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# Information Retrieval

## Ranking

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# Boolean model limits

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- Thus far, our queries have all been boolean.
  - Documents **either match or don't**.
- Good for **expert users** with precise understanding of their needs and knowledge of the collection.
  - Also good for applications: applications can easily make use of 1000s of results for further processing.
- Not good for the majority of users.
  - Most users incapable of writing Boolean queries (or they are, but they think it's too much work).
  - Most users don't want to wade through 1000s of results.
    - This is particularly true of web search.



# Boolean model limits: example

- Boolean queries often result in either **too few** (=0) or **too many** (1000s) results.
- Example: suppose a user is having troubles with his network card:
  - Query 1: “*standard user dlink 650*” → 200,000 hits
  - Query 2: “*standard user dlink 650 no card found*”: 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - **AND gives too few**
  - **OR gives too many**



# Ranked retrieval

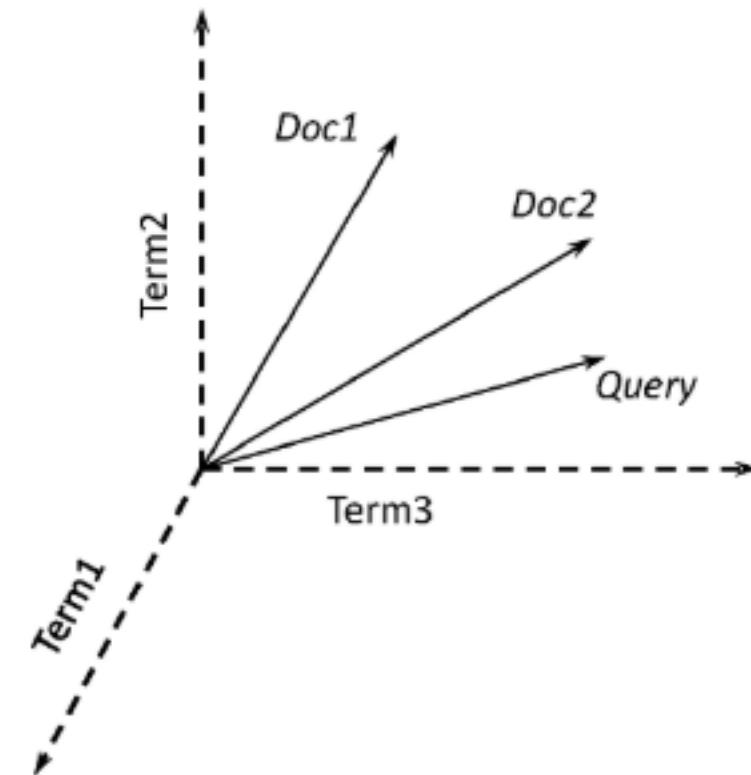
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- Rather than a set of documents satisfying a query expression, in **ranked retrieval**, the system returns an ordering over the **(top)** documents in the collection for a query
- When a system produces a ranked result set, large result sets are not an issue
  - Indeed, the size of the result set is not an issue
  - We just show the **top  $k$**  ( $\approx 10$ ) results
  - We don't overwhelm the user
  - Premise: the ranking model works
- Most known ranking model is Vector Space Model (G. Salton, 1975)



# Vector Space Model (VSM)

- Like in the bag-of-word model, documents are **represented by a vector** of “term weights”:
- Collection represented by a matrix of **term weights**
- $d_{ij}$  is the weight of Doc<sub>i</sub> for Term<sub>j</sub>



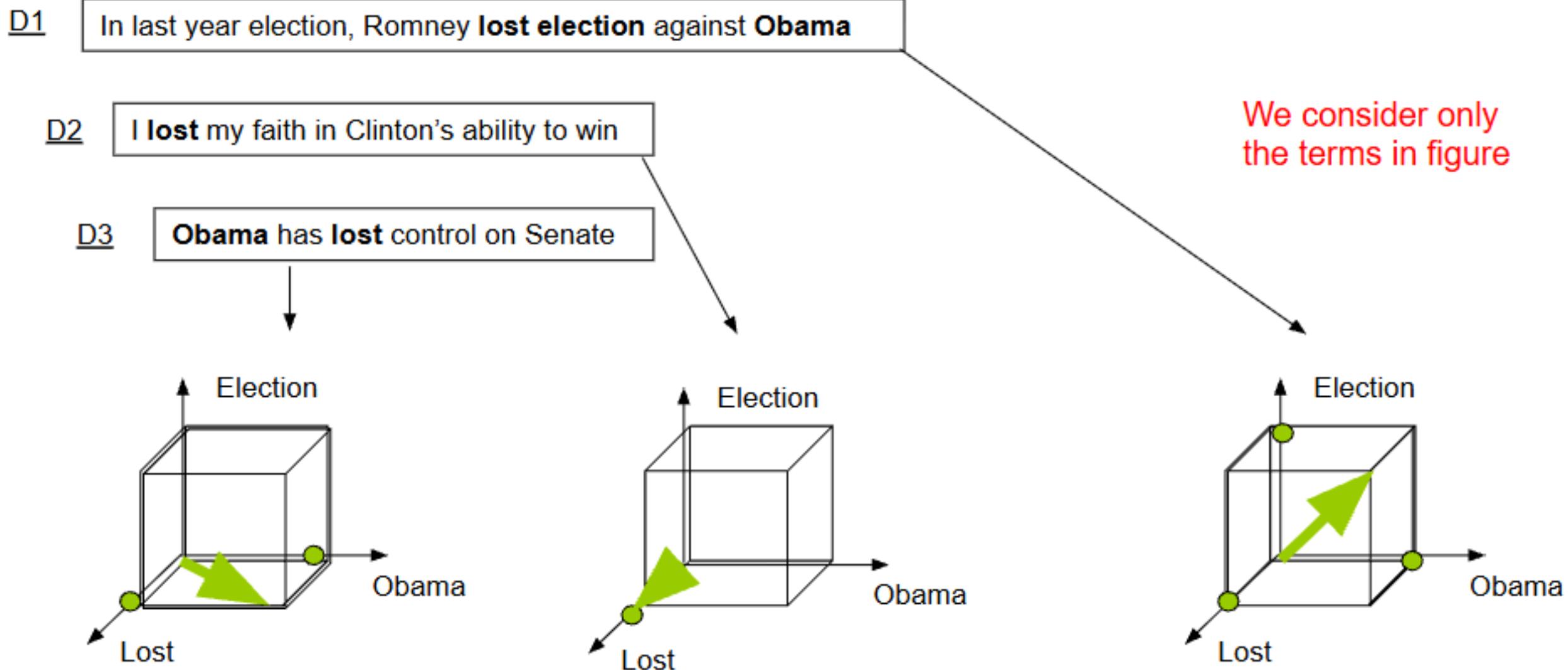
$$D_i = (d_{i1}, d_{i2}, \dots, d_{it}) \quad Q = (q_1, q_2, \dots, q_t)$$

	Term <sub>1</sub>	Term <sub>2</sub>	...	Term <sub>t</sub>
Doc <sub>1</sub>	$d_{11}$	$d_{12}$	...	$d_{1t}$
Doc <sub>2</sub>	$d_{21}$	$d_{22}$	...	$d_{2t}$
⋮	⋮	⋮		
Doc <sub>n</sub>	$d_{n1}$	$d_{n2}$	...	$d_{nt}$



# VSM: vector representation

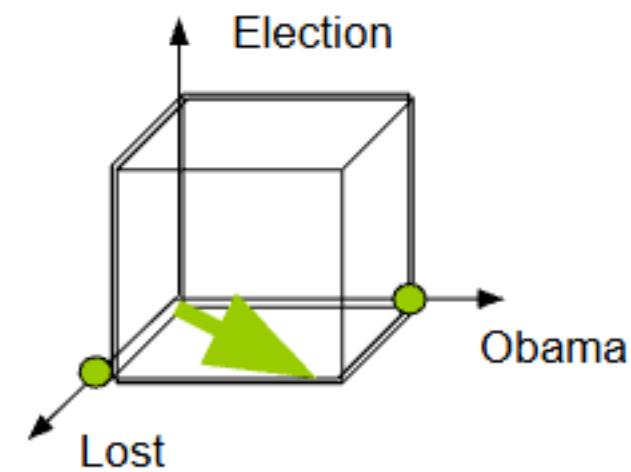
- An example with 3 terms (3-dimensional vector space) and 3 documents.





# Documents as vectors

- So we have a  $|V|$ -dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero.





# Queries as vectors

- **Key idea 1:** Do the same for queries: represent them as vectors in the space
- **Key idea 2:** Rank documents according to their proximity to the query in this space
- proximity = **similarity of vectors**
- proximity  $\approx$  inverse of distance
- **Reminder:** We do this because we want to get away from the in-or-out Boolean model.
- Instead: **rank more relevant documents** higher than less relevant documents



# Recap

- In the boolean model, a document is either relevant or non-relevant.
- Representing document and queries as vectors, we can compute the **similarity between the vector** of the document and the vector of the query:
  - This is an estimation of the relevance of the document with respect to the query.
- We need to define the similarity between vectors but...
  - Before going on we must introduce the **term frequency**.



# TF: Term Frequencies

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- So far, vectors were binary, representing the **presence or absence** of terms in a document.
- We would like to assign a **weight** for each term in a vector space:
  - The term frequency  $\text{tf}_{t,d}$  of term  $t$  in document  $d$  is defined as the number of times that  $t$  occurs in  $d$ .
  - We want to use term frequency (**tf**) when computing query-document match scores.



# Example: TF as term weight

Three dimensional pictures are useful, but can be misleading for high-dimensional space (more than 3 terms considered):

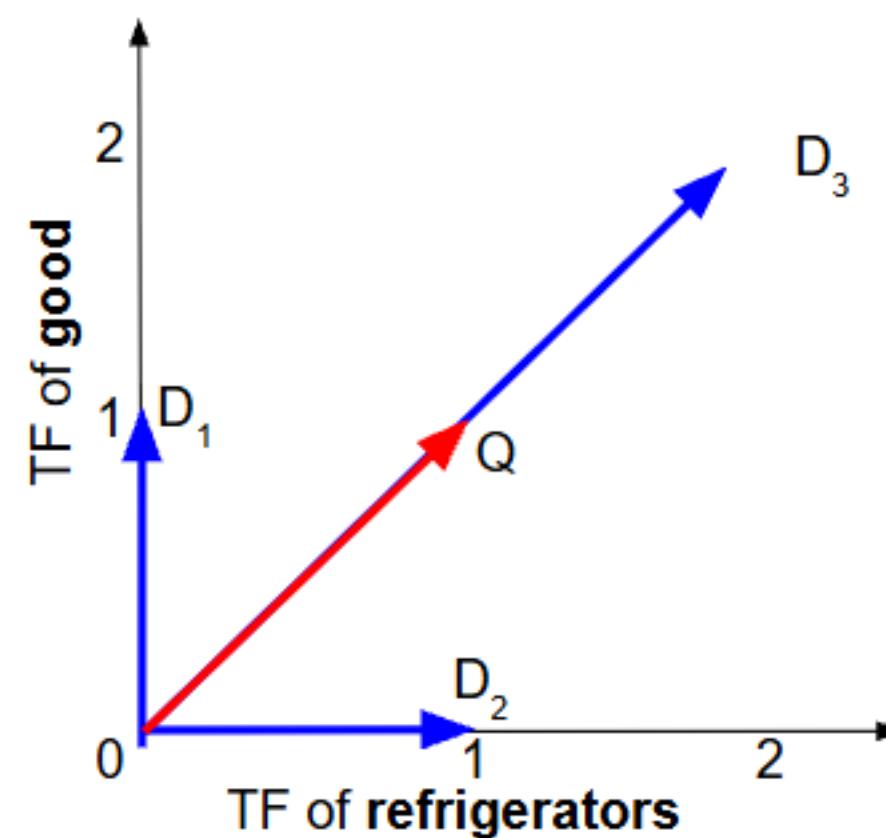
- D<sub>1</sub> Tropical Freshwater Aquarium Fish.
- D<sub>2</sub> Tropical Fish, Aquarium Care, Tank Setup.
- D<sub>3</sub> Keeping Tropical Fish and Goldfish in Aquariums, and Fish Bowls.
- D<sub>4</sub> The Tropical Tank Homepage - Tropical Fish and Aquariums.



Terms	Documents			
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
Aquarium	1	1	1	1
Bowl	0	0	1	0
Care	0	1	0	0
Fish	1	1	2	1
Freshwater	1	0	0	0
Goldfish	0	0	1	0
Homepage	0	0	0	1
Keep	0	0	1	0
Setup	0	1	0	0
Tank	0	1	0	1
Tropical	1	1	1	2

# Vector Space Distance

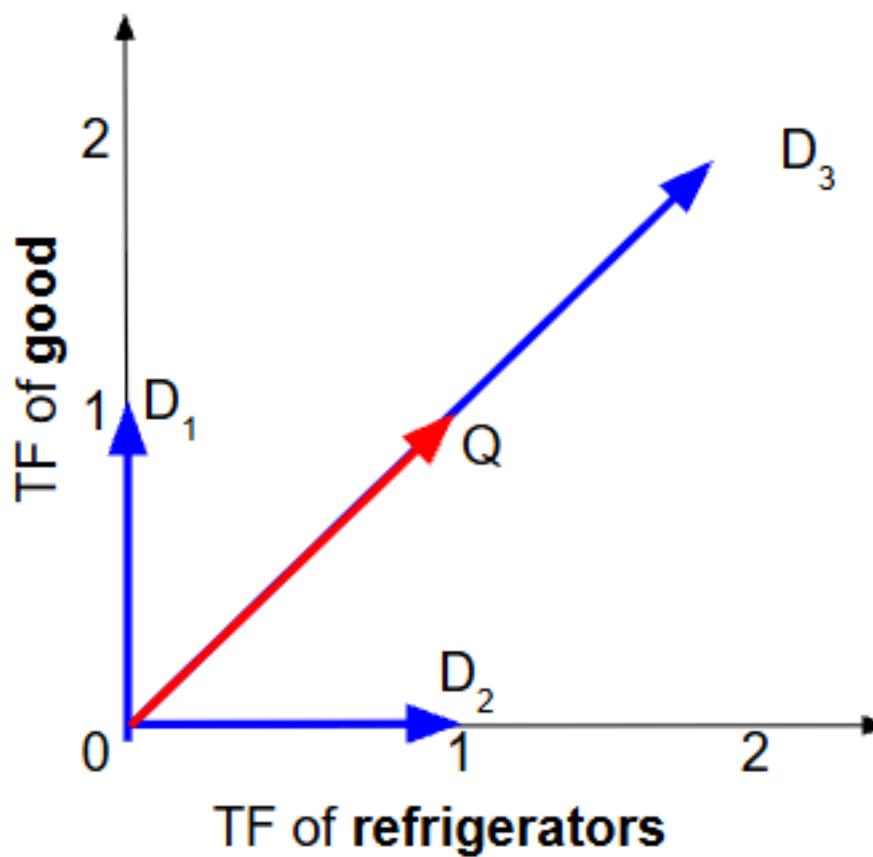
- How to formalize the proximity between two vectors when we have term frequencies?
- Example query Q: “Good refrigerators”
  - $D_1$ : “Good morning to all of you.”
  - $D_2$ : “Don’t put pizza in refrigerators”
  - $D_3$ : “Good refrigerators review: top five good refrigerators.”



**Note:** we are showing only the two interesting dimensions, “good” and “refrigerators”. There is a dimension for each indexed term (e.g. for pizza, morning, review...)

# First cut: euclidean distance?

- We could consider the straight line distance between the end points, i.e. the euclidean distance
  - Distance between Q and  $D_1$  is 1
  - Distance between Q and  $D_2$  is 1
  - Distance between Q and  $D_3$  is  $\sqrt{2} \approx 1.414$
- $D_3$  is the least similar, this doesn't seem right!



- Remember Q: "Good refrigerators"
  - $D_1$ : "Good morning to all of you."
  - $D_2$ : "Don't put pizza in refrigerators"
  - $D_3$ : "Good refrigerators review: top five good refrigerators."



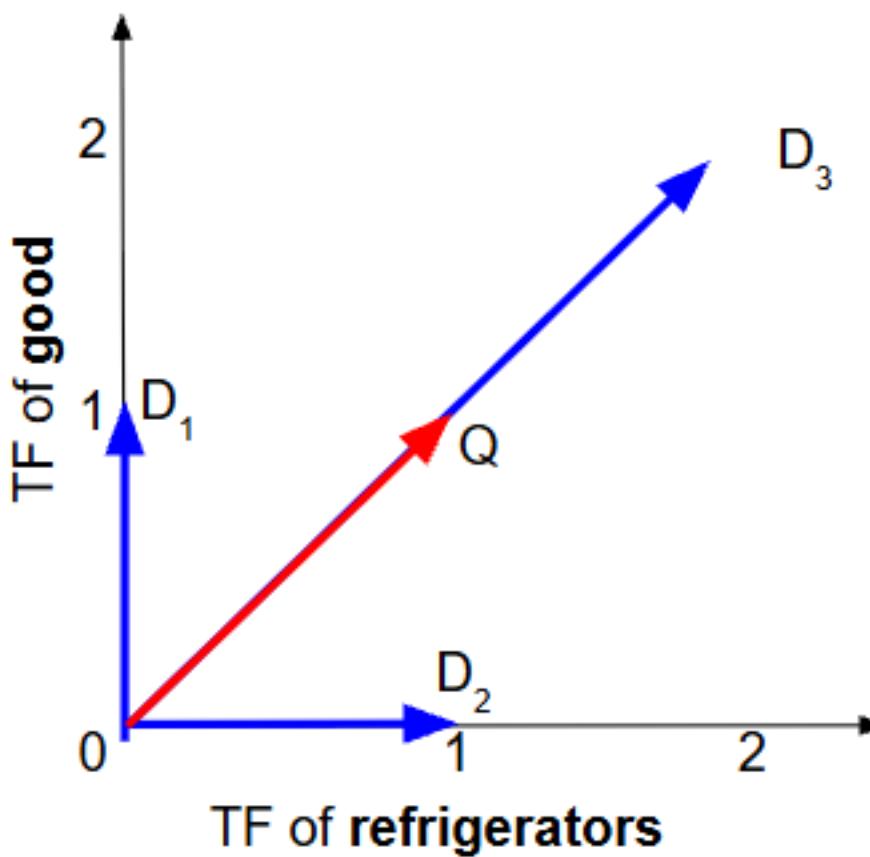
# Angle instead of distance

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- Thought experiment: take a document  $d$  and append it to itself many times. Call this longer document  $d'$ .
- “Semantically”  $d$  and  $d'$  have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.
- **Key idea:** Rank documents according to angle with query.

# Example: Using angles

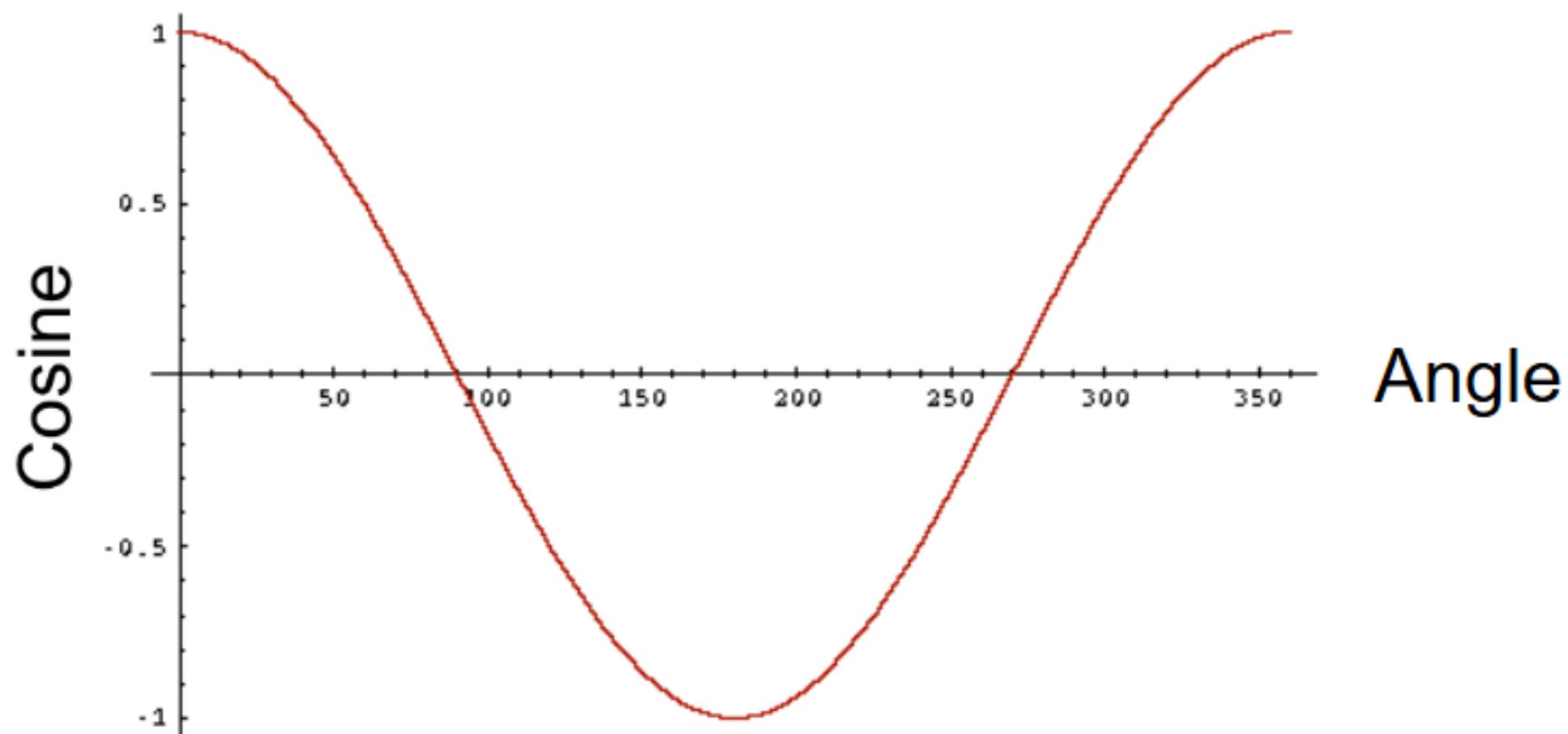
- We now consider the angle between vectors
  - Angle between  $Q$  and  $D_1$  is  $45^\circ$
  - Angle between  $Q$  and  $D_2$  is  $45^\circ$
  - Angle between  $Q$  and  $D_3$  is  $0^\circ$
- $D_3$  is the most similar, this is what we need!



- Remember  $Q$ : “Good refrigerators”
  - $D_1$ : “Good morning to all of you.”
  - $D_2$ : “Don’t put pizza in refrigerators”
  - $D_3$ : “Good refrigerators review: top five good refrigerators.”

# From angles to cosines

- The following two notions are equivalent.
  - Rank documents in decreasing order of the angle between query and document
  - Rank documents in increasing order of cosine(query, document)
- Cosine is a monotonically **decreasing** function for the interval  $[0^\circ, 180^\circ]$ , meaning that it's inversely proportional of the angle





# Cosine similarity

Dot product

Unit vectors

$$\cos(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{\|\vec{q}\| \|\vec{d}\|} = \frac{\vec{q}}{\|\vec{q}\|} \cdot \frac{\vec{d}}{\|\vec{d}\|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2} \sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

- $q_i$  is the tf weight of term  $i$  in the query
- $d_i$  is the tf weight of term  $i$  in the document

$\cos(\vec{q}, \vec{d})$  is the cosine similarity of  $\vec{q}$  and  $\vec{d}$  ... or,  
equivalently, the cosine of the angle between  $\vec{q}$  and  $\vec{d}$ .



# Example: Formula

	Election	Lost	Obama		Election	Lost	Obama
D1 [	1	1	1 ]	D2 [	0	1	0 ]
D3 [	0	1	1 ]	Q [	0	0	1 ]

$$\text{Sim}(Q, D1) = \frac{\sum_{i=1}^3 Q_i * D1_i}{\sqrt{\sum_{i=1}^3 Q_i^2} * \sqrt{\sum_{i=1}^3 D1_i^2}}$$

$$\text{Sim}(Q, D2) = \frac{\sum_{i=1}^3 Q_i * D2_i}{\sqrt{\sum_{i=1}^3 Q_i^2} * \sqrt{\sum_{i=1}^3 D2_i^2}}$$

$$\text{Sim}(Q, D3) = \frac{\sum_{i=1}^3 Q_i * D3_i}{\sqrt{\sum_{i=1}^3 Q_i^2} * \sqrt{\sum_{i=1}^3 D3_i^2}}$$



# Example: Instantiation

	Election	Lost	Obama		Election	Lost	Obama
D1 [	1	1	1 ]	D2 [	0	1	0 ]
D3 [	0	1	1 ]	Q [	0	0	1 ]

$$\text{Sim}(Q, D1) = \frac{0*1 + 0*1 + 1*1}{\sqrt{1} * \sqrt{3}} = 0.577$$

$$\text{Sim}(Q, D2) = \frac{0*0 + 0*1 + 1*0}{\sqrt{1} * \sqrt{1}} = 0.000$$

$$\text{Sim}(Q, D3) = \frac{0*0 + 0*1 + 1*1}{\sqrt{1} * \sqrt{2}} = 0.707$$



# How Search Engines Work



The user inputs a query

## Google

How search engines  
how search engines work  
how search engines work google  
how search engines rank  
how search engines see my site  
how search engines work for dummies  
how search engines operate  
how search engines work pdf  
how search engines work video  
how search engines see your site  
how search engines make money

Google Search I'm Feeling Lucky

How search engines work

Web Videos Images News Shopping More + Search tools

About 83,900,000 results (0.36 seconds)

You can find pages by following links from other pages but usually it is easier to search for things using a search engine. These are programs that search an index of the world wide web for keywords and display the results in order.

BBC Bitesize - How do search engines work?  
[www.bbc.co.uk/guides/zbtqj6f](http://www.bbc.co.uk/guides/zbtqj6f) British Broadcasting Corporation

Feedback

How Search Engines Work - The Beginners Guide to SEO ...  
[moz.com/beginners-guide-to-seo/how-search-engines-operate](http://moz.com/beginners-guide-to-seo/how-search-engines-operate) Moz  
Search engines have two major functions: crawling and building an index, and providing search users with a ranked list of the websites they've determined are ...

How Internet Search Engines Work - HowStuffWorks  
[computer.howstuffworks.com/internet/basics/search-engine.htm](http://computer.howstuffworks.com/internet/basics/search-engine.htm) Internet search engines do your research for you. Learn how internet search engines like Google work, how internet search engines build an index and what ...

Images for How search engines work

Report images

The search engine search the query terms in its very large **index**



About 83,900,000 results (0.36 seconds)

Millions of results found are **ranked** accordingly to a notion of **relevance** and are shown as a sorted list

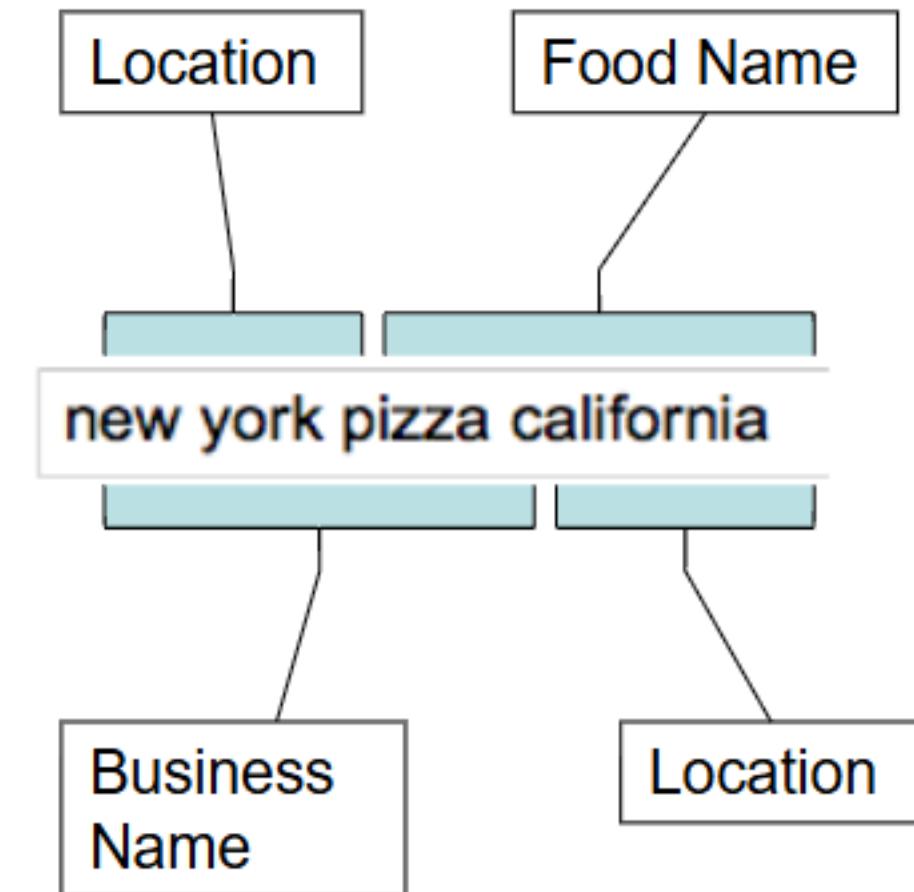
Automated programs (a.k.a. spiders or **crawlers**) scan all pages in the web to build an index



# What is Relevant?

- How can a search engine know what the user is looking for?
  - **Information need:** a desire to *locate and obtain information to satisfy a conscious or unconscious need* (Wikipedia)
  - **User query:** the set of consecutive terms formulated by a user to express his information need
  - **Query intent:** the task, goal or intent of a user, expressed by a query. The same query formulated by different users may be the result of different intents

Query intent can be ambiguous:





# Ranking: Google PageRank

- A relevance notion based solely on term frequencies is not enough to rank billions of documents
- Complex measures are applied to evaluate the quality, reliability and authority of web pages:
  - Measures based **on topological network properties** (such as Google PageRank)
  - Measures based **on different field boosting** (Title, subtitle, body have different weight)
  - Measures based **on semantics and time-space properties** (freshness of a page)
- PageRank set up a **rich-get-richer loop**, whereby few sites dominate the top ranks



# Search Engine Results Page

- A Search Engine Results Page (SERP) shows by default 10 results
- Of these results, on average, the first **5 are visible** without scrolling down
- We will take in consideration the **user behavior on the first SERP** of search engines (the first 10 results)



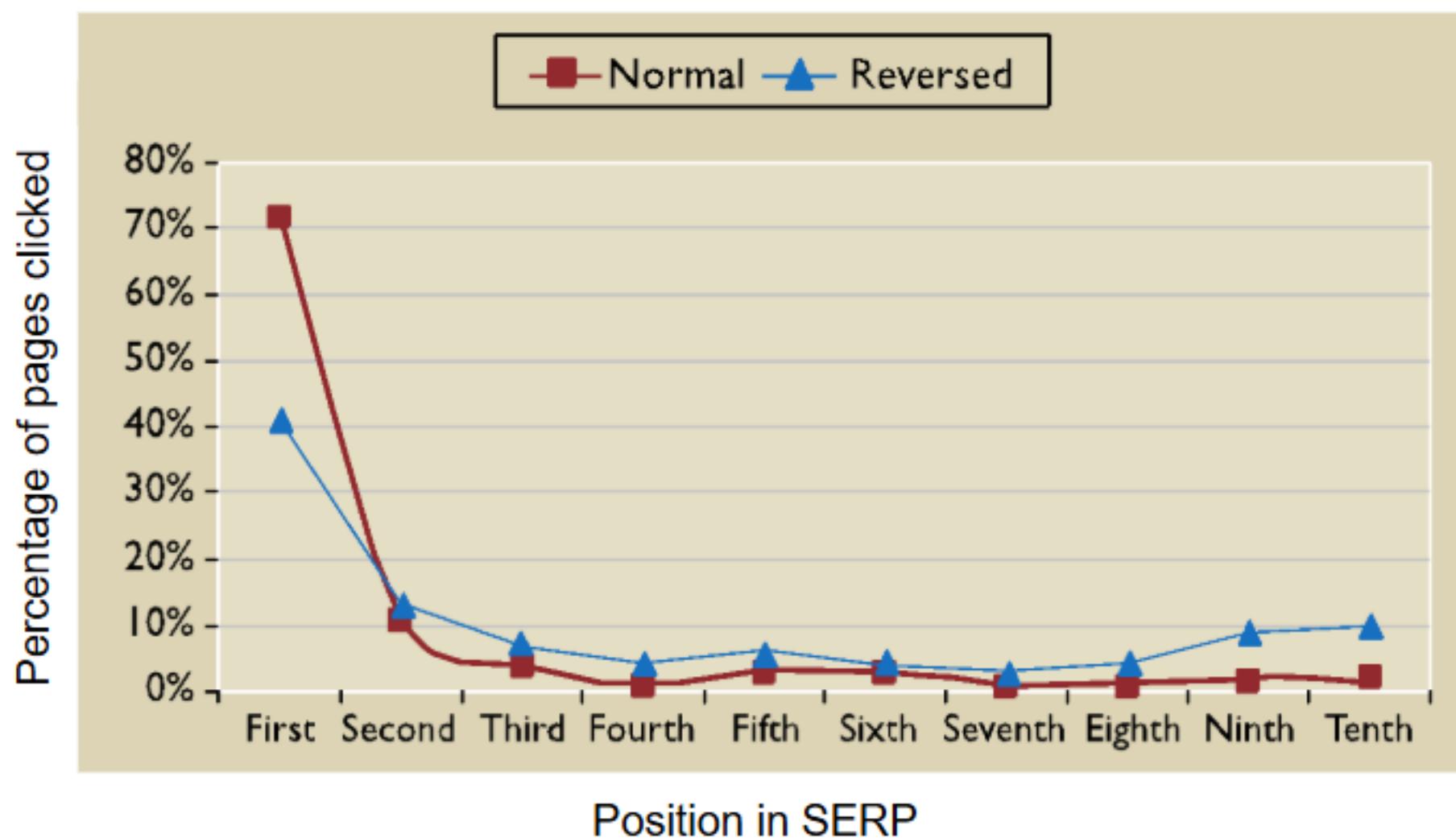
Visible  
Area

Scroll  
Area

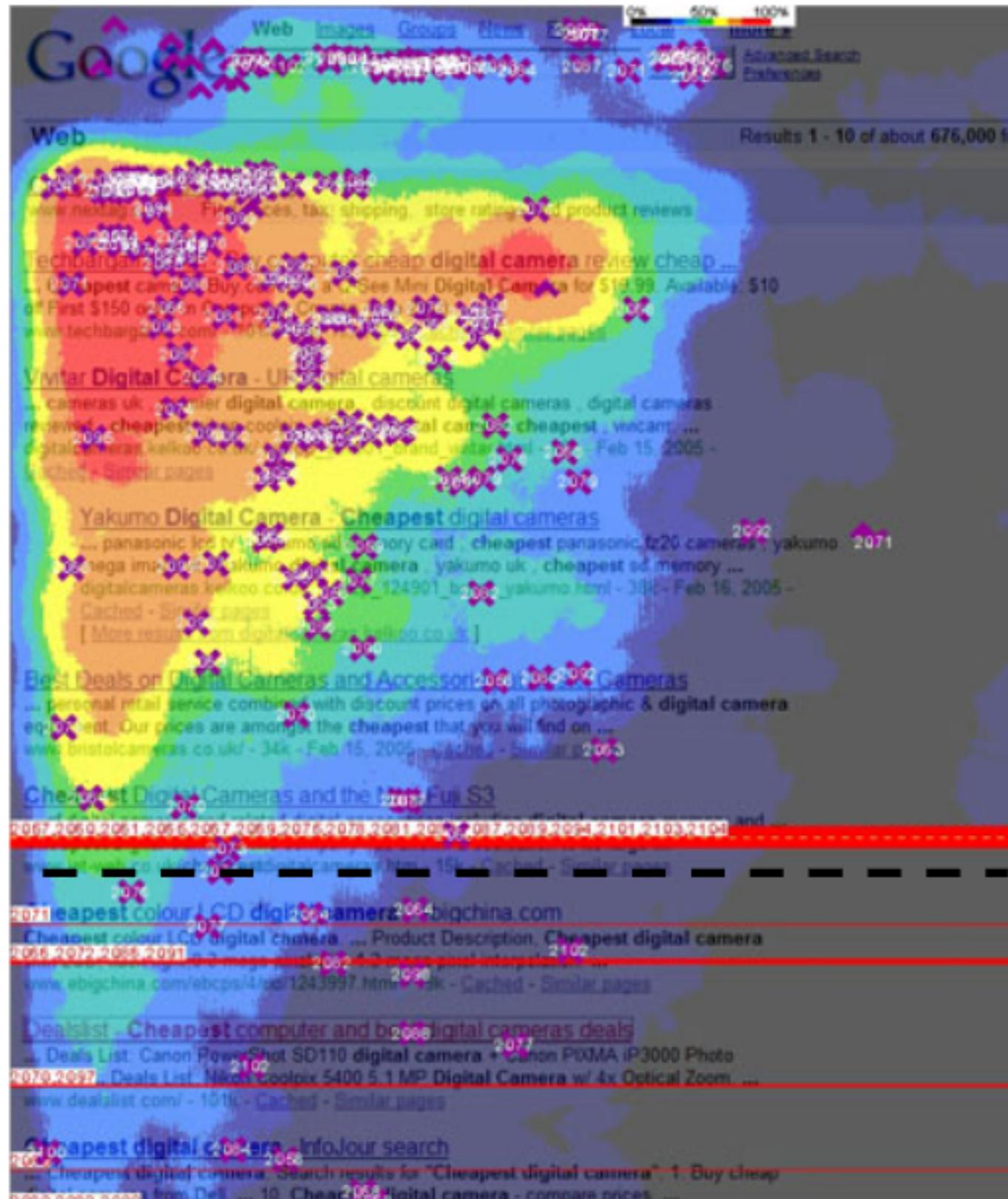


# Result positioning bias

- When searching for information, the average searchers do not judge systematically all the results, instead they **simply click on top results**
- Studies have been conducted comparing the users' responses when they received results lists in **normal ordering versus a reversed order**



# Eye-tracking: Golden Triangle

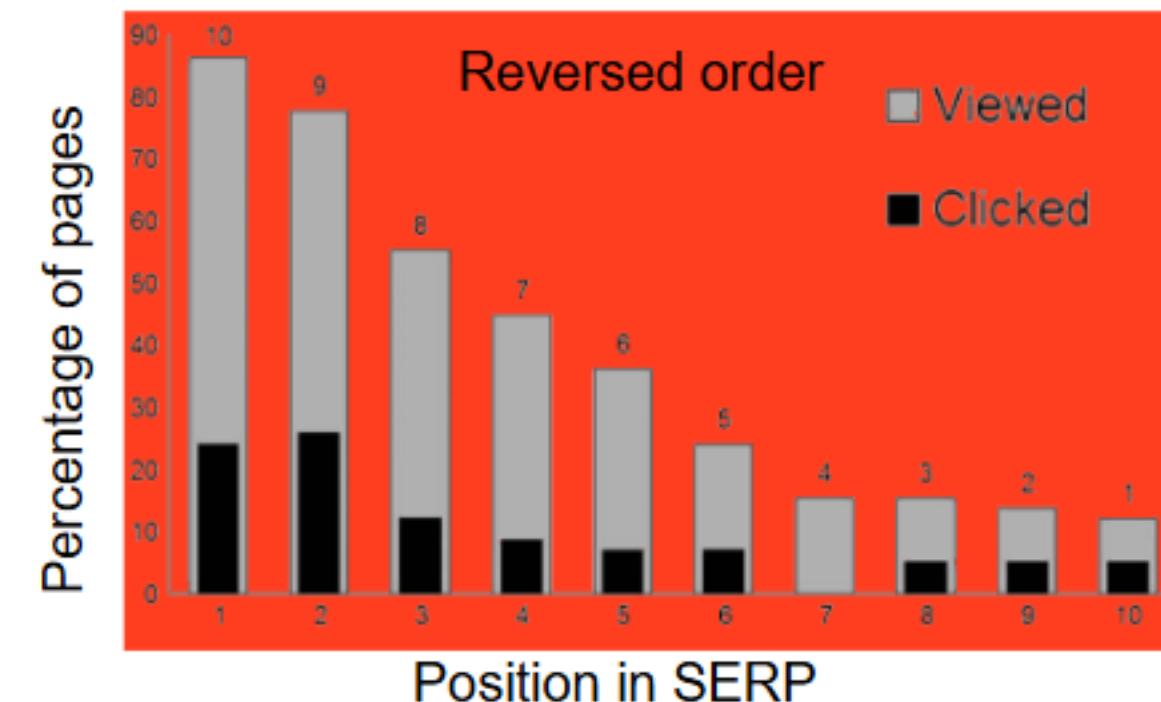
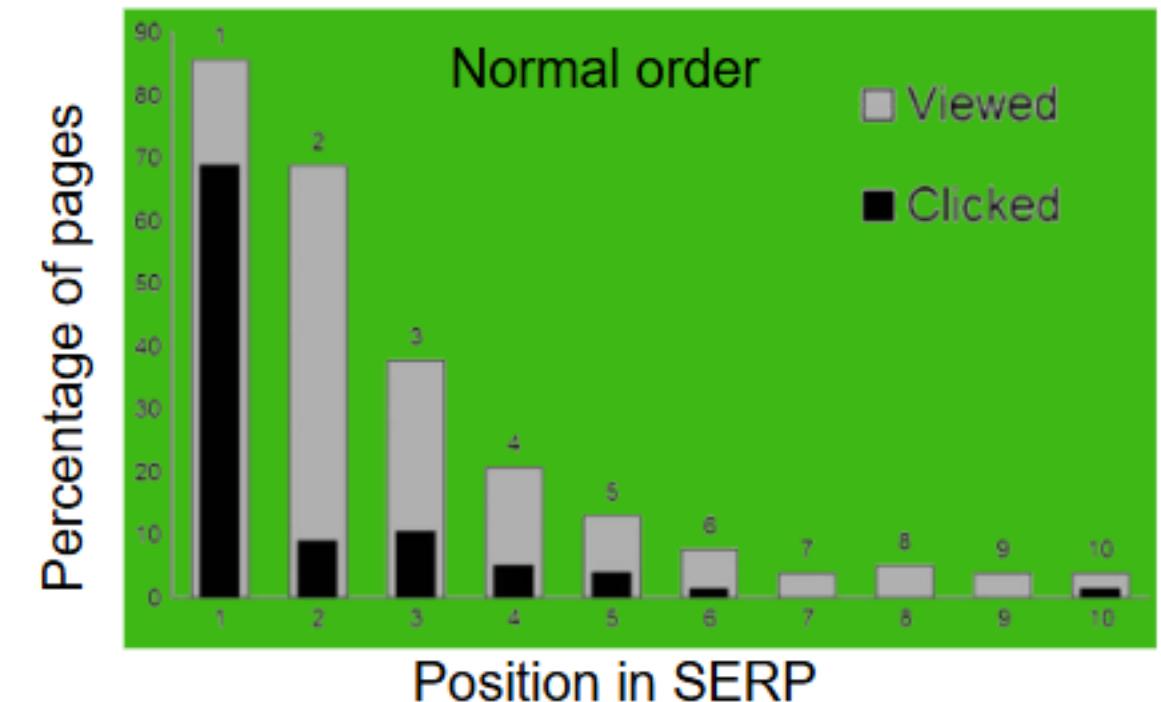


- Color 0% 50% 100% represents percentage of time spent looking the area.
- Purple X represents a mouse click on the page.
- Dotted line - - - represents where the page breaks on the computer screen (Visible area/Scroll area).
- Red lines — indicate how far down the page scrolled before leaving the page.

**Results in 1<sup>st</sup> position are clicked more than 10 times  
results on 6<sup>th</sup> position**

# Click and Eye-tracking biasy

- View percentage and mouse clicks are compared in normal vs. reversed order of the first 10 results
- Even though results show some user awareness, **excessive trust in ranking** algorithms may negatively affect smaller positioned websites





# Position impact on business

Web Images Videos Maps News Shopping Gmail more ▾

Google

shopping  
About 958,000,000 results (0.21 seconds)

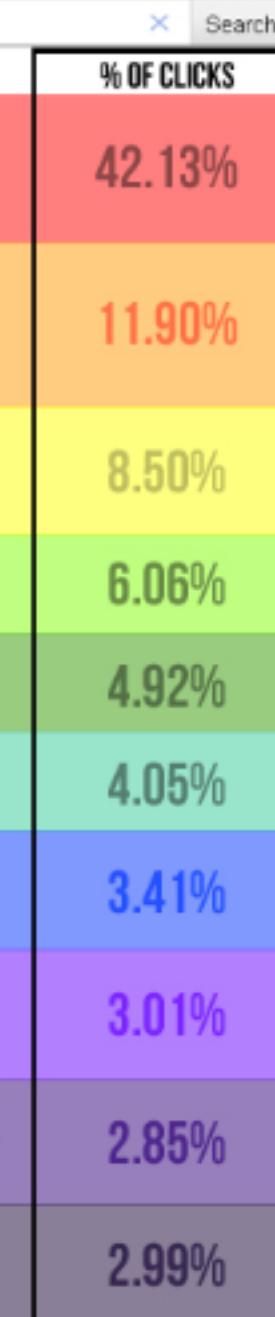
Everything  
Images  
Videos  
Shopping  
Blogs  
More

Columbus, OH  
 Change location

All results  
Related searches  
Wonder wheel  
Sites with images  
 More search tools

Something different  
nightlife  
art  
price comparison  
froogle

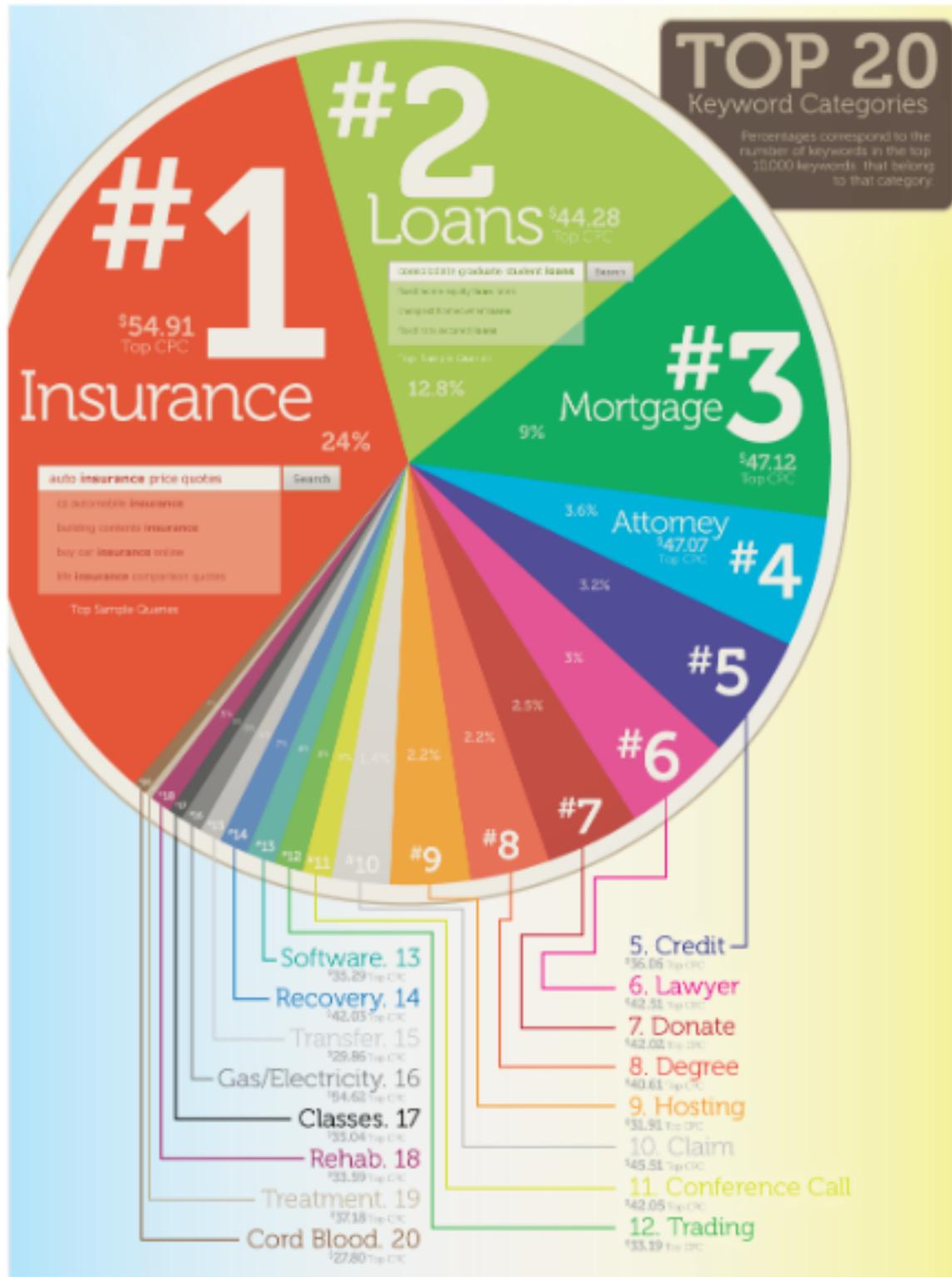
## 2006 AOL LEAKED DATA



- Results in 1<sup>st</sup> position are clicked more than 10 times results on 6<sup>th</sup> position
- Suppose company A and company B are respectively on the 1<sup>st</sup> and 6<sup>th</sup> position for a valuable business keyword
- B should buy 10 times the clicks that A gets to obtain the same traffic!
- **How much does a click cost?**



# Click economic value



- In web advertising, the cost of a single click, also known as Cost-Per-Click (CPC), is **1\$ on average**. However...
- On Google some keywords can be really expensive ([Google most expensive keywords 2011](#)):
  - **Insurance: 54.91\$**
  - **Mortgage: 47.12\$**
  - **Attorney: 47.07**
  - **Claim: 45.51\$**
  - **Loans: 44.28\$**
  - **Lawyer: 42.51\$**
- Bing is even more expensive ([Bing most expensive keywords 2015](#)):
  - **Lawyers: 109.21\$**
  - **Attorney: 101.77\$**
  - **Structured settlements: 78.39\$**



# Terms rarity

- Common terms are less informative than rare terms.
- Consider a query like “*good refrigerators*”.
  - With TF (term frequency) we consider the words frequency in each document
  - However, *good* is a very common word, that can be found in most documents, while *refrigerators* is less common and thus more informative for relevance
  - We **should weight more the rare words** than the common words
- We will use inverse document frequency (**idf**) to capture this, which in turn is based on document frequency (**df**)



# DF: Document Frequency

- $df_t$  is the document frequency of  $t$ : the number of documents that contain  $t$ 
  - Note: despite its name is *document* frequency, it involves the **whole collection of documents!**
  - It measures **how common a term is in the whole collection of documents.**
- $df_t$  is an **inverse** measure of the informativeness of  $t$ 
  - The **more common** is a word, the **less informative** will be for relevance (extreme case: stop words).
  - We use the **df** to formulate the **inverse** concept, in order to express the **rarity** of a word.



# IDF: Inverse Document Frequency

- We define the inverse document frequency (**idf**) of term  $t$  by

$$\text{idf}_t = \log_{10} (N/\text{df}_t)$$

- $N$  is the number of documents in the collection.
- We use **log** ( $N/\text{df}_t$ ) instead of  $N/\text{df}_t$  to smooth the effect of idf.
  - E.g. a common word like "*home*" can have a frequency **1 million bigger** than a rare word such as "*fenestration*"
  - We apply the logarithm to flatten this **exponential** phenomenon of linguistics (Zipf's law), otherwise very common words would have an idf of zero



# Effect of IDF on ranking

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- Does idf have an effect on ranking for one-term queries, like
  - iPhone ?
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
  - For the query **capricious person**, idf weighting makes occurrences of **capricious** count for much more in the final document ranking than occurrences of **person**
- IDF is never used alone, instead is used together with the TF for a better estimation of document relevance



# TF-IDF 1/2

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$w_{t,d} = \text{tf}_{t,d} \times \log_{10}(N / \text{df}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the “-” in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, **tf x idf**



# TF-IDF 2/2

$$w_{t,d} = tf_{t,d} \times \log_{10}(N / df_t)$$

- The tf-idf weight of a term:
  - Increases with the number of occurrences within a document (**tf**)
  - Increases with the rarity of the term in the collection (**idf**)



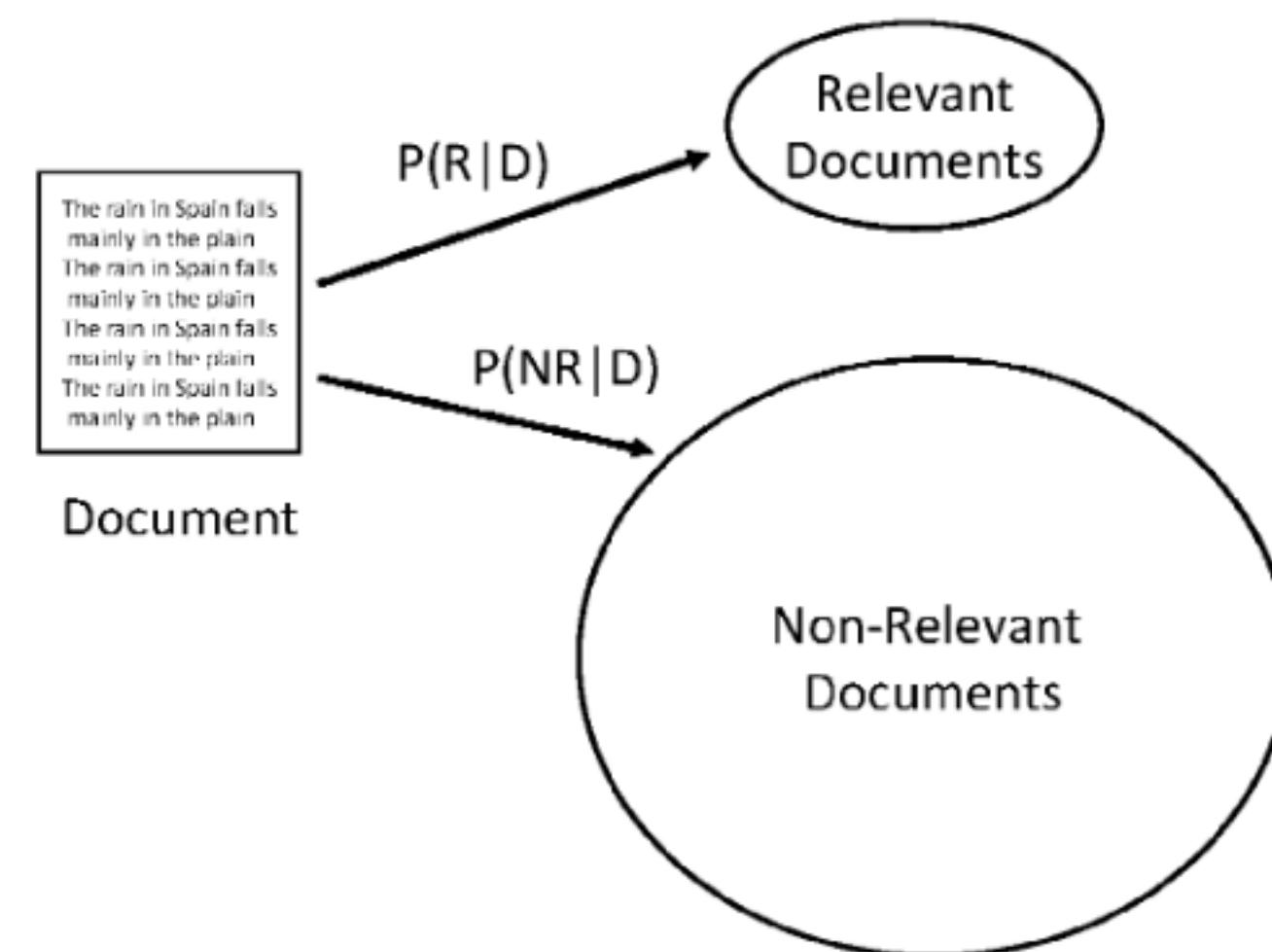
# Retrieval Models

- **Classic models**
  - Boolean retrieval
  - Vector Space model
- **Probabilistic Models**
  - BM25
  - Language models
- Progress in retrieval models has corresponded with improvements in effectiveness

# Probabilistic Models

- Probability Ranking Principle, Robertson (1977)
- The ranking is given by a probability of relevance
- Probability is estimated as accurately as possible using the available data → best effectiveness

“[...] If the **ranking of the documents is in order of decreasing probability of relevance** (where the probabilities are estimated as accurately as possible from the available data) **the overall effectiveness will be the best** that is obtainable on the basis of those data.”





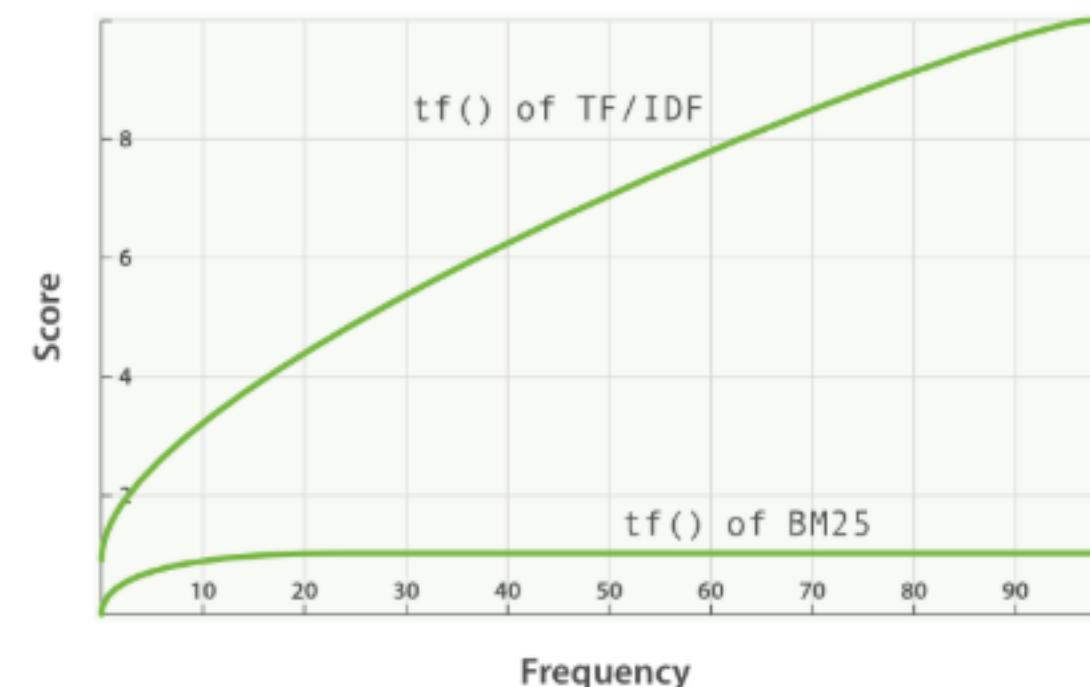
# Okapi BM25

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- The BM25 weighting scheme, often called *Okapi weighting*, after the system in which it was first implemented, was developed as a way of building a probabilistic model sensitive to
  - **term frequency**
  - **term rareness** in corpus (similarly to IDF)
  - **document length**.
- BM25 became a popular and effective ranking algorithm while relying on probabilistic foundations.
- The BM25 term weighting formulas have been used quite widely and quite successfully across a range of collections and search tasks.

# Okapi BM25: TF saturation

- In TF-IDF, a document similarity score could reach very high values if the term frequency of the query term is very high
- In Okapi BM25, the similarity score is always comprised between 0 and 1
  - At a certain point, highest values of term frequency do not affect the score





# Okapi BM25: parameters

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- Okapi BM25 has two parameters:
  - **k1**: this parameter controls how quickly an increase in term frequency results in term-frequency saturation. The default value is 1.2. Lower values result in quicker saturation, and higher values in slower saturation
  - **b**: this parameter controls how much effect document length normalization should have. A value of 0.0 disables normalization completely, and a value of 1.0 normalizes fully. The default is 0.75



# Language Model

- A statistical language model assigns a probability to a sequence of  $m$  words  $P(w_1, \dots, w_m)$  by means of a probability distribution.
- A separate language model is associated with each document in a collection. Documents are ranked based on the probability of the query  $Q$  in the document's language model  $P(Q | M_d)$ .
- Unigram language model
  - probability distribution over the words in a language
  - generation of text consists of pulling words out of a “bucket” according to the probability distribution and replacing them
- N-gram language model
  - some applications use bigram and trigram language models where probabilities depend on previous words



# Efficiency aspects of ranking

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- Computing the relevance score using non boolean models can be quite computationally-intensive:
  - Document is retrieved (has a relevance score) even if not all the query terms are found in it
  - Complex relevance formulas requires additional computations to compute score
- Strategies to control the efficiency:
  - The way we access documents affects computation time (one document at a time or one term at a time?)
  - Most of the time we are not interested in the least relevant documents



# Efficient scoring

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- A large fraction of the CPU work for each search query is devoted just to **compute the score** of relevance
  - Generally, we have a tight budget on latency (say, 250ms) to answer the query, otherwise the user will not be happy
  - CPU provisioning doesn't permit exhaustively scoring every document for every query
- Basic idea: avoid scoring docs that won't make it into the top K



# TAAT vs DAAT techniques

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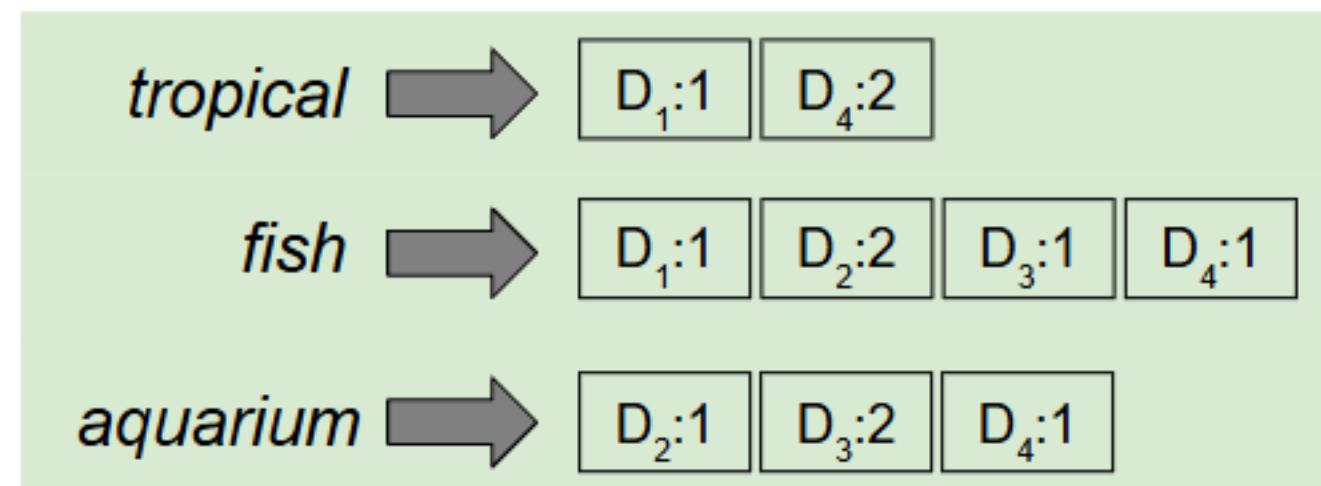
- **TAAT** : “Term At A Time”
  - Scores for all docs computed concurrently, one query term at a time
- **DAAT** : “Document At A Time”
  - Total score for each doc (including all query terms) computed, before proceeding to the next
- Each has implications for how the retrieval **index** is structured and stored



# TAAT: Example

Query: *tropical fish aquarium*

Inverted index:



Term frequency for each document:

$$D_1 = 1 + 1 = 2$$

$$D_2 = 2 + 1 = 3$$

$$D_3 = 1 + 2 = 3$$

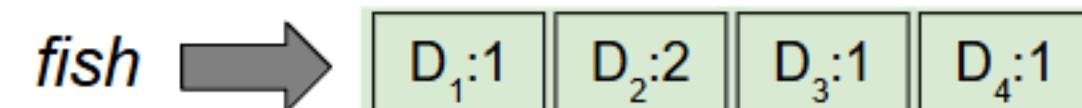
$$D_4 = 2 + 1 + 1 = 4$$



# DAAT: Example

Query: *tropical fish aquarium*

Inverted index:



Term frequency for each document:

$$D_1 = 1 + 1 = 2$$

$$D_2 = 2 + 1 = 3$$

$$D_3 = 1 + 2 = 3$$

$$D_4 = 2 + 1 + 1 = 4$$



# Safe ranking

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- The terminology “safe ranking” is used for methods that guarantee that the K docs returned are the **K absolute highest scoring documents**
  - (Not necessarily just under cosine similarity)
- Is it ok to be non-safe? Yes, the results would be approximated but good enough to satisfy the user.
- The goal is to run less computations while giving good results.



# Non-safe ranking

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- Non-safe ranking may be okay
  - Ranking function is only a proxy for user happiness
  - Documents close to top K may be just fine
- Non-safe: Index “elimination”
  - Only consider **high-idf (rare) query terms**, discard the common ones
  - Only consider **documents containing many query terms**, discard the ones with only few query terms.
- Non-safe: Champion lists
  - For each term, the scores of top relevant documents are pre-computed (before any query is issued)



# References

Christopher D. Manning, Prabhakar Raghavan and Hinrich Schütze

Introduction to Information Retrieval

Cambridge University Press.

2008

The book is also online for free:

- HTML edition (2009.04.07)
- PDF of the book for online viewing (with nice hyperlink features, 2009.04.01)
- PDF of the book for printing (2009.04.01)

