Introduction to Quantum Computing

Exercise Sheet 1

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Exercise 1. Express the following complex numbers in trigonometric and exponential forms:

1. $z = 1 - i$;
2. $z = \frac{2}{\sqrt{3}+i}$.

Exercise 2. Knowing that $\cos \frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}$, compute all 5-th roots of unity.

Exercise 3. Express the vectors of $\mathbb{C}^8$ corresponding to the following expressions:

$$|101\rangle \qquad |01\rangle \otimes |0\rangle \qquad \frac{1}{\sqrt{3}}(|101\rangle + |010\rangle + |111\rangle).$$

Exercise 4. Decide if the following vectors of $\mathbb{C}^8$ can be decomposed via tensors, or if they are entangled.

$$\frac{1}{\sqrt{3}}(|001\rangle + |111\rangle + |011\rangle), \qquad \frac{1}{\sqrt{2}}(|011\rangle + |100\rangle), \qquad \frac{1}{\sqrt{2}}(|011\rangle + |101\rangle).$$

Exercise 5. Compute the following vector in $\mathbb{C}^4$:

$$\frac{1}{\sqrt{6}}\Big(|01\rangle\langle 00| + 5|01\rangle\langle 01| + 3|10\rangle\langle 11| + 2i|10\rangle\langle 11|\Big)\frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big).$$

Exercise 6. Find an orthonormal basis composed of eigenvectors of the matrix below, as well as the associated eigenvalues.

$$M = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$