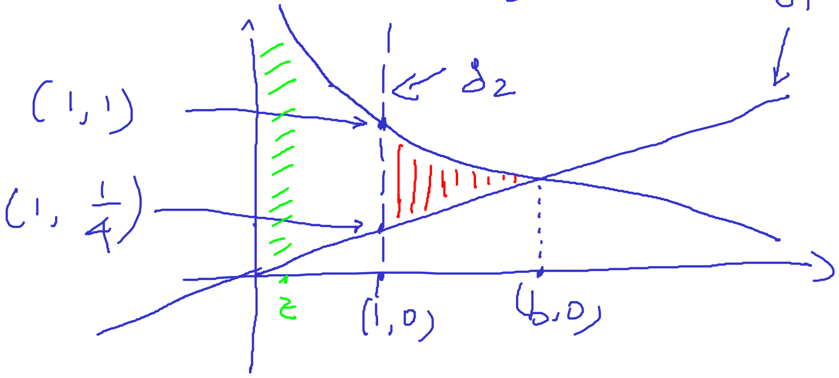
(1) $A\subseteq\mathbb{R}^2$, $A=\{(x,y)\in \overset{]0,+\infty[}{[1,+\infty[}\times\mathbb{R} \;/\; \underbrace{\frac{x}{4}}_{g_1(x)}\le y\le \underbrace{\frac{1}{x}}_{g_2(x)}\}$

$$\int_A x^2 e^{xy}\,dx\,dy$$

Studio $g_1(x)\le g_2(x)$

$$\Leftrightarrow \frac{x}{4}\le\frac{1}{x} \Leftrightarrow x^2=4$$

$$\Leftrightarrow \begin{cases} x=\pm 2\\ x\ge 1\end{cases} \Rightarrow b=2$$



$$\int_A x^2 e^{xy}\,dx\,dy=\int_1^2\left(\int_{x/4}^{1/x} x^2 e^{xy}\,dy\right)dx=\int_1^2 x^2\left(\int_{x/4}^{1/x} e^{xy}\,dy\right)dx$$

$$=\int_1^2 x^2\left[\frac{e^{xy}}{x}\right]_{y=x/4}^{y=1/x}dx=\int_1^2\left(x\underbrace{e^{x/x}}_{=e}-x\,e^{x^2/4}\right)dx$$

$$=\left[\frac{e}{2}x^2-2e^{x^2/4}\right]_1^2=\cancel{(2e-2e)}-\left(\frac{e}{2}-2e^{1/4}\right)$$

$$\frac{d}{dx}\frac{x^2}{4}=\frac{x}{2}$$

(2) $f(x,y)=y^2-[1+x(y+1)]^{1/2}$ punti critici e classificazione

$$\text{Dominio}=\{(x,y)\in\mathbb{R}^2 \;/\; 1+x(y+1)\ge 0\}$$

$$\nabla f(x,y)=\left(-\frac{1}{2}[1+x(y+1)]^{-1/2}(y+1),\; 2y-\frac{1}{2}[1+x(y+1)]^{-1/2}x\right)=(0,0)$$

$$\begin{cases} 1+x(y+1)>0\\ y+1=0\\ 2y-\frac{x}{2}[1+x(y+1)]^{-1/2}=0\end{cases} \Rightarrow \begin{cases} y=-1 \quad (\Rightarrow \text{prima condiz vera})\\ -2-\frac{x}{2}\underbrace{[1+0]^{-1/2}}_{=1}=0\end{cases}$$

$$\begin{matrix}x=-4\\ y=-1\end{matrix} \Rightarrow \underline{P(-4,-1)}$$

$$\partial_{xx}f=-\frac{y+1}{2}\cdot\left(-\frac{1}{2}[1+x(y+1)]^{-3/2}(y+1)\right)=\frac{(y+1)^2}{4}[1+x(y+1)]^{-3/2} \to =0 \text{ per } y=-1$$

$$\partial_{xy}f=-\frac{1}{2}\cdot\left(-\frac{1}{2}\right)[1+x(y+1)]^{-3/2}\underbrace{x(y+1)}_{=0}-\frac{1}{2}\underbrace{[1+x(y+1)]^{-1/2}}_{=1}\cdot 1\Big|_{y=-1}=-\frac{1}{2}$$

$\partial_{yy} f = \cdots$ $\quad Hf(-4,-1) = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & \boxed{*} \end{bmatrix} \longrightarrow$ ha det < 0

$\Leftarrow$ p. di sella.

$A = \begin{pmatrix} 0 & b \\ b & c \end{pmatrix} \qquad b, c \in \mathbb{R} \qquad \Rightarrow \underline{\det(A) = -b^2 < 0}$.

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③ $f(x,y,z) = x^2 z e^{y^2}$. Direz. massima crescita in $(-1, 2, 3)$. Taylor

$\nabla f(x,y,z) = \left(2xze^{y^2}, 2y\,x^2 z e^{y^2}, x^2 e^{y^2}\right)$

$\nabla f(-1,2,3) = \left(-6e^4, 12e^4, e^4\right) = \underline{e^4(-6, 12, 1)}$ $\quad \|$

$v_{max} = \dfrac{\nabla f(-1,2,3)}{|\nabla f(-1,2,3)|} = \dfrac{(-6,12,1)}{|(-6,12,1)|} = \dfrac{1}{\sqrt{181}}(-6,12,1)$

$\dfrac{\partial f}{\partial v_{max}}(-1,2,3) = \langle \nabla f(-1,2,3), v_{max} \rangle = \dfrac{|\nabla f(-1,2,3)|^2}{|\nabla f(\text{---})|} = |\nabla f(-1,2,3)|$

$= e^4 |(-6,12,1)| = e^4\sqrt{181}$

$\langle \nabla f(-), \dfrac{\nabla f(-)}{|\nabla f(-)|} \rangle = \dfrac{1}{|\nabla f|} \langle \nabla f, \nabla f \rangle = |\nabla f|$

$\mathbb{R}^n \qquad \langle x, y \rangle \underset{Def}{=} \sum_j x_j y_j$

$|x| := \sqrt{\langle x, x \rangle}$

ES $\quad f(x,y) = (x+1)^{1/2} e^{-x^2 y} \qquad v_{max}$ in $\left(1, \frac{1}{2}\right)$

Se $w \in \mathbb{R}^2$ ha norma unitaria e $w \perp v_{max}$, calcolare $\dfrac{\partial f}{\partial w}\left(1, \frac{1}{2}\right)$

$\nabla f(x,y) = \left(\dfrac{1}{2\sqrt{x+1}} e^{-x^2 y} + \sqrt{x+1}\cdot(-2xy)e^{-x^2 y}, \; -x^2\sqrt{x+1}\, e^{-x^2 y}\right)$

$\nabla f\left(1, \frac{1}{2}\right) = \left(\dfrac{1}{2\sqrt{2}} e^{-1/2} - 2\sqrt{2}\cdot\dfrac{1}{2} e^{-1/2}, \; -\sqrt{2}\, e^{-1/2}\right)$

$= e^{-1/2}\left(\dfrac{1}{2\sqrt{2}}(1-4), -\sqrt{2}\right) = e^{-1/2}\left(-\dfrac{3}{2\sqrt{2}}, -\sqrt{2}\right)$

$= \dfrac{e^{-1/2}}{2\sqrt{2}}(-3, -4) \quad \Rightarrow \dfrac{\nabla f}{|\nabla f|}\left(1, \frac{1}{2}\right) = \dfrac{(-3,-4)}{\sqrt{9+16}} = \left(-\dfrac{3}{5}, -\dfrac{4}{5}\right)$

Se $w \in \mathbb{R}^2$, $|w| = 1$. Formula gradiente

$$\frac{\partial f}{\partial w}\left(1,\tfrac12\right) = \left\langle \nabla f\left(1,\tfrac12\right), w \right\rangle = \left(w \perp \frac{\nabla f(1,\frac12)}{|\nabla f(1,\frac12)|} \right)$$

$$= 0$$

$A = \{ (x,y) \in \mathbb{R}^2 : x \in [0,2], \ |y| \le 2\sqrt{x}, \ y \ge \sqrt{x} \}$.

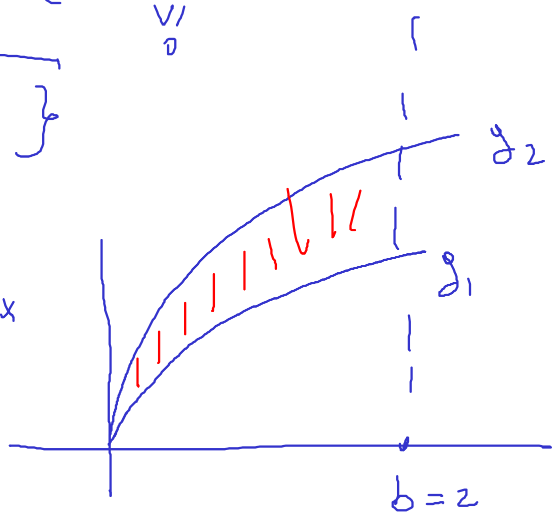
$$\int_A \sqrt{y^2 + x^2 y^2}\, dx\, dy$$

$$\begin{cases} -2\sqrt{x} \underset{(*)}{\le} y \le 2\sqrt{x} \\ y \underset{(**)}{\ge} \underbrace{\sqrt{x}}_{\ge 0} \end{cases}$$

< 0

inutile

$\{ (x,y) \ / \ x \in [0,2], \ \underbrace{\sqrt{x}}_{g_1} \le y \le \underbrace{2\sqrt{x}}_{g_2} \}$



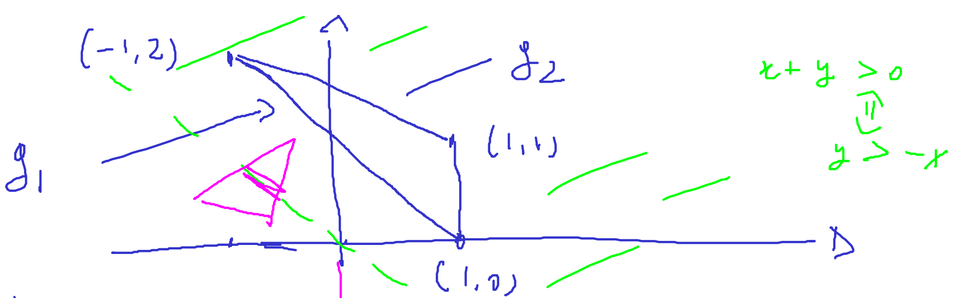
$$\int_A \sqrt{\underbrace{y^2 + x^2 y^2}_{y^2(1+x^2)}}\, dx\, dy = \int_0^2 \left(\int_{\sqrt{x}}^{2\sqrt{x}} y\sqrt{1+x^2}\, dy \right) dx$$

$$= \int_0^2 \sqrt{1+x^2} \left[\frac{y^2}{2} \right]_{y=\sqrt{x}}^{y=2\sqrt{x}} dx = \frac12 \int_0^2 \sqrt{1+x^2}\,(4x - x)\, dx$$

$$= \frac32 \int_0^2 \underbrace{x\sqrt{1+x^2}}_{f(x)}\, dx = \frac34 \int_0^2 2x\sqrt{1+x^2}\, dx = \frac34 \left[\frac{(1+x^2)^{3/2}}{3/2} \right]_0^2.$$

Es: Triangolo vertici $(1,0)$, $(-1,2)$ $(1,1)$

$$\int_T \frac{1}{x+y}\, dx\, dy$$



$g_1(x) = -x + 1$

$g_2(x) = g_2(1) + g_2'(1)(x-1) = 1 - \frac12 (x-1) = -\frac{x}{2} + \frac32$

$$\int_T \frac{1}{x+y}\, dx\, dy = \int_{-1}^{1} \left(\int_{-x+1}^{(3-x)/2} \frac{1}{x+y}\, dy \right) dx = \int_{-1}^{1} \Big[\ln|x+y| \Big]_{y=-x+1}^{y=\frac32 - \frac{x}{2}} dx$$

$$= \int_{-1}^{1} \left[\ln \left(x + \frac{3}{2} - \frac{x}{2} \right) - \cancel{\ln \left(x - x + 1 \right)} \right] dx \quad = \quad \text{int. per parti} \ldots$$