______ $\frac{7}{8}$

Closure

Today we compared the size of fractions -- we noticed that some fractions are bigger or smaller than other fractions.

What did you learn in this lesson?

Do you like fraction strips or pictures best for comparing fraction size?

Worksheet #1

FRAC-EFM: Exploring Fraction Magnitude

Investigate

DIRECTIONS

Label the fractions on the number lines to the right.

Before you get started, grab the **scaffold** at the back and fill it out. Use it to plot the fractions.

Then, use this number line to compare the **size or magnitude** of the fractions in the problems to the right.

Let's start with an easier problem:

Use the number line below to plot the following fractions:

$$\frac{1}{3},\ \frac{1}{4},\ \frac{2}{5},\ \frac{1}{4},\ \frac{2}{6},\ \frac{3}{6},\ \frac{2}{4},\ \frac{3}{4},\ \frac{1}{2},\ \frac{5}{6}$$

Now use the number line to solve the following problems. Use **= , < , or >** to compare the **size or magnitude** of the following numbers:

$\frac{1}{2}$ ______ $\frac{3}{6}$

$\frac{2}{5}$ ______ $\frac{5}{6}$

$\frac{1}{4}$ ______ $\frac{2}{5}$

$\frac{2}{6}$ ______ $\frac{1}{3}$

$\frac{2}{6}$ ______ $\frac{1}{2}$

$\frac{3}{4}$ ______ $\frac{5}{6}$

$\frac{1}{2}$ ______ $\frac{2}{4}$

$\frac{2}{5}$ ______ $\frac{2}{6}$

Worksheet #1

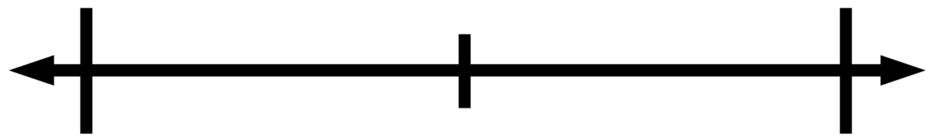
FRAC-EFM: Exploring Fraction Magnitude

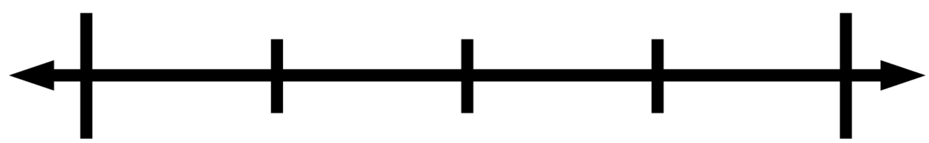
Scaffold

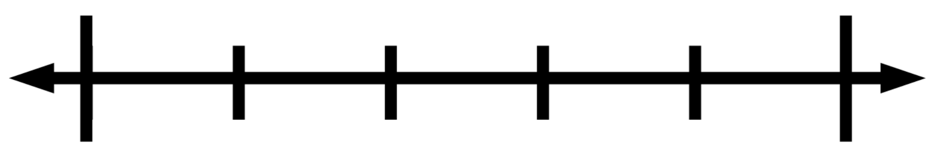
DIRECTIONS

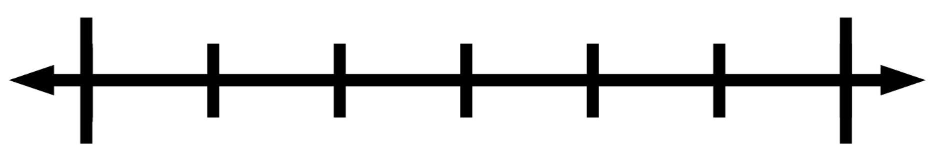
Fill in the endpoints as **0 and 1**. Then, label all the fractions.

You can use these number lines to help you solve the problems in this worksheet.









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Handout #1

MEAS-RULEIN: Using a 12 Inch Ruler

Into: How Big is a Fraction?

LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To measure lengths in inches using a 12-inch ruler.

- To compare certain fraction magnitudes using a ruler.

Talk About . . .

• **Feet and Inches.** For example: *You've probably used **feet and inches** many times before. For example, "How tall are you?"*

• **12 inch or 1 Foot Ruler.** For example: *A very common tool is a 12 inch ruler. Get out your 12 inch ruler now.*

• **Measurement and Fractions.** For example: *Very often we use fractions and mixed numbers when working with feet and inches. You often hear "two and a half inches" or "five and three-quarters of a foot".*

We're going to learn about adding numbers in our head. **true | false**

You often use fractions when measuring things. **true | false**

REVIEW

Represent **two and one-thirds** numerically. Then, draw a picture to represent this mixed number:

Draw a unit number line and label the end points as 0 and 1. Then, use it to plot the following fractions:

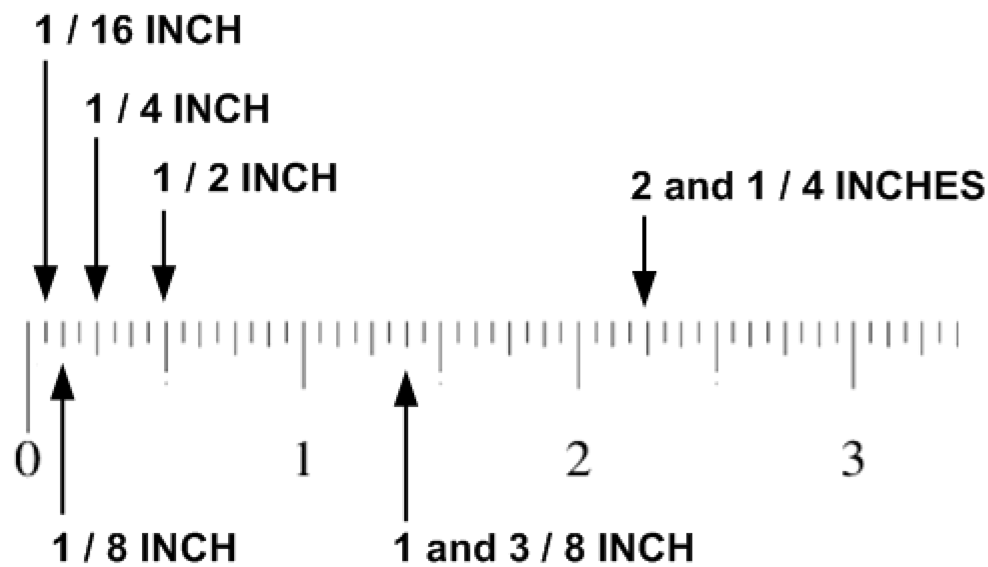
$$\frac{3}{4},\ \frac{1}{4},\ \frac{1}{2},\ \frac{2}{4},\ \frac{2}{2},\ 0$$

Handout #1

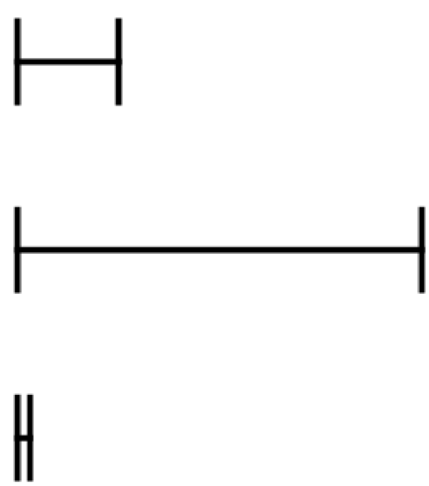
MEAS-RULEIN: Using a 12 Inch Ruler

How To: Read An Inch Rulef

Let's look at how to read a ruler:



Measure the following lines:

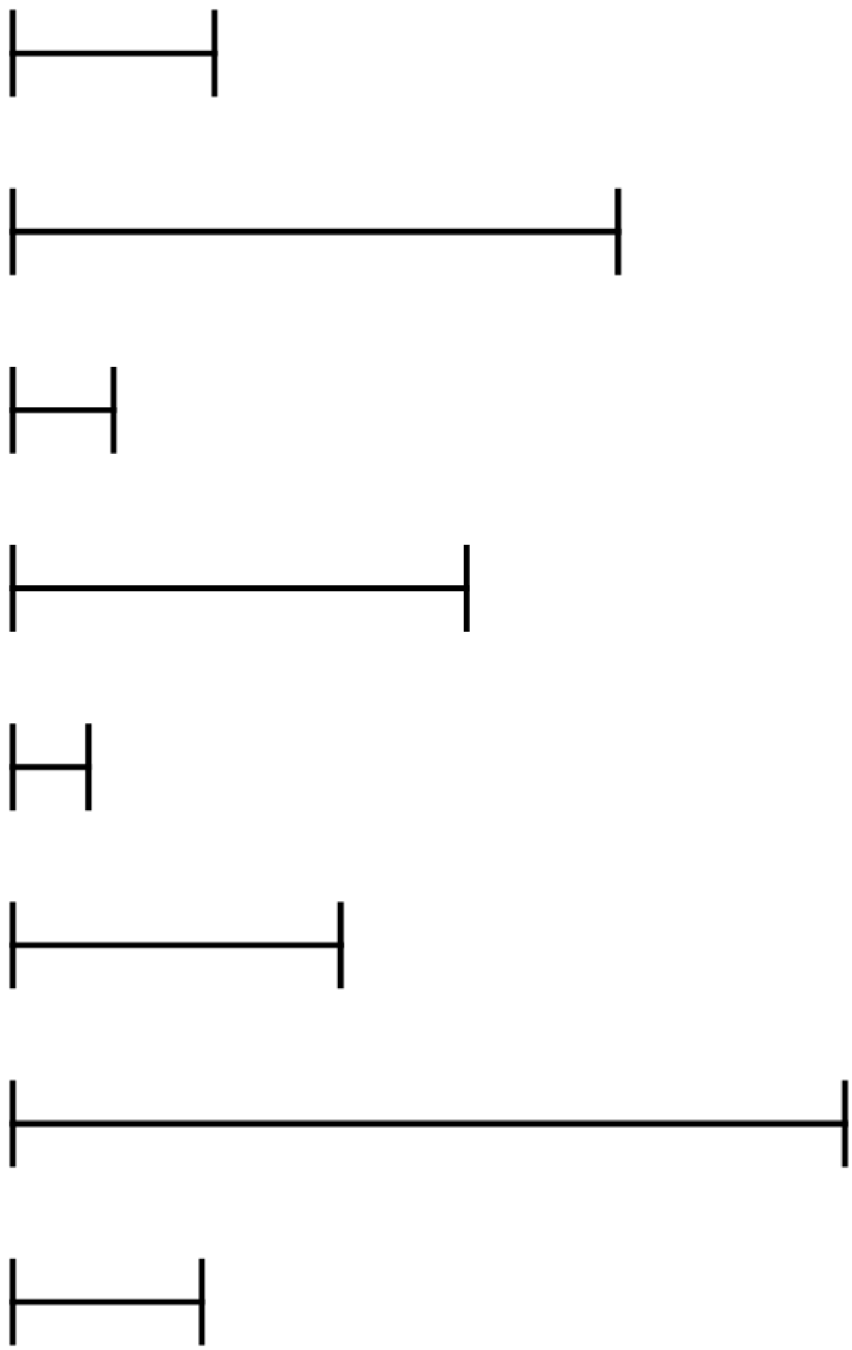


Handout #1

MEAS-RULEIN: Using a 12 Inch Ruler

Questions

Measure the following lines:



Handout #1

MEAS-RULEIN: Using a 12 Inch Ruler

Investigate: Using a Ruler to Order Fractions

DIRECTIONS

Use your ruler to draw each number **in inches**. Then use it to complete the following problems.

Use **= , < , or >** to compare the **size or magnitude** of the following numbers:

$2\frac{1}{2}$ ______ $1\frac{3}{4}$

$\frac{1}{4}$ ______ $\frac{8}{4}$

$\frac{15}{16}$ ______ 1

$1\frac{1}{2}$ ______ $1\frac{3}{4}$

Closure

Today we measure different lengths using a 12-inch ruler.

? What did you learn in this lesson?

? Did using a ruler help you better understand fractions?

Handout #1

FRAC-CFF: Calculating Equivalent Fraction Form

Into: Fraction Shape Shifters

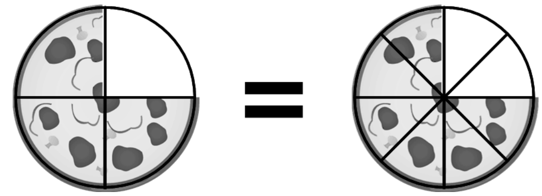
LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To be able to compare and order fraction magnitudes using pictures, number lines, and manipulatables.

Talk About . . .

- **.** For example: *Alright -- we've already seen that two fractions **can look different** but still **be equal**. That is, two equal fractions can have different **form**. We've used cutouts and pictures to show this.*



- **Changing A Fraction's Form Using Multiplication and Division.** For example: *But today we're going to make a connection with **a fraction's form** and **multiplication** and **division**.*

? Two fractions that look different are **never** equal: true | false

But before moving on, let's review a couple of concepts:

? List all the factors of **18**:

List all the factors of **24**:

List all the **common factors** of **18** and **24**:

? Using examples, prove the following true or false: 'Any whole number multiplied by 1 is equal to itself.'

? Is $\frac{1}{3}$ equal to $\frac{2}{6}$?

Justify your answer with a picture:

Handout #1

FRAC-CFF: Calculating Equivalent Fraction Form

How To: Change Fraction Form Using Multiplication and Division

We know that two fractions can look different (or **have different form**) but still be equal.

Now we're going to look at how to use **multiplication (×)** to change a fraction to a **higher form**.

Then, we'll look at how to use **division (÷)** to change a fraction to lower form.

Multiplication (x)

higher form

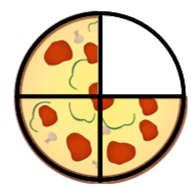
$$\frac{3}{4} = \frac{3}{4} \times \frac{2}{2} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

NOTE TO TUTOR

*It is **very important to stress** that when lowering (reducing) the form of the fraction, the divisor has to fit **evenly** into both the numerator and the denominator. Make up more examples to stress this point. Take 4/10 and show that 'dividing by 5' does not reduce this fraction.*

Division (÷)

lower form



*Notice! 2 goes into both **6** and **8** evenly!*

$$\frac{6}{8} = \frac{6}{8} \div \frac{2}{2} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

NOTE TO TUTOR

Be sure to stress the multiple way to draw the fraction 'line' as shown in this picture. Both are valid.

NOTE TO TUTOR

*Reinforce, with examples, that multiplication and division are **inverse** processes and this is a further example of this fact.*

[?] Change $\frac{2}{4}$ to a couple of **different** but equal forms.

Handout #1

FRAC-CFF: Calculating Equivalent Fraction Form

Compute It

TUTOR PROMPT

When coverting to lower form, ask your students:

'Can you think of a number that goes into both the top and bottom (numerator and denominator)?

Q1 Convert to a higher form:

$\frac{2}{3}$ $\frac{4}{10}$ $\frac{3}{9}$ $\frac{12}{24}$

Q2 Covert the following fractions to lower form:

$\frac{5}{10}$ $\frac{6}{9}$ $\frac{10}{20}$ $\frac{6}{8}$

Q3 Convert to a higher form:

$\frac{5}{5}$ $\frac{4}{5}$ $\frac{1}{3}$ $\frac{1}{2}$

Q4 Covert the following fractions to lower form:

$\frac{15}{20}$ $\frac{3}{9}$ $\frac{75}{100}$ $\frac{1,000}{2,000}$

? What's easier for you: converting a fraction to a lower form or converting a fraction to a higher form?

Handout #1

FRAC-CFF: Calculating Equivalent Fraction Form

Judge It

DIRECTIONS

To the right are a series of fractions that **look different** but are **equal in size or magnitude**.

Prove that they equal by using either **pictures** or **division**. You choose which method you want to use:

Show that $\frac{1}{2} = \frac{2}{4}$:

Show that $\frac{8}{10} = \frac{4}{5}$:

Show that $\frac{1}{3} = \frac{250}{750}$:

? To solve the problem '1/2 = 2/4' which method did you like better and why?

? To solve the problem '1/3 = 250/750' which method did you like better and why?

Handout #1

FRAC-CFF: Calculating Equivalent Fraction Form

Investigate: Lowest Form

Covert the following fractions to lower form:

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{4}{5}$ $\frac{5}{9}$

DEFINITION: LOWEST FORM

When a fraction cannot be reduced any lower, it is a fraction in **lowest** form.

Can you convert the fractions above to lower form? In your own words, answer why or why not?

NOTE TO TUTOR

The main point of this section to define 'lowest form' as the, you got it, the lowest form a fraction can take. Let them see on their own that some fractions can not be reduced any further (that is, they are in 'lowest form').

Covert the following fractions to lowest form:

$\frac{2}{4}$ $\frac{1}{3}$ $\frac{8}{10}$ $\frac{10}{20}$

$\frac{2}{6}$ $\frac{2}{2}$ $\frac{5}{10}$ $\frac{15}{30}$

$\frac{18}{20}$ $\frac{100}{200}$ $\frac{25}{75}$ $\frac{11}{33}$

$\frac{9}{9}$ $\frac{18}{36}$ $\frac{17}{34}$ $\frac{11}{99}$

NOTE TO TUTOR

*Here, connect finding lowest form to finding the **common factors** of two numbers.*

At first, let your student use whatever method is working for them.

*But then show them that **the lowest form can be found by dividing each number by the largest common factor between the two numbers**.*

Handout #1

FRAC-CFF: Calculating Equivalent Fraction Form

Prove It: Different but Equal?

Q5 In your own words, and using several examples, is the following statement true or false:

*Two fractions can look different (have a different **form**) but be equal in **size or magnitude***

Q6 In your own words, and using several examples, is the following statement true or false:

*Two fractions **that are in lowest form** can look different but be equal in **size or magnitude***

Closure

Did the fact 'when you change a fraction's form, you do not change its size (magnitude)' confuse you or not? If so, what was confusing:

? What does 'lowest form' mean:

Worksheet #1

FRAC-CFF: Calculating Equivalent Fraction Form

Compute It

Covert the following fractions to lowest form:

TUTOR PROMPT

Your students may find lowest form in multiple steps (that is, reduce the fraction to a lower form and realize they can 'keep going' to lowest form).

*Let them build this skill. At times, make up other examples and show them that **if they find the greatest** common factor of the numerator and the denominator they can reduce the fraction to lowest form in one step.*

$\frac{2}{4}$	$\frac{1}{3}$	$\frac{4}{4}$	$\frac{3}{6}$
$\frac{5}{8}$	$\frac{9}{10}$	$\frac{5}{10}$	$\frac{2}{10}$
$\frac{18}{20}$	$\frac{5}{15}$	$\frac{9}{9}$	$\frac{6}{9}$
$\frac{14}{18}$	$\frac{44}{44}$	$\frac{11}{44}$	$\frac{50}{100}$
$\frac{15}{16}$	$\frac{6}{8}$	$\frac{4}{6}$	$\frac{0}{5}$
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$
$\frac{100}{200}$	$\frac{500}{1,000}$	$\frac{250}{500}$	$\frac{80}{160}$

Worksheet #1

FRAC-CFF: Calculating Equivalent Fraction Form

Judge It: Finding Lowest Form

Reduce $\frac{\mathbf{8}}{\mathbf{10}}$ to lowest form using pictures:

Reduce $\frac{\mathbf{8}}{\mathbf{10}}$ to lowest form by using division:

Which method to like better -- pictures or division -- and why?

Worksheet #2

FRAC-CFF: Calculating Equivalent Fraction Form

Say It: Fraction Form Concept Web

DIRECTIONS

First, convert the fraction to the right to lower form using both pictures **and** on paper using division.

Then, convert the fraction below it to higher form using both pictures **and** on paper using multiplication.

Then, use these examples to connect this concept map together. Take a **red pencil** and connect all the words associated with **converting a fraction to higher form**.

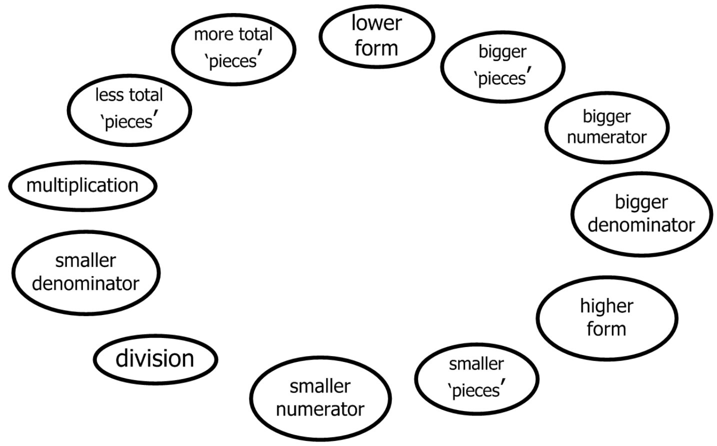
Then, take a **green pencil** and connect all the words associated with **converting a fraction to lower form**.

lower form →

$\frac{3}{6} =$

higher form →

$\frac{2}{5} =$



Circle the words so the sentence is correct:

When you convert a fraction to higher form, you **divide | multiply** both the numerator and denominator of the fraction by **the same number | different numbers**.

You then get a fraction where each piece is **smaller | larger** but you have **more total pieces | less total pieces** so the fraction remains equal.

When you convert a fraction to lower form, you **divide | multiply** both the numerator and denominator of the fraction by **the same number | different numbers**.

You then get a fraction where each piece is **smaller | larger** but you have **more total pieces | less total pieces** so the fraction remains equal.

Worksheet #2

FRAC-CFF: Calculating Equivalent Fraction Form

Prove It: Changing A Fraction to Higher Form Does Not Change The Size (Magnitude) of the Fraction

In your own words, and using several examples, is the following statement true or false:

Any whole number multiplied by the number 1 does not change that number's ***size or magnitude***

Convert to a lower form:

$\frac{4}{4}$ $\frac{5}{5}$ $\frac{10}{10}$ $\frac{229}{229}$

In your own words, and using several examples, is the following statement true or false:

Any fraction where the ***numerator equals the denominator*** *is equal to 1*

In your own words, and using several examples, is the following statement true or false:

Changing a fraction to higher form does not change its ***size or magnitude***

Worksheet #3

FRAC-CFF: Calculating Equivalent Fraction Form

Questions

To the right are a bunch of fractions -- each one looks different **but some** are equal in **size or magnitude**.

Put all fractions that are **equal in size or magnitude** in the same boxes.

Each time you use a fraction, put an X through it so you know you used it!

You can use either **division and convert the fractions to lowest form** or **use fractions strips, numberlines, pictures, or cutouts.**

You decide what works best for you!

NOTE TO TUTOR

At times, take an example and show that using fraction strips or a picture or division all leads to the same answer.

Then take another example, and have your student explain to you why all three methods work.

$\frac{1}{2}$ $\frac{2}{2}$ $\frac{2}{4}$ $\frac{2}{8}$ $\frac{0}{4}$ $\frac{0}{1}$ $\frac{2}{5}$ $\frac{2}{3}$ $\frac{4}{10}$ $\frac{0}{10}$ $\frac{4}{8}$ $\frac{0}{8}$

$\frac{0}{2}$ $\frac{0}{5}$ $\frac{1}{3}$ $\frac{4}{4}$ 1 $\frac{2}{6}$ $\frac{0}{9}$ 0 $\frac{1}{4}$

$\frac{4}{6}$ $\frac{6}{8}$ $\frac{3}{3}$ $\frac{3}{4}$ $\frac{3}{9}$ $\frac{5}{10}$ $\frac{0}{1}$ $\frac{6}{6}$

$\frac{0}{6}$ $\frac{1}{1}$ $\frac{6}{9}$ $\frac{10}{10}$ $\frac{0}{3}$ $\frac{9}{9}$ $\frac{8}{8}$ $\frac{5}{5}$

Which method did you use most often to compare fractions -- division, fraction strips, pictures or cutouts? Why?

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Worksheet #3

FRAC-CFF: Calculating Equivalent Fraction Form

Questions

To the right are a bunch of fractions -- each one looks different **but some** are equal in **size or magnitude**.

Put all fractions that are **equal in size or magnitude** in the same boxes.

Each time you use a fraction, put an X through it so you know you used it!

You can use either **division and convert the fractions to lowest form** or **use fractions strips, numberlines, pictures, or cutouts.**

You decide what works best for you!

$\frac{0}{7}$ $\frac{3}{5}$ $\frac{6}{10}$ $\frac{2}{2}$ $\frac{2}{8}$

$\frac{2}{5}$ $\frac{0}{4}$ $\frac{0}{8}$ $\frac{4}{5}$ $\frac{0}{1}$ $\frac{9}{15}$ $\frac{4}{10}$ $\frac{0}{10}$

$\frac{0}{2}$ $\frac{4}{4}$ 0 1 $\frac{1}{3}$ $\frac{0}{9}$ $\frac{2}{6}$ $\frac{0}{5}$

$\frac{1}{4}$ $\frac{3}{3}$ $\frac{3}{4}$ $\frac{0}{1}$ $\frac{6}{8}$ $\frac{3}{9}$ $\frac{6}{6}$ $\frac{12}{20}$ $\frac{12}{15}$

$\frac{10}{10}$ $\frac{0}{3}$ $\frac{1}{1}$ $\frac{5}{5}$ $\frac{8}{10}$ $\frac{9}{9}$ $\frac{8}{8}$ $\frac{0}{6}$ $\frac{7}{7}$

Handout #1

FRAC-EAS: Exploring Fraction Addition and Subtraction

Into: How Many Pieces Total?

LESSON OBJECTIVE

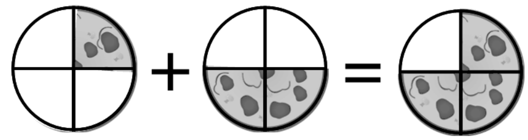
When you complete this lesson, you will be able:

- To add and subtract fractions that already have a common denominator.

- To represent fraction addition and subtraction using pictures, manipulatables, and number lines.

- To reduce an answer to lowest form.

Talk About . . .



- **Whole Number Addition.** For example: *It's easy to add whole numbers, right? Like if you have three apples and I have two apples, we have **5 apples total**.*
- **Fraction Addition.** For example: *But can we add fractions, like if you ate 1/2 a pizza and I ate 1/4 of the pizza, how much pizza did we eat together?*
- **Common Denominator.** For example: *But when adding fractions, we have to pay attention to the **denominator** -- we have to make sure both fractions have a **common denominator**.*

Today we'll learn the definition of the numerator: **true** | **false**

It is possible to add fractions: **true** | **false**

REVIEW

Define **numerator** and **denominator**. Make up a fraction and draw a picture to explain your answer:

Which fraction is bigger $\frac{2}{3}$ or $\frac{3}{5}$?

Use pictures, a number line, or use fraction strips to answer the problem:

Handout #1

FRAC-EAS: Exploring Fraction Addition and Subtraction

Investigate: How Many Pieces Did You Eat?

NOTE TO TUTOR

Symbolically means using the math symbols for fractions and addition.

Have them write this out as 1/4 + 2/4 = 3/4. Then, have them draw a picture over each fraction so they can see it pictorially.

Take the following situation:

*You order a large pizza and cuts it into **four equal pieces** (into **fourths**). For breakfast, you eat one piece. For lunch, you eat two pieces. How much of the pizza did you eat total? Use pictures to solve the problem:*

Now, write this as an **addition problem** and solve it **symbolically**. Also, draw a picture over each fraction to show what's going on:

Investigate: Adding Fractions -- Numerators and Denominators

Susan is solving a problem below, here is her work. If it is wrong, correct each step and say why it is wrong. Use pictures to justify your answer.

You split an orange into eight pieces. You eat three pieces and give two away to a friend. How much of your orange did you give away?

Step 1: Okay, this is an addition problem.

Step 2: So I set it up like this and solve it:

$$\frac{2}{8} + \frac{3}{8} = \frac{5}{16}$$

? What does this example teach us about adding fractions?

Handout #1

FRAC-EAS: Exploring Fraction Addition and Subtraction

Investigate: How Many Pieces of Pizza Do You Have Left?

NOTE TO TUTOR

Again, feel free to reinforce this using multiple approaches (such as fraction strips).

Take the following situation:

*You order a 16" pizza and cuts it into **six equal pieces** (into **sixths**). For breakfast, you eat 1 piece. How much of a pizza do you have left?*

Write this as a **subtraction problem** and solve it **symbolically**. Also, draw a picture over each fraction to show what's going on:

Kelly takes an orange and splits it into fifths. She gives three pieces away to a friend. How much of an orange does she have left? Solve this **both symbolically and using pictures:**

A cake is split into eight equal pieces (into **eighths**). You eat 3 pieces and your friend eats 2 pieces. How much of the cake did you and your friend eat? Solve this **both symbolically and using pictures:**

Handout #1

FRAC-EAS: Exploring Fraction Addition and Subtraction

Questions

DIRECTIONS

Draw pictures to solve each problem.

NOTE TO TUTOR

At times, get them to say the problem out loud.

*Stress questions like 'if you have **one**-fourth and you have **another** **two**-fourths, how many **fourths** do you have total?*

Draw pictures below each fraction to show what's going on:

Q1 $\frac{1}{2} + \frac{0}{2} =$	**Q2** $\frac{4}{5} - \frac{1}{5} =$
Q3 $\frac{1}{3} + \frac{1}{3} =$	**Q4** $\frac{2}{4} - \frac{1}{4} =$

Solve directly:

Q5 $\frac{5}{8} + \frac{2}{8} =$

Q6 $\frac{4}{5} - \frac{2}{5} =$

Q7 $\frac{7}{9} + \frac{1}{9} =$

Q8 $\frac{8}{9} - \frac{8}{9} =$

Handout #1

FRAC-EAS: Exploring Fraction Addition and Subtraction

Investigate: Reducing to Lowest Form

NOTE TO TUTOR

Be sure your student draws pictures of pizza for each step.

It's very important to show that:

1 We visualize the problem using pictures to perform the addition.

2 We can visualize reduction to lowest form using pictures.

DaShaun and Raul are both taking the same test. On the following question:

*You make a pizza cut it into eighths. You eat 1 piece and your friend eats 2 pieces. How much of the pizza did you and your friend eat **combined**?*

Here are both of the answers? Using pictures, correct each answer and say whether it is right or wrong:

Raul's Answer	**DaShaun's Answer**
$\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$ of a pizza	$\frac{1}{8} + \frac{3}{8} = \frac{\cancel{4}^{1}}{\cancel{8}_{2}} = \frac{1}{2}$ of a pizza

Now, let's change the problem slightly:

*You make a pizza cut it into eighths. You eat 1 piece and your friend eats 2 pieces. How much of the pizza did you and your friend eat **combined**? Be sure and reduce your answer to **lowest form**.*

Now correct each of their answers:

Raul's Answer	**DaShaun's Answer**
$\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$ of a pizza	$\frac{1}{8} + \frac{3}{8} = \frac{\cancel{4}^{1}}{\cancel{8}_{2}} = \frac{1}{2}$ of a pizza

? In your own words, is an answer wrong if you don't reduce to lowest form?

Why do most people always reduce to lowest form?

Handout #1

FRAC-EAS: Exploring Fraction Addition and Subtraction

Judge It

Let's say you eat **three-eighths** of a pizza and your friend also eats **three-eigths of a pizza**. How much did you eat total?

In the table below, solve this both directly or symbolically AND solve it using pictures.

DEFINITION: SYMBOLICALLY

Symbolically means using the math symbols to get to an answer.

Use Numbers	Use Pictures
1. Add The Fractions	 + =
2. Reduce to LOWEST form	

Which method did you like better: using math symbols or using pictures? Why?

Compute It

Solve **directly and reduce** to lowest form (only use a picture if you need to):

Q9 $\frac{2}{4} + \frac{2}{4} =$

Q10 $\frac{4}{5} - \frac{3}{5} =$

Q11 $\frac{0}{5} + \frac{2}{5} =$

Q12 $\frac{5}{8} - \frac{5}{8} =$

Q13 $\frac{7}{10} + \frac{1}{10} =$

Q14 $\frac{6}{6} - \frac{4}{6} =$

Q15 Tom puts one-eighth of a cup of flour and five-eighths of a cup of flour into a bowl. Draw a picture showing this situation and solve. Remember to reduce to lowest form when finished.

Q16 June Ping eats three-fifths of a cookie. How much of her cookie does she have left. Draw a picture showing this situation and solve. Remember to reduce to lowest form when finished.

Closure

What did you learn in this lesson?

Do you find addition and subtracting fractions hard or easy?

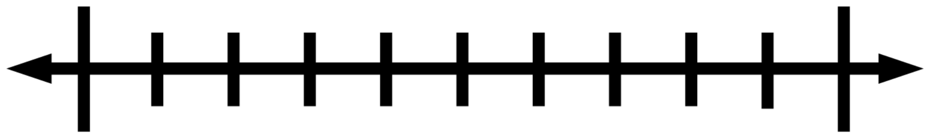
Is it harder than adding or subtracting whole numbers? If so, what's harder?

Worksheet #1

FRAC-EAS: Exploring Fraction Addition and Subtraction

Connections: Using a Number Line to Add Fractions

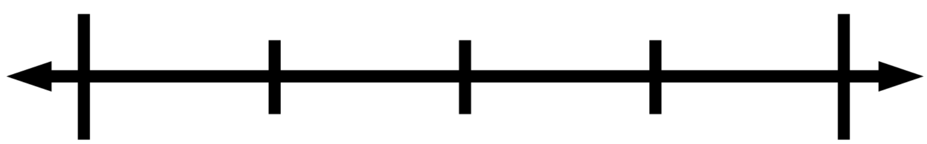
Label the endpoints of the number line below as **0 and 10** and fill in the tick marks.



Now use it to solve the following problems:

5 + 2 = **4 + 3 =** **9 - 2 =**

Label the endpoints of the number line below as **0 and 1** and fill in the tick marks.



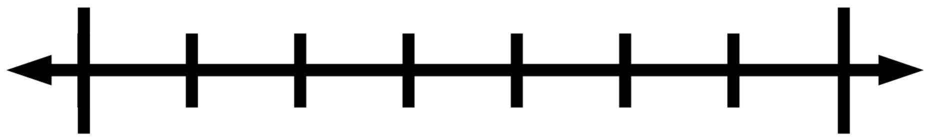
$\frac{1}{4} + \frac{1}{4} =$ $\frac{3}{4} - \frac{1}{4} =$ $\frac{2}{4} + \frac{2}{4} =$ $\frac{4}{4} - \frac{3}{4} =$

Worksheet #1

FRAC-EAS: Exploring Fraction Addition and Subtraction

Connections: Using a Number Line to Add Fractions

Label the endpoints of the number line below as **0 and 1** and fill in the tick marks.



$\frac{1}{7} + \frac{3}{7} =$ $\qquad$ $\frac{2}{7} - \frac{0}{7} =$ $\qquad$ $\frac{6}{7} + \frac{1}{7} =$ $\qquad$ $\frac{7}{7} - \frac{7}{7} =$

Handout #1

FRAC-CFM: Calculating Fraction Magnitude

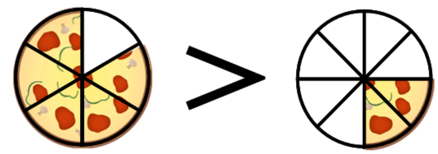
LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To directly compare and order fraction magnitudes by finding common denominators.

Into: Calculating Fraction Magnitude

Talk About . . .



• **Fractions Can Have Different Form.** For example: *We know two fractions can look different but be equal. That is, we know 1/2 is equal to 2/4 -- 2/4 is just in a higher form.*

• **Fractions Can Have Different Magnitude.** For example: *And we also know that a fraction can be bigger or smaller than another fraction. We've shown this in the past with by comparing pictures and cutouts.*

• **Using Direct Calculation to Compare Fraction Magnitude.** For example: *But today we're going to work on using direct calculation to find a common denominator between two fractions so we can compare them directly. This is often a lot faster and easier (once you learn it!) than using pictures or number lines to compare fraction magnitude.*

REVIEW

Convert to a higher form. Use both pictures and multiplication to justify your answer:

$\frac{2}{3}$ $\qquad$ $\frac{1}{1}$

Use pictures, fraction strips, or any method you want to solve the following problems. Use **= , < , or >** to compare the **size or magnitude** of the following numbers:

$\frac{1}{2}$ ______ $\frac{3}{6}$ $\qquad$ $\frac{3}{5}$ ______ $\frac{3}{4}$

$\frac{1}{2}$ ______ $\frac{2}{3}$ $\qquad$ $\frac{2}{3}$ ______ $\frac{4}{6}$

Handout #1

FRAC-CFM: Calculating Fraction Magnitude

Definitions: Common Denominator

We know we can **change a fraction** to higher form. That is, we can make both the numerator and the denominator bigger at the same time.

This keeps **size of the fraction the same** because **the number of total pieces gets bigger (the numerator gets bigger)** but **each piece also gets smaller (the denominator gets bigger)**.

We are going to change fractions to higher form now in order to **find a common denominator between them**. This makes **all the pieces the same size** and allows us to more easily compare the **size or magnitude** of different fractions.

DEFINITION: COMMON DENOMINATOR

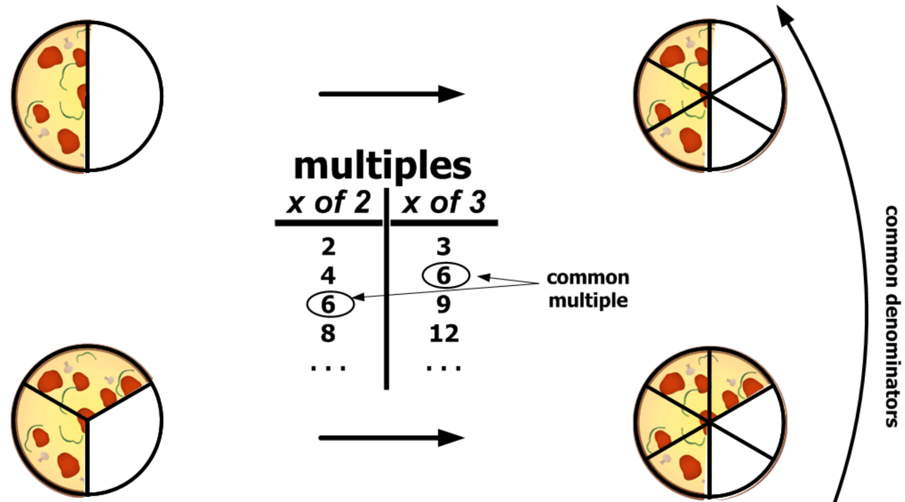
When two fractions have the **same** denominator.

Often, you have to **find** a common denominator of two fractions by **changing one or both fractions** to a **different form**.

DEFINITION: LOWEST COMMON DENOMINATOR

The smallest possible common denominator between two fractions.

$$\frac{1}{2} = \frac{1}{2} \times \frac{3}{3} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$



$$\frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Make a table (like in the picture) and list the first few multiples of the numbers **5** and **4**?

Now, use that table to write $\frac{3}{5}$ and $\frac{3}{4}$ with common denominators:

Which fraction is smaller?

Handout #1

FRAC-CFM: Calculating Fraction Magnitude

Compute It

Use **= , < , or >** to compare the **size or magnitude** of the fractions.
Compare the fractions by finding a **common denominator**.

NOTE TO TUTOR

At times (a couple of examples), draw a pictures like circles for your student to show that finding the common denominator is the same as a pictorial representation.

But allow them time to practice the steps of finding a common denominator.

$\frac{1}{2}$ ______ $\frac{3}{4}$ $\qquad$ $\frac{1}{3}$ ______ $\frac{2}{3}$

$\frac{1}{5}$ ______ $\frac{1}{10}$ $\qquad$ $\frac{5}{6}$ ______ $\frac{2}{3}$

$\frac{1}{2}$ ______ $\frac{1}{3}$ $\qquad$ $\frac{3}{4}$ ______ $\frac{7}{8}$

$\frac{6}{7}$ ______ $\frac{1}{14}$ $\qquad$ $\frac{3}{4}$ ______ $\frac{4}{5}$

$\frac{1}{2}$ ______ $\frac{2}{2}$ $\qquad$ $\frac{9}{10}$ ______ $\frac{1}{2}$

? Find a common denominator for all the fractions below and then order them from least to greatest:

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{4}, \frac{0}{3}, \frac{2}{2}$$

Handout #1

FRAC-CFM: Calculating Fraction Magnitude

DIRECTIONS

Solve the problem to the right in **three different ways**. Then talk with your tutor about which method you liked best and why.

Judge It

Which fraction is bigger $\frac{2}{3}$ or $\frac{3}{5}$?

Solve the problem using pictures:

Solve the problem using fraction strips:

Solve the problem by making the pieces equal by finding a **least common denominator**:

Closure

Today we learned how to find a common denominator for two different fractions. This allowed us to **directly compare their size or magnitude**.

What is a **common denominator** for two fractions?

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Handout #1

FRAC-DIV: Fractions as Division

Into: Fractions, Multiplication, and Division

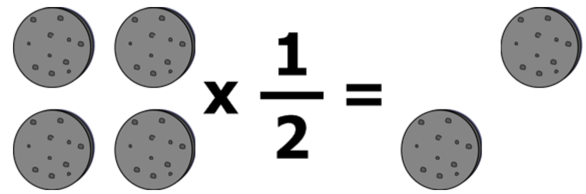
LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To compare a fraction with dividing by a whole number.

- To define an improper fraction.

Talk About . . .



- **What Are Fractions?.** For example: *Remember, fractions are used when we break up a whole into pieces. Then we can describe pieces of the whole.*

- **The Concept of a Whole.** For example: *In the past, we often used the whole to be 1 whole pizza or 1 whole cookie. But today, we're going to work on different kinds of wholes like a bag of cookies or a room full of people -- that's going to be our whole we're going to break up.*

REVIEW

Is $\frac{3}{6}$ equal to $\frac{1}{2}$?

Make sure you prove your answer:

Find a common denominator for all the fractions below and then order them from least to greatest:

$$\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{1}{3}, \frac{3}{4}$$

Handout #1

FRAC-DIV: Fractions as Division

Investigate: Fair Share

NOTE TO TUTOR

If your student is stuck, have them cut each burger into 3 pieces. Then, divvy up the 'thirds' among three people.

That will lead to an improper fraction of 8/3 burgers per person which is 2 and 2/3 a burger each -- have them explore this concept. This is formally introduced in the next section.

Consider the following problem:

You buy 8 hamburgers for a barbecue. 3 people show up. Assuming every one ***gets a fair share*** *how many hamburgers can each person eat? (Note, you can cut up the hamburgers.)*

Draw a picture to represent the problem and solve:

NOTE TO TUTOR

For the 'wall painting' problem, draw the ten gallons and break each gallon into fourths. Then, label the first 10 'fourths' with a '1' for wall one. Show them how the gallons can be equally spread among the four walls.

? You buy 10 gallons of paint to paint 4 walls. How many gallons of paint does each wall need? (Be sure also to draw a picture to show the 10 gallons and how you split them up.)

Handout #1

FRAC-DIV: Fractions as Division

Definitions: Mixed Numbers and Improper Fractions

You may have noticed in the last problem we had fractions **where the numerator** was bigger than the **denominator**. This is called an improper fraction.

This is very much **related to division** as we show in the picture to the right.

Mixed Number: $2\frac{3}{4}$

Improper Fraction: $\frac{11}{4}$

Using Division

$$\begin{array}{r} 2\ R\ 3 \\ 4\overline{)11} \\ \underline{-8} \\ 3 \end{array}$$

DEFINITION: IMPROPER FRACTION

A fraction where the numerator is bigger than the denominator.

These fractions are always bigger in **size of magnitude** than 1.

In your own words, what's a mixed number?

In your own words, what's a improper fraction?

Draw a picture using circles representing 4 2/3:

Write out a fraction representing **ten-thirds** -- then draw a picture to represent this improper fraction:

Handout #1

FRAC-DIV: Fractions as Division

Questions

TUTOR PROMPT

Prompt your student to draw pictures to represent the situation on a couple of problems.

Change the improper fraction to a mixed number **or vice versa**:

$\frac{14}{3}$ =

$\frac{7}{2}$ =

$3\frac{1}{5}$ =

$\frac{20}{7}$ =

$15\frac{1}{2}$ =

$\frac{9}{9}$ =

$\frac{1}{3}$ =

$5\frac{5}{6}$ =

$\frac{8}{5}$ =

$10\frac{1}{10}$ =

Handout #1

FRAC-DIV: Fractions as Division

Say It

Let's solve two very similar but different problems and then compare them. Make sure to draw a picture for each problem. Here's the first problem:

Q1: Alaysia gets out 24 pieces of chocolate. She then finds 6 plastic bags. How many pieces of chocolate go into each bag?

Here's the second problem:

Q2: Alaysia gets out 24 pieces of chocolate. She then finds 5 plastic bags. How many pieces of chocolate go into each bag?

? What is different about these problems?

What kind of number is your final answer in both problems?

Handout #1

FRAC-DIV: Fractions as Division

Questions

Change the improper fraction to a mixed number **or vice versa**:

$\frac{5}{2} =$ $3\frac{5}{7} =$

Q1: At a recent birthday party, there were 1 cake and 5 people. How much of a cake did each person get?

Q2: At a recent birthday party, there were 3 cakes and 5 people. How much of a cake did each person get?

Q3: At a recent birthday party, there were 11 cupcakes and 5 people. How much of a cake did each person get?

The U.S. Census Bureau estimates that there are 7 children for every 3 married couples in the US. On average, how many children does each married couple have?

Closure

Today, we learn about improper fractions, how to convert them to and from mixed numbers, and looking at the connections between fractions and division.

Does making a connection between fractions and division make sense to you?

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

Into: How Many Pieces Total?

LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To add and subtract fractions and mixed numbers and reduce to lowest form.

- To find common denominators between fractions if necessary.

- To represent fraction addition and subtraction problems using pictures.

Talk About . . .

• **Adding Fractions.** For example: *We already saw that we can add fractions, right? Just add the numerators (add up the pieces) and we know that **we keep the denominator the same on both fractions**.*

• **What If The Denominators Are Not The Same.** For example: *But what if **the pieces of each whole are different sizes**, then what? Can we still add or subtract them, or not?*

• **Mixed Numbers.** For example: *And what if we have mixed numbers as well as improper fractions? Can we still add or subtract them, or not?*

• **It's Most Important To Understand The 'Big Idea' And To Be Able To Represent The Problem With Pictures.** For example: *As we'll see, there can be a lot of steps when adding and subtracting mixed numbers and fractions. While it's important to learn some steps, it's more important that you can understand what's going on; to understand the basic logic and be able to represent the problem using pictures. Then, if you forget a step, you'll always be able to figure out a strategy to get to the right answer.*

? Today we'll working on multiplying and dividing fractions: **true | false**

When you adding two fractions, you add up the denominators: **true | false**

REVIEW

? Make up a fraction. Use it to define **numerator** and **denominator**. Draw a picture for your fraction and use the picture in your explanation:

? Rewrite $\frac{3}{4}$ and $\frac{5}{6}$ so they both have a common denominator:

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

Investigate: How Many Pieces Did You Eat?

Solve the following problem:

Sosa buys a large pizza and cut it into fourths. Sosa eats 1 piece of pizza. Rocio buys a large inch pizza and cuts it into sixths. Rocio eats 1 piece of pizza.

Draw a picture to **represent the situation** in the boxes below:

NOTE TO TUTOR

After your student has had time to construct a strategy, explicitly reinforce the concept of "denominator" and "piece size" and that "the pieces need to be the same size".

Sosa's Pizza	Rocio's Pizza

Now answer the following question:

How much of a pizza did Sosa and Rocio eat ***combined****?*

What do you notice about this problem?

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

How To: Add Fractions

There's a lot to do when adding fractions. Most of the time we:

1 Need to represent the fractions using numbers or pictures or both.

2 If we need to, we have to change the form of one or both of the fractions so they both have a common denominator.

3 Add the fractions.

4 If we need to, convert the answer to lowest form.

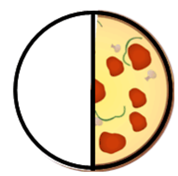
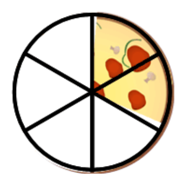
Let's use the steps to your left to solve the following problem:

You make a pizza and cut it into sixths. You eat two pieces.

*Your friend makes a pizza of the same size and cuts it in half and eats **one of those big pieces**.*

*How much pizza did you and your friend eat **combined**?*

Fill in the table below for each step and solve this problem both **numerically** and by **using pictures**:

Use Numbers	Use Pictures
1. Represent the fractions. $\frac{2}{6}$,	, 
2. Convert to common denominator. ,	
3. Add the fractions.	+ =
4. Reduce to lowest form.	

Do you find any of these steps hard?

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

Questions

$\frac{1}{4} + \frac{1}{2} =$

$\frac{1}{3} + \frac{1}{2} =$

$\frac{1}{6} + \frac{1}{3} =$

$\frac{3}{5} + \frac{1}{6} =$

TUTOR PROMPT

On one problem, draw a picture to represent the situation.

On another, prompt your student to draw a picture.

$\frac{1}{9} + \frac{1}{3} =$

$\frac{3}{5} + \frac{3}{10} =$

You have 3 cans of paint. Your friend has 5 cans of paint. How many cans of paint do you have combined?

You have two paint cans -- each one is a 1 gallon paint can. In one paint can, you have 1/3 of a gallon of paint left. In the other, you have 1/5 of a gallon of paint left. How many cans of paint do you have total?

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

Investigate: The Great Sandwich Giveaway

Take the following problem:

You have 5/8 of a sandwich. You then give half of your sandwich away. How much of your sandwich do you have left?

Draw a picture to represent the situation and solve.

Questions

$\mathbf{\frac{1}{3} - \frac{1}{6} =}$

$\mathbf{\frac{7}{8} - \frac{3}{4} =}$

$\mathbf{\frac{5}{9} - \frac{1}{3} =}$

$\mathbf{\frac{5}{10} - \frac{1}{2} =}$

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

Investigate: Mixing In Mixed Numbers

Take the following problem:

Yesterday, you ran 5/8 of a mile. Today, you ran 1/2 a mile. How many miles did you run total?

Draw a picture to represent the situation and solve:

? Sam buys two and one-half pounds of hamburger for dinner. Realizing he doesn't have enough, he goes back and buys another one and one-third pounds. How many pounds did he buy total?

? When making some noodles, you first put in **six and one-half** cups of water. Then, you notice you put in too much. So you take out **one and one-fourth** cups of water. How much water is left in the pan?

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

Compute It

Solve **directly and reduce to lowest form** :

Q1 $3\frac{1}{2} + 2\frac{1}{4} =$

Q2 $3\frac{1}{2} - 2\frac{1}{4} =$

NOTE TO TUTOR

Grab the cutouts at the back of this handout.

Every now and then, use them to represent the a problem.

Also, every now and then, draw a picture for your student or have them do the same.

Remember, you can keep making up problems!

Q3 $\frac{3}{4} + \frac{1}{2} =$

Q4 $1\frac{2}{3} - \frac{1}{2} =$

Q5 $\frac{0}{6} + \frac{5}{5} =$

Q6 $\frac{9}{10} - \frac{4}{5} =$

Q7 $11\frac{2}{3} + 5\frac{1}{6} =$

Q8 $3\frac{5}{6} - 2\frac{1}{3} =$

Q9 $5\frac{1}{6} + \frac{2}{5} =$

Q10 $\frac{8}{9} - \frac{2}{3} =$

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

Questions

Q11 Last year Harry grew five and one-half inches. This year Harry grew four and two-fifth inches. How many inches did Harry grow over the last two years?

TUTOR PROMPT

Get your student to draw pictures to represent the problem.

Q12 Sarah has three and two-thirds pizzas. She then gives one and one-sixth pizzas to her friends. How many pizzas does Sarah have left?

Closure

Today we worked a lot more on adding and subtracting mixed numbers and fractions. There can be a lot to do and there are often many steps.

What did you learn in this lesson?

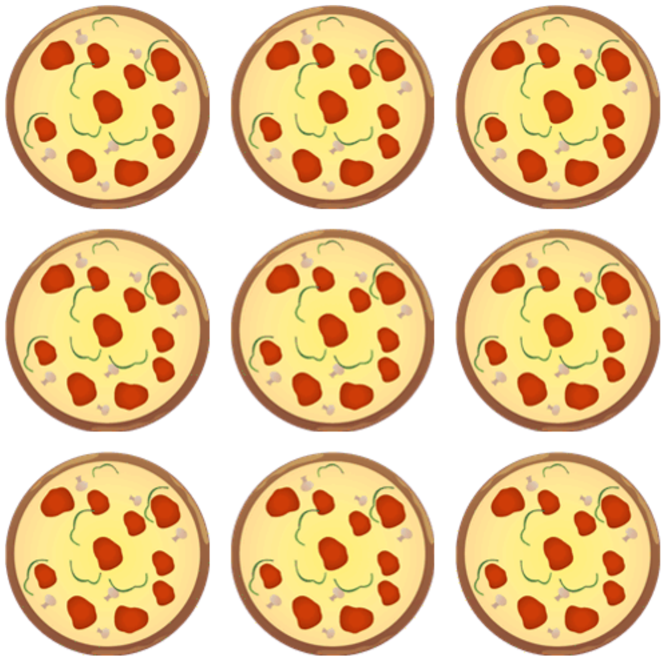
What did you find hard to do?

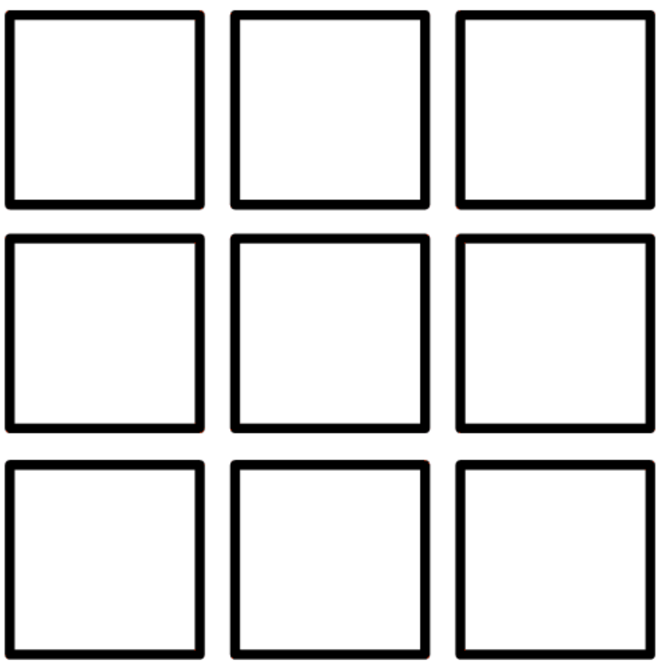
What did you find easy to do?

Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

-- cutouts --

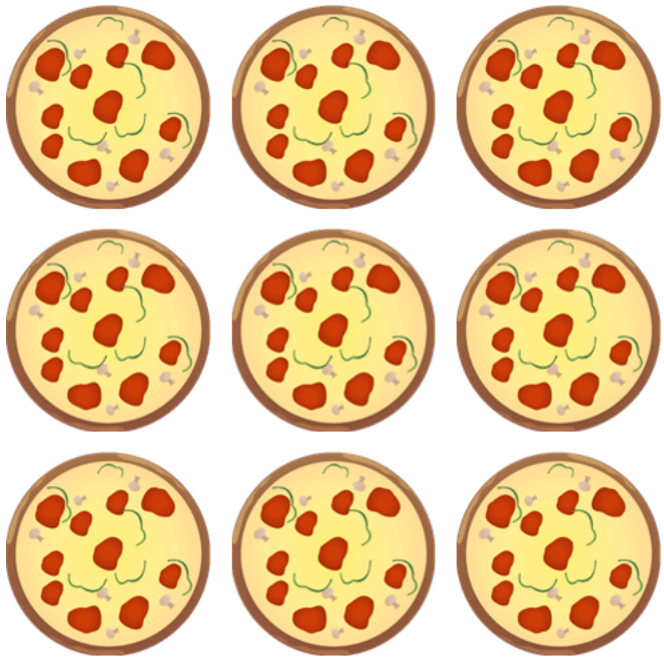


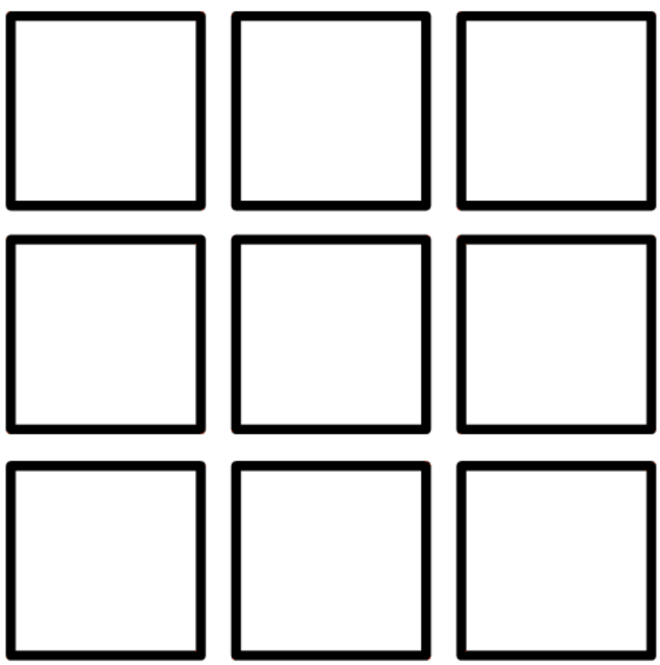


Handout #1

FRAC-CAS: Calculating Fraction Addition and Subtraction

-- cutouts --





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Handout #1

FRAC-MW: Multiplying Whole Numbers and Fractions

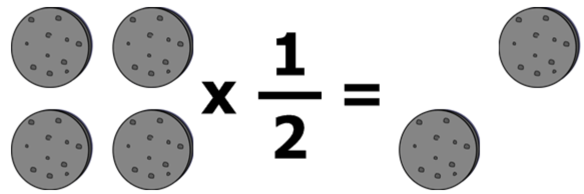
LESSON OBJECTIVE

When you complete this lesson, you will be able:

- To multiply a whole number by a fraction and reduce to lowest form.

Into: Fractions, Multiplication, and Division

Talk About . . .



• **What Are Fractions?.** For example: *Remember, fractions are used when we break up a whole into pieces. Then we can describe pieces of the whole.*

• **The Concept of a Whole.** For example: *In the past, we often used the whole to be 1 whole pizza or 1 whole cookie. But today, we're going to work on different kinds of wholes like a bag of cookies or a room full of people -- that's going to be our whole we're going to break up.*

REVIEW

? Is $\frac{3}{6}$ equal to $\frac{1}{2}$?

Make sure you prove your answer both directly (symbolically) and by using pictures:

? Find a common denominator for all the fractions below and then order them from least to greatest:

$$\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{1}{3}, \frac{3}{4}$$

Handout #1

FRAC-MW: Multiplying Whole Numbers and Fractions

Investigate: How Many Cookies Did You Eat

DIRECTIONS

Get the "little" cookies attached to his handouts.

They are colored on two sides -- use the light side to mean "A Cookie You Ate" and the dark side to mean "A Cookie You Have Left"

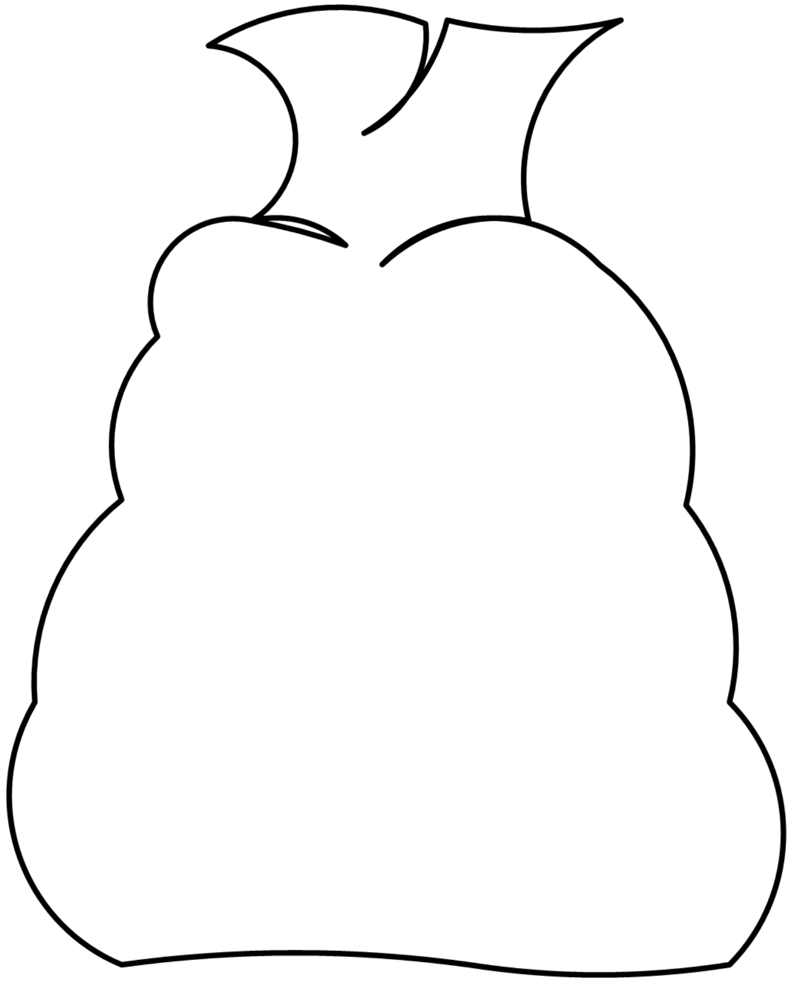
Use this bag and the cookies to solve the problems below:

TUTOR PROMPT

Keep making up examples. Use evenly divisible fractions -- for example for eight cookies use 1/4, 1/2, and 3/4. For six cookies use 1/3 and 2/3, etc.

The bag of cookies is "the whole" in these problems. So, in this problem, you "ate 3/4 ***of the bag*** *of cookies."*

Reinforce the similarity between breaking up a circle and breaking up this "bag of cookies" which is our "whole amount" in this problem.



Let's say you eat **three-fourths** of the bag of cookies. Show that with the cutouts.

? How many cookies did you eat?

How many cookies do you have left?

How **much of the bag** of cookies did you eat?

? In these problems, what is "the whole?"

Handout #1

FRAC-MW: Multiplying Whole Numbers and Fractions

How To: Multiply By A Fraction

Take the following problem: *You buy a bag of cookies. Each bag contains 8 cookies. You then eat three-fourths of a bag of cookies. How many cookies did you eat?*

Let's look at how to solve this using pictures and by multiplying by a fraction:

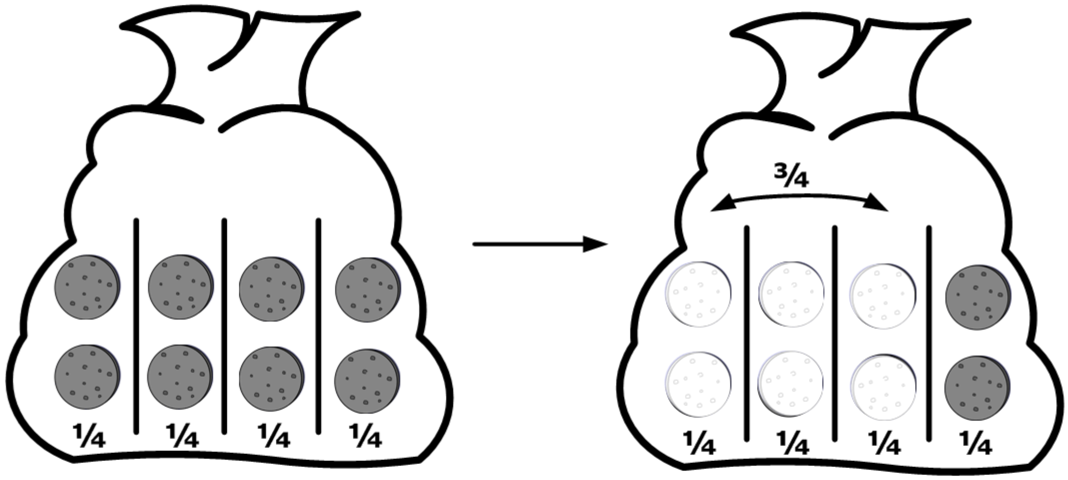
TUTOR PROMPT

Point out the use of the word "of". We need three-fourths of a bag.

¾ of 8 cookies

TUTOR PROMPT

Point out -- "How much is 1/4 of a bag? -- it's 2 cookies. So how many fourths do we have? three-fourths. So we ate 2 x 3 = 6 cookies.



TUTOR PROMPT

Show them there's more than one way to solve a problem. First, multiply to get 24/4. Then reduce to get 6/1 = 6.

Cancel!

$$8 \times \frac{3}{4} = \frac{\cancel{8}^{\,2}}{1} \times \frac{3}{\cancel{4}_{\,1}} = \frac{6}{1} = 6$$

? At the start of your birthday party there are ten people at your house. By 7pm, two-fifths of them have gone home. How many people went home?

Solve this with pictures and by multiplying by a fraction.

Handout #1

FRAC-MW: Multiplying Whole Numbers and Fractions

Compute It

NOTE TO TUTOR

At times, use the cookies to represent the problem.

Also, at times represent the problem using pictures. Sometimes draw a picture for your students and vice versa.

Q1 $9 \times \frac{1}{3} =$

Q2 $10 \times \frac{1}{5} =$

Q3 $\frac{1}{4} \times 16 =$

Q4 $15 \times \frac{2}{3} =$

Q5 $\frac{3}{5} \times 10 =$

Q6 $8 \times \frac{0}{2} =$

Q7 $\frac{3}{7} \times 14 =$

Q8 $14 \times \frac{7}{7} =$

Handout #1

FRAC-MW: Multiplying Whole Numbers and Fractions

Investigate

Q: Soran buys 8 pounds of hamburger at the store. That night, he uses one-third of his hamburger for dinner. How many pounds of hamburger did Soran use for dinner?

Make sure to draw a picture to represent the problem. Then solve:

NOTE TO TUTOR

Make sure your student writes out the multiplication problem "8 x 1/3 = ".

A roll of carpet contains 14 square yards. To cover a hall wall in his house, Joe needs one-fifth a roll of carpet. How many square yards of carpet does he need?

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Handout #1

FRAC-MW: Multiplying Whole Numbers and Fractions

How To: Use Fraction Multiplication Strategies

There are **many ways** to multiply fractions.

Here's a common strategy used by many people:

1. First, write both numbers as fractions and line up the two numbers.
2. Then, cross cancel out anything you can.
3. Multiply the numerator and the denominators.
4. Reduce the fraction to a mixed number in lowest form.

But remember, the most important thing is that **you understand** what's going on. You can use any strategy you want, as long as everything is making sense.

These are just steps many people use because **they are usually the fastest**.

> REMEMBER!
>
> It's more important that you understand what's going on and can draw a picture to represent the problem than remember these steps.
>
> You might forget the steps, but if you understand what's going on you'll always get to the right solution, even if it takes a little longer.

Let's take a look at how to **multiply 8 by five-sixths**:

Cancel by 2!

$$8 \times \frac{5}{6} = \frac{\cancel{8}^{\,4}}{1} \times \frac{5}{\cancel{6}_{\,3}} = \frac{20}{3} = 6\frac{2}{3}$$

REMEMBER!!! Sometimes you <u>can't</u> cancel anything.

Change Improper Fraction to Mixed Number

Now let's try some practice:

Q9 $8 \times \frac{5}{6} =$

Q10 $15 \times \frac{3}{10} =$

Q11 $11 \times \frac{1}{2} =$

Q12 $15 \times \frac{5}{6} =$

Handout #1

FRAC-MW: Multiplying Whole Numbers and Fractions

+−×÷ Compute It

Q13 $9 \times \frac{1}{2} =$

Q14 $10 \times \frac{1}{3} =$

Q15 $12 \times \frac{1}{5} =$

Q16 $5 \times \frac{2}{3} =$

Q17 $14 \times \frac{2}{3} =$

Q18 $9 \times \frac{5}{6} =$

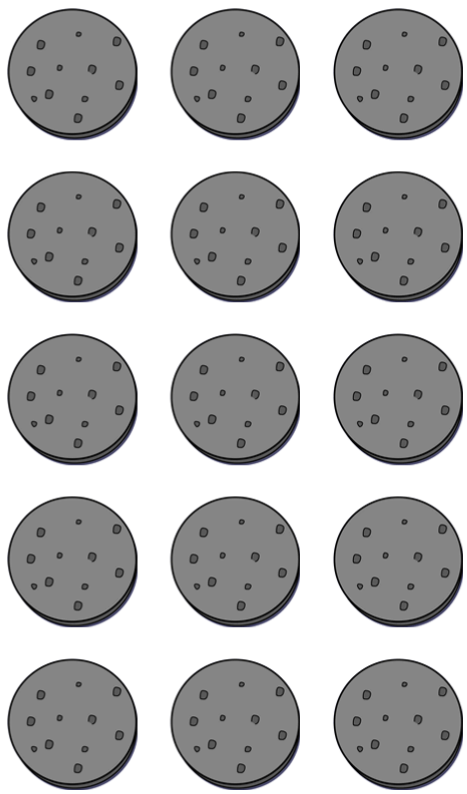
? You buy a ten-gallon bucket of paint and use one-third of the bucket. How many gallons of paint did you use?

Closure

What did we learn about today?

What do you find hard about multiplying fractions?

-- cutouts --



-- cutouts --