



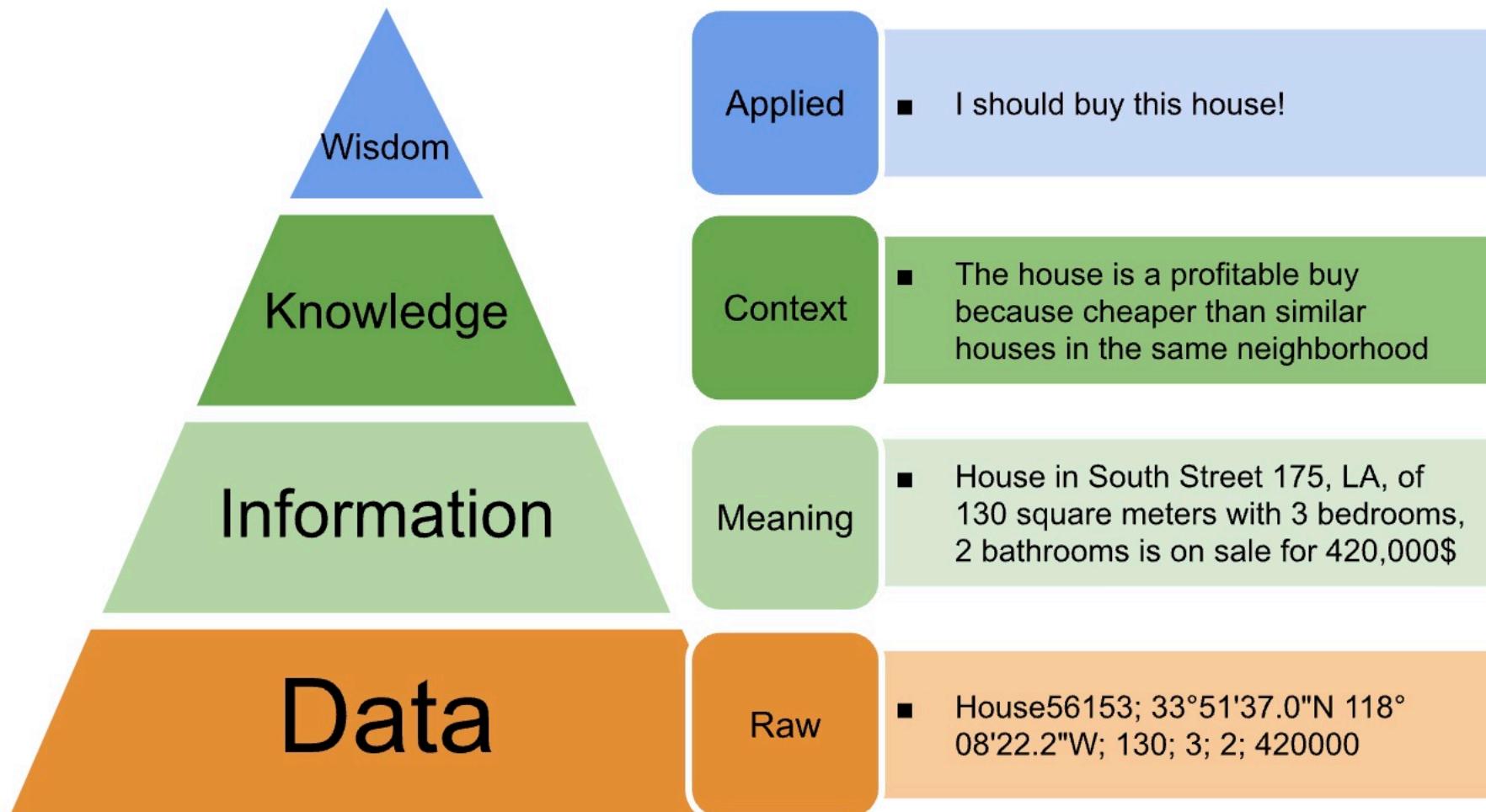
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# Data Analytics

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# From data to wisdom

**DIKW Pyramid:** Typically information is defined in terms of data, knowledge in terms of information, and wisdom in terms of knowledge



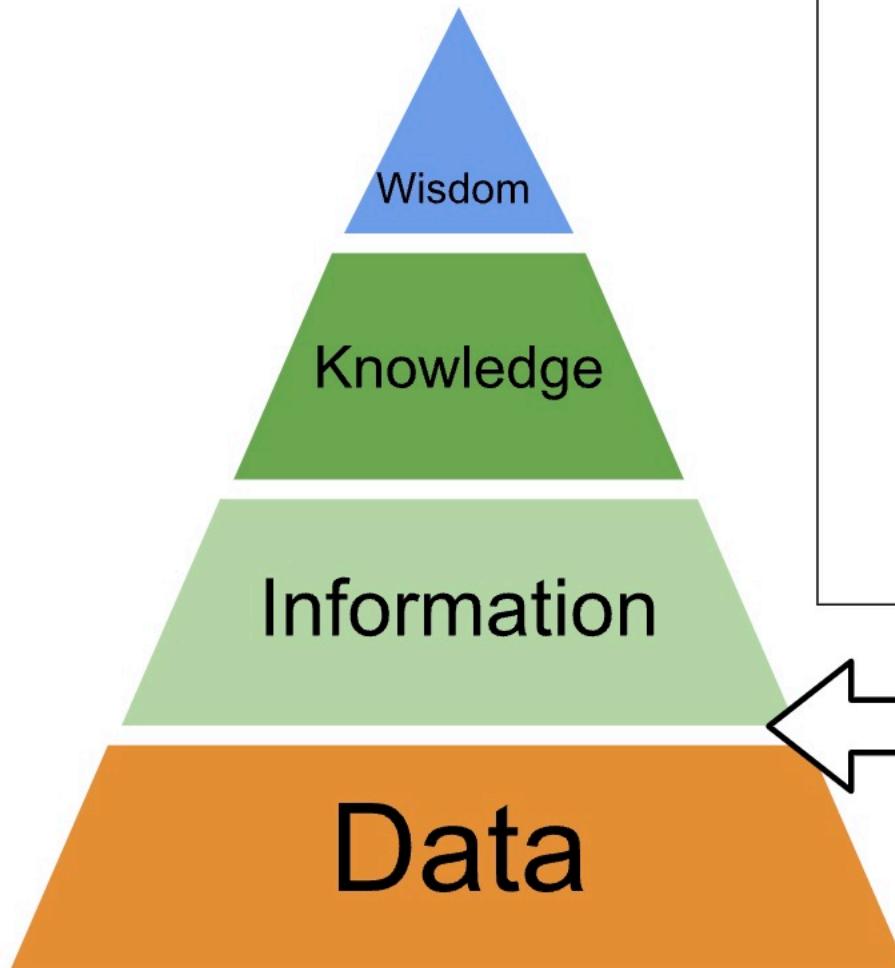


# So far in this course

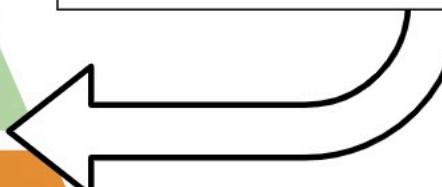
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- **Semi-structured data**
  - Storing and querying data without having a rigid schema.
  - How semi-structured data relates to structured data and can be queried using a query language for structured data (SQL)
- **Information retrieval**
  - Retrieve a subset of documents with respect to user's information need
  - Searching in the WWW
  - Ranking documents by their relevance to a text query and user's feedback

# Where in the pyramid?

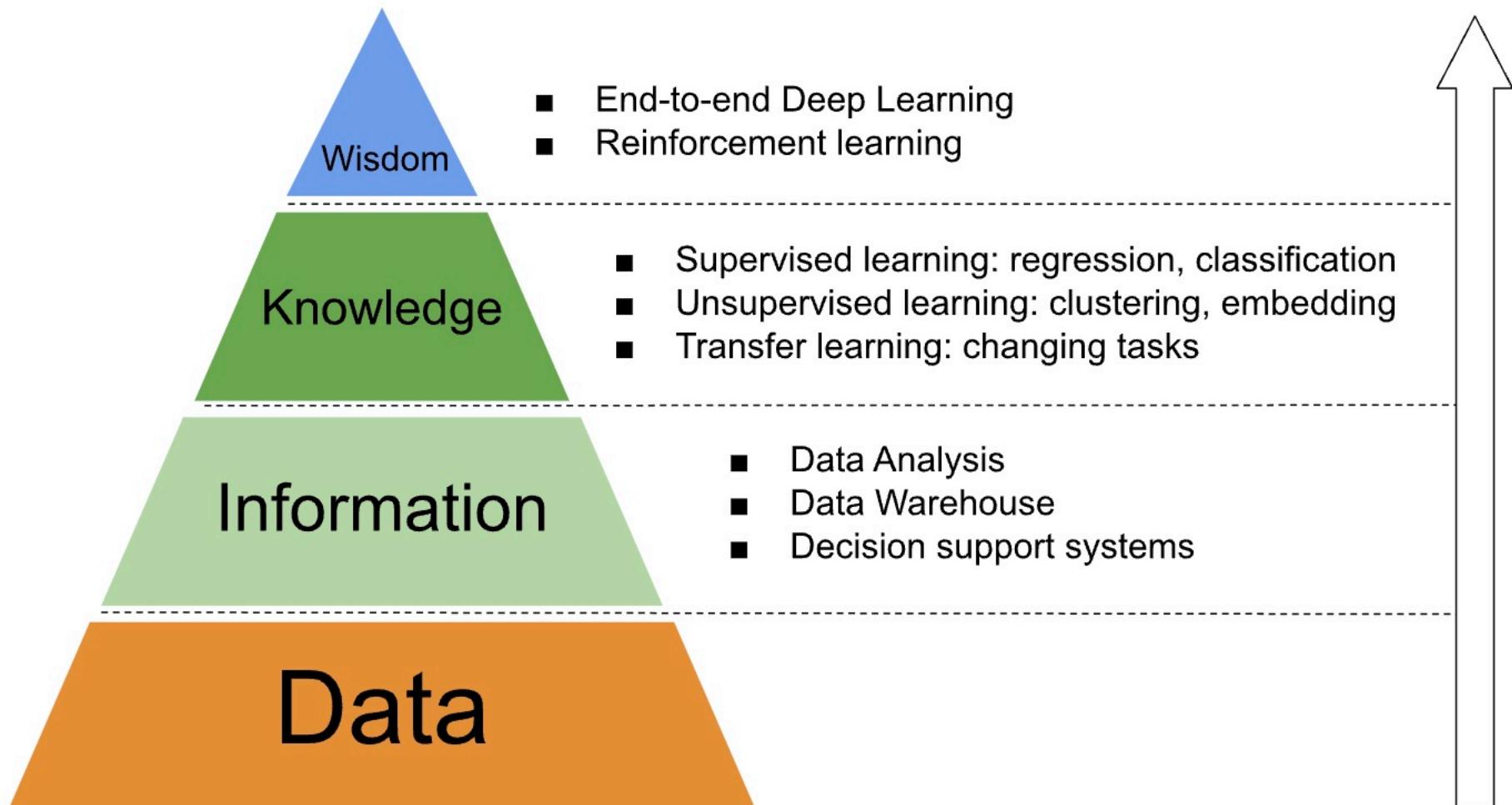


- We were still in a low level:
  - Semi-structured data can provide information only by manually defining complex queries.
  - Information retrieval searches and ranks information without extracting it from data: documents are already “information” in natural language.



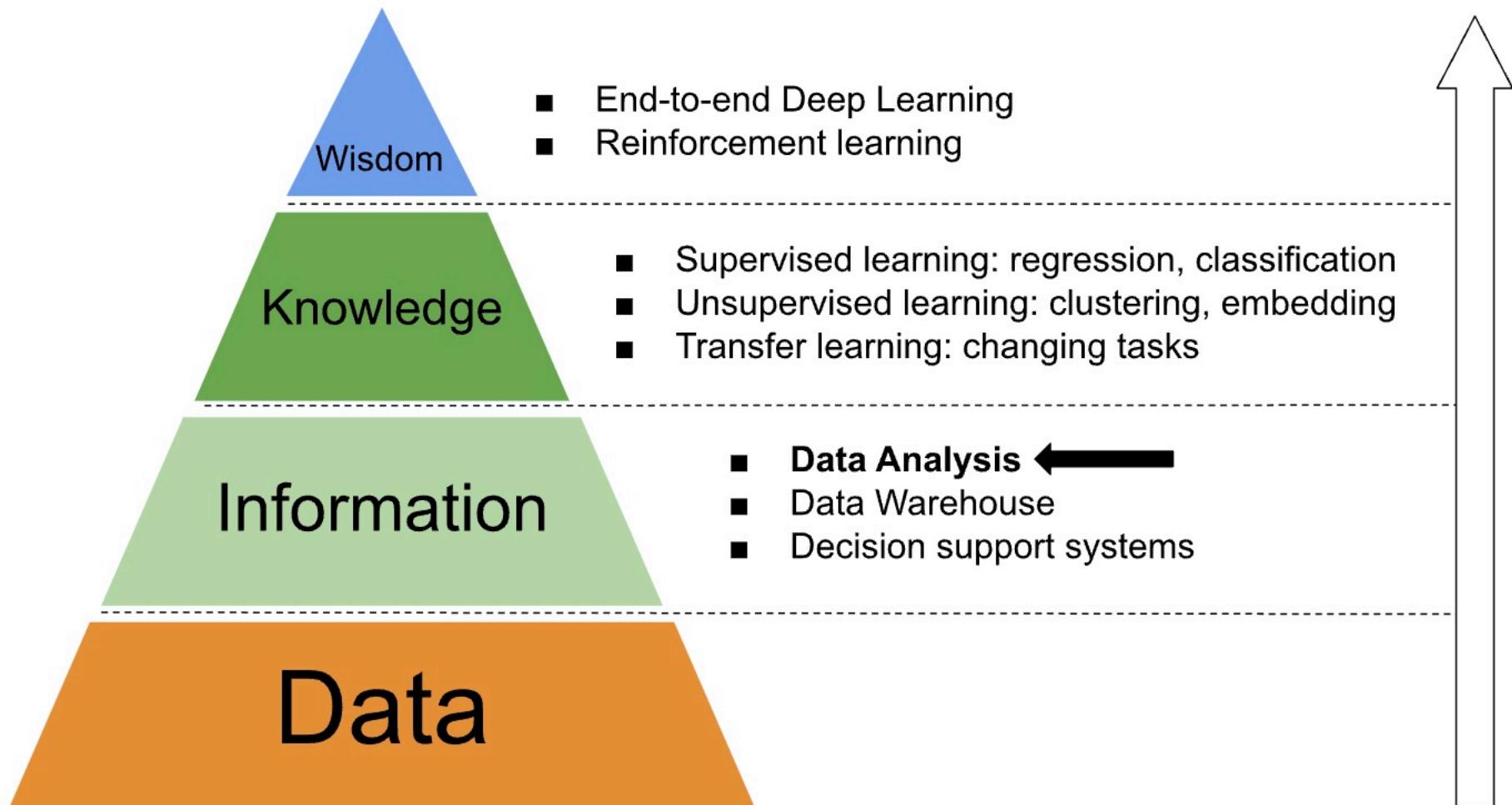
# What we will see

Starting from the bottom:



# What we will see

Starting from the bottom:



# Data Analysis

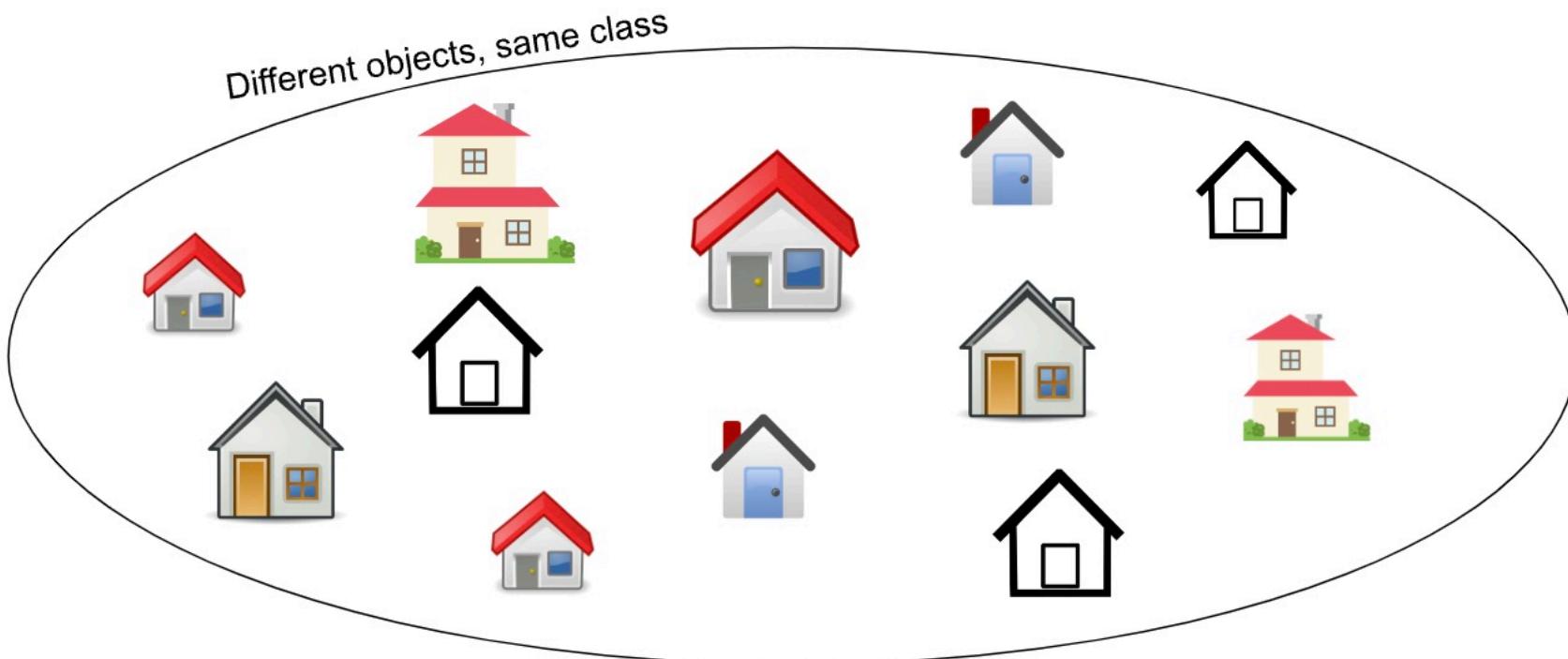
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- The scope of Data Analysis is to **extract** basic **information** from collections of data.
- Extracted information can be:
  - Summarized information: e.g. the average from a set of numerical values
  - Association information: e.g. the relation between two sets of values (houses' price vs. square meters)
- Conceptual foundation is descriptive **statistics**



# Concepts: Population

- A **population** is a collection of objects we are interested in, for example:
  - All the houses in Los Angeles
  - All the students in the university
  - All the receipts from a grocery shop





# Concepts: Record

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- A **record** (or observation, case) is a tuple of values that characterize an element of a population

City	Latitude	Longitude	Bedrooms	SquareMeters	Price
Los Angeles	33°51'37.0"N	118°08'22.2"W	3	130	420000

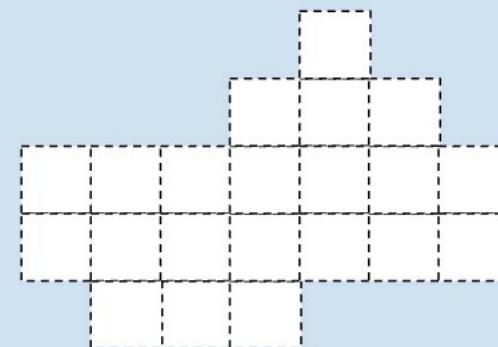


# Concepts: variable

- A **variable** (or field, feature) it's the name for a record's value and has a common meaning and type for all the records in the population



**SquareMeters**  
(Area of the property, Real  
value)





# Concepts: type of variable 1/2

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- We can classify variable depending on the type of the values they can take
- Most important distinction is between
  - **numerical** variables (quantitative): if we can apply arithmetic operations on them
  - **categorical** variables (qualitative): otherwise
- Example:
  - The city (e.g. Los Angeles, New York, Rome) is a **categorical** variable
  - The price (e.g. 420000) is a **numerical** variable



# Concepts: type of variable 2/2

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- Numerical variables can be:
  - **Discrete**, if values can be counted
  - **Continue**, if they are the results of a continuous measure
- Categorical variables can be:
  - **Ordinal**, if a natural order exists on the possible values (e.g. school grades: A,B,C,D)
  - **Nominal**, otherwise (e.g. colors)



# A dataset

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- Finally, the collection of records (a dataset) takes the form of a single table

City	Latitude	Longitude	Bedrooms	SquareMeters	Price
Los Angeles	33°51'37.0"N	118°08'22.2"W	3	130	420000
Los Angeles	33°50'17.7"N	118°09'12.6"W	2	60	380000
Los Angeles	33°49'32.3"N	118°08'44.1"W	5	230	2500000
...	...	...	...	...	...
Albuquerque	35°12'08.1"N	106°58'31.1"W	2	105	190000
Albuquerque	35°15'17.0"N	106°59'26.8"W	4	225	440000
Albuquerque	35°14'22.0"N	106°26'26.2"W	2	140	220000
Albuquerque	35°32'23.0"N	106°38'21.2"W	3	150	250000

# Descriptive statistic

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- Descriptive statistic provides synthesizing indicators to identify, with a single value, **statistical properties** of a population..
- ...with respect to a **single variable**:
  - Centrality indicators: arithmetic mean, mode, median
  - Variation indicator: variance, standard deviation
- ...with respect to **multiple variables**:
  - Covariance
  - Correlation



# Centrality: arithmetic mean

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- Let  $X$  be a **numerical** variable of our dataset (we can't extract the mean from categorical values!)
- $n$  is the **number of records** in our population
- $X_i$  is the  $i$ -th record

$$mean = \frac{\sum_{i=1}^n X_i}{n}$$



# Arithmetic mean: properties

- Suppose you have a record with a missing data (e.g. the price)

City	Latitude	Longitude	Bedrooms	SquareMeters	Price
Los Angeles	33°51'37.0"N	118°08'22.2"W	3	130	???

- You can keep the record without affecting the variable mean:
  - A solution is to **replace the missing data with the mean** for that variable
  - Adding a record with a mean value will not change the arithmetic mean for the whole dataset

# Centrality: median

- Given a population of *sorted* values (e.g. the column “SquareMeters” sorted by its value):

$$(30, 34, 37, 37, \dots, 91, 91, 91, 91, 91, \dots, 525, 600, 670)$$
 $x_1$  $x_{n/2}$  $x_n$ 

- The *median* is the value in central position ( $x_{n/2}=91$ )
- Median age in Italy



# Median: properties

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- Median is a **robust** indicator: anomalies such as very large or very small values do not affects much the median value.
- This was not true for mean, which is much more sensitive to anomalies.
- Consider the following example:

2,3,3,4,5,6,6            {Median:4; Mean:4}

2,3,3,4,5,6,80    {Median:4; Mean:14.6}

- Median is still 4 (value in the central position) while mean has shifted from 4 to 14.6!



# Mode: properties

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- Unlike mean and median, mode also makes sense on **categorical** data:  
mode(Rome,Rome,Los Angeles,Albuquerque): Rome
- In a **voting** system (e.g. a set of many different classifiers) the mode determines the winning final result.
- While robust to **anomalies** like the median, it makes sense also when there is **no linear order** on the possible values (e.g. points in the plane).



# Centrality: mode

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- Given a set of values of a variable (e.g. the column “bedrooms”):  
 $(1,2,4,2,5,3,2,2,3,4,1,3,4,2,6,2,1,3,1,2)$
- First we count the occurrences of a value, that is the **frequency** of that value
  - e.g. “1” is repeated 4 times, “2” is repeated 7 times, “3” is repeated 4 times ...
- The *mode* is the value with higher frequency in the set of observations (i.e. the value “2” in the above example)



# Centrality: comparison

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- We consider the following set of observations (values) on the variable bedrooms:

$$(1, 2, 2, 3, 4, 7, 9)$$

- **Arithmetic mean** (sum of values of a data set divided by number of values):  
 $(1+2+2+3+4+7+9)/7=4$
- **Median** (middle value):  $(1, 2, 2, 3, 4, 7, 9) = 3$
- **Mode** (most frequent value):  
 $(1, 2, 2, 3, 4, 7, 9) = 2$



# Centrality: in summary

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- We have seen three **centrality** indicators: arithmetic mean, median and mode.
- All these indicators provides a different way of “**summarizing**” a set of values into a single, synthesizing value
  - **Arithmetic mean** is also useful to replace missing or wrong data without changing its overall distribution, but its value it's not drawn from the available data and it's sensitive to anomalies.
  - **Median** is an actual value from the observations and it's robust to anomalies but needs ordinal data.
  - **Mode** is also an actual value, the most frequent one. Robust to anomalies, does not need ordinal data and can be applied on categorical variables.



# Variation: squared deviation

- Squared deviation: it measures the difference between each value  $x_i$  and the mean of the observations  $\bar{x}$

$$dev = \sum_{i=1}^n (x_i - \bar{x})^2$$

- The more the values are far from the mean, the higher the deviation. In the following sample the mean is 15:

$$dev(4,6,10,40) = \sum_{i=1}^n (x_i - 15)^2$$

$$\begin{aligned} &= (4-15)^2 + (6-15)^2 + (10-15)^2 + (40-15)^2 = 121 + 81 + 25 + 625 \\ &= 852 \end{aligned}$$



# Variation: variance

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- The squared deviation is **affected by the number of observations**: the more values we have, the higher the deviation tend to be.
- The **variance** (often represented with  $s^2$ ,  $\sigma^2$ , or  $\text{Var}$ ) normalizes squared deviation by the number of observations:

$$s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{1}{n} \text{dev}$$

- In the previous example:  $s^2(4,6,10,40) = 852/4 = 213$



# Variation: standard deviation

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- Variance and squared deviation consider the **squared difference between values and the mean**, in order to have non-negative differences.
- This leads to large values that do not reflect the estimated deviation from the mean.
- **Standard deviation** (often represented with  $s$ ,  $\sigma$ , or  $Stdev$ ) is the **square root** of the variance.

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

- Unlike squared deviation and variance, standard deviation is expressed in **the same units** as the data.



# Variation: comparison

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- We use the same example on the three variation indicators to observe their difference:

**(4,6,10,40)**

- Square deviation:  $\text{dev}(4,6,10,40) = 852$
- Variance:  $\text{Var}(4,6,10,40) = 213$
- Standard deviation:  $\text{Stdev}(4,6,10,40) = 14.6$



# Other single variable indicators

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- **Minimum** (min): it's the minimum value in the observations
- **Maximum** (max): it's the maximum value of the observations
- **Range**: it's the difference between the maximum and the minimum value



# Multiple variables indicators

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- In descriptive statistics, the *association measures* allow to describe the **relation between two variables**, looking for associations.
- For example, they are useful to determine:
  - If the **price** of the houses is associated with the **square meters**
  - If the smoke is associated with heart diseases
  - If the budget on advertising is associated with the number of sales
- We will see two measures:
  - **Covariance**: measure the strength of the relation between two variables
  - **Correlation**: measure the strength of the relation between -1 and 1



# Association measures: covariance

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- The **covariance** quantifies the strength of the relation between two variables X and Y
- More specifically, it is the mean of the products of the values deviations:

$$\text{cov}(X,Y) = \frac{\sum_{i=1}^n (X_i - \bar{X}) \times (Y_i - \bar{Y})}{n}$$

# Association measures: covariance

- The **covariance** quantifies the strength of the relation between variables X and Y
- More specifically, it is the **mean** of the **products** of the **values deviations**:

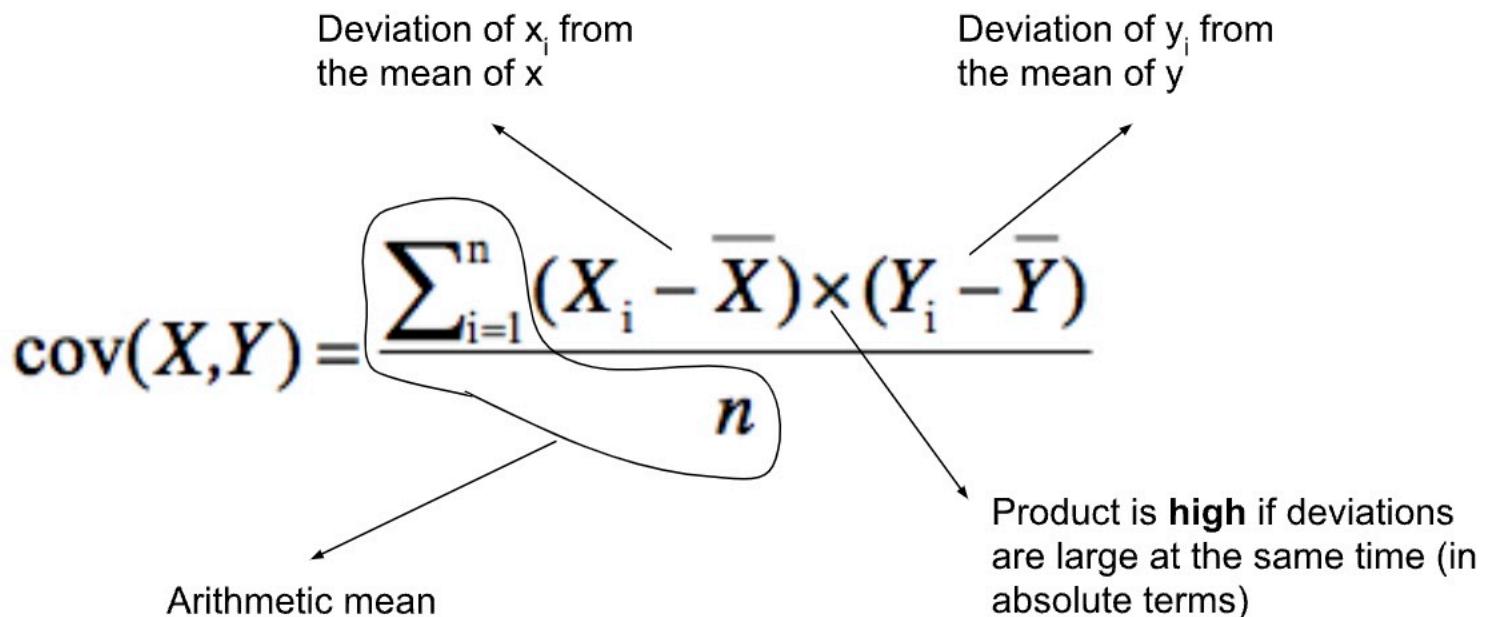
$$\text{cov}(X,Y) = \frac{\sum_{i=1}^n (X_i - \bar{X}) \times (Y_i - \bar{Y})}{n}$$

Deviation of  $x_i$  from the mean of x

Deviation of  $y_i$  from the mean of y

Arithmetic mean

Product is **high** if deviations are large at the same time (in absolute terms)





# Meaning of covariance

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- The idea behind: the sum of products of the two deviation is high if, every time X deviate from its mean, Y also deviate from its mean **accordingly**.
- Why?
  - If Y does not deviate, its deviation is low and so will be the product
  - If Y deviates randomly (sometimes in a direction, sometimes in another direction) the summation will “cancel out”
- Note: a **positive** covariance means X and Y deviates in the **same direction** (directly proportional), a **negative** covariance means they deviates in **opposite directions** (inversely proportional)



# Covariance limit

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- A problem with covariance is that its value is affected by the **unit of measure**:
  - If values are large, the covariance tends to be large (even if X and Y are not much related)
  - If values are small, the covariance tends to be small (even if X and Y are strongly related)
- For example, given **the same set** of prices, we can **decrease** the covariance by a factor 1000, by simply expressing the price as **thousands of \$**!



# Correlation

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- In order to overcome the problem of the unit measure, we use the **correlation**.
- The correlation solve this problem producing a result which is independent from unit measure, because it takes into account the standard deviations of X and Y:

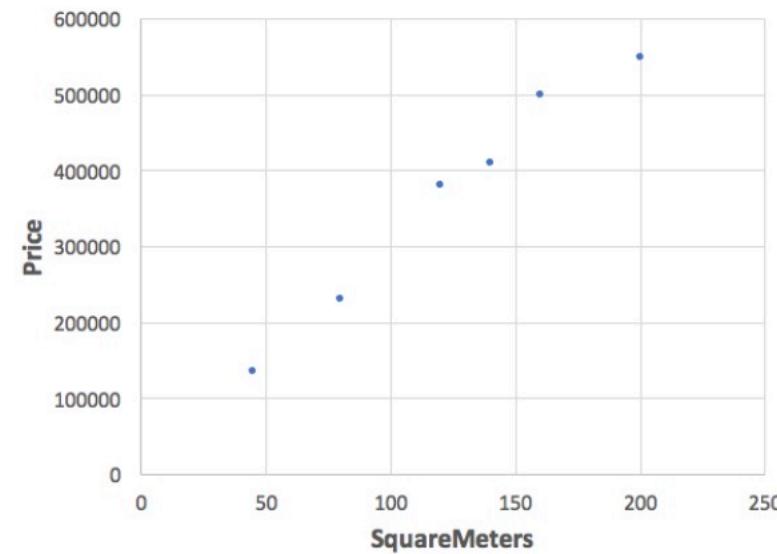
$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\text{Stdev}(X) \times \text{Stdev}(Y)}$$

- Dividing the covariance by the product of the two standard deviations we ensure a value between -1 and 1.

# Scatterplot and regression

- Apart from computing association measures, it is useful to visualize the pairs of values from the two variables in an XY plane:

SquareMeters	Price
120	380,000
200	550,000
80	230,000
160	500,000
45	135,000
140	410,000



- While association measures tell us **if** an association exists (correlation here is 0.99!!), **regression** models estimate the **actual function** that relates the two variable: in this case  
$$\text{Price}(\text{SquareMeters}) = 3000 * \text{SquareMeters}$$
- But hold on! We will/have see regression models, in the machine learning section.



# Meaning of correlation

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- A value of correlation is close to -1 if the two variables tend to vary in opposite direction (inversely proportional)
- A value of correlation is close to 1 if the two variables tend to vary in the same direction (directly proportional)
- A value of correlation is close to 0 if the two variables have independent variations (at least for **linear** relations!)



# Association measures: summary

- Looking at two different variables at the same time help us understand if there is any relation between the two:
  - Covariance tell us if and how much X and Y vary accordingly
  - Correlation, in addition, it's not affected by the variables unit measures.
- **Warning:** this association measures works only if the relation is **linear** (i.e. the points form a straight line in the XY plane)
  - Correlation can be 0 even if exists a strong but **non-linear** relation between X and Y.



# References

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Jackie Nicholas

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